4. If $x = \frac{5y}{z - 4y}$, find the value of y when $x = 3$ and $z = 2$. [4]

5. **(a)** Subtract $2\frac{1}{2}$ from the sum of $3\frac{1}{6}$ and $4\frac{5}{8}$. [3]

(b) Subtract the sum of $(2x^2 + 5x - 3)$ and $(4x^3 + 3x - 9)$ from the product of $2x$ and $(3x^2 - 5x - 4)$. [4]

Section B (28 marks)

6. **(a)** The sum of three consecutive odd numbers is 141, find the largest of the three numbers. [4]

(b) Five teachers took a group of students for a movie. Each adult ticket cost \$7.20 and students' tickets were sold at half price. If the total cost for the group was \$212.40, calculate the number of children in the group. [3]

7. **(a)** Solve the equation $x - 2 = \frac{x - 4}{3}$. [4]

(b) Copy and complete the following number sequence:

7, 4, 8, 6, 11, 10, 16, 16, 23, 24, ____, ____ [2]

8. **(a)** Similar bars of soap are sold in packs of 3 for \$2 or packs of 8 for \$5.20. Find the difference in the price for 48 bars of soap. [3]

(b) Mr Tan gave \$240 to his wife, $\frac{2}{5}$ of the remainder of his money to his son and kept the rest. If he had \$195 left, how much money did he have originally? [3]

(c) The temperature in Arrowtown for six days are −4°C, −12°C, −18°C, 2°C, 4°C and −8°C. Find the average temperature for these six days. [2]

9. **(a)** The result of adding 90 to a number is the same as multiplying that number by 6. Find the number. [3]

(b) John is 4 years older than David and Joe is 2 years younger than David. If the sum of their ages is 41, how old will John be in 8 years' time? [4]

CHAPTER 9

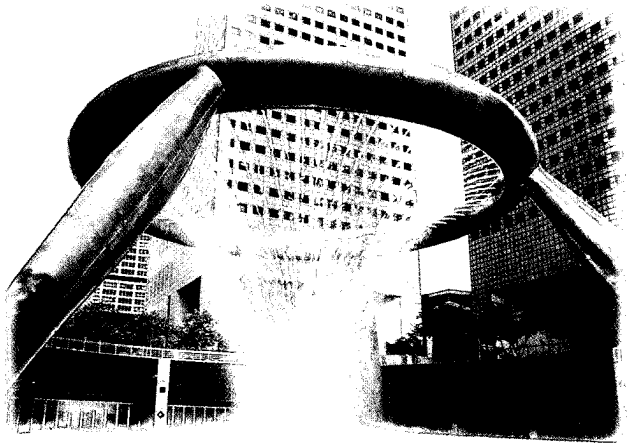
Perimeter and Area of Simple Geometrical Figures

In this chapter, you will learn how to

- △ find the perimeter and area of simple geometrical figures;
- △ solve problems involving these figures and figures related to them.

Preliminary Problem

The picture shows the famous "Fountain of Wealth', the world's largest fountain. Located at Suntec City in Singapore, it has attracted many visitors from all over the world. The bronze fountain has a circular ring of perimeter 66 m and a base area of 1 683 m^2.



Perimeter

The perimeter of a closed plane figure is the distance to go along one round of the plane.

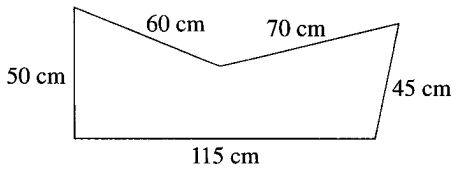
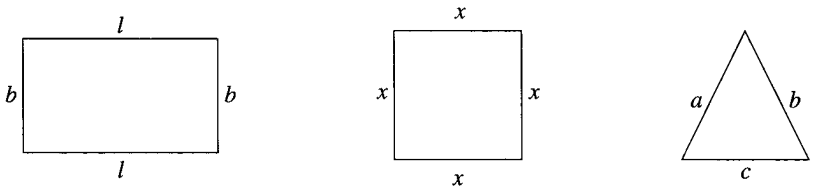


Fig. 9.1

In Fig. 9.1, the perimeter of the closed figure = (50 + 60 + 70 + 45 + 115) cm
= 340 cm



(a) (b) (c)

Fig. 9.2

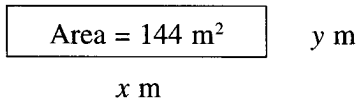
The perimeter of the rectangle in Fig. 9.2(a) is **$P = (2l + 2b)$ units**
$= 2(l + b)$ units

The perimeter of the square in Fig. 9.2(b) is **$P = 2(x + x)$ units**
$= 4x$ units

The perimeter of the triangle in Fig. 9.2(c) is **$P = (a + b + c)$ units**

In-Class Activity

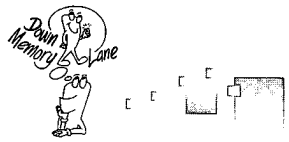
Adam intends to build a house of floor area 144 m^2.



Complete the following table to get some possible different perimeters, but the same area, of the floor:

x (m)	1	2	4	6	8	12	16	18	24	36	72	144
y (m)	144	72	36	24	18	12	9	8	6	4	2	1
Perimeter (m)	290	148	80	60	52	48	50	52	60	80	148	290

(a) Which design do you think is the cheapest to build? What is its perimeter?
(b) Which is the most expensive design to build?



Units of Length or Distance

The metric system of units was first introduced in the nineteenth century. It is essentially a simple system based on the decimal system. As such, it allows easy conversion from one unit to another. The metric system of units involves measurements of length, area, mass, capacity and volume; all these units being related through the decimal notation.

We often use the following units to measure lengths or distances.

Metre (m): The basic unit of length is the metre.
This is normally used to measure distance between two places within a small compound. For example, the distance between your school gate and the school hall is measured in metres.

Centimetre (cm): This is a unit used to measure the length of small objects or the distance between two neighbouring points. For example, the length of your desk is measured in centimetres.

Millimetre (mm): This is normally used for measuring small lengths or thickness. For example, the thickness of a page of this book is given in millimetres.

Kilometre (km): This is used to measure the distance between two places far away from each other. For example, the distance between Hongkong and Singapore is measured in kilometres.

The above units are related as shown below:

1 centimetre (cm) = 10 millimetres (mm)
1 metre (m) = 100 centimetres (cm)
1 kilometre (km) = 1 000 metres (m)

In-Class Activity

Work with a partner.

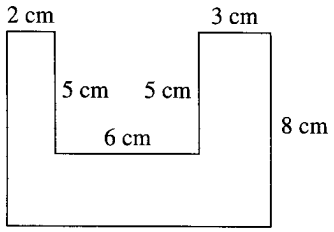
Use a measuring tape to measure and then record the perimeter of your
(a) classroom blackboard; (b) classroom floor.

Example 1

Find the perimeter of the given figure.

Solution

Perimeter of the figure
= [2 + 2(5) + 6 + 3 + 2(8) + (2 + 6 + 3)] cm
= (2 + 10 + 6 + 3 + 16 + 11) cm
= 48 cm

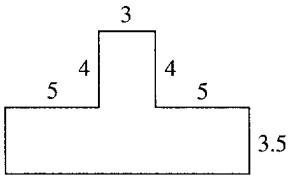


Exercise 9a

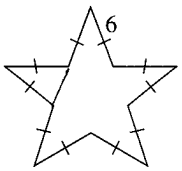
1. Find the perimeter of each of the following geometrical figures:
   (a) A triangle of sides 8 cm, 9 cm and10 cm.
   (b) A rectangle with length 9 cm and breadth 7 cm.
   (c) A square of side 7 cm.

2. Find the perimeter of each of the following figures. All measurements are in cm:

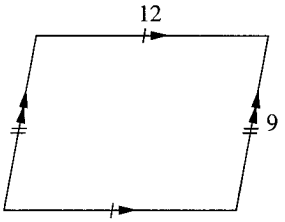
(a)



(b)



(c)



3. A piece of wire is bent to form a square of side 8 cm. It is then reshaped to form a rectangle of length 10 cm and breadth x cm. Find x.

4. A boy is asked to run 15 times round the edge of a rectangular field measuring 30 m by 25 m. Find the total distance the boy ran.

5. The length of a rectangular plot of land is twice its breadth. If its perimeter is 102 m, calculate its breadth.

6. Find, in cm, the perimeter of a rectangle measuring a m by b cm.

Area of Simple Figures

Which is bigger — a football field or a basketball court? The football field is bigger because it covers a larger surface than the basketball court. In other words, the football field has a larger area.

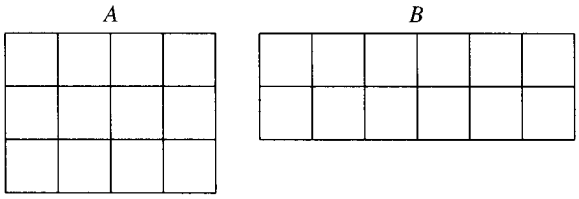


Fig. *A* and Fig. *B* consist of squares of the same size. Count the number of squares in each. What do you notice? Can we say that *A* and *B* cover the same amount of space, i.e., they have the same area?

Area is the measure of the amount of surface covered.

Units of Area

We often use the following units to measure area.

Square centimetre (cm²): We usually use unit squares to compare areas. A square of side 1 cm is used as a standard unit. We call this unit area 1 square centimeter (1 cm²). See Fig. 9.3.

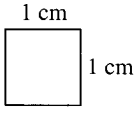


Fig. 9.3

Square millimetre (mm²): In Fig. 9.4, each small square has an area of 1 square millimetre (1 mm²).

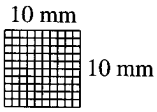


Fig. 9.4

$1 \text{ cm} = 10 \text{ mm}$

$\therefore \quad 1 \text{ cm}^2 = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$

Square millimetres are used to measure the areas of very small shapes.

Square metre (m²): The square metre is used to measure the area of large surfaces such as the floor area of a flat.

$1 \text{ m} = 100 \text{ cm}$

$\therefore \quad 1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$

Hectare (ha): The hectare is used to measure large land areas such as farms.

$1 \text{ ha} = 10\,000 \text{ m}^2$

Square km (km²): The square kilometre is used to measure the area of a very large surface such as the area of a country.

$1 \text{ km} = 1\,000 \text{ m}$

$\therefore \quad 1 \text{ km}^2 = 1\,000 \text{ m} \times 1\,000 \text{ m}$

$= 1\,000\,000 \text{ m}^2 = 100 \text{ ha}$

The British used feet, inches, yards, furlongs, miles, etc. to measure length, and acre to measure area.

Find out what these units are and compare them with the SI units.

The largest freshwater lake in the world is Lake Superior, one of the Great Lakes of North America. It covers an area of 82 350 km², roughly 130 times the size of Singapore.

Example 2

Express (a) 975 cm² in m²; (b) 2.65 km² in m²; (c) 48 000 m² in ha; (d) 5 mm² in cm².

Solution

(a) $975 \text{ cm}^2 = \_\_\_\_ \text{ m}^2$

$1 \text{ cm} = \frac{1}{100} \text{ m}$

$1 \text{ cm}^2 = \frac{1}{100} \text{ m} \times \frac{1}{100} \text{ m}$

$= \frac{1}{10\,000} \text{ m}^2$

$\therefore \quad 975 \text{ cm}^2 = 975 \times \frac{1}{10\,000} \text{ m}^2$

$= 0.097\,5 \text{ m}^2$

(b) $2.65 \text{ km}^2 = \_\_\_\_ \text{ m}^2$

$1 \text{ km} = 1\,000 \text{ m}$

$1 \text{ km}^2 = 1\,000 \times 1\,000 \text{ m}$

$= 1\,000\,000 \text{ m}^2$

$\therefore \quad 2.65 \text{ km}^2 = 2.65 \times 1\,000\,000 \text{ m}^2$

$= 2\,650\,000 \text{ m}^2$

(c) $48\,000 \text{ m}^2 = \_\_\_\_ \text{ ha}$

$10\,000 \text{ m}^2 = 1 \text{ ha}$

$1 \text{ m}^2 = \frac{1}{10\,000} \text{ ha}$

$\therefore \quad 48\,000 \text{ m}^2 = 48\,000 \times \frac{1}{10\,000} \text{ ha}$

$= 4.8 \text{ ha}$

(d) $5 \text{ mm}^2 = \_\_\_\_ \text{ cm}^2$

$1 \text{ mm} = \frac{1}{10} \text{ cm}$

$1 \text{ mm}^2 = \frac{1}{10} \text{ cm} \times \frac{1}{10} \text{ cm}$

$= \frac{1}{100} \text{ cm}^2$

$\therefore \quad 5 \text{ mm}^2 = 5 \times \frac{1}{100} \text{ cm}^2 = 0.05 \text{ cm}^2$

Area of a Rectangle

Consider a rectangle of length l units and breadth b units (see Fig. 9.5). The rectangle is made up of b rows, each with l unit squares.

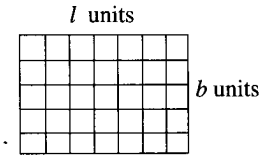


Fig. 9.5

No. of unit squares in the rectangle = $l \times b$

$\therefore$ area of rectangle = $(l \times b)$ unit^2

In general,

area of a rectangle = length × breadth

Hence, $\text{length} = \frac{\text{area}}{\text{breadth}}$, $\text{breadth} = \frac{\text{area}}{\text{length}}$

You are given 4 rectangular pieces of wood. Two of these measure 4 cm by 3 cm while the other two measure 13 cm by 1 cm. Use these 4 pieces of wood to enclose an area as large as you possibly can.

Problem Solving — Draw a Diagram

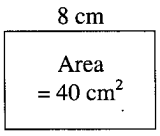
Many problems can be made easier by **drawing a diagram**.

Example 3

The area of a rectangle is 40 cm^2 and one of its sides is 8 cm long. Find the breadth and the perimeter of the rectangle.

Solution

Draw a simple diagram like this



Breadth of the rectangle = $\frac{40\ \text{cm}^2}{8\ \text{cm}} = 5$ cm

Perimeter of the rectangle = 2(8 + 5) cm = 26 cm

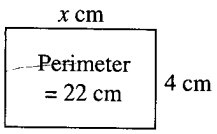
Example 4

The perimeter of a rectangle is 22 cm and its breadth is 4 cm. Find its area.

Solution

We can **draw a diagram** and then **form an equation** to solve the problem.

Let the length of the rectangle be x cm.



Then $2(x + 4) = 22$

$2x + 8 = 22$

$2x = 14$

$x = 7$

$\therefore$ the area of the rectangle $= (7 \times 4)\ \text{cm}^2 = 28\ \text{cm}^2$

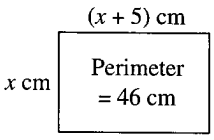
Example 5

The width of a rectangle is 5 cm less than its length, and its perimeter is 46 cm. Find its width and its area.

Solution

Let the width of the rectangle be x cm.

Then the length is $(x + 5)$ cm.



Perimeter $= 2[x + (x + 5)]\ \text{cm} = 46\ \text{cm}$

$4x + 10 = 46$

$\therefore\ 4x = 36$

$x = 9$

$\therefore$ Its width is 9 cm and its length is 14 cm.

$\therefore$ Its area $= (9 \times 14)\ \text{cm}^2$

$= 126\ \text{cm}^2$

Example 6

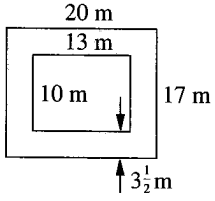
A rectangular field is 13 m long and 10 m wide. It has a cement path $3\frac{1}{2}$ m wide around it. What is the area of the cement path?

Solution

Area of the field and cement path $= (20 \times 17)\ \text{m}^2 = 340\ \text{m}^2$

Area of the field $= (13 \times 10)\ \text{m}^2 = 130\ \text{m}^2$

$\therefore$ area of cement path $= (340 - 130)\ \text{m}^2 = 210\ \text{m}^2$



Exercise 9b

1. Copy and fill in the missing numbers:

(a) $8.5\ \text{cm}^2 = \_\_\_\_\_\ \text{m}^2$

(b) $2.5\ \text{mm}^2 = \_\_\_\_\_\ \text{cm}^2$

(c) $6.3\ \text{m}^2 = \_\_\_\_\_\ \text{cm}^2$

(d) $40.6\ \text{cm}^2 = \_\_\_\_\_\ \text{mm}^2$

(e) $44.4\ \text{km}^2 = \_\_\_\_\_\ \text{ha}$

(f) $3.1\ \text{ha} = \_\_\_\_\_\ \text{m}^2$

(g) $53.7\ \text{m}^2 = \_\_\_\_\_\ \text{km}^2$

(h) $0.28\ \text{km}^2 = \_\_\_\_\_\ \text{m}^2$

(i) $53\,200\ \text{mm}^2 = \_\_\_\_\_\ \text{m}^2$

(j) $69\,450\ \text{cm}^2 = \_\_\_\_\_\ \text{m}^2$

(k) $3.4\ \text{ha} = \_\_\_\_\_\ \text{km}^2$

(l) $462\ \text{m}^2 = \_\_\_\_\_\ \text{ha}$

2. Copy and complete the table below for each given rectangle:

	Length	*Breadth*	*Perimeter*	*Area*
(a)	6 m	4 m		
(b)	8 m			48 m^2
(c)		2.2 m		8.8 m^2
(d)	4.5 m		23 m	
(e)		26 mm	98 mm	

3. Find the number of 15-centimetre square tiles required to cover a rectangular floor 5.4 m long and 4.05 m wide.

4. Find the area, in square centimetres, of a rectangular strip of board 3.28 m long and 75 mm wide.

5. A square cardboard of side 20 m has a 4 m wide border round three of its sides. Find the area of the border.

6. A paper box without a lid is 25 cm long. 16 cm wide and 5 cm deep. How many square centimetres of paper have been used to make the box?

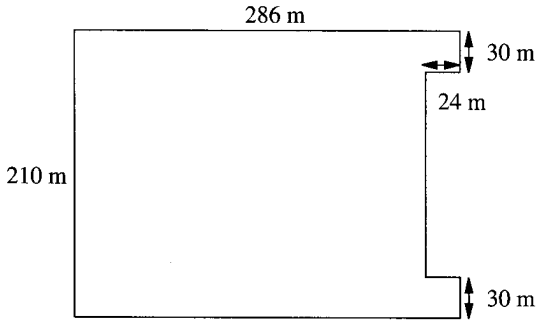
7. Find the cost required to carpet a hall 8 m by 5.5 m if a rectangular section 2 m by $1\frac{1}{2}$ m is taken out to provide for the fireplace and the carpet costs \$52.50 per m^2.

8. A swimming pool 25 m by 10 m has a concrete border all round. Find the area of the concrete border if it is 2.5 m wide at the sides and 5 m at the ends.

9. The perimeter of a square is 36 cm. Find its area.

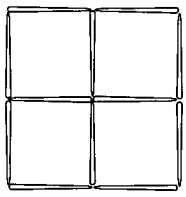
***10.** Find the total area of cardboard used in making a match box, complete with the sliding portion, 4 cm long, 2.5 cm wide and 1.2 cm deep (ignore the thickness of the cardboard).

11. Find, in hectares, the area of the figure shown below. Give your answer correct to 2 decimal places.



12. The length of a rectangle is 8 cm more than its width. If its perimeter is 56 cm, find its length and its area.

12 toothpicks are arranged as shown below. Remove only 2 toothpicks so as to leave only 2 squares.



Area of a Parallelogram

A **parallelogram** is a quadrilateral in which the opposite pairs of sides are parallel.

We can obtain a rectangle from a parallelogram. To do so, draw on a piece of paper a parallelogram *ABCD* as shown in Fig. 9.6.

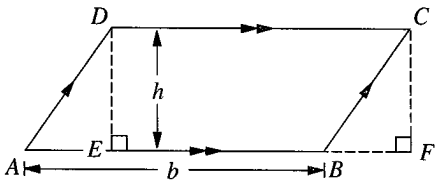


Fig. 9.6

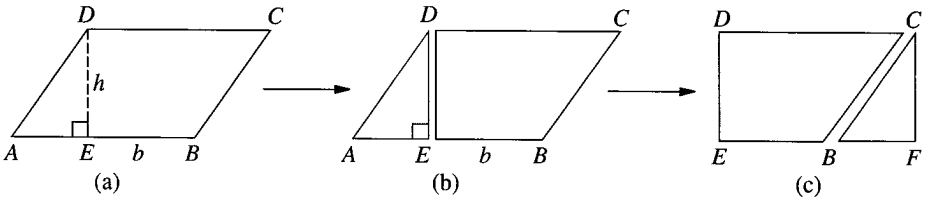


Fig. 9.7

Cut off $\triangle AED$ and place it in the position BFC (Fig. 9.7(c)). A rectangle $EFCD$ is obtained. Do you agree that the parallelogram $ABCD$ and the rectangle $EFCD$ have the same area?

Area of the parallelogram $ABCD$ = Area of the rectangle $EFCD$
$= DC \times DE$

But $DC = AB$ (opposite sides of a parallelogram)

$\therefore$ area of parallelogram $ABCD = AB \times DE$
$= b \times h$
= base × height

In general,

area of a parallelogram = base × height = $b \times h$

Area of a Triangle

Look at the square $ABCD$, the rectangle $PQRS$ and the parallelogram $HIJK$ in Fig. 9.8.

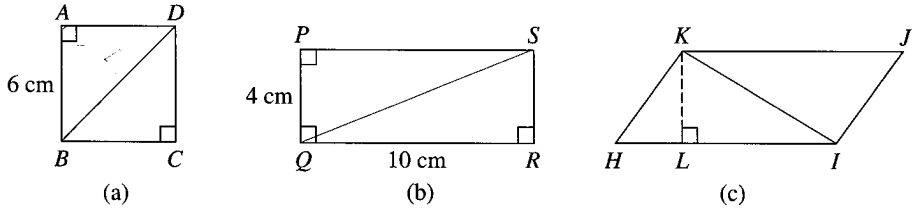


Fig. 9.8

The square $ABCD$ in Fig. 9.8(a) is cut into two halves by the diagonal BD. Similarly, the diagonal QS cuts the rectangle $PQRS$ in Fig. 9.8(b) into two equal right-angled triangles PQS and SQR.

Area of square $ABCD = BC \times CD$

$\therefore$ area of $\triangle BCD = \frac{1}{2} \times BC \times CD$

Since BC is the base and CD is the height of $\triangle BCD$,

$\therefore$ area of $\triangle BCD = \frac{1}{2} \times$ base × height.

How would you deduce that in Fig. 9.8(b), area of $\triangle QRS = \frac{1}{2} \times QR \times RS = \frac{1}{2} \times$ base × height?
If QS is taken as the base, where should the height of $\triangle QRS$ be?

The diagonal KI cuts the parallelogram $HIJK$ in Fig. 9.8(c) into two identical triangles which are not right-angled.

Area of $HIJK = HI \times KL$

$\therefore$ area of $\triangle HIK = \frac{1}{2} \times HI \times KL$

HI is the base and KL is the height of $\triangle HIK$.

$\therefore$ area of $\triangle HIK = \frac{1}{2} \times$ base $\times$ height

In general

$$\textbf{area of a triangle} = \frac{1}{2} \times \textbf{base} \times \textbf{height}$$
$$= \frac{1}{2} bh$$

Area of a Trapezium

A **trapezium** is a quadrilateral with one pair of parallel sides. Fig. 9.9 shows a trapezium $ABCD$ in which AD is parallel to BC with a height of AH. The trapezium is divided into $\triangle ABC$ and $\triangle ACD$.

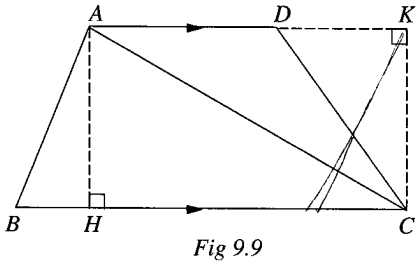


Fig 9.9

Area of trapezium $ABCD$ = area of $\triangle ABC$ + area of $\triangle ACD$

$$= \left(\frac{1}{2} \times BC \times AH\right) + \left(\frac{1}{2} \times AD \times CK\right)$$

Note: $AH = CK$

Area of trapezium $ABCD = \frac{1}{2} AH\ (BC + AD)$

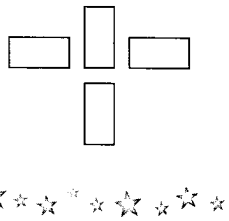
$= \frac{1}{2} \times$ height $\times$ sum of parallel sides

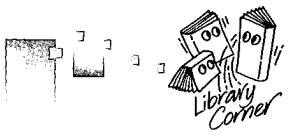
In general,

$$\textbf{area of a trapezium} = \frac{1}{2} \times \textbf{height} \times \textbf{sum of parallel sides}$$

Problems

Four rectangular cards of identical size are arranged as shown below. You are to move only one card so as to form a square.



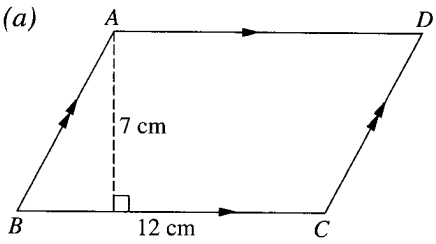


Due to land reclamation, the area and perimeter of Singapore have increased in size. Find out the area of Singapore and its perimeter in 1970, 1980, 1990 and today.

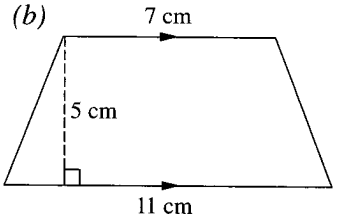
Example 7

Find the areas of the following figures:

(a)



(b)



Solution

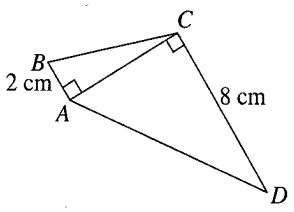
(a) Area of parallelogram $ABCD$ = base × height
$= (12 \times 7)\ \text{cm}^2 = 84\ \text{cm}^2$

(b) Area of trapezium $= \frac{1}{2} \times$ height × sum of parallel sides
$= \left[\frac{1}{2} \times 5 \times (7 + 11)\right]\text{cm}^2 = 45\ \text{cm}^2$

Example 8

In the figure below, the sides AB and DC of the quadrilateral ABCD are both perpendicular to the diagonal AC. Given AB = 2 cm, DC = 8 cm and the area of $\triangle ABC$ = 6 cm², calculate

(a) the length of AC;
(b) the area of the quadrilateral ABCD.



Solution

(a) Area of $\triangle ABC = \frac{1}{2} \times AB \times AC$

$\therefore \quad \frac{1}{2} \times AB \times AC = 6$

$\frac{1}{2} \times 2 \times AC = 6$

$\therefore \quad AC = 6\ \text{cm}$

(b) Area of $\triangle ACD = \frac{1}{2} \times AC \times CD$

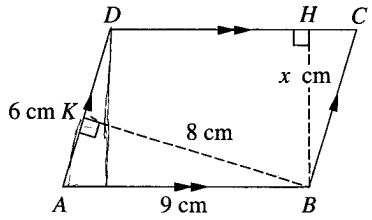
$= \left(\frac{1}{2} \times 6 \times 8\right)\text{cm}^2 = 24\ \text{cm}^2$

$\therefore$ the area of the quadrilateral $ABCD = (6 + 24)\ \text{cm}^2 = 30\ \text{cm}^2$

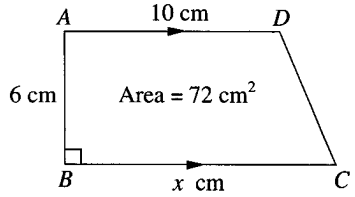
Example 9

Find the value of x in the following figures:

(a)



(b)



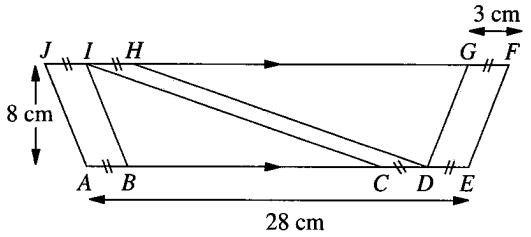
Solution

(a) Area of parallelogram $ABCD$
$= AB \times BH = AD \times BK$
$9 \times x = 6 \times 8$
$\therefore \quad x = \frac{6 \times 8}{9} = 5\frac{1}{3}$

(b) Area of trapezium $ABCD = \frac{1}{2} \times AB \times (AD + BC)$
$72 = \left[\frac{1}{2} \times 6 \times (10 + x)\right]$
$24 = 10 + x$
$\therefore \quad x = 14$

Example 10

Find the total area of the shaded parts in the diagram.



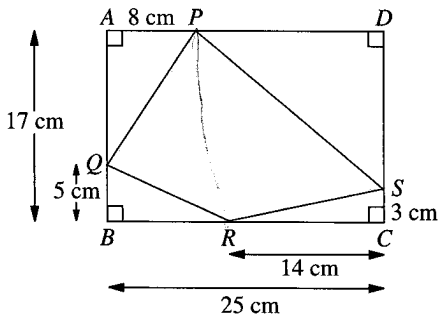
Solution

The total area of the shaded parts is made up of 3 parallelograms of the same base length (3 cm) and of the same height (8 cm).

$\therefore$ area of shaded region $= [3(3 \times 8)]\ \text{cm}^2 = 72\ \text{cm}^2$

Example 11

In the figure on the right, AB = 17 cm, BC = 25 cm, AP = 8 cm, BQ = 5 cm, CR = 14 cm and CS = 3 cm. Find the area of the shaded region.



Solution

Area of shaded region $PQRS$ = area of $ABCD$ – area of triangles (APQ + PDS + CSR + BQR)

Area of $ABCD = (17 \times 25)\ \text{cm}^2 = 425\ \text{cm}^2$

Area of $\triangle APQ = \frac{1}{2} \times AP \times AQ = \left[\frac{1}{2} \times 8 \times (17 - 5)\right]\text{cm}^2 = 48\ \text{cm}^2$

Area of $\triangle PDS = \frac{1}{2} \times PD \times DS = \left[\frac{1}{2} \times (25 - 8) \times (17 - 3)\right]\text{cm}^2 = 119\ \text{cm}^2$

Area of $\triangle SRC = \frac{1}{2} \times RC \times SC = \left(\frac{1}{2} \times 14 \times 3\right)\text{cm}^2 = 21\ \text{cm}^2$

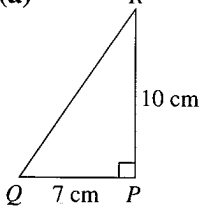
Area of $\triangle BRQ = \frac{1}{2} \times BR \times BQ = \left[\frac{1}{2} \times (25 - 14) \times 5\right]\text{cm}^2 = 27.5\ \text{cm}^2$

$\therefore$ Area of $PQRS = (425 - 48 - 119 - 21 - 27.5)\ \text{cm}^2 = 209.5\ \text{cm}^2$

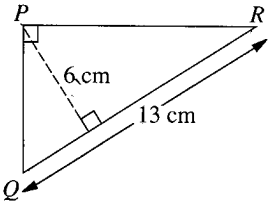
Exercise 9c

1. Find the area of the triangle PQR in the following cases:

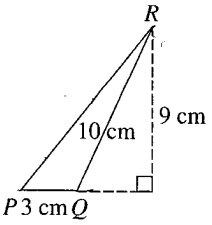
(a)



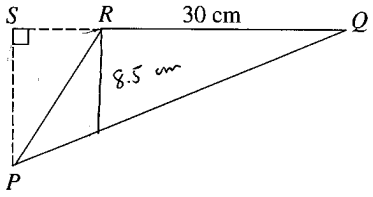
(b)



(c)



2. In the diagram below, the area of $\triangle PQR$ is 255 cm^2 and the length of QR is 30 cm. Find the length of PS.

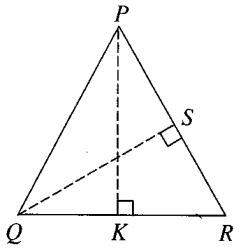


3. For questions (a) to (c), refer to the figure in which QS is perpendicular to PR and PK is perpendicular to QR.

(a) Find the area of $\triangle PQR$ if $PR = 17$ cm and $QS = 12$ cm.

(b) Find the area of $\triangle PQS$ if $QS = 7$ cm, $PR = 14$ cm and $SR = 9$ cm.

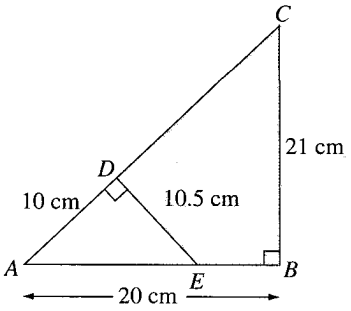
(c) Find QS if $PR = 14$ cm and the area of $\triangle PQR = 147\ \text{cm}^2$.



4. Copy and complete the following table for each parallelogram:

	(a)	(b)	(c)
Base (cm)	12		7.8
Height (cm)	7	6	
Area (cm^2)		42	42.9

5. In the diagram, $AB = 20$ cm, $BC = 21$ cm, $AD = 10$ cm and $DE = 10.5$ cm. Angles ABC and ADE are right angles. If $\triangle ADE$ is removed from $\triangle ABC$, what is the area of the shaded region that remains?



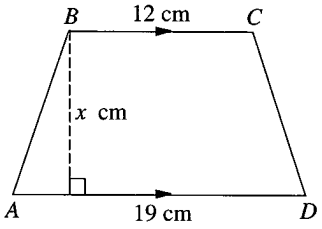
6. Copy and complete the following table for each trapezium:

	(a)	(b)	(c)
Height (cm)	6	14	8
Parallel side 1 (cm)	7	8	
Parallel side 2 (cm)	11		5
Area (cm^2)		126	72

***7.** What is the cost of spraying insecticide on a field measuring 2 000 m by 3 200 m if the cost is \$22 per hectare? (1 ha = 10 000 m^2)

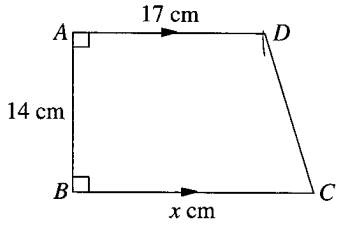
***8.** Find the unknowns, marked x, in the following figures:

(a)



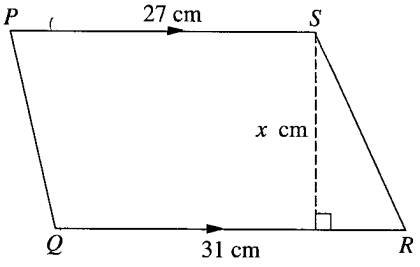
Area of $ABCD$ = 124 cm^2

(b)



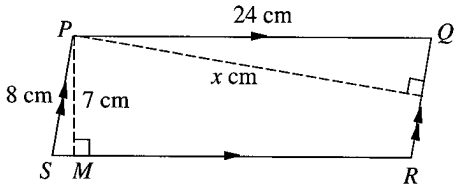
Area of $ABCD$ = 280 cm^2

(c)



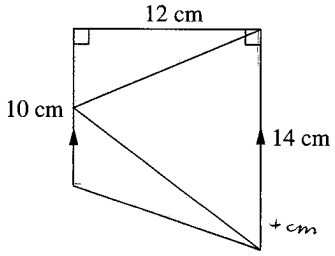
Area of $PQRS$ = 348 cm^2

(d)

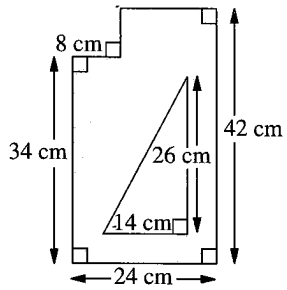


9. Find the areas of the following shaded parts:

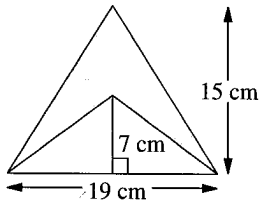
(a)



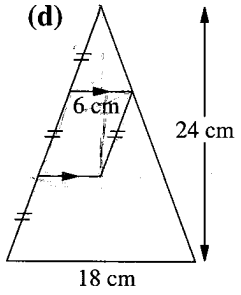
(b)



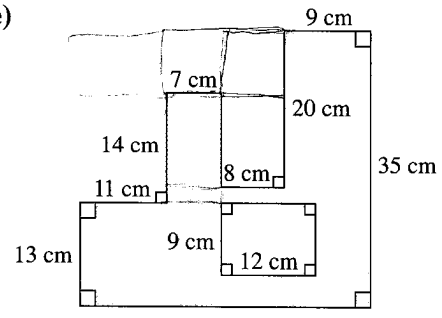
(c)



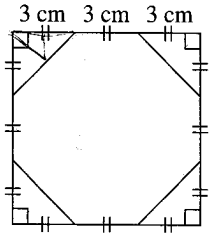
(d)



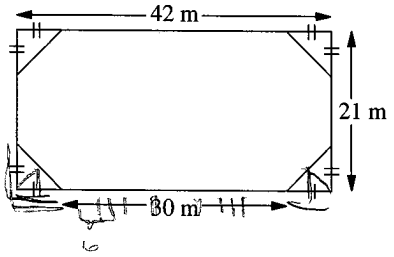
***(e)**



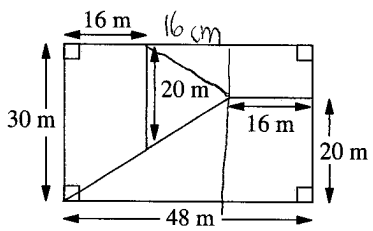
(f)



(g)

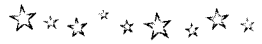


(h)

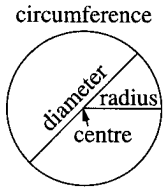


Perimeter of a Circle

The circumference of the Earth's equator is approximately 40 000 km. An imaginary belt of 40 000 km will fit the equator nicely. If we increase the length of the belt by 1 m, will it be possible for a cat to squeeze through? How far above the surface of the equator will the belt be?



A circle consists of points that are all equidistant from a particular point called the **centre**. The **perimeter** of a circle, or the length of its boundary, is called the **circumference**. The distance from the centre of a circle to any point on its circumference is called the **radius**. The **diameter** of the circle is twice the length of its radius.



The diameter of a tin can be found by placing it between two books as shown in Fig. 9.10.

To find the circumference of the tin can, simply wrap a strip of paper round the top as shown in Fig. 9.11. Then measure the length of the strip of paper to get the circumference.

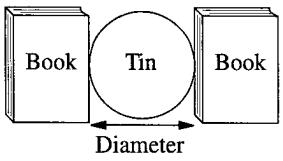


Fig 9.10

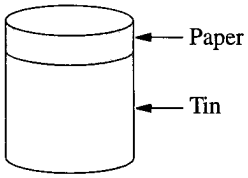


Fig 9.11

The table below shows the circumferences and diameters of several tin cans found by the above methods.

Tin Can	*Circumference (c)*	*Diameter (d)*	$\frac{c}{d}$
A	48.5 cm	15.5 cm	
B	40.0 cm	12.7 cm	
C	31.3 cm	9.9 cm	
D	26.1 cm	8.3 cm	

In each case, find the value of $\frac{c}{d}$ correct to two decimal places. What do you notice?

As a matter of fact, the ratio $\frac{c}{d}$ is the same for all circles. This ratio $\frac{c}{d} = \frac{\text{circumference}}{\text{diameter}}$ is called **pi** and is denoted by the symbol π. Usually π is taken to be approximately equal to 3.14, $\frac{22}{7}$ or 3.142.

To find the circumference of a circle, we use $\frac{\text{circumference}}{\text{diameter}} = \frac{c}{d} = \pi$.

Therefore $c = \pi d$.

Since $d = 2r$ where r denotes the radius of the circle, $c = 2\pi r$.

Hence,

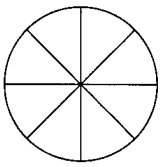
circumference of a circle, $c = \pi d$ or $2\pi r$, where d = diameter and r = radius.

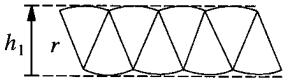
Area of a Circle

Is the circle perfectly round?

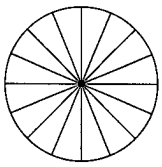
Look at the following figures:

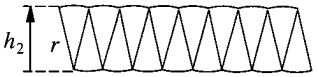
(a)





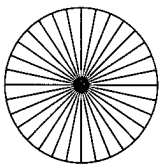
(b)

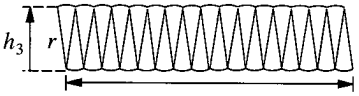




(h gets closer and closer to r)

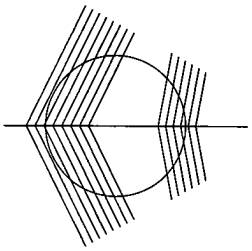
(c)

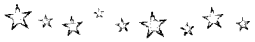




The length of the base gets closer and closer to πr.

Fig. 9.12





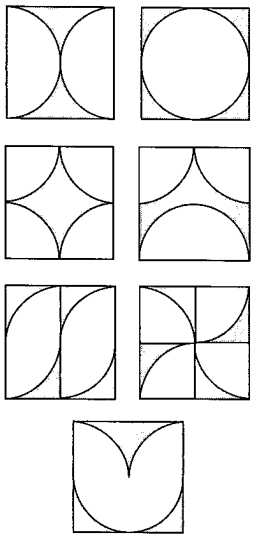
In Fig. 9.12(a), a circle is divided into 8 equal parts and rearranged as shown. In Fig. 9.12(b) and (c), the circles are divided into 16 and 32 equal parts respectively. In each case, the parts are rearranged in a straight line as shown. Notice that the figures resulting from the rearrangements of the parts tend to look like parallelograms. As the number of equal parts increases, the area of the resulting figure, which is the same as the area of the **original circle**, will be closer and closer to the area of a parallelogram. Notice also that the height h of the parallelogram gets closer and closer to r, the radius of the circle, and the length of the base gets closer and closer to πr, which is half of the circumference of the circle.

$\therefore$ area of the parallelogram = base $\times$ height

$= \pi r \times r = \pi r^2$

Area of a circle = πr^2, where r = radius.

Which of the following shaded figures has the greatest area? The squares are of the same length and the curved lines are all arcs of circles.



Example 12

A circle has a radius of 7 m. Find its area and circumference. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution

Area of circle $= \pi r^2 = \left(\frac{22}{7} \times 7 \times 7\right) \text{m}^2$

$= 154 \text{ m}^2$

Circumference of circle $= 2\pi r = \left(2 \times \frac{22}{7} \times 7\right) \text{m}$

$= 44 \text{ m}$

Example 13

The area of a circle is 78.5 cm². Calculate the circumference of the circle. (Take $\pi = 3.14$)

Solution

Area of circle $= \pi r^2$

$78.5 = 3.14r^2$

$r^2 = \frac{78.5}{3.14} = 25$

$r = \sqrt{25} = 5$

$\therefore$ circumference of circle $= 2\pi r$

$= 2(3.14)(5) \text{ cm}$

$= 31.4 \text{ cm}$

Example 14

The diameter of the wheel of a car is 0.35 m. Find the number of revolutions made by the wheel per minute when the car is travelling at 33 km/h. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution

In 60 minutes, the car travels $(33 \times 1\,000)$ m.

In 1 minute, the car travels $\frac{33 \times 1\,000}{60}$ m.

Number of revolutions made per minute $= \frac{\text{distance travelled}}{\text{circumference of wheel}}$

$= \frac{33 \times 1\,000}{60} \times \frac{1}{\pi d}$

$= \frac{33 \times 1\,000}{60} \times \frac{7}{22} \times \frac{100}{35}$

$= 500$

Example 15

In the figure, ABCD is a rectangle of length 24 cm and breadth 16 cm. Given that CQ = PB = $\frac{1}{2}PQ$, calculate the area of the trapezium PQRS.

Solution

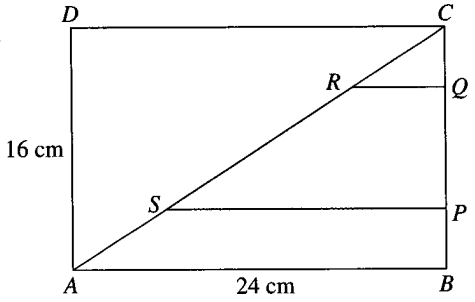
Strategy 1: Use an equation

$PQ = 8$ cm, $RQ = \frac{1}{4}AB = 6$ cm and $PS = \frac{3}{4}AB = 18$ cm

Using the formula $\frac{1}{2} \times$ height $\times$ sum of parallel sides = area of trapezium,

area of $PQRS$ $= \frac{1}{2} \times 8 \times (6 + 18)$

$= 96$ cm^2

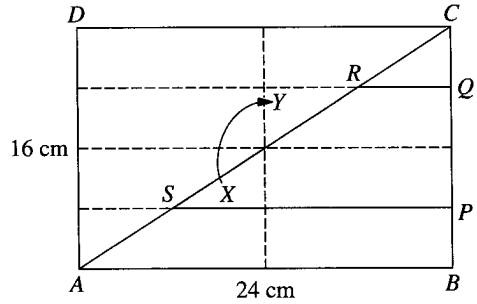


Strategy 2: Draw a diagram

Divide the rectangle into 8 equal parts as shown. If we move the shaded triangle X onto Y, the total shaded area is equal to $\frac{1}{4}$ of the big rectangle.

$\therefore$ area of $PQRS = \frac{1}{4} \times 16 \times 24$

$= 96$ cm^2



Exercise 9d

Take π to be $\frac{22}{7}$ for this exercise unless otherwise stated.

1. Copy and complete the following table below for each circle:

	(a)	(b)	(c)	(d)
Radius	10 m			
Diameter				3.6 m
Circumference		176 mm		
Area			616 cm^2	

2. Calculate the circumference and area of each circle, given its diameter:

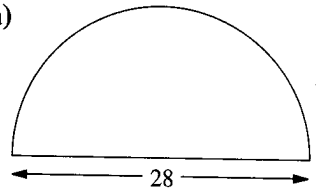
(a) 70 mm **(b)** 28 cm **(c)** 35 cm **(d)** $\frac{14}{3}$ cm

3. Calculate the circumference and area of each circle, given its radius (take $\pi = 3.14$), giving your answer correct to 2 decimal places:

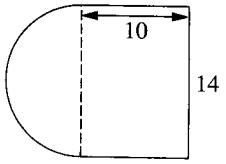
(a) 3.5 cm **(b)** 13.8 m **(c)** 0.37 m **(d)** 5.25 cm

★**4.** Find the perimeter and area of each of the following figures. All dimensions are given in cm and the circular portions are semicircles.

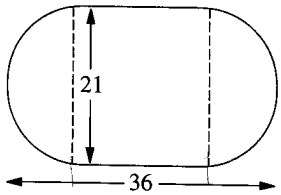
(a)



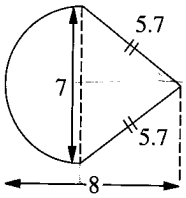
(b)



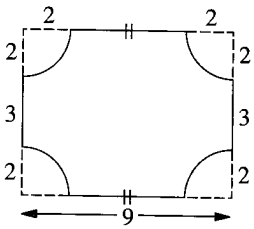
(c)



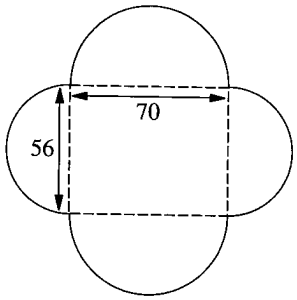
★**(d)**



(e)



(f)



5. Two wire circles of diameters 12 cm and 8 cm are cut and then joined to make one large circle. Find the diameter of this larger circle.

6. As many 8-cm diameter discs as possible are cut from a sheet of rectangular cardboard measuring 170 cm by 90 cm. Find the area of the sheet that is left.

7. If the minute hand of a big clock is 1.12 m long, find the rate at which its tip is moving in centimetres per minute.

8. Find the speed of a point on the rim of a 24-cm diameter fly-wheel which is turning at 2 800 revolutions per minute. Give your answer in metres per second.

★**9.** A lorry travels at 50 km/h. Given that the diameter of its wheel is 88 cm, find how many revolutions per minute the wheel is turning. Give your answer to the nearest whole number.

10. Find the difference between the perimeter of a square of area 1 m^2 and the circumference of a circle of the same area.

Summary

1. For a rectangle with length l units and breadth b units, the perimeter = $2(l + b)$ units and the area = $(l \times b)$ $units^2$.

2. Area of a parallelogram = base × height

3. Area of a triangle = $\frac{1}{2}$ × base × height

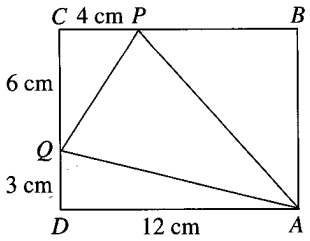
4. Area of a trapezium = $\frac{1}{2}$ × height × sum of parallel sides

5. For a circle with radius r units, the circumference = $2\pi r$ units and the area = πr^2 $unit^2$.

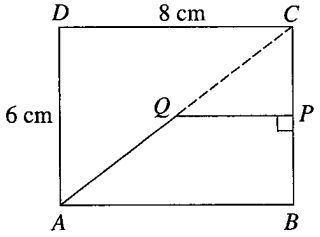
Review Questions 9

Take π to be $\frac{22}{7}$ for this exercise.

1. $\triangle APQ$ is enclosed within the rectangle $ABCD$ as shown in the figure below. Calculate the area of $\triangle APQ$.



2. In the figure, $ABCD$ is a rectangle of length 8 cm and breadth 6 cm. If $BP = CP$, calculate the area of trapezium $ABPQ$, where AQC is a diagonal of the rectangle.



3. A piece of wire 48 cm long is bent to form a rectangle whose length is twice its width. Calculate its area.

4. The length of a rectangle is 4 cm longer than its width and its perimeter is 44 cm. Find the length and area of the rectangle.

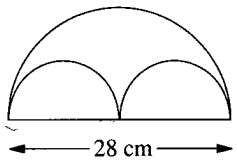
5. A rectangular driveway 12 m long and $4\frac{1}{2}$ m wide is to be covered by similar square tiles of side 25 cm each. Find the number of tiles needed to cover the driveway.

6. The area of a trapezium is 36 cm^2 and the perpendicular distance between its parallel sides is 6 cm. If the lengths of these parallel sides are x cm and y cm, find the value of $(x + y)$. Given further that x is twice as big as y, find the values of x and y.

7. A bucket of water is brought up from a well 9.68 m deep by a rope which winds round a drum 22 cm in diameter. How many turns of the handle are required to bring up a bucket from the bottom of the well?

8. A racing track is a circular ring with inner diameter 140 m and track 7 m wide. How much further does a motorist on the outside rim travel, when he goes round the circuit once, than another who goes round the circuit on the inside rim?

***9.** The diagram shows three semicircles. Calculate the perimeter and area of the shaded region.



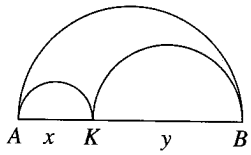
(Take $\pi = \frac{22}{7}$)

1. A single turn of wire wound onto a 5-cm diameter transformer has a mass of 5.5 g. What is its length if the mass of the complete coil of the wire is $1\frac{3}{4}$ kg?

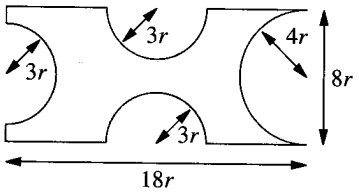
2. A goat, tethered by a rope 1.5 m long, eats a square metre of grass in 14 minutes. Find the time taken if it is to eat all the grass within its reach.

3. A metal disc of radius 6 cm costs 66 cents. Find the cost of 3 square metres of the metal.

4. In the figure below, AB is the diameter of the big semicircle. AK and BK are the diameters of the two smaller semicircles. Given $AK = x$ cm and $BK = y$ cm, find, in terms of x and y, the area of the shaded region enclosed by the three semicircles.

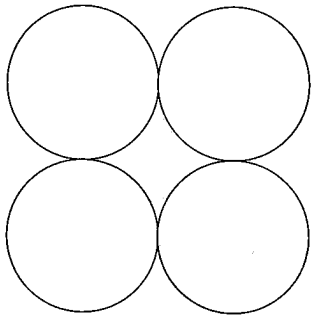


5. The figure shows a rectangular cardboard with 4 semicircles being cut off. Find the area of the remaining cardboard in terms of r.



6. Kumar walks round a rectangular field the length of which is twice its width. He then walks round another rectangular field half as wide but having the same perimeter as the first field. If the difference in area between the two fields is 432 m^2, find the length of the second field.

7. The diagram shows 4 circles of equal radius touching each other. If the radius of each circle is 12 cm, calculate the area of the shaded region.



CHAPTER 10

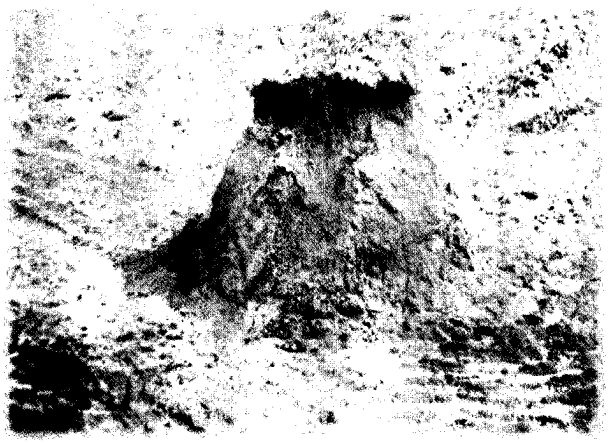
Volume and Surface Area

In this chapter, you will learn how to

- find the volume and surface area of cubes, cuboids, prisms and cylinders;
- solve problems involving volumes made up of the above solids;
- solve problems involving density.

Preliminary Problem

The picture shows the land being cleared for the construction of infrastructure for a new township. The contractor has deliberately left some heaps of soil behind. Do you know that the purpose of this is to estimate the volume of soil taken from the site?



Volume

The volume of an object is the amount of space it occupies. The object that occupies more space is said to have a greater volume.

Fig. 10.1 shows a piece of wood, a brick and a matchbox.

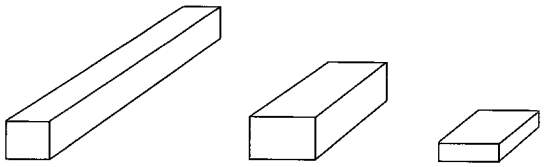
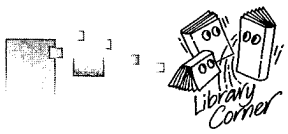


Fig. 10.1

Which one of them occupies the least amount of space? Obviously, the matchbox occupies the least space. But which of the other two, the piece of wood or the brick, occupies more space? To answer this question, we first have to make some measurements and then obtain the volume of each object.



The British System of measure uses pints, gallons, quarts and barrels as units for volume. Find out what these units are and compare them with the SI units.

Units of Volume

As with the case of the area of a plane figure, we compare the volume of an object with a standard unit. A standard unit for volume is a cube with side 1 cm (see Fig. 10.2). We call this 1 cubic centimetre, written as 1 cm^3.

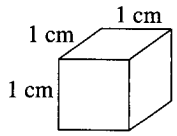


Fig. 10.2

Similarly, a cube with side 1 mm will have a volume of 1 mm^3 and that with side 1 m will have a volume of 1 m^3.

Volume of a Cuboid

Fig. 10.3 shows a rectangular block with dimensions 5 cm by 1 cm by 1 cm. The block contains 5 unit cubes, each of volume 1 cm^3. So the volume of the block is $(5 \times 1 \times 1)$ $cm^3 = 5$ cm^3.

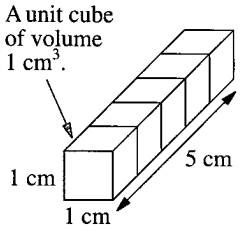


Fig. 10.3

Fig. 10.4 shows a rectangular block with dimensions 5 cm by 2 cm by 1 cm. It contains 10 unit cubes. Hence, the volume of the block is $(5 \times 2 \times 1)$ cm^3 = 10 cm^3.

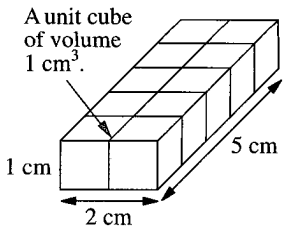


Fig. 10.4

The rectangular block in Fig. 10.5 has dimensions 5 cm by 2 cm by 4 cm. It contains 4 layers of the block shown by Fig. 10.4. Hence, it is made of (4×10) unit cubes = 40 unit cubes and its volume is $(5 \times 2 \times 4)$ cm^3 = 40 cm^3.

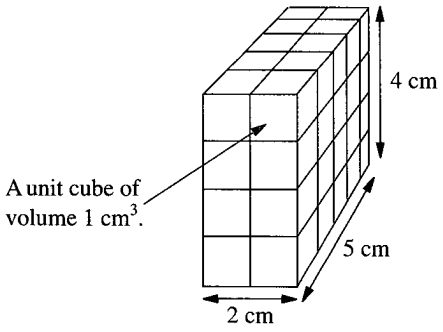


Fig. 10.5

A prism is a solid figure with a flat base and parallel upright edges. A glass prism breaks up white light into different colours.

NB: Each of the rectangular blocks in Figs. 10.3, 10.4 and 10.5 is called a **rectangular prism** or a **cuboid**.

From the above discussion, we see that we can find the volume of a cuboid by multiplying together the length, width and height, which must all be measured in the same units. That is, the volume, V, of a cuboid L units long, W units wide and H units high is given by

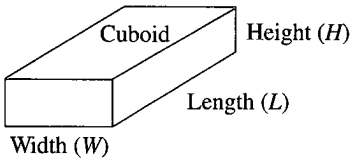


Fig. 10.6

$$V = (\underbrace{L \times W}_{\text{Area of the base}} \times H) \text{ unit}^3$$

A cube can be considered as a special cuboid whose length, width and height are equal, i.e., $L = W = H$. The volume of a cube whose side is L units long is given by

$$V = (L \times L \times L) \text{ unit}^3$$
$$= L^3 \text{ unit}^3$$

Surface Area of a Cuboid

If we unfold a cardboard cuboid, we get a *net* of the cuboid as shown in Fig. 10.7. This net will help us find the total surface area of the cuboid.

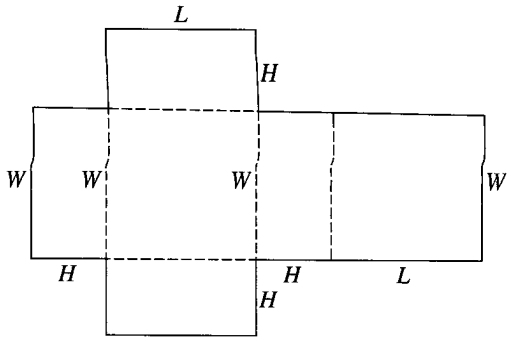
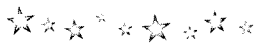


Fig. 10.7

Which area is larger and by how much — a half-km square or a half square km?



There are several different ways of unfolding the same cardboard cuboid to obtain different nets of the same solid. Fig. 10.8 shows another net of the same cuboid. Can you draw another net for the same cuboid?

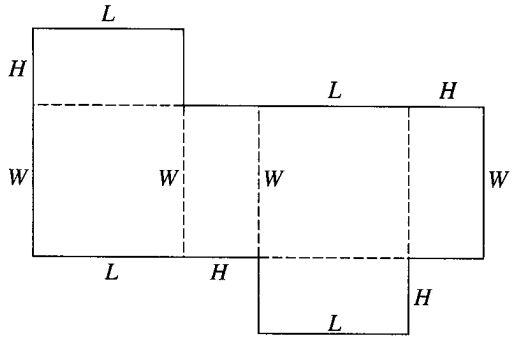


Fig. 10.8

From Fig. 10.7, the surface area of a cuboid of length L units, width W units and height H units
$= [2(L \times W) + 2(L \times H) + 2(W \times H)]$ units^2
$= 2(L \times W + L \times H + W \times H)$ units^2.

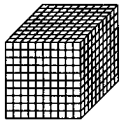
In the case of a cube where the length, width and height are all equal, i.e. $L = W = H$, the total surface area
$= 2(L \times L + L \times L + L \times L)$
$= 2(L^2 + L^2 + L^2)$ units^2
$= 6L^2$ units^2.

Example 7

Express (a) 1 cm^3 in mm^3 and (b) 1 m^3 in cm^3.

Solution

(a) Since 1 cm = 10 mm, a cube with side 10 mm has a volume 1 cm^3.
i.e., 1 cm^3 = (10 × 10 × 10) mm^3
= 1 000 mm^3



(b) Similarly, since 1 m = 100 cm.
1 m^3 = (100 × 100 × 100) cm^3
= 1 000 000 cm^3

Example 2

The figure below shows a rectangular prism 9 cm long and 7 cm wide. Given that the volume of the prism is 315 cm³, find

(a) the height of the prism;
(b) its surface area.

Solution

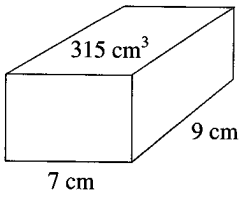
(a) $L = 9$, $W = 7$, $H = ?$

Volume of the prism $= L \times W \times H$

$315 = 9 \times 7 \times H$

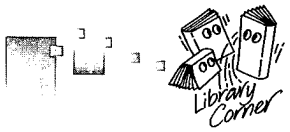
$$H = \frac{315}{9 \times 7} = 5$$

$\therefore$ the height of the prism is 5 cm.



(b) The surface area of the prism $= 2(9 \times 7 + 9 \times 5 + 7 \times 5)$ cm²

$= 286$ cm²



Find out the names and capacities of the largest and the smallest reservoirs in Singapore. Also, find out the average daily consumption of water in Singapore in 1999.

Volume of Fluids

The volume of fluids, or liquids, is measured using special units. These units are the millilitre (m*l*), the litre (*l*) and the kilolitre (k*l*). Normally, we buy milk and petrol by the litre and we take medicine by the millilitre.

1 m*l* = 1 cm³

1 litre = 1 000 m*l* = 1 000 cm³

1 kilolitre = 1 000 litres = 1 m³

Example 3

A container is in the form of a cuboid 20 cm long, 3 cm wide and 14 cm high. Find the volume of the liquid, in litres, that the container can hold (i.e., the capacity of the container).

Solution

The volume of the container $= (20 \times 3 \times 14)$ cm³ $= 840$ cm³

1 000 cm³ = 1 litre

$\therefore$ the volume of the liquid $= \frac{840}{1\,000}$ litre $= 0.84$ litre

Example 4

Express (a) 3 600 000 mm³ in (i) cm³ and (ii) ml;
(b) 0.7 m³ in (i) cm³ and (ii) litres.

Solution

(a) (i) $10 \text{ mm} = 1 \text{ cm}$

$1\,000 \text{ mm}^3 = 1 \text{ cm}^3$

$3\,600\,000 \text{ mm}^3 = \frac{3\,600\,000}{1\,000} \text{ cm}^3$

$= 3\,600 \text{ cm}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ ml}$

$3\,600 \text{ cm}^3 = 3\,600 \text{ ml}$

(b) (i) $1 \text{ m} = 100 \text{ cm}$

$1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$

$0.7 \text{ m}^3 = (0.7 \times 1\,000\,000) \text{ cm}^3$

$= 700\,000 \text{ cm}^3$

(ii) $1\,000 \text{ cm}^3 = 1 \text{ litre}$

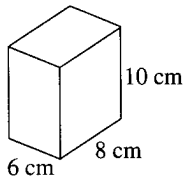
$700\,000 \text{ cm}^3 = \frac{700\,000}{1\,000}$ litres

$= 700$ litres

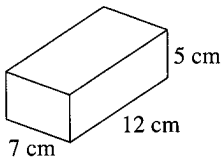
Exercise 10a

1. Find the volume and surface area of the following cuboids, and draw its net:

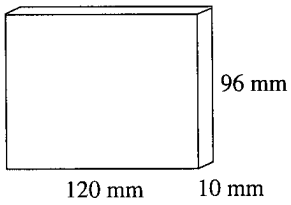
(a)



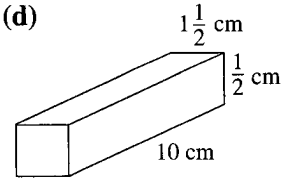
(b)



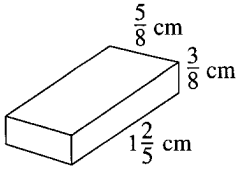
(c)



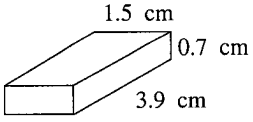
(d)



(e)



(f)



2. Copy and complete the following table for each cuboid:

	Length	*Width*	*Height*	*Volume*	*Surface Area*
(a)	24 mm	18 mm	5 mm		
(b)	5 cm	3 cm		120 cm³	
(c)		6 cm	$3\frac{1}{2}$ cm	52.5 cm³	
(d)	12 m		6 m	576 m³	
(e)	$2\frac{1}{4}$ cm	8 cm		$58\frac{1}{2}$ cm³	
(f)	9 cm	12 cm			426 cm²

3. Find the capacity of each of the following rectangular tanks, giving your answer in litres:

(a) Height = 3.6 m, length = 5.5 m, width = 3.5 m.

(b) Height = 2.7 m, length = 4.75 m, width = 2.6 m.

(c) Height = 0.15 m, length = 0.24 m, width = 0.19 m.
(d) Height = 38 cm, length = 52 cm, width = 18 cm.

4. Find the total surface area of a solid cube of volume 64 cm^3.

5. A man sells sugarcane juice in 200 m*l* cups. How many cups of sugarcane juice can he dispense from his big rectangular tank of length 65 cm, width 40 cm and height 54 cm?

*6. An open water tank with length 20 cm and width 15 cm holds 4.8 litres of water. Calculate the height of the water level in the tank and the total surface area of the cuboid in contact with the water.

7. A rectangular tank measures 4 m long, 2 m wide and 4.8 m high. Initially it is half filled with water. Find the depth of water in the tank after 4 000 litres more of water are added to it.

8. A rectangular water tank of length 60 cm and width 40 cm contains water up to a depth of 30 cm. A piece of ice measuring 20 cm by 15 cm by 12 cm is dropped into the tank of water. Calculate the new depth of water when the ice melts completely, assuming its volume decreases by $\frac{1}{10}$.

9. It took two and a half years and 2.85 million m^3 of earth to fill the disused Sin Seng quarry at Rifle Range Road.
   (a) If each truck can carry a maximum load of 6.25 m^3 of earth per trip, how many trips are needed to fill the entire quarry?
   (b) If the cost of transport, material and administration for each truck load is \$55, how much would it cost to fill the quarry?
   The quarry site now provides an area of approximately 3 hectares for future development. Calculate the cost of one m^2 of the land. (1 hectare = 10 000 m^2)

10. In November 1998, the government announced in Parliament a \$10.5 billion package to help the country overcome the Asian economic crisis. If the \$10.5 billion is to be issued in \$2 notes, what will be the volume of all the \$2 notes, assuming that a \$2-note has a length of 13.3 cm, a width of 6.4 cm and a thickness of 0.15 mm. Give your answer in m^3.

Density

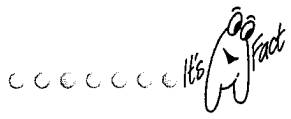
Which is heavier, 1 kg of iron or 1 kg of feathers?

The density of a substance is defined as the mass of one unit volume of the substance.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

We usually express density in g/cm^3 or kg/m^3.

If 1 cm^3 of a certain substance weighs 3.5 g, we say that the density of the substance is 3.5 g per cm^3 or 3.5 g/cm^3. Similarly, if the mass of 1 m^3 of a substance is 500 kg, then the density of the substance is 500 kg/m^3.



Mercury is the liquid with the greatest density. Its density is 13.6 g/cm³, while that of water at 4°C is only 1 g/cm³.

Example 5

If 15 cm³ of a solid weighs 42 g, find the density of the solid.

Solution

15 cm³ of the solid weighs 42 g.

1 cm³ of the solid weighs $\frac{42}{15}$ g = 2.8 g.

∴ the density of the solid is 2.8 g/cm³.

Example 6

The density of a substance is 2.5 g/cm³. If the substance has a volume of 32 cm³, find its mass.

Solution

Mass = density × volume
= (2.5 × 32) g = 80 g

∴ the mass of the substance is 80 g.

If the population of the world is 5 × 10⁹, what is the length of the edge of a cubical box that could hold this many people assuming that the volume of an average person is 5.4 × 10⁻² m³?

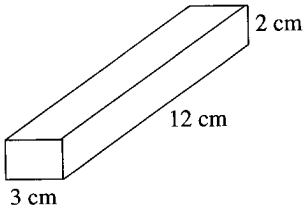
Example 7

The diagram shows a rectangular solid weighing 360 g. Find (a) its volume, and (b) its density in (i) g/cm³ and (ii) kg/m³.

Solution

(a) Volume of cuboid = $L \times W \times H$
= (12 × 3 × 2) cm³ = 72 cm³

∴ the volume of the solid is 72 cm³.



(b) (i) Density = $\frac{\text{mass}}{\text{volume}} = \frac{360}{72}$ g/cm³ = 5 g/cm³

(ii) 5 g = $\frac{5}{1\,000}$ kg

1 cm³ = $\left(\frac{1}{100} \times \frac{1}{100} \times \frac{1}{100}\right)$ m³ = $\frac{1}{1\,000\,000}$ m³

Density = $\left(\frac{5}{1\,000} \div \frac{1}{1\,000\,000}\right)$ kg/m³ = 5 000 kg/m³

∴ the density of the solid is 5 g/cm³ or 5 000 kg/m³.

Exercise 10b

1. Find the density of a metal if 25 g of it has a volume of 8 cm^3.

2. Calculate the density of a solid if 40 cm^3 of it weighs 96.4 g.

3. If 12 cm^3 of a liquid weighs 15.6 g, find the density of the liquid.

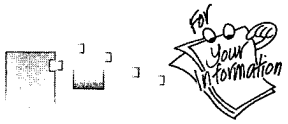
4. Calculate the mass of a piece of solid of volume 26 cm^3 and density 2.8 g/cm^3.

5. Calculate the volume of a piece of cork of mass 105 g and density 0.84 g/cm^3.

6. Calculate the volume of a liquid of mass 3.4 kg and density 13.6 g/cm^3.

7. A rectangular block, 12 cm by 8 cm by 7 cm, has a density of 2.8 g/cm^3. Find
   (a) its volume; **(b)** its mass.

8. A rectangular block, 14 cm by 22 cm by 4 cm, has a mass of 9.4 kg. Find
   (a) its volume; **(b)** its density.



Right Prisms

In general, a right prism is a solid which has two parallel planes of the same shape and size. Also, its lateral surface are perpendicular to its parallel ends.

Cut out a large number of identical triangles from a piece of cardboard and pile them up as shown in Fig. 10.8. A solid is formed. This solid is called a triangular prism. The two parallel planes, PQR and $P'Q'R'$, are triangular in shape and the *triangular prism* takes its name from these planes.

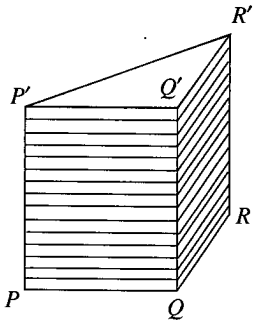


Fig. 10.8

A cuboid is a prism with rectangular planes (see Fig. 10.9). Hence, it is called a rectangular prism. Notice that, in Fig. 10.8, the other three surfaces, which are called the lateral surfaces of the triangular prism, are all rectangular, and that PP', QQ' and RR' are all perpendicular to the planes PQR and $P'Q'R'$. Similarly, in Fig. 10.9, the four lateral surfaces of the cuboid are rectangular and AA', BB', CC' and DD' are perpendicular to the planes $ABCD$ and $A'B'C'D'$. These prisms are called **right prisms**.

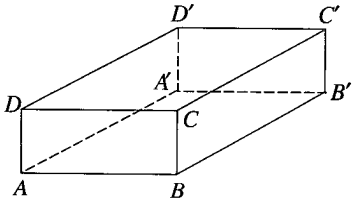


Fig. 10.9

Fig. 10.10 and Fig. 10.11 show a right pentagonal and a hexagonal prism respectively.

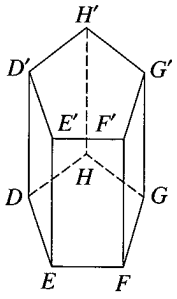


Fig. 10.10

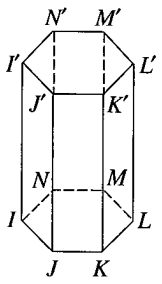


Fig. 10.11

A right prism has a **uniform cross-section**, i.e., the cross-section of the prism is identical to the two parallel ends (see Fig. 10.12).

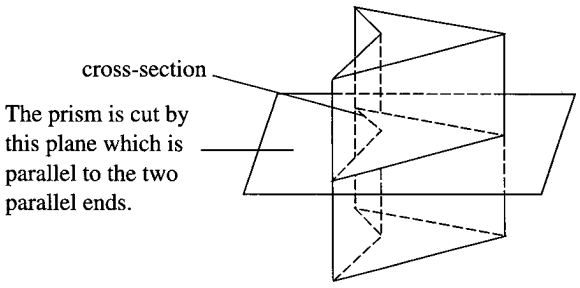


Fig. 10.12

Volume of a Prism

The three prisms shown in Fig. 10.13 are obtained by stacking up a large number of respective identical shapes cut out from cardboards.

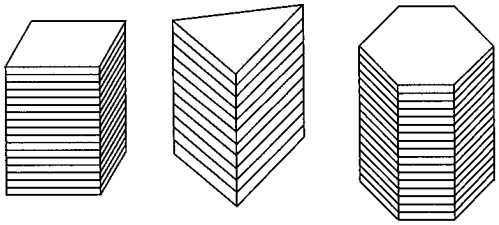


Fig. 10.13

The volume of the right rectangular prism or cuboid
= area of base × height
= area of an identical cardboard × height of rectangular stack
= area of rectangular cross-section × distance between parallel rectangular ends

The volume of a right triangular prism
= area of triangular cross-section × distance between parallel triangular ends

Try to obtain a similar formula as the ones above for the volume of a hexagonal prism.

In general, for a right prism, the volume is given by

volume = area of cross-section × distance between parallel ends
= base area × height

Surface Area of a Prism

Fig. 10.14 shows a right prism whose base is a polygon.

Let A denote the surface area of the prism. Suppose the height of the prism is H and the lengths of the sides of the base are L_1, L_2, L_3, L_4, L_5 and L_6.

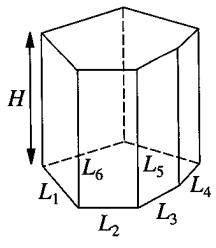
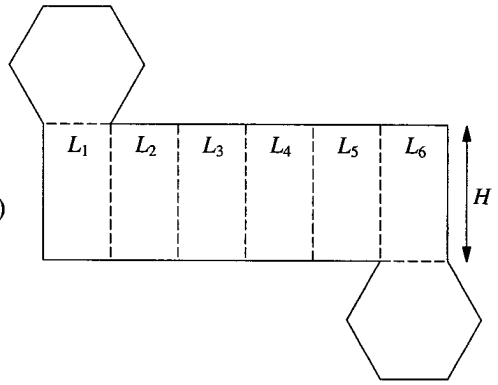


Fig. 10.14

A net of the prism is shown on the right.

The dotted lines indicate the folds.



The area, A, is given by

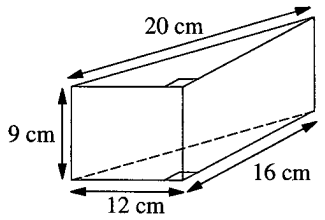
$A = L_1H + L_2H + L_3H + L_4H + L_5H + L_6H + 2(\text{base area})$
$= (L_1 + L_2 + L_3 + L_4 + L_5 + L_6)H + 2(\text{base area})$
$= \text{perimeter of base} \times \text{height} + 2(\text{base area})$

In general,

surface area of a right prism = perimeter of the base × height + 2(base area)

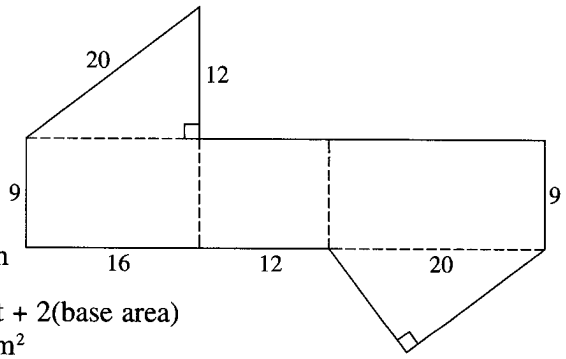
Example 8

Draw a net of the right prism shown on the right and then find its volume and surface area.



Solution

Here is a net of the right prism.



Area of the base $= \left(\frac{1}{2} \times 12 \times 16\right) \text{cm}^2 = 96 \text{ cm}^2$

Volume of the solid = area of the base × height
$= (96 \times 9) \text{ cm}^3 = 864 \text{ cm}^3$

Perimeter of the base = (12 + 16 + 20) cm = 48 cm

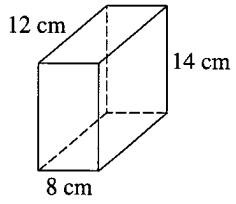
Total surface area = perimeter of the base × height + 2(base area)
$= [48 \times 9 + 2(96)] \text{ cm}^2 = 624 \text{ cm}^2$

Example 9

Find the volume and surface area of the right prism shown.

Solution

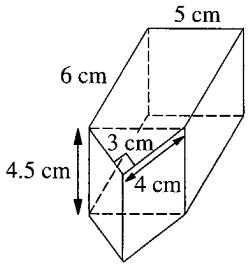
Volume of the right prism = $(12 \times 8 \times 14) \text{ cm}^3$
$= 1\,344 \text{ cm}^3$



Surface area of the right prism
$= 2(12 \times 8 + 8 \times 14 + 14 \times 12) \text{ cm}^2$
$= 2(96 + 112 + 168) \text{ cm}^2 = 752 \text{ cm}^2$

Example 10

Find the volume and surface area of the right pentagonal prism shown.



Solution

Area of the base $= \left[(6 \times 5) + \left(\frac{1}{2} \times 3 \times 4\right)\right]$ cm^2 = 36 cm^2

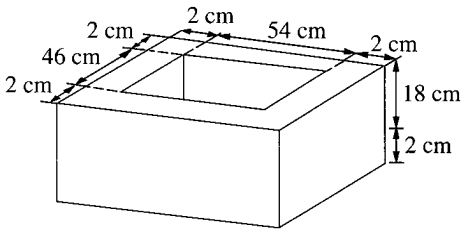
$\therefore$ volume of the prism = (36 × 4.5) cm^3
= 162 cm^3

Area of the lateral surfaces = perimeter of the base × height
= [(6 + 5 + 6 + 4 + 3) × 4.5] cm^2
= 108 cm^2

$\therefore$ total surface area of the prism = [108 + 2(36)] cm^2
= 180 cm^2

Example 11

Find the volume of wood used in making an open rectangular box 2 cm thick, given that its internal dimensions are 54 cm long, 46 cm wide and 18 cm deep.



Solution

External length = (54 + 2 + 2) cm = 58 cm

External breadth = (46 + 2 + 2) cm = 50 cm

External height = (18 + 2) cm = 20 cm

External volume = (58 × 50 × 20) cm^3 = 58 000 cm^3

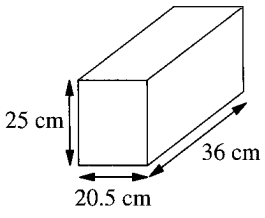
Internal volume = (54 × 46 × 18) cm^3 = 44 712 cm^3

$\therefore$ volume of wood used = (58 000 – 44 712) cm^3
= 13 288 cm^3

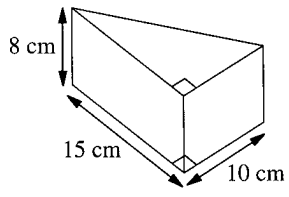
Exercise 10c

1. Draw a net of each of the following right prisms and find its volume.

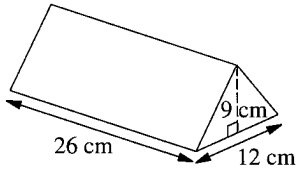
(a)



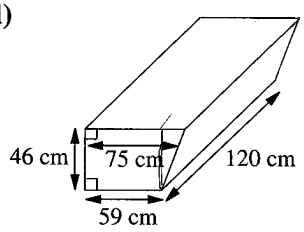
(b)



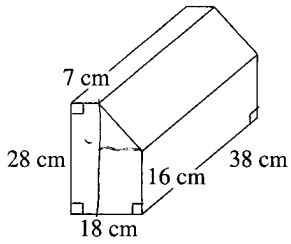
(c)



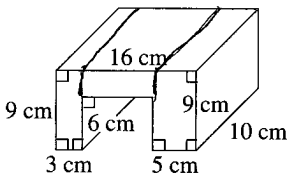
(d)



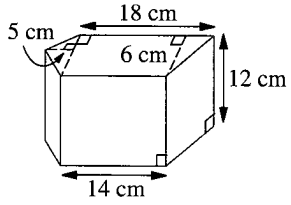
(e)



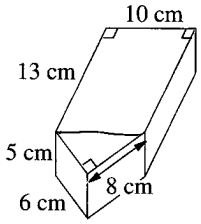
(f)



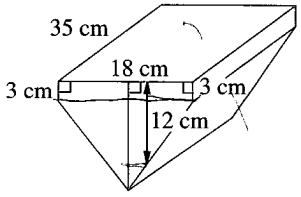
(g)



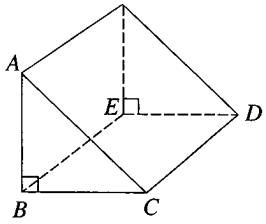
(h)



(i)



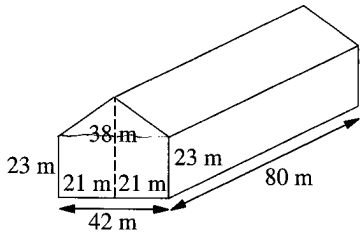
2. The figure shows a right prism standing on a horizontal, rectangular base *BCDE*. The triangle *ABC* is a vertical cross-section of the solid prism.



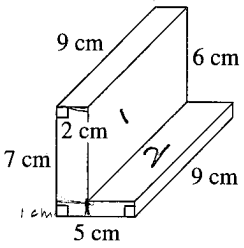
Copy and complete the table below:

	AB	*BC*	*CD*	*Area of* $\triangle ABC$	*Volume of prism*
(a)	3 cm	4 cm	7 cm		
(b)	9 cm		11 cm	63 cm^2	
(c)		15 cm	300 cm		72 000 cm^3
(d)	24.6 cm	7.8 cm			38 376 cm^3

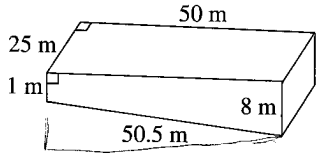
***3.** Not taking into consideration the thickness of the walls and roof, find the air space in the hall with the dimensions given in the figure:



***4.** Find the volume and the surface area of the solid, which is in the shape of a right prism, as shown:



***5.** A swimming pool is 50 m long and 25 m wide. It is 1 m deep at the shallow end and 8 m deep at the other end. Find the volume of water in the pool when it is full as well as the total area of the pool which is in contact with the water (refer to figure).



6. Find the volume and surface area of a right prism of height 20 cm whose base is a square of side 15 cm.

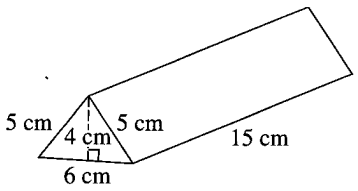
7. $4\frac{1}{2}$ litres of oil are poured into a rectangular container whose cross-section is a square of side 12 cm. What is the depth of the oil in the container?

8. A closed box is 135 cm long, 80 cm wide and 60 cm deep internally. It is to be lined on its sides and bottom with cedar veneer of negligible thickness. Find, in square metres, the area of veneer needed.

9. The parallel ends of a right prism, 15 cm long, are isosceles triangles with measurements shown below. Find

(a) the volume;

(b) the surface area of the prism.



10. The internal dimensions of an open concrete tank are 1.8 m long, 0.8 m wide and 1.2 m high. Find the capacity of the tank in litres. If the concrete is 0.1 m thick, find also, in cubic metres, the volume of concrete used.

11. A trough, in the form of an open rectangular box, is 1.85 m long, 45 cm wide and 28 cm deep externally. If the trough is made of wood 2.5 cm thick, find, in cubic centimetres, the volume of wood required.

12. A tin which is 12 cm long, 9 cm wide and 4 cm deep holds 120 g of tea. If 1 kg of the same tea is packed into a tin which has a 12-cm square base, how tall will the tin have to be?

***13.** The cross-section of a drain is a rectangle 30 cm wide. If water 3.5 cm deep flows along the drain at a rate of 22 cm per second, how many litres of water will flow through each minute?

Formally, the cylinder shown here is called a right circular cylinder. In this book, we use 'cylinder' to represent a 'right circular cylinder'.

Cylinders

We can form a cylindrical solid by vertically stacking up a pile of 50-cent coins as shown in the figure. This solid is called a right circular prism or simply a right cylinder. Its cross-sectional area is a circle. Steel pipes, oil drums and many tin containers for liquids and preserved food are all common examples of cylinders. Can you name other objects which are cylindrical in shape?

a stack of 50-cent coins

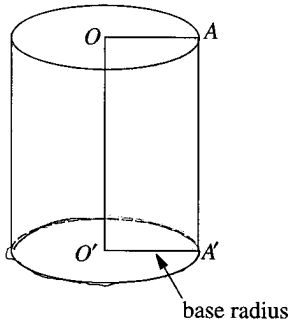
Volume of a Cylinder

Since a cylinder is a right prism with uniform cross-section, we can find its volume by applying the same method used in finding the volume of a right prism,

i.e., **volume of a cylinder = base area × height**

Thus, the volume of a cylinder of base radius r and height h is given by

$$\textbf{volume} = \pi r^2 h$$



Surface Area of a Cylinder

Fig. 10.15(a) shows two equal circles of radius r and a rectangle of height h. It is a net of a cylinder. To form the cylinder shown in Fig. 10.15(b), we roll up the rectangle and bring the two edges AB and CD together. The two equal circles formed will become the top and base circles of the cylinder. Obviously, the length of the rectangle is equal to $2\pi r$, the circumference of each circle.

∴ the area of the curved surface of the cylinder = area of rectangle $ABCD = 2\pi rh$

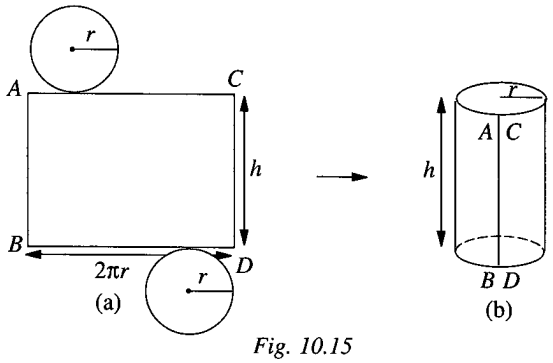


Fig. 10.15

Surface area of a solid cylinder = the area of curved surface + 2 × the area of the base circle
$$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

A man wishes to take 4 litres of water out of a big tank of water. But he has only one 5-litre and one 3-litre jar. How can he do it?

Example 12

The diameter of the base of a right circular cylinder is 14 cm and its height is 10 cm. Find the volume and surface area of the solid cylinder. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution

$r = \frac{14}{2} = 7$, $h = 10$

Volume $= \pi r^2 h$

$$= \left(\frac{22}{7} \times 7^2 \times 10\right) \text{cm}^3$$
$$= 1\,540 \text{ cm}^3$$

∴ the volume is 1 540 cm³.

Surface area $= 2\pi r(h + r)$

$$= \left[2 \times \frac{22}{7} \times 7 \times (10 + 7)\right] \text{ cm}^2$$

$$= 748 \text{ cm}^2$$

$\therefore$ the surface area is 748 cm^2.

The standard unit of measure of weight, or size, of precious stones like diamond is the "carat". A carat is equal to 0.2 gram or 200 milligrams (mg). A carat is further divided into 100 points. Thus, a 20-point diamond has a weight of 40 mg.

However, the carat is a measure of purity as far as gold jewellery is concerned. 24 carat gold refers to pure gold and 18 carat gold is $\frac{18}{24} \times 100\%$, i.e., 75% pure gold. If you are given a choice of a 12 carat diamond or 500 g of 12 carat gold bar, which would you choose?

Example 13

Find the mass, in kg, of a cylindrical metal bar 1.2 m long and 1.4 cm in radius. (The density of the metal is 7.5 g/cm^3.)

Solution

$r = 1.4$ cm, $h = 1.2$ m $= 120$ cm

Volume $= \pi r^2 h$

$$= \left(\frac{22}{7} \times 1.4 \times 1.4 \times 120\right) \text{ cm}^3$$

$$= 739.2 \text{ cm}^3$$

Mass = density × volume

$= (7.5 \times 739.2)$ g

$= 5\,544$ g

$= 5.544$ kg

$\therefore$ the mass of the bar is 5.544 kg.

Example 14

If water flows through a 56-mm diameter pipe at the rate of 3 m/s, what volume of water, in litres, is discharged per minute? $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution

$r = 28$ mm $= 2.8$ cm, $h = 3$ m $= 300$ cm

Volume of water discharged per second $= \pi r^2 h$

$$= \left(\frac{22}{7} \times 2.8 \times 2.8 \times 300\right) \text{ cm}^3$$

$$= 7\,392 \text{ cm}^3$$

Volume of water discharged per minute $= (7\,392 \times 60)$ cm^3

$= 443\,520$ cm^3

$= 443.5$ litres (correct to 1 decimal place)

$\therefore$ 443.5 litres of water are discharged per minute.

Example 15

A circular metal sheet 30 cm in diameter and 0.25 cm thick is melted and then recast into a cylindrical bar of diameter 5 cm. Find the length of the bar.

Solution

Volume of circular sheet = $\left(\pi \times 15 \times 15 \times \frac{1}{4}\right)$ cm^3

Let the length of the bar be x cm.

Volume of bar = $\left(\pi \times \frac{5}{2} \times \frac{5}{2} \times x\right)$ cm^3

Volume of bar = Volume of circular sheet

$$\pi \times \frac{5}{2} \times \frac{5}{2} \times x = \pi \times 15 \times 15 \times \frac{1}{4}$$

$$x = \frac{\pi \times 15 \times 15 \times \frac{1}{4}}{\pi \times \frac{5}{2} \times \frac{5}{2}} = 9$$

$\therefore$ the bar is 9 cm long.

In this example, no numerical value is used for π.

Hollow Cylinders

Imagine a solid cylinder of radius R and height h. Suppose another cylinder of smaller radius r (i.e. $r < R$) but of the same height h is scooped out from it. This results in a tube, or a hollow cylinder, as shown in Fig. 10.16. The volume of the hollow cylinder is the difference between the volumes of the two solids.

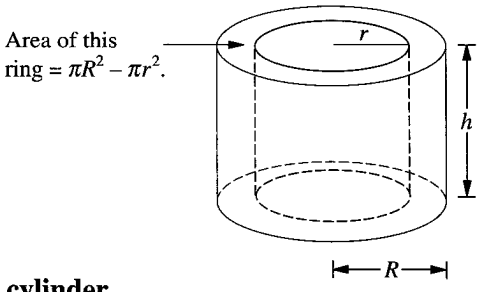


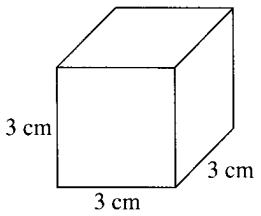
Fig. 10.16

$\therefore$ **Volume of a hollow cylinder**
$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$

NB: An open cylinder refers to one with a base but without a lid. A closed cylinder refers to one with a base and a lid.

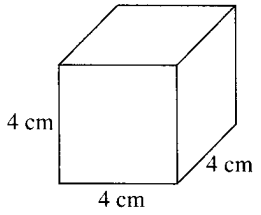
Do not substitute the value of π until it is clear that such a substitution is necessary.

A cube of side 3 cm is painted green on all its 6 faces. It is to be cut into 27 1-cm cubes. How many cuts do you need to make?



How many small cubes have
(1) no painted faces at all;
(2) 1 face painted green;
(3) 2 faces painted green;
(4) 3 faces painted green;
(5) 4 faces painted green?

You are now given a 4-cm cube which is also painted green on all its faces. How many cuts do you need to make to reduce it to 64 1-cm cubes?



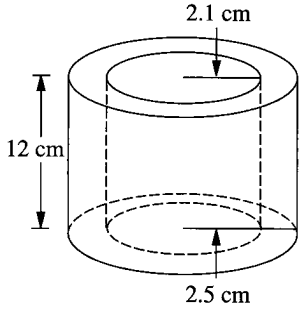
How many small cubes have
(1) 4 faces painted green;
(2) 3 faces painted green;
(3) 2 faces painted green;
(4) 1 face painted green;
(5) no painted faces at all?

Example 16

The figure on the right shows a section of a steel pipe. Given the internal radius of the pipe is 2.1 cm, the external radius is 2.5 cm and the length of the pipe is 12 cm, find

(a) the volume of steel used; *(b) its total surface area. (Take $\pi = 3.14$)*

Solution



(a) $R = 2.5$, $r = 2.1$, $h = 12$

The cross-section of the pipe is a ring.

Area of ring $= [\pi(2.5)^2 - \pi(2.1)^2]$ cm^2

$= 1.84\pi$ cm^2

Volume of pipe $= (18.4\pi \times 12)$ cm^3

$= (1.84 \times 3.14 \times 12)$ cm^3

$= 69.3$ cm^3 (correct to 1 decimal place)

$\therefore$ the volume of steel used $= 69.3$ cm^3.

(b) Total surface area of pipe

= areas of internal and external curved surfaces + area of 2 rings

$= [(2\pi \times 2.1 \times 12) + (2\pi \times 2.5 \times 12) + (2 \times 1.84\pi)]$ cm^2

$= (50.4\pi + 60\pi + 3.68\pi)$ cm^2

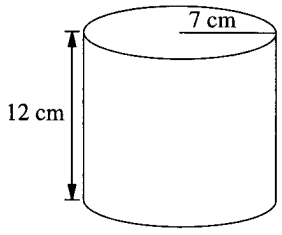
$= 358.2$ cm^2 (correct to 1 decimal place)

Exercise 10d

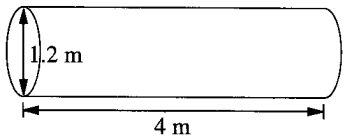
In this exercise, take π to be $\frac{22}{7}$ unless otherwise stated.

1. Find the volume and total surface area of each of the following cylindrical solids:

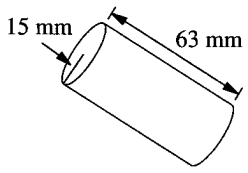
(a)



(b)



(c)



2. Find the diameters of the cylinders given the following:

(a) volume 704 cm^3, height 14 cm;

(b) volume 12 320 cm^3, height 20 cm.

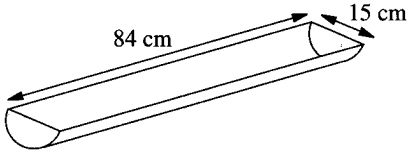
3. Find the heights of the cylinders given the following:

(a) volume 528 cm^3, diameter 4 cm;

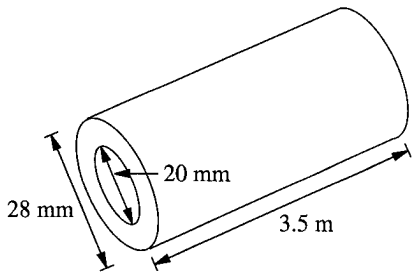
(b) volume 1 056 m^3, radius 4 m.

4. A cylindrical can of radius 5 cm and height 8 cm is used to pour water into a larger cylinder of radius 20 cm and height 2 m. How many times must this be done to fill the larger cylinder?

5. The diagram shows a drinking trough in the shape of a half-cylinder with dimensions as shown. Find its capacity in litres.



6. The diagram shows a metal pipe with an outer diameter of 28 mm and an inner diameter of 20 mm. Its length is 3.5 m. Find the volume, in cm^3, of the metal used in making the pipe.



7. In a toy factory, 200 wooden solid cylinders 7 cm long and 35 mm in diameter have to be painted. What is the total surface area, in cm^2, that needs to be painted?

8. 500 cylindrical cans, without top lids and each 14 cm high with diameter 8 cm, are to be made from a sheet of metal. Find, in m^2, the total area that needs to be painted externally. (Take $\pi = 3.14$)
Correct your answer to one decimal place.

9. A railway tunnel 147 m long is to be bored with a circular cross section of radius 5 m. What volume of soil has to be excavated? If the soil is to be taken away in wagons of capacity 75 m^3 each, how many wagons are needed?

10. A beer cask has a height of 63 cm and a diameter of 50 cm. Find its capacity in litres. How many glasses full of beer can it serve if the capacity of each glass is 0.6 litre?

11. A cylindrical solid, whose base radius and height are 10 cm and 14 cm respectively, has a density of 8.6 g/cm^3. Find
(a) its volume; **(b)** its mass.

12. A cylindrical solid with a base radius of 7 cm and a height of 20 cm has a mass of 2.6 kg. Find
(a) its volume; **(b)** its density.

13. Assuming that a \$1 coin is cylindrical with a diameter of 2.24 cm and a thickness of 2.5 mm, find the volume of the coin, giving your answer in cm^3. If the density of the coin is 5.4 g/cm^3, find its weight, correct to 2 decimal places.

The Singapore government announced a \$10.5 billion recovery package in Parliament in 1998 to help the country overcome the Asian economic crisis. If the \$10.5 billion is to be given out in \$1 coins, what is the volume of all the coins? (Give your answer in m^3, correct to 1 decimal place.) Also find the weight of all the \$10.5 billion coins, giving your answer in tonnes, correct to the nearest tonnes.

Summary

1. The volume of an object is the amount of space it occupies. A standard unit for volume is 1 cm^3, which is the volume of a cube of side 1 cm.

2. **(a)** Volume of a cuboid L units long, W units wide and H units high = $(L \times W \times H)$ $unit^3$.
(b) Volume of a cube with side L units long = L^3 $unit^3$.
(c) Volume of a right prism = base area × height.

3. **(a)** Surface area of a cuboid, L units long, W units wide and H units high
$= 2(L \times W + L \times H + W \times H)$ $unit^2$.
(b) Surface area of a cube with side L units long = $6L^2$ $unit^2$.

4. For a cylinder of base radius r and height h, curved surface area = $2\pi rh$, total surface area = $2\pi rh + 2\pi r^2$ or $2\pi r(h + r)$ and volume = $\pi r^2 h$.

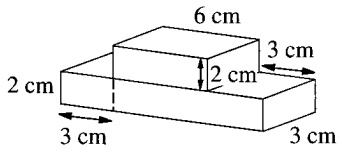
5. The volume of a hollow cylinder with external radius R, internal radius r and height h is given by $V = h(\pi R^2 - \pi r^2)$.

Review Questions 10

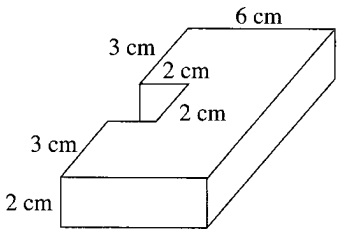
Take π to be $\frac{22}{7}$, where necessary, for the following questions:

*1. (a) Find the volume and surface area of each of the following right prisms:

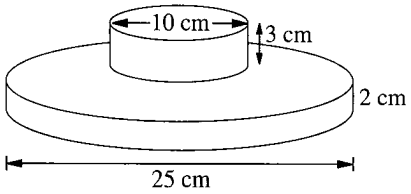
(i)



(ii)

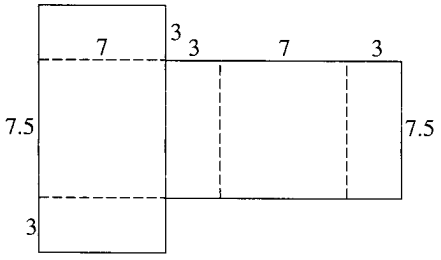


(b) Find the volume and surface area of the following solid, which is made up of two cylinders:

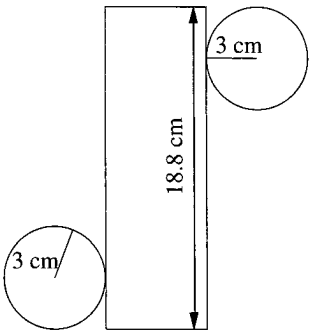


2. The following shows the nets of certain solids. State the name of each of the solids formed and draw a sketch of the solid.

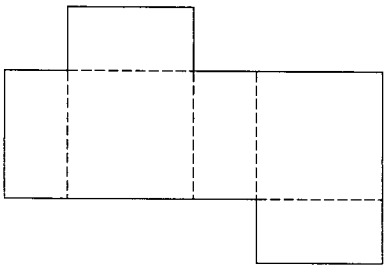
(a)



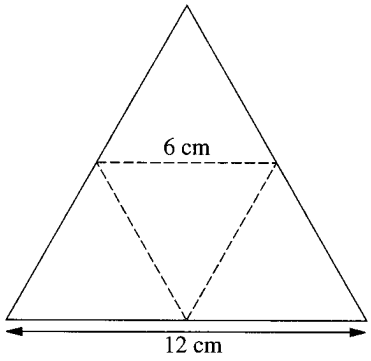
(b)



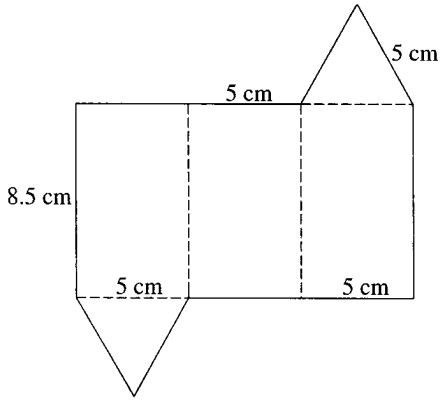
(c)



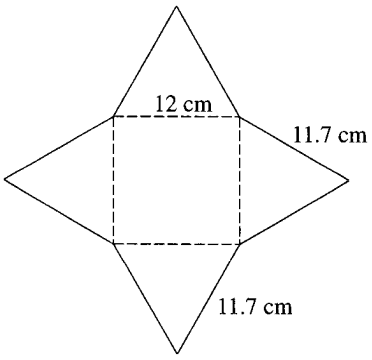
(d)



(e)



(f)



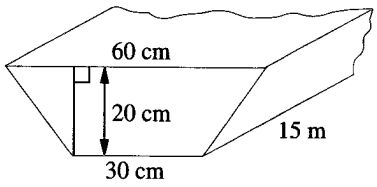
3. A room 8 m long and 5.5 m wide contains 123.2 m^3 of air. Find the height of the room.

4. A brick measures 18 cm by 9 cm by 6 cm. Find the number of bricks that will be needed to build a wall 4.5 m wide, 18 cm thick and 3.6 m high.

5. A water tank, 0.8 m long, 0.8 m wide and 2.4 m deep is half-full of water. How many times can a watering-can be filled if its capacity is approximately 12 litres?

6. How many matchboxes, each 80 mm by 75 mm by 18 mm, can be packed into a box 72 cm by 60 cm by 45 cm internally?

7. The following figure shows a trough 15 m long. Its cross-section is a trapezium. Find the amount of water that the trough can hold in litres.



8. A slab of marble is 2.4 m long, 28 cm wide and 5 cm thick. If the density of the marble is 3.1 g/cm^3, what is its mass?

9. A rectangular wooden beam is 24 cm by 16 cm in cross-section and 6 m long. Find the mass of the beam if the wood has a density of 750 kg/m^3.

10. Find the mass of the water that has fallen onto a flat roof 10.4 m long and 6.5 m wide, when 25 mm of rain is recorded. (The mass of 1 cm^3 of water is 1 g.)

11. How many cubic metres of concrete are needed to surround a rectangular pond $4\frac{1}{4}$ m by 4 m with a border $\frac{1}{4}$ m wide and 18 cm thick?

12. 132 litres of oil is poured into a cylindrical drum, 40 cm in diameter. What is the depth of the oil in the drum?

13. Ten open cylindrical containers are to be painted on the outside, including the base. Each container has a radius of 30 cm and a height of 28 cm. Given that 150 g of paint is needed to paint an area of 1 m^2, find the amount of paint required to paint the ten cylinders. Give your answer in kg.

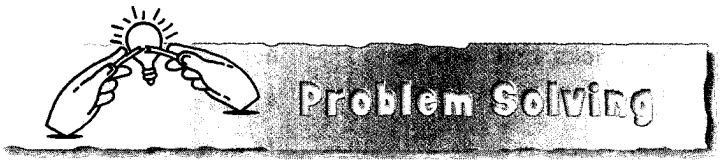
14. A cylindrical barrel 70 cm in diameter and 80 cm in height is filled with water. A leak at the bottom drains away 0.2 litres of water every minute. How long will it take for the water level to drop by 6 cm?

15. The Singapore Expo has an exhibition area of 60 000 m^2, making it the largest exhibition centre in the region. If the average height of the exhibition centre is 4.85 m, find the volume of air in the centre. If the density of air is approximately 1.26 kg/m^3, find the mass of air contained in the centre.

16. It took two and half years and 2.85 million m^3 of earth to fill the disused Sin Seng quarry in Rifle Range Road.

(a) If each truck can carry 5.75 m^3 of earth per trip, how many trips are needed to fill the quarry?

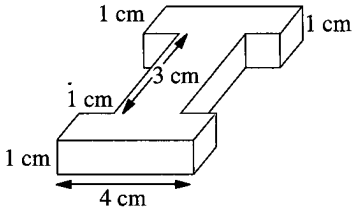
(b) Taking one year to be 365 days, find the number of truck loads ferried per day for the above project, giving your answer correct to the nearest whole number.



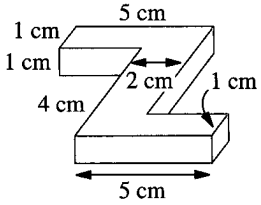
Take π to be $\frac{22}{7}$, where necessary, for the following questions.

1. **(a)** Calculate the volume and surface area of each of the following right prisms:

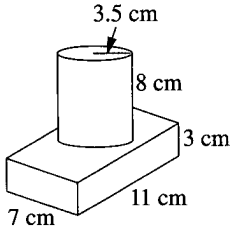
(i)



(ii)

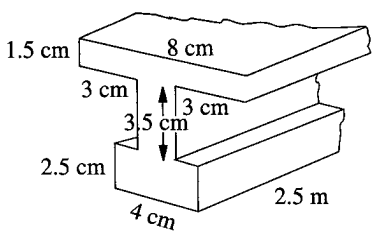


(b) Calculate the volume and surface area of the following solid, which is made up of a cylinder and a right prism:



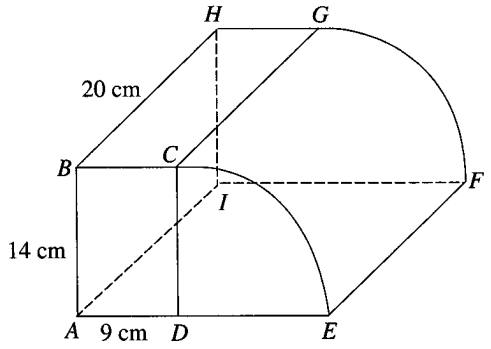
2. The figure shows the dimensions of the cross-section of a girder which is 2.5 m long. Find

(a) the volume of the girder;
(b) the surface area;
(c) its weight if the material weighs 7.8 g per cm^3.



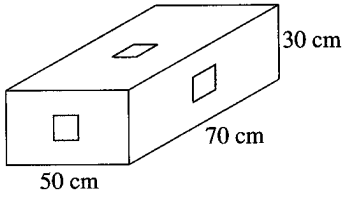
3. The figure shows a closed container of uniform cross-section. The cross-section consists of rectangle *ADCB* and a quadrant *DEC* of a circle, centre *D*. Given *AB* = 14 cm, *AD* = 9 cm and *BH* = *CG* = *EF* = *AI* = 20 cm, calculate

(a) the area of the cross-section *ADECB*;
(b) the volume of the container;
(c) the area of the surface *BCEFGH*.



4. A section of a metal pipe has internal diameter 4.2 cm and external diameter 5.0 cm. If the length of the metal used for the pipe is 8.9 cm, calculate the volume of the metal used for making the pipe. If the metal costs \$8 per kg and 1 m^3 of the metal has a mass of 2 700 kg, find the cost of the pipe.

5. A cuboid of dimension 70 cm by 50 cm by 30 cm has "square holes" measuring 10 cm by 10 cm in the centre of three faces of the cuboid, as shown. Calculate the volume and the surface area of the remaining solid.



CHAPTER 11

Ratio, Rate and Proportion

In this chapter, you will learn how to

- find the ratio of two or more quantities;
- recognize and use common measures of rate;
- solve problems involving rate;
- use direct and inverse proportions;
- solve problems involving ratios and proportions.

Preliminary Problem

The picture shows an ancient sun-dial used in China for measuring the time of the day.

The measurement of time is essential to calculate the speed of a moving object. It is also needed in many other situations such as measuring the rate of one's heart beat, the rate of change of force in the measure of power and the rate at which your body digests food.

Ratio

In a secondary one class of 45 pupils, 15 of the pupils are girls. We can compare the number of boys and the number of girls in the class using two different ways:

(1) There are 15 more boys than girls in the class. Here, we are comparing the number of boys and the number of girls in the class by finding their difference.
(2) The number of boys in the class is twice that of girls. Here, we are comparing the number of boys and the number of girls by finding a fraction consisting of the number of boys over the number of girls. The fraction is thus $\frac{30}{15}$.

The fraction obtained in (2) is an example of a **ratio** which is used to compare two quantities of the same kind.

A ratio may be written with two dots in between the numbers. In our example, the boy-girl ratio in the class is expressed as 30 : 15, or $\frac{30}{15}$.

30 : 15 means 'the ratio of 30 to 15'.

In general, the ratio of a to b, where a and b represent two quantities and b is not zero, is written as $\boldsymbol{a : b}$, or $\frac{a}{b}$.

In-Class Activity

If the ratio of the length of Fatimah's hair to that of Fandi's hair is 3 : 1, can we make the following conclusions?

1. Fatimah's hair is 3 times as long as Fandi's hair;
2. Fatimah's hair is very long;
3. Fatimah is 3 times as old as Fandi;
4. Fatimah is 3 times taller than Fandi.

Expressing a ratio in its simplest form is the same as reducing it to its lowest term. Thus 25 : 15 in its simplest terms is 5 : 3.

We usually express a ratio in its simplest form.

Equivalent Ratios

We know that $\frac{30}{15} = \frac{2}{1}$, thus 30 : 15 = 2 : 1. We say that 30 : 15 and 2 : 1 are **equivalent ratios.**

NB: The **order** in which the ratio is expressed is important.
Using the previous example of the class of 45 pupils, the boy-girl ratio is 30 : 15 = 2 : 1, or $\frac{2}{1}$, while the girl-boy ratio is 15 : 30 = 1 : 2, or $\frac{1}{2}$.

A ratio has **no units**. It is merely a number which indicates how many times one quantity is as great as the other or what fraction one quantity is of another. For example, the boy-girl ratio of 2 : 1 indicates that the number of boys is twice that of girls, and the girl-boy ratio of 1 : 2, or $\frac{1}{2}$, indicates that there are half as many girls as there are boys.

Example 1

Find the ratio of (a) 50 g to 200 g and (b) 700 g to 1 kg.

Divide both terms of the ratio by the HCF of the terms, i.e., 50.

Solution

(a) The ratio of 50 g to 200 g can be found using two different methods.

Method 1 $50 : 200 = \frac{50}{200} = \frac{1}{4} = 1 : 4$

Method 2 $50 : 200 = \frac{50}{50} : \frac{200}{50} = 1 : 4$

$\therefore$ the ratio of 50 g to 200 g is 1 : 4.

(b) 700 g and 1 kg are of different units and thus we have to express them in the same units first.

It is easier to express 1 kg as 1 000 g.

$\therefore$ the ratio of 700 g to 1 kg is 700 : 1 000 or 7 : 10.

Ratios can be used to compare more than two quantities. For example, three men, *A*, *B* and *C*, share the profit of a business. They receive \$4 000, \$3 000 and \$1 000 respectively. The ratio of their share of the profit is then 4 000 : 3 000 : 1 000 or 4 : 3 : 1.

Exercise 11a

1. Copy and complete the following equivalent ratios:

(a) 2 : 3 = ☐ : 9 **(b)** ☐ : 8 = 12 : 32

(c) 6 : 24 = 3 : ☐ **(d)** 12 : ☐ = 36 : 21

2. Express each of the following ratios in its simplest form:

(a) 6 : 10 **(b)** 44 : 8

(c) 3.6 : 4.5 **(d)** 0.4 : 20

(e) $1\frac{1}{2} : 2$ **(f)** 32 : 40 : 24

(g) $1\frac{1}{3} : \frac{2}{3} : \frac{1}{6}$ **(h)** 1.2 : 2 : 2.8

(i) $1.4 : \frac{2}{5} : \frac{1}{3}$ **(j)** $6\frac{2}{5} : 9.6 : 16$

3. Express each of the following as a ratio of the first quantity to the second, in its lowest term, **(i)** in the form a : b, **(ii)** as a fraction:

(a) 25 cents, 80 cents

(b) 210°, 360°

(c) 250 cm, 1 m

(d) 80 cents, \$1.20

(e) 1 kg 250 g, 3 kg

(f) 3 min 30 s, 1 h

4. In a carpark, the ratio of red cars to green is 5 : 6, while that of green cars to blue is 3 : 10. Find the ratio of red cars to blue cars.

5. A school has an enrolment of 630 local students and 120 foreign students. Find the ratio of foreign students to local students.

6. A man earns \$1 200 and spends \$450 per month. Find the ratio of **(a)** his income to his expenditure and **(b)** his savings to his income.

7. Three people, A, B and C, share \$416 among themselves. A receives \$169 and B receives \$156. Find the ratio in which the sum of money is shared.

8. The interior angles of a quadrilateral are 40°, 60°, 120° and 140°. Find the ratio of these angles according to the order given.

9. The sides of two squares are 4 cm and 6 cm. Find the ratio of **(a)** their areas and **(b)** their perimeters.

10. The table below shows how 117 people travel to work.

Taxi	*MRT Train*	*Bus*	*Car*
9	21	72	15

Find the ratio of people using the four different modes of transport.

Increase and Decrease in Ratio

If the number of teachers in a school is increased from 45 to 55, then the ratio no. of present staff : no. of previous staff = 55 : 45 = 11 : 9.

$$\frac{\text{no. of present staff}}{\text{no. of previous staff}} = \frac{55}{45} = \frac{11}{9}$$

$\frac{11}{9}$ is an improper fraction.

We say that the number of teachers has been **increased in the ratio** 11 : 9, or $\frac{11}{9}$. In other words, the number of present staff is $\frac{11}{9}$ times that of previous staff. Hence, we have no. of present staff = $\frac{11}{9}$ × no. of previous staff.

Notice that when a number x is multiplied by an improper fraction, its value is increased.

Example 2

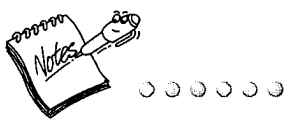
Increase \$20 in the ratio 6 : 5; what is the result?

Solution

The new value is \$20 × $\frac{6}{5}$ = \$24.

A newspaper agent orders 84 copies of newspapers everyday. During the holidays, he decreases his order to 63 copies. The ratio no. of copies ordered during the holidays : usual no. of copies ordered = 63 : 84 = 3 : 4, or $\frac{3}{4}$.

We say that the number of copies ordered per day has been **decreased in the ratio** 3 : 4, or $\frac{3}{4}$, during the holidays. In other words, the number of copies ordered per day during the holidays is $\frac{3}{4}$ of the usual number of copies ordered.



$\frac{3}{4}$ is a proper fraction.

i.e., new no. of copies ordered during the holidays = $\frac{3}{4}$ × usual no. of copies ordered

To decrease a number x, we multiply it by a proper fraction.

Example 3

Find the result of decreasing 56 m in the ratio 7 : 8.

Solution

The new length is $\left(56 \times \frac{7}{8}\right)$ m = 49 m.

Example 4

In what ratio must 40 m^3 be decreased to become 24 m^3?

Solution

The required ratio = new value : old value
= 24 m^3 : 40 m^3
= 24 : 40 = 3 : 5

Exercise 11b

1. Increase 96 in the ratio 7 : 4; what is the result?

2. Decrease \$288 in the ratio 2 : 9; what is the result?

3. Find the result of increasing or decreasing the quantities in the given ratios:
(a) 40 kg, 5 : 8 **(b)** 56 m, 8 : 7
(c) 35 hectares, 2.5 : 1
(d) 2.5 cm^2, 2 : 5

4. **(a)** In what ratio must 35 be increased to become 49?
(b) In what ratio must 72 kg be increased to become 96 kg?

5. **(a)** In what ratio must 105 be decreased to become 75?
(b) In what ratio must 144 kg be decreased to become 108 kg?

6. The price of petrol drops from \$1.20 per litre to 95 cents per litre. Find the ratio in which the price decreases.

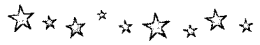
7. Two sums of money are in the ratio 5 : 8. The smaller amount is \$65. Find the larger amount.

8. A photograph measuring 5.5 cm by 9 cm is enlarged in the ratio 7 : 5. Find the dimensions of the enlarged photograph.

9. The cost of mutton has increased in the ratio 9 : 7. If the original price was \$5.60 per kg, what is the new price?

10. Due to import duty, the price of a car increases in the ratio 11 : 8. What is the new price of a car which originally cost \$25 600?

If it takes four minutes to boil one egg, how long will it take to boil three eggs?



Rate

Let us consider the following questions:

A : If one dozen eggs cost \$1.80, what is the cost of 30 eggs?

B : If eight tins of a certain brand of tonic food beverage cost \$26.40, what is the cost of 18 tins?

C : A car travels 570 km on 60 litres of petrol. If the car travels 190 km, what will the petrol consumption be in litres?

D : A boy works 5 hours and is paid \$12.50. How much will he be paid if he works 12 hours?

To answer each of the above questions, we must first find

A : The cost of 1 egg = $\$\frac{1.80}{12}$ ← dollars / ← eggs or $\frac{180}{12}$ ← cents / ← eggs

B : The cost of 1 tin = $\$\frac{26.40}{8}$ ← dollars / ← tins

C : The petrol consumption for 1 km = $\frac{60}{570}$ ← litres / ← km

D : The pay for 1 hour = $\$\frac{12.50}{5}$ ← dollars / ← hours

Each of above results is different from a ratio in that it involves two quantities of different kinds. Each of them is called a **rate**.

A : The rate = $\$\frac{1.80}{12}$ = \$0.15 per egg

∴ the cost of 30 eggs = \$0.15 × 30 = \$4.50.

B : The rate = $\$\frac{26.40}{8}$ = \$3.30 per tin

∴ the cost of 18 tins = \$3.30 × 18 = \$59.40.

Understand the problem by asking the questions:

1. *How far can a car travel on 1 litre of petrol?*
2. *How much petrol is needed to travel 1 km?*
3. *How many litres of petrol are required to travel 260 km?*

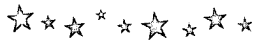
C : The rate = $\frac{60}{570} = \frac{2}{19}$ litre per km

∴ if the car travels 190 km, the petrol consumption = $\frac{2}{19} \times 190$ = 20 litres.

D : The rate = $\$\frac{12.50}{5}$ = \$2.50 per hour

∴ he will be paid \$2.50 × 12 = \$30.00 for working 12 hours.

NB: We normally use the word "per" or the symbol "/" to denote a rate. Thus we have \$2.50 per hour or \$2.50/hour.



Example 5

How far can a car travel on 15 litres of petrol if it can travel 91 km on 7 litres of petrol? How much does the owner of the car spend on petrol, which costs $1.10 per litre, when he travels 260 km?

Solution

Distance travelled on 1 litre of petrol = $\frac{91}{7}$ km = 13 km.

$\therefore$ distance travelled on 15 litres of petrol = (13×15) km = 195 km.

Consumption of petrol per km = $\frac{7}{91}$ litre = $\frac{1}{13}$ litre.

Consumption of petrol for 260 km = $\left(\frac{1}{13} \times 260\right)$ litres = 20 litres.

$\therefore$ the owner spends $\$1.10 \times 20 = \$22$ on petrol when he travels 260 km.

Exercise 11c

1. Copy and complete the following:

(a) If a typist types 900 words in 1 hour, her rate of typing is _____ words per minute.

(b) If a man pays $600 rent for 3 months, the rental rate is _____ dollars per month.

(c) If $14.06 is charged for 74 units of electricity, the rate is _____ cents per unit.

(d) A machine is used to stamp bottle caps. If 150 bottle caps can be stamped in 30 seconds, the rate is _____ caps per second.

2. A man earns $250 in a five-day week. What is his pay for 3 days?

3. A car uses 40 litres of petrol to travel 340 km. How far can it travel if it has only 32 litres of petrol?

4. A machine stamps 720 bottle caps in 2 minutes. How many bottle caps can it stamp in 40 seconds?

5. A wire 22 cm long has a mass of 374 g. What is the mass of 13 cm of this wire?

6. A shopkeeper buys 72 articles for $82.80. How much will he have to pay if he buys 150 such articles?

7. 40 cm of a certain type of piping cost $2.00. What is the cost of 1 km of such piping?

8. The cost of a long-distance call lasting 4 minutes and 20 seconds was $23.40. At this rate, what was the cost of a call lasting 6 minutes 30 seconds?

9. 250 cm^3 of a liquid weighs 125 g. Find the weight of 1 000 cm^3 of the liquid.

10. 200 g of fertilizer is required for a land area of 8 m^2. At this rate,

(a) how many grams of fertilizer are needed for a land area of 1 m^2?

(b) how many grams of fertilizer are required for a land area of 14 m^2?

(c) for what land area will 450 g of fertilizer be sufficient?

***11.** A cook uses fifteen 2-kg bottles of cooking-oil over a 4-week period. If he decides to buy 5-kg tins of oil instead, how many tins of cooking oil will he use over a 10-week period if the rate of using it remains unchanged?

*12. In a certain company, the amount of travelling expenses an employee may claim is calculated as follows:

If the distance travelled exceeds 20 km, claimable amount = 20 × rate + (number of km − 20) × \$0.70.

Otherwise,

claimable amount = number of km × rate, where the rate in both instances is \$0.55 per km if the engine capacity of the employee's car exceeds 1 000 cc, otherwise it is \$0.50 per km.

Find the travelling expenses allowed in each case:

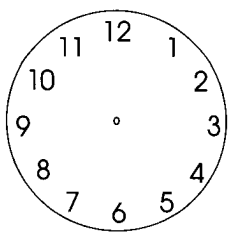
(a) **(i)** 18 km;
(ii) 28 km travelled in a 1 298 cc car;

(b) **(i)** 16 km;
(ii) 25 km travelled in a 998 cc car.

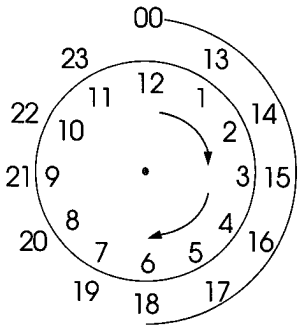
a.m. stands for "ante meridiem" (Latin word) meaning "before midday". p.m. stands for "post meridiem" meaning "after midday".

Time

To record the time of the day, we can either use the 12-hour clock or the 24-hour clock. In the 12-hour clock, morning (from midnight to just before noon) is denoted by a.m.; afternoon, evening and night are denoted by p.m. In the 24-hour clock, four digits are used to indicate time. The first two digits denote hours and the last two denote minutes.



12-hour clock



24-hour clock

The table below shows some examples.

Time	*12-hour clock*	*24-hour clock*
2 o'clock early morning	2.00 a.m.	02 00
5 to 11 in the morning	10.55 a.m.	10 55
Noon	12.00 p.m.	12 00
Half past 12 early afternoon	12.30 p.m.	12 30
Quarter to 3 in the afternoon	2.45 p.m.	14 45
5 past 8 in the evening	8.05 p.m.	20 05
One minute to midnight	11.59 p.m.	23 59
Midnight	12.00 a.m.	00 00
One minute past midnight	12.01 a.m.	00 01

Example 6

A journey starting at 08 40 takes $4\frac{3}{4}$ hours. Find the time the journey ends.

Solution

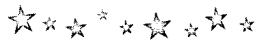
	h	min	
Starting time	08	40	$\left(4\frac{3}{4}\text{ h}\right)$
Journey time	04	45	
Arrival time	13	25	

40 + 45 = 60 + 25 = (1) h (25) min

(8 + 4 + 1)

∴ the journey ends at 13 25, or 1.25 p.m.

A cashier of a bank is given one million one-cent coins to count. How long will she take if she can count five coins in one second?



26 indicates that the car arrives in Kuala Lumpur the next day.

Example 7

A car leaves Singapore at 21 15 on Wednesday and arrives in Kuala Lumpur $5\frac{1}{2}$ hours later. At what time and day does the car arrive in Kuala Lumpur?

Solution

	h	min	
Starting time	21	15	$\left(5\frac{1}{2}\text{ h}\right)$
Journey time	05	30	
Arrival time	(26)	45 → (02)	45

−24

∴ the car arrives in Kuala Lumpur at 02 45, or 2.45 a.m., on Thursday.

Example 8

A train left Singapore at 07 35 and arrived in Seremban at 13 05. How long was the train journey?

Solution

	h	min
	12	65
Arrival time	~~13~~	~~05~~
Departure time	07	35
	5	30

∴ the train journey was 5 h 30 min long.

One hour is converted to 60 minutes.

24 h is added so that the arrival time is measured from 00 00 on Saturday.

Example

A bus leaves Town A on Saturday night and is supposed to arrive at Town B at 08 17 on Sunday morning. If the estimated journey time is 10 h 43 min, at what time should the bus leave Town A?

Solution

		h	min	
		31	77	
Arrival time 08 17 (Sunday) =	+24	~~32~~	~~17~~	(Saturday)
Journey time		10	43	
Departure time		21	34	(Saturday)

$\therefore$ the bus should leave Town *A* at 21 34, or 9.34 p.m., on Saturday.

Exercise 11d

1. Convert the following times to 24-hour clock notation:

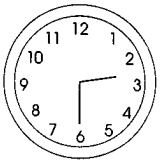
(a) 8.00 a.m. **(b)** 2 p.m.
(c) 5.30 p.m. **(d)** 9.42 p.m.
(e) noon **(f)** 12.45 a.m.
(g) midnight **(h)** 2.42 a.m.

2. Convert the following times to 12-hour clock notation:

(a) 03 30 **(b)** 15 00
(c) 23 12 **(d)** 19 15
(e) 09 23 **(f)** 12 00
(g) 00 05 **(h)** 24 00

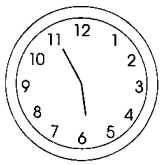
3. Write down, using the 24-hour clock notation, the times shown:

(a)



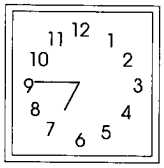
a.m.

(b)



p.m.

(c)



a.m.

4. Copy and complete the following table:

	Departure time	*Journey time*	*Arrival time*
(a)	15 45	5 hours	
(b)	02 40	55 minutes	
(c)	08 45	$9\frac{3}{4}$ hours	
(d)	22 35	8 hours	
(e)	15 45		17 50
(f)	11 50		15 15
(g)	09 48		22 16
(h)	20 35 (Tue)		07 15 (Wed)
(i)		$1\frac{1}{4}$ hours	23 50
(j)		$17\frac{3}{4}$ hours	12 45 (Fri)

5. A train left a station at 8.35 a.m. and arrived at its destination at 3.12 p.m. How long did the journey take?

6. An overnight train left at 21 55 on a journey that took 9 h 18 min. Find the time at which it arrived at its destination.

7. A car arrived at a town at 15 06 after travelling for $4\frac{1}{4}$ hours. Find the time the car started its journey.

8. According to a timetable, a coach was due to leave a station at 22 55 and arrive at its destination at 06 05 the next day. How long would the journey take? If the train actually arrived 35 minutes early, at what time did it arrive?

***9.** Lessons in a certain school start at 7.45 a.m. and end at 3.45 p.m., with an hour's break at lunchtime and 20 minutes morning recess. If there are altogether 8 lessons of equal length, how long is each lesson?

***10.** Shown below is the schedule of the arrival and departure times of a long-distance express overnight coach.

Destination	*Arrival*	*Departure*
Singapore	—	21 30
Johor Baru	22 15	22 30
Seremban	02 25	02 30
Kuala Lumpur	03 50	04 20
Ipoh	07 50	08 00
Taiping	09 20	09 30
Butterworth	10 45	—

Find the time taken for the coach to travel from

(a) Singapore to Seremban;
(b) Johor Baru to Ipoh;
(c) Seremban to Taiping;
(d) Kuala Lumpur to Butterworth;
(e) Singapore to Butterworth.

Average Speed

Two cyclists, A and B, travel 90 km, in a race, in 5 hours and $4\frac{1}{2}$ hours respectively. Which cyclist travels faster?

Cyclist B travels faster since he takes less time to complete the race.

We can also find the **speed** at which each cyclist travels to find out who travels faster. As you can see, speed is a special kind of rate.

$$\text{Cyclist } A\text{'s speed} = \frac{90\text{ km}}{5\text{ h}} = 18\text{ km/h}$$

$$\text{Cyclist } B\text{'s speed} = \frac{90\text{ km}}{4\frac{1}{2}\text{ h}} = 20\text{ km/h}$$

Cyclist B travels faster since he travels at a greater speed.

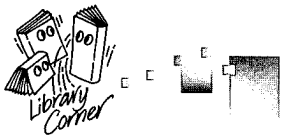
When calculating the speed of each cyclist, we assume that one travels at the same speed all the time. In reality, each cyclist will have difficulty cycling at the same speed all the time. For example, he may slow down when he is cycling up a slope or he may speed up when he is going down a slope. Thus, the speed calculated for each cyclist is not his exact speed at a particular instant. Instead, it is his average speed. For

If a cyclist is equipped with a speedometer, which gives his speed at a particular instant, the readings from the speedometer will change from time to time.

example, the average speed of cyclist A is 18 km/h. This means that on the average, he travels 18 km every hour. The average speed can be obtained by using the formula:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

NB: We can also express average speed in m/s.



The highest speed limit for cars on Singapore roads is 90 km/h. How many demerit points will a motorist be awarded if he is caught speeding on the expressway at

(a) 100 km/h;
(b) 120 km/h;
(c) 160 km/h?

Example 10

A car travelled 510 km in 6 hours. Find the average speed of the car for the whole journey.

Solution

Distance travelled = 510 km
Time taken = 6 hours

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}} = \frac{510 \text{ km}}{6 \text{ h}} = 85 \text{ km/h}$$

Convert km to m and hour to seconds.

Example 11

A cyclist is travelling at an average speed of 18 km/h.

(a) Express his average speed in m/s.
(b) Find the distance he travels in 3 hours.
(c) Find how far he travels in 25 seconds.

Solution

(a) 18 km = 18 × 1 000 m, 1 h = (60 × 60) s

$$\therefore \quad 18 \text{ km/h} = \frac{18 \text{ km}}{1 \text{h}} = \frac{(18 \times 1\,000) \text{ m}}{(60 \times 60) \text{ s}} = 5 \text{ m/s}$$

A sports car leaves Singapore for Kuala Lumpur at the same time as a bus, which leaves Kuala Lumpur for Singapore. They travel along the same road, the sports car at 110 km/h and the bus at 55 km/h. Which vehicle is further away from Singapore when they meet?

(b) In 3 hours, the cyclist travels (18 × 3) km = 54 km

Average speed in km/h — Time taken in hours

(c) In 25 seconds, he travels (5 × 25) m = 125 m

Average speed in m/s — Time taken in seconds

In general,

Distance travelled = Average speed × Time taken

Example 12

A train travels at an average speed of 15 m/s.

(a) Express its average speed in km/h.
(b) Find the time taken by the train to travel 750 m.
(c) If the train sets off from Station A at 8.00 a.m., find the arrival time of the train at Station B which is 36 km away.

Solution

(a) $15 \text{ m} = \frac{15}{1\,000}$ km, $1 \text{ h} = 3\,600$ s

In 1 second, the train travels $\frac{15}{1\,000}$ km.

In 1 hour, the train travels $\left(\frac{15}{1\,000} \times 3\,600\right)$ km = 54 km.

$\therefore$ its average speed is 54 km/h.

(b) $15 \text{ m/s} = \frac{15 \text{ m}}{1 \text{ s}}$

In 1 second, the train travels 15 m.

The time taken to travel 1 m is $\frac{1}{15}$ s.

$\therefore$ the time taken by the train to travel 750 m $= \frac{1}{15} \times 750$ s

$= \frac{750}{15}$ s (750: Distance travelled in m.; 15: Average speed in m/s.)

$= 50$ s

(c) Similarly in 1 hour, the train travels 54 km.

The time taken to travel 1 km is $\frac{1}{54}$ h.

$\therefore$ the time taken by the train to travel 36 km $= \frac{1}{54} \times 36$ h

$= \frac{36}{54}$ h (36: Distance travelled in km.; 54: Average speed in km/h.)

$= \frac{2}{3}$ h

$= \frac{2}{3} \times 60 \text{ min} = 40 \text{ min}$

$\therefore$ the arrival time of the train at Station *B* is 8.40 a.m.

In general,

$$\textbf{Time taken} = \frac{\textbf{Distance travelled}}{\textbf{Average speed}}$$

Fact

1. *The cheetah is the fastest land animal. It can acquire a speed of 110 km/h in a matter of seconds.*
2. *The men's world record for the 100 m sprint is approximately 36.7 km/h.*
3. *The speed of sound is about 34 times faster than the speed of the fastest 100 m human sprinter.*

Exercise 11e

1. Copy and complete the following. The first one has been done for you.

	Distance travelled	*Time taken*	*Average speed*
(a)	180 km	$1\frac{1}{2}$ h	120 km/h
(b)	200 m	25 s	
(c)	400 m	1 min	
(d)		$5\frac{1}{2}$ h	80 km/h
(e)		$\frac{1}{3}$ min	25 m/s
(f)	100 m		20 m/s

2. Express the following in m/s:

(a) 18 km/h **(b)** 72 km/h
(c) 90 km/h

3. Express the following in km/h:

(a) 10 m/s **(b)** 35 m/s
(c) $\frac{1}{2}$ km/s

4. How long will a man take to run, once, round a circular track of radius 28 m at an average speed of 8 m/s? $\left(\text{Take } \pi = \frac{22}{7}\right)$

5. A cyclist begins on a 24-km journey at 09 23. When will he complete his journey if he travels at an average speed of 16 km/h?

6. A train leaves Town X at 12 57 and arrives at Town Y 45 minutes later.

(a) At what time does the train arrive in Town Y?
(b) What is the average speed of the train, in km/h, if the distance between the two towns is 84 km?

7. A car travels at an average speed of 24 km/h. Find, in metres, the distance travelled by the car in 12 seconds.

8. A car travelled on a B class road for 20 minutes at an average speed of 57 km/h. It then travelled a distance of 55 km in 30 minutes on an expressway. Find

(a) the distance the car travelled on the B class road;
(b) the average speed, in km/h, of the car when it travelled on the expressway.

9. A man cycles for two hours at an average speed of 16 km/h and then walks for 3 hours at an average speed of 6 km/h. Find his average speed for the whole journey.

10. A train travels 68 km at an average speed of 51 km/h. It then travels another 20 km at an average speed of 40 km/h before reaching its destination. Calculate the average speed for the whole journey.

***11.** Two points, X and Y, are 120 m apart. M is the mid-point of X and Y. An object travels from X to M in 12 seconds and then from M to Y at an average speed of 15 m/s. Calculate

(a) the average speed of the object from X to M;
(b) the time taken to travel from M to Y;
(c) the average speed for the whole journey from X to Y.

***12.** Three points, L, M and N, lie on a straight line with LN = 160 m. An object travels from L to M at an average speed of 10 m/s in 6 seconds and then from M to N at an average speed of 25 m/s. Calculate

(a) the distance from L to M;
(b) the time taken to travel from M to N;
(c) the average speed for the whole journey from L to N.

Direct Proportion

Have you ever borrowed books from the National Library? If you have, be thoughtful and return them before they are overdue. If you are late in returning books, as of September 1999, you are liable to a fine of 20 cents per day for each overdue book. The table below shows the possible fines for one overdue book.

No. of days (x)	1	2	3	4	5	6	7	8	9	10
Fine (y cents)	20	40	60	80	100	120	140	160	180	200

Clearly, the **longer** the book is overdue, the **greater** is the fine. When the number of days the book is overdue is doubled, is the fine also doubled? When the number of days the book is overdue is tripled, is the fine also tripled?

We notice that 4 days : 2 days = 80 cents : 40 cents

$$4 : 2 = 80 : 40 = 2 : 1$$

or $$\frac{4}{2} = \frac{80}{40} = \frac{2}{1}$$

and 6 days : 2 days = 120 cents : 60 cents

$$6 : 2 = 120 : 60 = 3 : 1$$

or $$\frac{6}{2} = \frac{120}{60} = \frac{3}{1}$$

We also notice that $\frac{x}{y} = \frac{1}{20} = \frac{2}{40} = \frac{3}{60} = \frac{4}{80} = \frac{5}{100} = \frac{6}{120} = \frac{7}{140} = \frac{8}{160} = \frac{9}{180} = \frac{10}{200}$.

In general, x_1 days : x_2 days = y_1 cents : y_2 cents, i.e.

$x_1 : x_2 = y_1 : y_2$ or $\frac{x_1}{x_2} = \frac{y_1}{y_2}$ or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$, where x_1 and x_2 are any two values of x while y_1 and y_2 are the corresponding values of y.

Thus, x and y are two quantities such that when one increases (or decreases), so does the other, and that the two quantities are always in the same ratio. We say that the two quantities x and y are **directly proportional**. In this case the fine is **directly proportional** to the number of days a book is overdue.

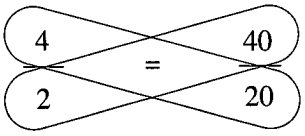
A **proportion** is a statement expressing the equivalence of two ratios. Hence, $\frac{x_1}{x_2} = \frac{y_1}{y_2}$, or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$, is a statement of proportion, and x_1, x_2, y_1 and y_2 are said to be **proportional**.

Let $\frac{a}{b} = \frac{c}{d}$ be a statement of proportion. If we multiply both sides of the proportion by the common denominator bd, we have

$$\frac{a}{b} \times bd = \frac{c}{d} \times bd, \quad \text{i.e., } ad = bc.$$

Dividing both sides of $ad = bc$ by d, we obtain $a = \frac{bc}{d}$.

In the proportion



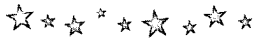
we have $ad = 4 \times 20 = 80$
and $bc = 2 \times 40 = 80$
i.e., $ad = bc$

This is a mathematical technique which is very useful in problem solving. It is sometimes called **cross-multiplication** and can be used to check whether a proportion is true.

For example, $\frac{75}{100} = \frac{3}{4}$ is true since $75 \times 4 = 3 \times 100 = 300$

but $\frac{6}{7} \neq \frac{18}{22}$ since $6 \times 22 = 132 \neq 18 \times 7 = 126$.

If Mr Tan travels from his home to his office at an average speed of v km/h. On his return journey, his average speed is u km/h. Can we conclude that his average speed for the whole journey is therefore $\frac{u+v}{2}$ km/h?



Example 13

Find x if 60 m : 100 m = 9 kg : x kg.

Solution

60 m : 100 m = 9 kg : x kg

$$\frac{60}{100} = \frac{9}{x} \quad \text{i.e.,} \quad \frac{3}{5} = \frac{9}{x}$$

$\therefore$ $3x = 45$ and $x = 15$

Example 14

Find the cost of 13 kg of biscuits if 6 kg of them cost $27.

Solution

Strategy 1: Use increase in ratio

6 kg of biscuits cost \$27.

$\therefore$ 13 kg of biscuits cost $\$27 \times \frac{13}{6} = \$58.50$ (increase in the ratio 13 : 6)

Strategy 2: Use proportion

Let \$$x$ be the cost of 13 kg of biscuits.

Then \$$x$: \$27 = 13 kg : 6 kg

i.e., $\frac{x}{27} = \frac{13}{6}$

$$x = \frac{13}{6} \times 27 = 58.50$$

$\therefore$ 13 kg of biscuits cost \$58.50.

Exercise 11f

1. Find x in each of the following cases:
   (a) $4 : 7 = x : 5$ **(b)** $18 : 7 = 10 : x$
   (c) $9 : x = 24 : 88$ **(d)** $x : 8 = 99 : 44$
   (e) x m : 12 m = \$42 : \$63
   (f) 1 km : 32 m = 250 g : x g

2. Copy and complete the following:
   (a) If $\frac{18}{7} = \frac{10}{a}$, then $18a =$ _____
   (b) If $\frac{x}{y} = \frac{u}{v}$, then $xv =$ _____

3. Find the ratio of $x : y$ in each of the following cases:
   (a) $5x = 7y$ **(b)** $3.2x = 1.2y$
   (c) $2\frac{1}{2}x = 4\frac{1}{2}y$ **(d)** $1.2x = 2\frac{3}{4}y$

4. The lengths of two pieces of wire are in the ratio 4 : 7. If the length of the longer piece is 3.5 m, what is the length of the shorter piece?

5. Find the cost of
   (a) 8 books when 6 books cost \$48, given that the price of each book is the same;
   (b) 10 kg of tea when 3 kg of tea cost \$18;
   (c) a kg of sugar when b kg of sugar cost c dollars.

6. $\frac{5}{9}$ of a piece of metal weighs 7 kg. What is the weight of $\frac{2}{7}$ of the metal?

7. In a bookstore, 60 books of the same kind occupy 1.5 m of shelf length. How much shelf length is required for 300 such books? If a shelf is 80 cm long, how many such books are needed to fill the shelf?

8. A pile of 108 identical books weighs 30 kg. Find
   (a) how many books weigh 20 kg and
   (b) the weight of 150 books.

Inverse Proportion

The time taken by a car to travel a distance of 120 km at various speeds is displayed in the table below.

Speed (x km/h)	20	30	40	60	120
Time taken (y hours)	6	4	3	2	1

Clearly, the **greater** the speed of the car, the **shorter** the time taken to cover the distance, i.e., when the quantity x increases, the quantity y decreases, or when the quantity x decreases, the quantity y increases.

Notice also that the product of the two quantities x and y is always the same, i.e.

$$xy = 20 \times 6 = 30 \times 4 = 40 \times 3 = 60 \times 2 = 120 \times 1 = 120.$$

We say that the two quantities x and y are **inversely proportional**. In this case, the time taken is **inversely proportional** to the speed.

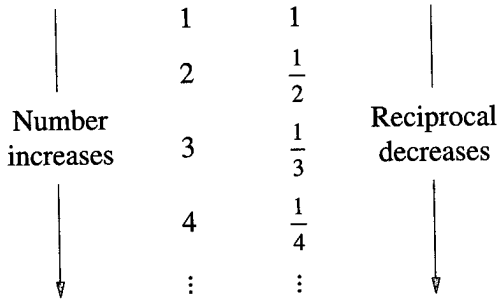
Reciprocals

Any two numbers whose product is 1 are called **reciprocals** of each other.

For example, 5 and $\frac{1}{5}$ are a pair of reciprocals. We say that $\frac{1}{5}$ is the reciprocal of 5 and vice versa.

Other examples of pairs of reciprocals are $\frac{6}{7}$ and $\frac{7}{6}$, -3 and $-\frac{1}{3}$, $-\frac{3}{2}$ and $-\frac{2}{3}$.

The diagram below shows some numbers and their corresponding reciprocals.



We see that the value of a number is inversely proportional to its reciprocal. Thus, in general, when a number gets bigger, its reciprocal becomes smaller. Conversely, the smaller the number, the bigger its reciprocal.

Example 15

Ten men can dig a trench in 4 hours. How long will 5 men take to dig the same trench? (Assume all the men are working at the same rate.)

Solution

Strategy 1: Use logical deduction

10 men take 4 hours

1 man will take (4×10) hours = 40 hours. Can you understand why?

5 men will take $(40 \div 5)$ hours = 8 hours.

Strategy 2: Use inverse proportion

Let x be the number of hours required.

10 men $\longleftrightarrow$ 4 hours
5 men $\longleftrightarrow$ x hours

Since product of two quantities, which are inversely proportional, is always the same,

$$5x = 10 \times 4$$
$$x = 8$$

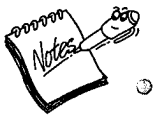
Alternatively,

$$x \text{ hours} : 4 \text{ hours} = 10 \text{ men} : 5 \text{ men}$$

$$\frac{x}{4} = \frac{10}{5}$$

$$x = \frac{10}{5} \times 4 = 8$$

$\therefore$ 5 men will take 8 hours to dig the trench.



Order of 4 and x is reversed.

5 men will take a longer time than 10 men. To increase 4 hours, we multiply it by an improper fraction $\frac{10}{5}$.

Strategy 3: Use increase in ratio

10 men can dig the trench in 4 hours.

5 men can dig the trench in $\left(4 \times \frac{10}{5}\right)$ hours = 8 hours.

When there are more cattle, will the same feed last longer or shorter?

Example 16

A farmer has enough feed to last his 40 cattle 35 days. If he buys 10 more cattle, how long can the same feed last? Assume the cattle finish the feed at the same rate.

Solution

Strategy 1: Use inverse proportion

$$\begin{aligned} 40 \text{ cattle} &\longleftrightarrow 35 \text{ days} \\ (40 + 10) \text{ cattle} = 50 \text{ cattle} &\longleftrightarrow x \text{ days} \end{aligned}$$

Using the fact that the product of two quantities that are inversely proportional is always the same, we have

$$50x = 40 \times 35$$

$$x = \frac{40 \times 35}{50} = 28$$

Alternatively,

$$x \text{ days} : 35 \text{ days} = 40 \text{ cattle} : 50 \text{ cattle}$$

$$\frac{x}{35} = \frac{40}{50}$$

$$x = \frac{40}{50} \times 35 = 28$$

$\therefore$ the same feed can last 28 days.

The number of days the feed will last decreases as there are more cattle to feed. To decrease the number of days in the ratio 40 : 50, we multiply it by a proper fraction $\frac{40}{50}$.

Strategy 2: Use decrease in ratio

40 cattle have feed for 35 days.

50 cattle have feed for $\left(35 \times \frac{40}{50}\right)$ days = 28 days.

Exercise 11g

1. Which of the following are in inverse proportion?
   (a) The number of pencils you buy and their total cost.
   (b) The number of pipes filling a tank and the time taken to fill it.
   (c) The number of men doing a job and the time taken to finish it.
   (d) The number of cattle to be fed and the time taken to finish a certain amount of the feed.

2. Four pipes can fill a tank in 70 minutes. How long will it take to fill the tank if 7 pipes are used?

3. A school librarian has enough money to order 8 paperback books at \$5.50 each. If the librarian decides instead to order books with hard covers at \$8.80 each, how many books can the librarian buy?

4. Thirty-five workers build a house in 16 days. How many days will 28 workers working at the same rate take to build the same house?

5. An aircraft flying at an average speed of 770 km/h takes 15 hours to complete a journey. Find the time taken for the aircraft to complete the same journey if its average speed is 660 km/h.

6. A consignment of fodder lasts 1 260 cattle for 50 days. Given that the cattle consume the fodder at a constant rate, find
   (a) the number of cattle an equal consignment of fodder lasts for 75 days;
   (b) the number of days an equal consignment of fodder lasts 1 575 cattle.

★7. A contractor estimates that he would need 56 workers to complete a job in 21 days. If he is asked to complete the job in 14 days, find the additional number of workers he has to employ.

★8. At a scouts' camp, there is sufficient food to last 72 scouts for 6 days. If 18 scouts do not turn up for the camp, how much longer can the food last for the other scouts?

★9. It takes 12 men to make 12 tables in 9 hours. How long will it take 8 men to make 32 tables?

Proportional Parts, Scales and Mixtures

Study the examples below.

Example 17

Suppose a sum of money is divided among 3 people, John, Mary and Peter, in the ratio 2 : 3 : 7.
This means that John's share: Mary's share: Peter's share = 2 : 3 : 7.
If the sum of money is \$192, how much will each of them receive?

Solution

Strategy 1: Use proportion

Consider the sum of money divided into (2 + 3 + 7) = 12 equal parts.
John, Mary and Peter will get 2, 3 and 7 parts respectively.

John's share: \$192 = 2 : 12

$\therefore$ John's share = $\frac{2}{12} \times \$192 = \$32$

Similarly,

Mary's share $= \frac{3}{12} \times \$192 = \$48$

and Peter's share $= \frac{7}{12} \times \$192 = \$112$

9 hens can lay 9 eggs in 9 days. How many eggs can 3 hens lay in 3 days?

Check: John's share + Mary's share + Peter's share
$= \$(32 + 48 + 112) = \$192$

Strategy 2: Use an equation

Let John's share be $\$(2x)$.

$\therefore$ Mary's share $= \$(3x)$ and Peter's share $= \$(7x)$

$$\$(2x + 3x + 7x) = \$192$$
$$\text{i.e.,} \quad 12x = 192$$
$$x = 16$$

$\therefore$ John's share $= \$(2 \times 16) = \$32$
Mary's share $= \$(3 \times 16) = \$48$
and Peter's share $= \$(7 \times 16) = \$112$

Example 18

(a) Edward makes a model plane using a scale of 1 : 20. The model plane has an overall length of 1.2 m and its wingspan is 65 cm. Find the wingspan and the overall length of the actual plane in metres.

(b) On another occasion, he makes a model of another plane whose actual overall length of 35 m using a scale of 1 : 25. What is the overall length of the model plane in centimetres?

Solution

(a) Wingspan of the actual plane : wingspan of model plane = 20 : 1
i.e. wingspan of the actual plane : 65 cm = 20 cm : 1 cm

$\therefore$ wingspan of the actual plane $= \frac{20 \text{ cm}}{1 \text{ cm}} \times 65 \text{ cm} = 1\,300 \text{ cm} = 13 \text{ m}.$

The overall length of the model plane = 1.2 m = 120 cm

Thus, the overall length of the actual plane $= \frac{20 \text{ cm}}{1 \text{ cm}} \times 120 \text{ cm} = 2\,400 \text{ cm} = 24 \text{ m}$

(b) Length of the model : length of the actual plane = 1 : 25
i.e. length of the model : 35 m = 1 m : 25 m

$\therefore$ length of the model plane $= \frac{1 \text{ m}}{25 \text{ m}} \times 35 \text{ m} = 1.4 \text{ m} = 140 \text{ cm}.$

Example 19

Two chemicals, A and B, are combined in the ratio 5 : 11 by volume to form another solution X. A second solution Y is formed by combining chemicals A, B and C in the ratio 2 : 3 : 15 by volume. Calculate

(a) the volume of each chemical in 400 litres of solution X;
(b) the volumes of chemicals A and C in 400 litres of solution Y;
(c) the ratio of the three chemicals in a solution Z formed by mixing equal volumes of solutions X and Y;
(d) the volume of chemical C in 400 litres of solution Z.

Solution

(a) Volume of chemical A in 400 l of solution $X = \left(\frac{5}{16} \times 400\right) l$
$= 125\ l$

Volume of chemical B in 400 l of solution $X = (400 - 125)\ l$
$= 275\ l$

(b) Volume of chemical A in 400 l of solution $Y = \left(\frac{2}{20} \times 400\right) l$
$= 40\ l$

Volume of chemical C in 400 l of solution $Y = \left(\frac{15}{20} \times 400\right) l$
$= 300\ l$

(c) Volume of chemical B in 400 l of solution $Y = (400 - 40 - 300)$
$= 60\ l$

In 800 l of solution Z, the volumes of chemicals A, B and C are $(125 + 40) = 165\ l$, $(275 + 60) = 335\ l$ and $300\ l$ respectively.

$\therefore$ the ratio of chemicals A, B and C in solution Z is 165 : 335 : 300, i.e., 33 : 67 : 60.

(d) $33 + 67 + 60 = 160$

$\therefore$ volume of chemical C in 400 l of solution $Z = \left(\frac{60}{160} \times 400\right)$
$= 150\ l.$

Exercise 11h

1. Divide \$15 in the ratio 1 : 4 : 5; what is the result?

2. Divide 4 m in the ratio 1 : 3 : 4; what is the result?

3. Divide 110 g of salt in the ratio 5 : 8 : 9; what is the result?

4. Find the result of dividing 3 m 8 cm of ribbon in the ratio 2 : 3 : 6.

5. Three partners of a firm divide the profit of \$6 720 among themselves in the ratio 2 : 3 : 7. What is the biggest share of the profit?

6. A man leaves \$12 800 to his three children, A, B and C, in the ratio 4 : 5 : 7. How much does B receive?

7. A sum of money is divided among three persons, X, Y and Z, in the ratio 10 : 7 : 5. If Y gets \$14 more than Z, how much will X get and what is the total sum of money?

8. Three families, P, Q and R, share 480 kg of rice. Q receives twice as much rice as P, and R receives half as much rice as Q. How much rice does each family get?

9. The model of an aircraft is in the scale 1 : 80.

(a) If the wingspan of the model is 25 cm, what is the wingspan of the actual aircraft in metres?
(b) If the real aircraft is 40 metres long, what is the length of the model in centimeters?

10. An architect's model of a block of flats is in the scale 1 : 50.

(a) If the model is 0.8 metres wide, how wide is the actual block of flats?
(b) If the block of flats is 30 metres tall, how tall is the model?

11. The scale of a map is given as 4 cm : 1 km.

(a) Rewrite the ratio as simply as possible.
(b) What is the length of a river, which is measured as 3 cm on the map?
(c) The distance between two towns is 8 km. How far apart are the towns on the map?

12. Tea at \$2.40 per kg is mixed with tea at \$3.20 per kg in the ratio 1 : 3.

(a) Calculate the weight of each type of tea in 40 kg of the mixture.
(b) Calculate the price per kg of the mixture.

13. A certain solution is to be prepared by combining chemicals X, Y and Z in the ratio 18 : 3 : 2.

(a) Calculate the volume of each chemical, X, Y and Z, in 69 litres of the solution.
(b) How many litres of the solution can be prepared by using 36 litres of X?

***14.** A shopowner blends three types of coffee, A, B and C, in the ratio 3 : 5 : 7.

(a) Calculate the weight of each type of coffee in 45 kg of the blended mixture.
(b) Given type A coffee costs \$7 per kg, type B coffee costs \$10 per kg and type C coffee costs \$13 per kg, calculate the cost per kg of the blended mixture.

***15.** Two types of coffee, P and Q, are blended in the ratio 3 : 13 by weight to form a standard blend X. A second standard blend, Y, is formed by blending type P coffee, type Q coffee and type R coffee in the ratio 1 : 7 : 12 by weight. Calculate

(a) the weight of each type of coffee in 800 g of X;
(b) the weights of type P coffee and type R coffee in 800 g of Y;
(c) the ratio of the three types of coffee in a third standard blend Z, formed by mixing equal weights of X and Y;
(d) the weight of type P coffee in 800 g of Z.

Strategies in Problem Solving

In this section, we shall look at some further examples of how some simple and effective strategies are used in problem solving.

Example 19

A farmer keeps some chickens and goats on his farm. One day, his son wants to know how many animals there are on the farm. The farmer wants his son to guess and tells him that there are altogether 50 heads and 140 legs of animals. How many goats and chickens are there?

Solution

The problem can be solved by either **making a systematic list, making a supposition, simplifying the problem** or **using equations**.

Strategy 1: Make a systematic list

Number of goats	*Number of chickens*	*Number of heads*			*Number of legs*		
		Goats	*Chickens*	*Total*	*Goats*	*Chickens*	*Total*
50	0	50	0	50	200	0	200
40	10	40	10	50	160	20	180
30	20	30	20	50	120	40	160
(20)	(30)	20	30	(50)	80	60	(140)

Thus, there are 20 goats and 30 chickens.

Strategy 2: Simplify the problem

Alternatively, imagine that all the goats stand on their hind legs. There were 50 heads counted. When the goats stand on their hind legs there would also be 50 pairs of legs on the ground. But 140 or 70 pairs of legs were counted. Therefore, the number of goat legs in the air must be 70 – 50 = 20 pairs.

Hence, there must be 20 goats.

$\therefore$ there were 50 – 20 = 30 chickens.

Strategy 3: Use an equation

Suppose x goats and $50 - x$ chickens were counted,

$$\begin{aligned} 4x + 2(50 - x) &= 140 \\ 4x + 100 - 2x &= 140 \\ 2x &= 40 \\ x &= 20 \\ \therefore \quad 50 - x &= 30 \end{aligned}$$

$\therefore$ 20 goats and 30 chickens were counted.

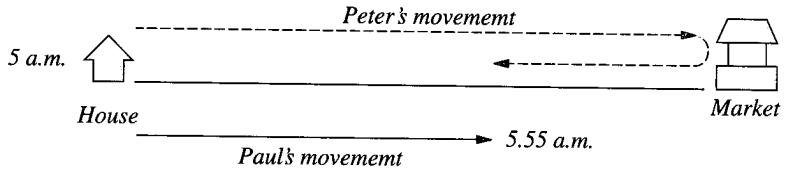
Example 20

Two brothers, Peter and Paul, are vegetable sellers. One morning, they left home together at 5 o'clock for the market, each pushing a cart full of vegetables. Peter travelled at a constant speed of 100 m/min and Paul a constant speed of 60 m/min. After arriving at the market, Peter spent 5 minutes unloading the vegetables and immediately returned along the same route to help Paul. If Peter met Paul at 5 minutes to 6 o'clock, what is the distance between their house and the market?

Solution

Strategy: Use a diagram or a model

The diagram drawn below shows Peter's movement and Paul's movement between their house and the market.



Peter travelled $100 \times 50 = 5\,000$ m in 50 min.

NB: Peter spent 5 minutes unloading.

Paul travelled $60 \times 55 = 3\,300$ m in 55 min.

They travelled a total distance of $5\,000 + 3\,300 = 8\,300$ m.

From the diagram, the distance between their house and the market is half of the total distance travelled.

$\therefore$ the distance between their house and the market $= \frac{1}{2} \times 8\,300 = 4\,150$ m.

Exercise 11i

***1.** Five identical balls are marked with the numbers, 7, 3, 4, 2, 1 and are placed in a box. The balls are thoroughly mixed before three balls are drawn out all at one time from the box. A score is obtained by adding the numbers on the three balls drawn. Find how many different scores are possible and the possible scores.

***2.** A passenger train travelled at a speed of 72 km/h. A man on the passenger train observed a goods train travelling at a speed of 54 km/h in the opposite direction. If the goods train passed him in 8 seconds, find the length of the goods train. (*Hint:* model the problem and draw a diagram).

***3.** Steven left Town *A* and walked towards Town *B* at a speed of 100 m/min. At the same time, Jason and Melvin started from Town *B* and walked towards Town *A* at a speed of 80 m/min and 75 m/min respectively. If Steven met Melvin six minutes after passing Jason, find the distance between Town *A* and Town *B*.

***4.** A man can lift 200 kg, excluding the bar, of the dumbbell. There are weights in four sizes, 5 kg, 10 kg, 25 kg and 40 kg, which he can put on the bar. Assuming that he has unlimited weights in these four sizes and that the individual weights on both sides must be identical, find how many different combinations of weights he can put on the bar to make up the 200 kg.

Summary

1. A **ratio** is expressed as a fraction of the first quantity over the second. To find the ratio of two quantities, we must first express them in the same units. A ratio has no unit.

2. A **rate** expresses a relationship involving two quantities of different kinds.

3. Average speed of a moving object is given by the formula:

   $$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}, \quad \text{Time taken} = \frac{\text{Distance travelled}}{\text{Average speed}}$$

   Also, Distance travelled = Average speed $\times$ Time taken.

4. A **proportion** is a statement that two ratios are equivalent.
   Two quantities are in **direct proportion** when one quantity is doubled, the other quantity is also doubled; when one quantity increases x times, the other quantity also increases x times.
   Two quantities are in **inverse proportion** when one quantity is doubled, the other is halved; when one quantity increases y times, the other quantity becomes $\frac{1}{y}$ of the original.

5. Any two numbers whose product is 1 are called reciprocals of each other. When a number gets bigger, its reciprocal becomes smaller. Conversely, the smaller the number, the bigger its reciprocal.

Review Questions 11

1. The bill for domestic power in a home is reduced from \$120 to \$90 per month. By what ratio has the bill decreased?

2. A poster measuring 150 cm by 180 cm is enlarged in the ratio 8 : 5. Find the length and breadth of the enlarged poster.

3. A manufacturing firm plans to increase its output in the ratio 2.25 : 1 next year. Its present output is 148 000 articles. How many articles does it hope to produce next year?

4. Goat's milk contains 27 g of protein, 30 g of fat and 36 g of carbohydrate. Find the ratio of protein to fat to carbohydrate in goat's milk.

5. Pupils in a class are told to choose one out of three sport options: tennis, basketball and badminton. Given that the pupils choose the options in the ratio 4 : 2 : 3 and that 20 choose tennis, find
   (a) the number of pupils in the class;
   (b) the number of pupils who choose badminton.

6. A student took 18 minutes to walk from home to school.
   (a) Given that he arrived in school at 07 05, find the time at which he left home.
   (b) Given also that he walked to school at an average speed of 4 km/h, calculate how far he had to walk.

7. A car took 2 hours and 15 minutes to travel 198 km. If it arrived at its destination at 12 06, find

(a) the time it started on its journey;
(b) the average speed of the car, giving your answer in kilometres per hour.

8. The cost of 1 kg of fish is \$4.12.

(a) Find the cost of $4\frac{1}{2}$ kg of fish.

(b) How many kilograms of such fish can be bought for \$20.60?

9. (a) Given that a typist can type 575 words in 25 minutes, how long will she take to type
(i) 3 680 words?
(ii) 8 855 words?

(b) On another occasion, the typist started to type a report at 10 35 and finished it at 11 28.
(i) How long did she take to type the report?
(ii) Assuming she made no mistakes and typed non-stop at her usual speed, find the number of words in the report.

10. A rope is cut into three pieces in the ratio 1 : 3 : 5. Given that the length of the longest piece is 35 m, find

(a) the length of the original rope;
(b) the length of the shortest piece of rope.

11. A car travelled 100 km with half the distance at 40 km/h and the other half at 80 km/h. Find the average speed of the car for the whole journey.

12. (a) A photography competition offers \$2 100 in prize money. Given that the prize money is divided among the first, second and third prize winners in the ratio 7 : 5 : 2, find the amount each prize winner gets.

(b) It was decided later to introduce a fourth and fifth prize. The ratio of the first, second, third, fourth and fifth prize is then adjusted to 7 : 5 : 2 : 1 : 1. Given that the prize money is increased to \$2 800, find how much each prize winner gets.

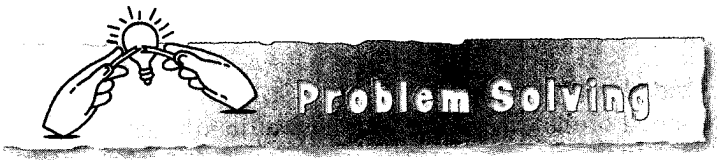
13. A model ship is $\frac{1}{2}$ metre long, and the actual ship is 30 metres long. What is the scale of model?

***14.** A lorry leaves a factory on a journey of 195 km at 08 45, travelling at an average speed of 52 km/h.

(a) Find the time at which the lorry arrives at its destination.
(b) On the return journey, the lorry leaves at 14 55 and arrives at the factory at 18 15. Calculate the time taken and the average speed of the lorry on the return journey.

***15.** A man parked his car in a carpark at 08 30 and retrieved it at 15 45 on the same day.

(a) How long did he park his car in the carpark?
(b) If the parking charges are \$1.50 for the first hour and 80 cents for each subsequent half hour or part thereof, how much must he pay on parking his car there?



1. Peter cycles to visit his grandmother and then returns home by the same route. He always cycles at 4 km/h when going uphill, 12 km/h when going downhill, and 6 km/h when on level ground. If his total cycling time is 2 hours and 20 minutes, what is the total distance he cycles in km?

2. Two candles of the same height are lit at the same time. Each candle burns at a constant rate and the first candle takes 5 hours while the second candle takes 4 hours to burn completely. Find the time, in hours, taken for the height of the first candle to be four times that of the second candle.

3. A container is filled with 56 litres of pineapple juice. 8 litres of pineapple juice are extracted and the container is refilled with mango juice. The content of the container is thoroughly mixed and 8 litres of the mixture are then extracted and the container is again refilled with mango juice. What is the ratio of mango juice to pineapple juice in the final mixture?

4. Twelve men take 6 hours to finish a piece of work. After the 12 men have worked for 1 hour, the contractor decides to call in 8 more men so that the work can be completed earlier. How many more hours would 20 men take to complete the remaining work?

5. Ahmad and Kumar together can paint a house in 12 days. Kumar and Chong Beng together can complete the same job in 15 days while Ahmad and Chong Beng together will take 20 days to paint the same house. How many days will it take Ahmad, Kumar and Chong Beng to complete the job together?

CHAPTER 12

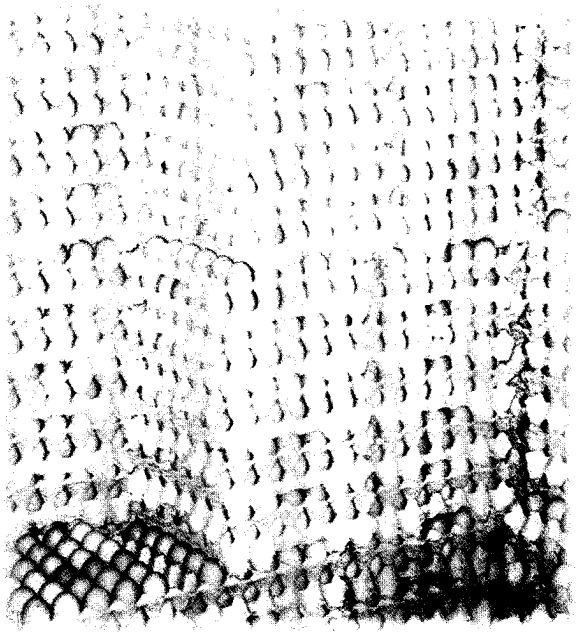
Arithmetical Problems

In this chapter, you will learn how to

- convert percentages into fractions;
- convert percentages into decimals;
- manipulate percentages and solve problems involving percentages;
- solve problems on personal and household finance and simple financial transactions.

Preliminary Problem

Singaporeans consumed about 1 240 million eggs in 1999. Assuming that the population of Singapore was 3.85 million, how many eggs on average did each Singaporean consume in 1999?



Percentages

We often see phrases like "Up to 75% off all items", "90% Housing Loan with Low Interest Rates" and "Fantastic Savings: 10–40% Genuine Discounts" in advertisements. 75%, 90%, 10% and 40% are examples of percentages.

Suppose there are 400 pupils in a school. During one afternoon, 120 pupils remain in school to participate in extra-curricular activities. We say that 120 out of 400 pupils participate in ECA that afternoon. The fraction of pupils who participate in ECA is equal to $\frac{120}{400}$, which can also be written as $\frac{30}{100}$. We say that 30 **per cent** (or percent) of the pupils participate in ECA. The expression "per cent" simply means **for every hundred**, or **out of every hundred**. Thus, a percentage is a fraction whose denominator is **100**. We use the symbol % to represent **per cent**.

So if a shop offers a customer 75% off an item which originally costs \$600, then for every \$100 of the original price, the customer will pay \$75 less.

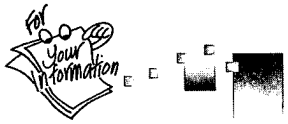
Similarly, if a bank provides a 90% housing loan to a customer who is buying a house, then the customer will get a loan of \$90 for every \$100 of the price of the house he is buying.

In another example, a mathematics examination paper is marked out of 50. If Meiling obtains 35 marks, Xiuyu 43 marks and Weiyun 32 marks, then we say that

Meiling gets $\frac{35}{50}$ of the marks or $\frac{70}{100}$, i.e., 70% of the marks.

Xiuyu gets $\frac{43}{50}$ of the marks or $\frac{86}{100}$, i.e., 86% of the marks.

Weiyun gets $\frac{32}{50}$ of the marks or $\frac{64}{100}$, i.e., 64% of the marks.



The word per cent originated from the Latin word "per centum" meaning "per hundred".

Changing Percentages to Decimals

We have seen that a percentage is simply a fraction with a denominator of 100. Thus, a percentage can be converted to a decimal by dividing the numerator of the fraction by 100.

Example 1

Change each percentage to a fraction and then to a decimal:

(a) 25% *(b) 65%* *(c) 18%* *(d) 4%*

Solution

(a) $25\% = \frac{25}{100}$

$= 0.25$ (25 hundredths)

(b) $65\% = \frac{65}{100} = 0.65$ (65 hundredths)

(c) $18\% = \frac{18}{100} = 0.18$ (18 hundredths)

(d) $4\% = \frac{4}{100} = 0.04$ (4 hundredths)

To change a percentage to a decimal, express it as a fraction with a denominator of 100. Then convert it to a decimal.

Example 2

Express each percentage as a decimal:

(a) 52% *(b) 36%* *(c) 125%* *(d) 4.8%*

(e) 0.75% *(f) 100%*

Solution

(a) $52\% = \frac{52}{100} = 0.52$ (b) $36\% = \frac{36}{100} = 0.36$ (c) $125\% = \frac{125}{100} = 1.25$

(d) $4.8\% = \frac{4.8}{100} = 0.048$ (e) $0.75\% = \frac{0.75}{100} = 0.007\ 5$ (f) $100\% = \frac{100}{100} = 1$

Example 3

Express each percentage as a decimal:

(a) $13\frac{1}{2}\%$ *(b)* $8\frac{1}{4}\%$ *(c)* $134\frac{3}{4}\%$ *(d)* $\frac{2}{5}\%$

Solution

(a) $13\frac{1}{2}\% = 13.5\% = 0.135$ (First write the fraction as a decimal)

(b) $8\frac{1}{4}\% = 8.25\% = 0.082\ 5$

(c) $134\frac{3}{4}\% = 134.75\% = 1.347\ 5$

(d) $\frac{2}{5}\% = 0.4\% = 0.004$

Changing Decimals to Percentages

We can also change a decimal to a percentage.

For example, $0.25 = \frac{25}{100}$ (25 hundredths)

$= 25\%$

To change a decimal to a percentage, write it as a fraction with denominator 100, then as a percentage.

Example 4

Express each decimal as a percentage:

(a) 0.24 *(b) 0.72* *(c) 0.09* *(d) 0.136* *(e) 1.12* *(f) 3*

Solution

(a) $0.24 = \frac{24}{100} = 24\%$ (b) $0.72 = \frac{72}{100} = 72\%$ (c) $0.09 = \frac{9}{100} = 9\%$

(d) $0.136 = \frac{13.6}{100} = 13.6\%$ (e) $1.12 = \frac{112}{100} = 112\%$ (f) $3 = \frac{300}{100} = 300\%$

Changing Fractions to Percentages

To change a fraction to a percentage, multiply it by 100%. Alternatively, change it to a decimal followed by expressing this decimal as a percentage.

Example 5

Change each fraction to a percentage:

(a) $\frac{7}{40}$ *(b)* $\frac{5}{8}$ *(c)* $1\frac{2}{3}$

Solution

(a) $\frac{7}{40} = \frac{7}{40} \times 100\% = 17\frac{1}{2}\%$ or 17.5%

(b) $\frac{5}{8} = \frac{5}{8} \times 100\% = 62\frac{1}{2}$ or 62.5%

(c) $1\frac{2}{3} = \frac{5}{3} \times 100\% = 166\frac{2}{3}\%$

Alternatively,

(a) $\frac{7}{40} = 0.175 = 17.5\%$

(b) $\frac{5}{8} = 0.625 = 62.5\%$

(c) $1\frac{2}{3} = 1.667$ (correct to 3 decimal places)

$= 166.7\%$

For part (c), where the fraction is non-terminating, we may use the second method if we are not required to give the exact answer.

Changing Percentages to Fractions

To change a percentage to a fraction, reverse the process of converting a fraction to a percentage.

Example 6

Convert each percentage to a fraction:

(a) 15% *(b) 62.5%* *(c) 215%*

Solution

(a) $15\% = \frac{15}{100}$ (Divide by 100%) or $15\% = 0.15$ (Change to decimal)

$= \frac{3}{20}$ $= \frac{15}{100} = \frac{3}{20}$

(b) $37.5\% = \frac{37.5}{100} = \frac{375}{1\,000} = \frac{3}{8}$ or $37.5\% = 0.375 = \frac{375}{1\,000} = \frac{3}{8}$

(c) $215\% = \frac{215}{100} = \frac{43}{20} = 2\frac{3}{20}$ or $215\% = 2.15 = 2\frac{15}{100} = 2\frac{3}{20}$

Exercise 12a

1. Express the following percentages as decimals:

(a) 6% **(b)** 11% **(c)** 22%
(d) 63% **(e)** 179% **(f)** 0.27%
(g) 28.7% **(h)** 134.6% **(i)** $3\frac{1}{2}\%$
(j) $5\frac{1}{4}\%$ **(k)** 0.074% **(l)** 54.37%
(m) 0.0063% **(n)** $1\frac{1}{8}\%$ **(o)** $\frac{7}{8}\%$
(p) $50\frac{3}{4}\%$

2. Write each decimal as a percentage:

(a) 0.17 **(b)** 0.575 **(c)** 0.83
(d) 2.36 **(e)** 0.09 **(f)** 0.025
(g) 0.008 **(h)** 2.564 **(i)** 0.000 5
(j) 1.2 **(k)** 4 **(l)** 6.25

3. Change each fraction to a percentage:

(a) $\frac{3}{4}$ **(b)** $\frac{9}{10}$ **(c)** $\frac{17}{20}$ **(d)** $\frac{6}{125}$
(e) $\frac{6}{5}$ **(f)** $\frac{12}{25}$ **(g)** $1\frac{6}{25}$ **(h)** $2\frac{2}{5}$

4. Express each fraction as a percentage, giving the answer to 1 decimal place:

(a) $\frac{2}{3}$ **(b)** $\frac{4}{7}$ **(c)** $\frac{2}{9}$ **(d)** $\frac{7}{12}$
(e) $1\frac{1}{3}$ **(f)** $\frac{5}{6}$ **(g)** $2\frac{3}{7}$ **(h)** $\frac{10}{11}$

5. Copy and complete the following table.

Percentage	*Fraction*	*Decimal*
	$\frac{3}{5}$	
Eleven per cent		
		0.175
		0.095
$78\frac{1}{2}\%$		

6. The forests of Singapore cover about $4\frac{1}{6}\%$ of the total land area. What fraction is this?

7. An electronic firm finds that $\frac{3}{64}$ of the resistors it makes are defective. What percentage is this?

8. Arrange the following in ascending order:
   (a) 0.39, $\frac{12}{32}$, $4\frac{1}{2}\%$ **(b)** 0.222, 22%, $\frac{2}{9}$
   (c) 64%, 0.6, $\frac{2}{3}$

Expressing One Quantity as a Percentage of Another

In a secondary school, 56 out of 70 teachers are female. What percentage of the teachers are female? What percentage of them are male?

We know that the fraction of female teachers in the school is $\frac{56}{70}$. Changing this fraction to percentage, we have $\frac{56}{70} \times 100\% = 80\%$. Hence 80% of the teachers in the school are female. The percentage of male teachers in the school = 100% – 80% = 20%.

In general, to express one quantity, a, as a percentage of another quantity, b, we

1. write a as a fraction of b,

2. multiply the fraction $\frac{a}{b}$ by 100% to convert it to a percentage.

Example 7

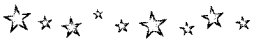
A pupil scored 108 out of 150 marks in Mathematics and 96 out of 160 marks in English. Find the percentage mark for each subject.

Solution

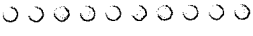
The pupil's percentage mark for Mathematics $= \frac{108}{150} \times 100\%$
$= 72\%$

The pupil's percentage mark for English $= \frac{96}{160} \times 100\%$
$= 60\%$

Ninety-nine boys and one girl are in a lecture theatre. How many boys must leave the theatre so that the percentage of boys becomes 98%?



About 70% of the earth's surface is covered by water.



Finding the Percentage of a Number

If you are told that 75% of the pupils in a class of 40 passed a Mathematics test, how many of them failed?

The number of pupils who passed the test = 75% of the 40 pupils

$$= \frac{75}{100} \times 40$$
$$= 30$$

$\therefore$ number of pupils who failed the test = 40 – 30 = 10

Alternatively, since 75% of the pupils passed the test, (100% – 75%) or 25% failed the test.

Number of pupils who failed the test = 25% × 40

$$= \frac{25}{100} \times 40 = 10$$

Example 8

Find (a) 25% of \$21.60, (b) $37\frac{1}{2}$% of 1.60 m.

Solution

(a) 25% of \$21.60 $= \frac{1}{4} \times \$21.60 \quad \left(25\% = \frac{1}{4}\right)$

$= \$5.40$

(b) $37\frac{1}{2}$% of 1.60 m $= \frac{75}{2}\% \times 1.60$ m

$= \frac{1}{2} \times 75\% \times 160$ cm $\quad \left(\frac{75}{2}\% = \frac{1}{2} \times 75\%;\ 1.60 \text{ m} = 160 \text{ cm}\right)$

$= \frac{1}{2} \times \frac{3}{4} \times 160$ cm $\quad \left(75\% = \frac{3}{4}\right)$

$= 60$ cm

Exercise 12b

1. **(a)** Express 30 cents as a percentage of \$1.
(b) Express 45 m as a percentage of 1 km.
(c) Express 1 kg as a percentage of 800 g.

2. Express the first quantity as a percentage of the second.

(a) 45 min, 1 h **(b)** 4 mths, 1 yr
(c) 335 cm, 5 m **(d)** 60°, 360°
(e) 15 mm, 1 m **(f)** 63¢, \$2.10

3. **(a)** In a test, 30 of the 36 students obtained passing grades.
(i) What fraction of the students passed?
(ii) What percentage of the students passed?
(b) In the same test, John scored 65 marks out of a possible total of 80 marks.
(i) Write his score as a fraction.
(ii) Write his score as a percentage.

4. A pupil saved \$7.20 of his weekly pocket money of \$24. What percentage did he save?

5. During the economic downturn, a company retrenched 24 out of its 400 employees. What percentage of employees was retrenched?

6. In a survey, 120 pupils were asked which learning site they would like to visit as part of the National Education learning journey.

24 said National Heritage Board
36 said Battle Box
54 said Singapore Discovery Centre
The rest said they would like to visit the People's Association.

Write these results as percentages.

7. Calculate the following:

(a) $15\frac{1}{2}\%$ of \$640

(b) 6.5% of 5 000 people

(c) 80% of 4.50 m

(d) 125% of 50 cm

(e) 30.6% of 300 *l*

(f) 60.5% of 8 hrs

8. In a certain constituency, there are 8 500 voters and on election day, 15% of them failed to vote. Calculate the number of people who voted.

9. A company finds that $4\frac{1}{4}\%$ of the tyres made are defective. The company made 28 000 tyres. How many tyres were defective?

10. **(a)** Soyabeans on an average contain 39.5% protein. A bushel of soyabeans weighs 120 kg. How many kilograms of protein are contained in a bushel?

(b) A 100-acre field yields 50 bushels per acre. How many kilograms of protein does the field yield?

11. In this question, take the Singapore population to be about 4 000 000.

(a) Figure out, mentally, what 10% of the Singapore population is.

(b) Use the answer in (a) to figure out the following percentages of the population:

(i) 20% **(ii)** 30%

(iii) 40% **(iv)** 50%

Percentage Change

The change in the value of an item can be expressed as a percentage increase or decrease in the original value. An **increase** of, say, 5% in the salary of a worker means that for every \$100 in the **original** salary, there is an increase of \$5, i.e., each \$100 in the original salary becomes \$105 in the **new** salary. Suppose the original salary for the worker is \$1 600, how much will his new salary be after a 5% increase?

New salary : Original salary = 105 : 100

$$\frac{\text{New salary}}{\text{Original salary}} = \frac{105}{100}$$

$$\text{New salary} = \frac{105}{100} \times \text{Original salary}$$

$$= \frac{105}{100} \times \$1\,600 = \$1\,680$$

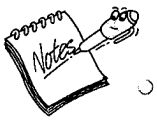
On the other hand, a **decrease** of, say, 5% in the salary means that for every \$100 in the **original** salary, there is a decrease of \$5, i.e., each \$100 in the original salary becomes \$95 in the **new** salary. In the above case,

New salary : Original salary = 95 : 100

$$\frac{\text{New salary}}{\text{Original salary}} = \frac{95}{100}$$

$$\text{New salary} = \frac{95}{100} \times \text{Original salary}$$

$$= \frac{95}{100} \times \$1\,600 = \$1\,520$$



1. *Original salary increases in the ratio 105 : 100 and the new salary is 105% of the original salary.*
2. *Original salary decreases in the ratio 95 : 100 and the new salary is 95% of the original salary.*

Example 9

The workers of an electronic company were given an increase amounting to 8% in their monthly salaries.

(a) If Peter earned $1 800 per month originally, find his monthly salary after the increment.
(b) If Paul's monthly salary after the increment is $1 728, find his original monthly salary.

Solution

(a) Peter's monthly salary after the increment is 100% + 8% = 108% of his original monthly salary.

His monthly salary after the increment = 108% of $1 800

$= 1.08 \times \$1\ 800 = \$1\ 944$

or $= \frac{108}{100} \times \$1\ 800 = \$1\ 944$

Alternatively, the increment = 8% of $1 800 = $144

and thus Peter's new monthly salary = $1 800 + $144 = $1 944

(b) **Method 1: Use proportion**

Paul's salary after the increment : Paul's original salary = 108 : 100

Paul's original salary $= \frac{100}{108} \times \$1\ 728 = \$1\ 600$

Method 2: Use an equation

Let Paul's original salary be $\$x$.

His monthly salary after the increment = 108% of $\$x = \$\frac{108}{100}x$.

$\therefore \quad \frac{108}{100}x = 1\ 728$

$x = 1\ 728 \times \frac{100}{108} = 1\ 600.$

$\therefore$ Paul's original salary is $1 600.

Example 10

After 6% of a bill has been deducted, $282 remains to be paid. How much was the original bill?

Solution

Strategy 1: Use proportion

Following a discount of 6%, 94% of the bill remains to be paid.

Original bill : $282 = 100 : 94

$\therefore$ original bill $= \frac{100}{94} \times \$282 = \$300$

Strategy 2: Use an equation

Let his original bill be $\$x$.

The deduction = $\$(x - 282)$

$\therefore$ 6% of $\$x$ = $\$(x - 282)$

i.e., $\frac{6}{100}x = x - 282$

or $\frac{94}{100}x = 282$

$$x = 282 \times \frac{100}{94} = 300$$

$\therefore$ the original bill is \$300.

1. *In 1998, a man was given a pay increase of 10%. In 1999, he suffered a pay cut of 10%. Was he better off or worse off as compared to 1997?*
2. *In 1998, a man was given a pay cut of 10%. In 1999, he was given a pay increase of 10%. Was he better off or worse off as compared to 1997?*

Example 11

The cost of a television set is raised from \$600 to \$624. Find the percentage increase.

Solution

The increase = \$624 – \$600 = \$24

The percentage increase = $\frac{\text{Increase}}{\text{Original cost}} \times 100\%$

$$= \frac{\$24}{\$600} \times 100\% = 4\%$$

Example 12

The cost of a piece of furniture is calculated as follows:

Wood \$300; Paint \$200; Wages \$200.

If the costs of the wood and paint are increased by 12% and 7% respectively, while the wages are decreased by 10%, find the percentage increase or decrease in the cost of the furniture.

Solution

	Original cost (\$)	*Percentage change*	*New cost (\$)*
Wood	300	+12%	$\frac{112}{100} \times 300 = 336$
Paint	200	+7%	$\frac{107}{100} \times 200 = 214$
Wages	200	–10%	$\frac{90}{100} \times 200 = 180$
Furniture	700		730

The percentage increase in the cost of the furniture

$$= \frac{\text{Increase}}{\text{Original cost}} \times 100\% = \frac{730 - 700}{700} \times 100\% = \frac{30}{700} \times 100\% = 4\frac{2}{7}\%$$

Exercise 12c

1. Increase 28 by 125%; what is the result?

2. Decrease 216 by $37\frac{1}{2}$%; what is the result?

3. The result of a number, when increased by 15%, is 161. Find this number.

4. The result of a number, when decreased by 20%, is 192. Find this number.

5. A man spends \$880 in a month. Out of this, 26% goes to his rent. How much is his rent?

6. A flat costs 36% more today than when it was built. If the original cost of the flat was \$90 000, find its price today.

7. After spending 88% of his income, a man has \$216 left. Find his income.

8. If 10% is deducted from a bill, \$58.50 remains to be paid. How much is the bill?

9. A property company sold 20% more houses in 1999 than it did in 1998. If the company sold 426 houses in 1999, calculate how many houses it sold in 1998.

10. The height of a tree was 4.8 m. After one year, the height of the tree was increased by 12.5%. Find its new height.

11. A machinist is hired at \$7.20 per hour.
    (a) On a particular day, he clocked in at 8.00 a.m. and clocked out at 3.45 p.m. Find his wage for that day.
    (b) At the end of a one-year probation period, his wage will increase by $27\frac{1}{2}$%. If the machinist successfully completes the apprenticeship,
        (i) what will his pay be per hour?
        (ii) what will his pay be per week, if he starts with a forty-hour week?

12. The production cost of a printer is as follows: overheads — \$80; wages — \$120; raw materials — \$100.

    If the cost of overheads increases by 20%, wages by 15% and raw materials by 11%, find the percentage increase in the production cost of the printer.

13. A new car costs \$120 000. After 1 year, its value decreases by 20%. For the second year, its value decreases a further 10%. What is the value of the car after 2 years?

14. James is 8% taller than John, and Wilson is 10% shorter than John. By what percentage is James taller than Wilson?

15. In 1998, a train carried 8% more passengers than in 1997. In 1999, it carried 8% more passengers than in 1998. What was the total percentage increase in the number of passengers from 1997 to 1999?

16. During 1998, a swimming lesson at a particular school lasted 45 minutes.
    (a) In 1999, the lesson time will be 50 minutes. Find the increase in the lesson time from 1998 to 1999 as a percentage of the lesson time in 1998.
    (b) The lesson time in 1999 will be 25% more than it was in 1997. Find how long the lesson lasted in 1997.
    (c) At present, James takes 240 seconds to swim 100 metres. In a year's time, he would have **improved** his swimming **time** by 25%. Calculate the time he will take to swim 100 metres in a year's time.
    (d) Robert, at present, also takes 240 seconds to swim 100 metres. In a year's time, he would have **increased** his swimming **speed** by 25%. Calculate the time he will take to swim 100 metres in a year's time.
    (e) Who will be the faster swimmer, James or Robert, in a year's time?

Profit and Loss

A manufacturer produces goods at a certain cost. If the goods are sold at a **higher** price than the cost price, then the manufacturer makes a **profit** or **gain**. But if, for some reason, the manufacturer sells the goods at a **lower** price than the cost price, he suffers a **loss** on the transaction. Thus,

Profit = Selling price – Cost price

Loss = Cost price – Selling price

Percentage Profit and Percentage Loss

For comparison, we usually express the actual profit or loss as a percentage of the cost price. For example, a shopkeeper sold an article costing \$50 for \$60 and another article costing \$100 for \$110. We note that in each transaction, the shopkeeper made a profit of \$10. It seems that both transactions are equally favourable. However,

the percentage profit for the first transaction is $\frac{\$10}{\$50} \times 100\% = 20\%$

and the percentage profit for the second transaction is $\frac{\$10}{\$100} \times 100\% = 10\%$.

Hence, the percentage profit gives a better comparison.

Example 13

A bag costing \$28 is sold for \$35. Find the percentage profit.

Solution

Profit = Selling price – Cost price
= \$35 – \$28 = \$7

$\therefore$ percentage profit = $\frac{7}{28} \times 100\% = 25\%$

Example 14

A vase costing \$60 is sold for \$50. Find the percentage loss.

Solution

Loss = Cost price – Selling price
= \$60 – \$50 = \$10

$\therefore$ percentage loss = $\frac{10}{60} \times 100\% = 16\frac{2}{3}\%$

Example 15

A bookseller gains 30% by selling a book for \$65. Find the cost of the book.

Solution

Strategy 1: Use proportion

Selling price = Cost price + Profit

If cost price = 100%, then selling price = (100% + 30%) of the cost price
= 130% of the cost price

Cost price : Selling price = 100 : 130

$$\text{Cost price} = \frac{100}{130} \times \text{Selling price}$$

$$= \frac{100}{130} \times \$65 = \$50$$

Strategy 2: Use an equation

Let the cost price be $\$x$.

The profit = 30% of $\$x = \frac{30}{100} \times \$x = \$\frac{3x}{10}$

Profit = Selling price – Cost price

$$\frac{3x}{10} = 65 - x$$

$$3x = 650 - 10x$$

$$13x = 650$$

$$x = 50$$

$\therefore$ the cost price of the book is \$50.

Example 16

Desmond receives 400 kg of bananas, for which he pays \$0.75 per kg. On average, 8% of the bananas will spoil. Find the selling price per kg to obtain a 75% profit on cost.

Solution

The cost of the bananas = \$0.75 × 400 = \$300.

The total selling price = 1.75 × \$300 = \$525 (to obtain a 75% profit on cost)

He expects to sell (100 – 8)% = 92% of the bananas or 0.92 × 400 = 368 kg of the bananas

$$\text{Selling price per kg} = \frac{\text{Total selling price}}{\text{Number of kg of bananas expected to sell}} = \frac{\$525}{368}$$

$$\simeq \$1.427 \text{ or } \$1.43$$

Desmond must sell the bananas for \$1.43 per kg to receive the profit he desires. He will receive additional profit if he sells more than 92% of the bananas.

Exercise 12d

1. Find the gain or loss per cent in the following cases:
   (a) cost price = \$40, gain = \$5;
   (b) selling price = \$30, gain = \$2;
   (c) selling price = \$60, loss = \$20;
   (d) cost price = \$16.25, selling price = \$18.50.
2. A gold chain is sold for \$635 at a gain of 27 per cent. Find the profit.
3. By selling a book for \$16.50, a bookseller loses 12%. What is the cost price of the book?
4. Peter bought an antique chest for \$600 and was forced to sell it for \$500. Find the percentage loss.
5. If Susan sells her car at a loss of 6%, what is her selling price when she paid \$18 400 for it?
6. To make a profit of $33\frac{1}{3}\%$, a bicycle must be sold for \$240. What is the cost price of the bicycle?
7. The profit on a certain refrigerator is 35% of the cost price. If the profit is \$280, find
   (a) the cost price and
   (b) the selling price of the refrigerator.
8. The retail price of a television set is \$840. If this is 140% of the wholesale price, find the wholesale price.
9. A man buys a dozen cameras for \$1 800. He sells them at a profit of \$36 each. Find his profit percentage.
10. A florist bought 360 roses at \$10 per dozen. If he sold them at \$1.10 each, what is his percentage profit?

*11. Mr Lin buys an article and sells it to Mr Chen at a gain of 25%. Mr Chen sells the article to Mr Ang at a gain of 20%. How much money did Mr Lin pay for the article, if Mr Ang pays \$360 for it?

*12. A shopkeeper buys 300 identical articles at a total cost of \$1 500. He fixes the selling price of each article at 20% above the cost price and sells 260 articles at this price. As for the remaining articles, he sells them at 50% of the selling price. Calculate the shopkeeper's total profit.

*13. Simon ordered 200 boxes of Fuji apples from China. He paid \$28 per box for the apples. There were 60 fruits in each box and he expected 15% of them to spoil. If he wants to make a profit of 80% on cost, what should be the selling price per fruit?

Discount

A discount
A reduction in the price of an article or commodity for payment in cash.

Very often, retailers cannot sell defective merchandise, overstocked items, discontinued models and so on at the retail selling prices. To clear the merchandise in stock, the retailers usually sell the items at a lower price, called the sale price. The difference between the original selling price or the **marked price** and the cheaper price or the **sale price** is called the **discount**.

Discount = Marked price – Sale price

Discount is often expressed as a percentage of the original price.

Example 17

A watch priced at \$160 is sold for \$140. Find the percentage discount.

Solution

Discount = Marked price – Selling price
= \$160 – \$140 = \$20

$\therefore$ percentage discount = $\frac{20}{160} \times 100\% = 12\frac{1}{2}\%$

Example 18

The charges for processing and printing a roll of film in a shop are as shown in the table.

Mr Lee sends 3 rolls of 36 prints and 6 rolls of 24 prints to a shop for processing and printing.

(a) How much does it cost him?
(b) How much will it cost him if a 15% discount is given?

Charges per roll	
24 prints	\$9.00
36 prints	\$12.00

Solution

(a) $3 \times 12 + 6 \times 9 = 36 + 54 = 90$
$\therefore$ it costs him \$90.

(b) Discounted price : original price = 85 : 100
$\therefore$ discounted price = $\frac{85}{100} \times \$90 = \$76.50$. It will cost him \$76.50.

Alternatively, discount = 15% of the original price = $15\% \times \$90 = \$13.50$
$\therefore$ it will cost him \$90 – \$13.50 = \$76.50.

Example 19

A ladies' bag selling for \$175 is marked down 20% for a special promotion. It is later marked down further by $17\frac{1}{2}\%$ of the sale price. Since it still has not sold, it is marked down further to a price 60% off the original selling price.

(a) What are the two sale prices of the bag?

(b) What is the final selling price of the bag?

Solution

(a) The first sale price of the bag = (100 – 20)% of \$175 = $\frac{80}{100} \times \$175 = \$140$.

The second sale price of the bag = $\left(100 - 17\frac{1}{2}\right)\%$ of \$140 = $\frac{165}{2} \times \frac{1}{100} \times 140 = \$115.50$

(b) The final selling price of the bag = (100 – 60)% of \$175 = $\frac{40}{100} \times \$175 = \$70$.

Commission

A **commission** is the payment an agent gets for selling or buying something on behalf of another person. It is usually given as a percentage of the cost price or the selling price.

Example 20

A flat was bought for \$220 000 by an agent who received a commission of $1\frac{1}{2}$%. How much commission did he receive?

Solution

$$1\frac{1}{2}\% \text{ of } \$220\,000 = \frac{3}{2} \times \frac{1}{100} \times \$220\,000$$
$$= \$3\,300$$

$\therefore$ the agent received \$3 300 as commission.

Exercise 12e

1. Find the percentage discount of the following, given the marked prices and the sale prices:

	Marked Price	Sale Price
(a)	\$100	\$88
(b)	\$580	\$464

2. A supermarket gives a 10% discount on the marked prices of all items during a sale. Find for the following marked prices
   (i) the amount of discount given and
   (ii) the sale price:
   (a) \$45 **(b)** \$110

3. Find the marked prices of the following, given the amount of discount:
   (a) A 7% discount which is \$49.
   (b) A $33\frac{1}{3}$% discount which is \$270.

4. Find the marked prices of the following, given the percentage discount and the sale prices:
   (a) 12% discount, \$77
   (b) 25% discount, \$123

5. At a sale, the price of a washing machine was reduced by 12% to \$440. What was the original price of the washing machine?

6. A shopkeeper marks the price of an article at \$80. Find the selling price of the article if
   (a) he gives a discount of 10%,
   (b) he gives two successive discounts of 5% each.

7. During a sale a shopkeeper reduced the prices of all his goods by 15%. Calculate the original selling price of a calculator, which was sold for \$23.80 during the sale.

8. John bought an air-conditioner priced at \$800, but was given a discount of $12\frac{1}{2}$%. Calculate the price he paid.

9. A camcorder originally marked to sell for \$2 000 was reduced 20% during a special promotion. The price was then reduced an additional 30% to clear the stock. What was the sale price for each reduction?

10. The cost of a colour film for 36 photographs is usually \$3.60. Additionally, the cost of processing and printing the 36 photographs is usually \$12.60.
    (a) Show that the **total** cost of each photograph is usually 45 cents.

(b) A shop keeps the price of the film at \$3.60, and offers a reduction of 20% off the cost of processing and printing. Calculate the total cost of each photograph at this shop.

(c) Calculate the percentage reduction in the total cost, compared to the usual cost, of a photograph obtained from this shop.

11. A property agent charges a commission of 5% on the first \$10 000 and $2\frac{1}{4}\%$ on the remainder of the selling price. Find the amount of commission he will receive if he sells a piece of property for \$46 000.

12. Mr. Goh's monthly income consists of \$500 plus a commission of 4% on all his sales. Find his total sales for a particular month if his income was \$1 220 that month.

13. A tour guide earns commission by bringing tourists to patronise a certain handicraft shop. Given that the commission he receives is 3% of the total sales, calculate his commission on a particular day when the 12 tourists he brought to the shop spent an average of \$250 each.

14. Three salesmen went to 400 households to sell a certain brand of rice cookers costing \$60 each. 15% of the households bought a rice cooker each from them. If the three salesmen sold the rice cookers in the ratio 3 : 4 : 5, find the commission each salesman received if the commission is 4% of the total sales.

Simple Interest

When you deposit money into a bank, you receive interest for allowing the bank to use your money. Similarly, when you borrow money from the bank, you must pay a certain interest for using its money. The interest in both cases is calculated as a percentage (called the **rate**) of the capital (called the **principal**) deposited or borrowed. Interest is usually calculated at a fixed yearly rate (called **rate per annum**).

Sometimes, interest rates are calculated on half-yearly or quarterly period, monthly or even daily basis. The amount of interest depends on the length of time the money is deposited or borrowed. If interest is always calculated on the original principal, it is called **simple interest**. When the interest is added to the principal, the sum is called the **amount**.

The principal is taken to be the same; i.e., \$100 for each year.

Example 21

A man borrows \$100 for 3 years at a rate of 6% per annum. What is the simple interest he has to pay?

Solution

The principal is \$100.

The interest on \$100 for 1 year is $\frac{6}{100} \times \$100 = \$6$

$\therefore$ the interest on \$100 for 3 years is $3 \times \$6 = \$18$.

From the above, we know that the interest payable (or earned) depends on

(i) the amount borrowed or lent, i.e., the principal,
(ii) the rate of interest charged, i.e., the rate %,
(iii) the period of the loan or deposit, i.e., the time.

For a sum of $\$P$ deposited in a bank at $R\%$ simple interest per annum

for T years, the simple interest (in \$) is $\boldsymbol{I = \frac{PRT}{100}}$

Example 22

Mary needed capital for her bakery. She borrowed \$60 000 for 4 years at a simple interest rate of 8% per year.

(a) How much interest must be paid?
(b) How much money will Mary pay at the end of 4 years?

Solution

(a) P = \$60 000, R = 8 and T = 4

Simple interest $I = \frac{PRT}{100} = \frac{60\,000 \times 8 \times 4}{100} = 19\,200$

$\therefore$ \$19 200 of interest must be paid.

(b) The amount of money Mary will pay at the end of 4 years = principal + interest
= 60 000 + 19 200 = \$79 200.

Example 23

To save money for a bookshop, Ronald invested \$2 500 at 6% per annum simple interest. How long will it take for the amount to add up to \$3 400?

Solution

Simple interest = \$3 400 – \$2 500 = \$900

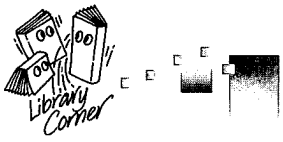
I = \$900, R = 6 and P = \$2 500

$$I = \frac{PRT}{100}$$

$$900 = \frac{2\,500 \times 6 \times T}{100}$$

$$T = \frac{900 \times 100}{2\,500 \times 6} = 6$$

$\therefore$ the time taken is 6 years.



Credit cards are widely used nowadays. Find out the rate of interest charged by credit card companies on card holders who do not pay up their bills in time. How much higher is it as compared to the rate of interest on savings account?

Example 24

Simon wanted to borrow some money to expand his fruit shop. He was told he could borrow a sum of money for 30 months at 12% simple interest per year and pay $1 440 in interest charges. How much money could he borrow?

Solution

$$T = \frac{30}{12} = 2.5 \text{ year} \qquad I = \frac{PRT}{100}$$

$$1\,440 = \frac{P \times 12 \times 2.5}{100}$$

$$P = \frac{1\,440 \times 100}{12 \times 2.5} = \$4\,800$$

Simon could borrow $4 800.

Example 25

To buy a car, Raymond borrowed $20 000 for $3\frac{1}{2}$ years and paid $5 880 simple interest on the loan. What rate of interest did he pay?

Solution

$$I = \frac{PRT}{100}$$

$$5\,880 = \frac{20\,000 \times R \times 3.5}{100}$$

$$R = \frac{5\,880 \times 100}{20\,000 \times 3.5} = 8.4$$

He paid 8.4% simple interest per year.

Exercise 12f

1. Copy and complete the following table:

	Principal	*Interest rate*	*Time*	*Simple interest*	*Amount*
(a)	$12 000	8%	7 years		
(b)	$500	11%		$220	
(c)		9%	4 years	$108	
(d)	$3 000		10 years	$1 200	
(e)			2 years	$360	$3 960
(f)	$1 800		18 months	$189	
(g)	$4 500		2 years		$5 040
(h)		5%		$90	$1 290

2. Kenneth's shoe repair shop borrowed \$6 600 from a bank at 8% simple interest per annum. How much did he owe the bank at the end of 11 months?

3. A finance company charges \$55 simple interest on a sum of money which is borrowed for five months. Given that the rate of interest is 12% per annum, find the sum of money.

4. A bank charges 2.25% per month simple interest on personal loans. If John borrows \$6 400 for a period of 2 years 1 month, find the total interest he has to pay.

5. Mrs. Lee invests \$800 at 6% per annum and \$1 200 at 7% per annum. What is her total annual interest on these two investments?

6. How long would \$1 250 have to be deposited at 6% per year simple interest to gain \$750 simple interest?

7. Andrew lent Roger \$4 800 for 7 months. At the end of this period Roger had to pay Andrew an interest of \$119. What was the rate of simple interest per annum?

8. In a certain year, James puts \$600 in a bank at the end of March and \$400 in the same bank at the end of June. The bank offers 3% per annum simple interest rate. Find the total amount that James receives from the bank at the end of December in that year.

9. A bank increased the rate of interest, which it paid to depositors from 3.5% to 4% per annum. Find how much more interest Susan would receive if she deposited \$6 400 in the bank for 6 months at the new interest rate.

10. Mrs. Jasmine invested \$4 000 in a Building Society which paid simple interest at a rate of $7\frac{1}{4}\%$ per annum to its investors. After 2 years, the rate was increased to 7.6% per annum. Find the amount she had at the end of 7 years.

11. Mr. Chen deposits a certain sum of money in a bank. If the interest rate of the bank decreases from $3\frac{3}{4}\%$ per annum to $3\frac{1}{2}\%$ per annum, Mr. Chen's interest will decrease by \$50 in a year. Find the sum of money he deposits.

Compound Interest

The interest is not always calculated based on the original principal. Suppose John deposits \$2 000 in his savings account in a bank for 2 years at 5% per annum and the interest due to him is calculated as follows:

First year: $P = \$2\,000$, $R = 5$, $T = 1$, $I = \dfrac{\$2\,000 \times 5}{100} = \mathbf{\$100}$.

Second year: $P = \$2\,000 + \mathbf{\$100} = \$2\,100$, $R = 5$, $T = 1$, $I = \dfrac{\$2\,100 \times 5}{100} = \mathbf{\$105}$.

The total interest for 2 years = **\$100** + **\$105** = **\$205**.

In the above computation, the interest of **\$100** due to him at the end of the first year is **compounded** with, i.e. added on, to the principal of \$2 000. This amount \$2 100 becomes the principal for the second year and is used to obtain the interest due to him at the end of the second year.

The total interest of **$205** is called the **compound interest** and the sum $2 000 is said to be deposited at compound interest **compounded** annually.

If the sum is deposited at 5% per annum simple interest for 2 years, calculate the simple interest. Which interest is larger, the simple interest or the compound interest?

Example 26

Find the compound interest on $600 for 3 years at $3\frac{1}{2}\%$ per annum, compounded annually.

Solution

First year, $P = \$600$, $R = 3\frac{1}{2} = \frac{7}{2}$, $T = 1$ year

$$I = \$600 \times \frac{7}{2} \times \frac{1}{100} = \$21$$

By the end of the first year, the principal is $600 + $21 = $621.

Second year, $P = \$621$, $R = \frac{7}{2}$, $T = 1$ year

$$I = \$621 \times \frac{7}{2} \times \frac{1}{100} = \$21.735$$

By the end of the second year, the principal is $621 + $21.735 = $642.735.

Third year, $P = \$642.375$, $R = \frac{7}{2}$, $T = 1$ year

$$I = \$642.735 \times \frac{7}{2} \times \frac{1}{100} = \$22.495725.$$

By the end of the third year, the compound interest is

$$\$21 + \$21.735 + \$22.495725 = \$65.230725 = \$65.23$$ (correct to the nearest cent.)

Example 27

Find the compound interest on $2 500 for 1 year at 4% per annum compounded half-yearly.

Solution

Since interest is calculated half-yearly, the rate of interest becomes 2% per half-year.

Principal for the first half-year = $2 500, interest at 2% = $2 500 × 0.02 = $50.

Principal for the second half-year = $2 550, interest at 2% = $2 550 × 0.02 = $51.

$\therefore$ total interest = $50 + $51 = $101.

Alternatively, *first half-year*: $P = \$2\,500$, $R = 4$, $T = \frac{1}{2}$ and $I = \$2\,500 \times \frac{4}{100} \times \frac{1}{2} = \$50$;

second half-year: $P = \$2\,550$, $R = 4$, $T = \frac{1}{2}$ and $I = \$2\,550 \times \frac{4}{100} \times \frac{1}{2} = \$51$;

$\therefore$ total interest = $50 + $51 = $101.

Exercise 12g

1. Find the compound interest on
   (a) \$450 for 2 years at 10% per annum compounded yearly;
   (b) \$700 for 3 years at 11% per annum compounded yearly;
   (c) \$5 000 for 2 years at $11\frac{3}{4}\%$ per annum compounded yearly;
   (d) \$1 200 for 3 years at 4% per annum compounded yearly;
   (e) \$10 000 for 2 years at $7\frac{1}{2}\%$ per annum compounded annually.
2. Wilson invests \$5 000 at $5\frac{1}{4}\%$ per annum compound interest compounded annually. Find the amount at the end of the third year.
3. Joan invests \$800 at $12\frac{1}{2}\%$ per annum compound interest compounded half-yearly. What is the amount at the end of the first year?
4. Mr. Sim invests \$9 000 at 2% per annum compound interest compounded daily. What is his amount at the end of the third day?

Hire Purchase

Hire Purchase *is a method of buying goods in which payment of purchase price is spread over a specified period by payment of an initial deposit followed by regular instalments.*

The following is an advertisement for the sale of a bedroom set.

For Sale: \$4 500
\$1 000 deposit
Balance 7%, 2 years

After a deposit of \$1 000 is paid, you can take the bedroom set home. However, you are not the owner of the set. You are merely the hirer. The ownership of the set will be transferred to you only after you have paid off the balance of \$3 500 plus the simple interest on \$3 500 for 2 years at 7% per annum by making 24 equal monthly payments. This method of purchasing goods is called **hire purchase transactions**. Each monthly payment is known as an **instalment**.

In the above, interest $= \frac{\$3\,500 \times 7 \times 2}{100} = \$490$

Total amount to be paid off = \$3 500 + \$490 = \$3 990

$\therefore$ each monthly instalment $= \frac{\$3\,990}{24} = \$166.25$

NB: The deposit of \$1 000 is called the **down payment**. Sometimes, no down payment is required so the whole cost is paid by instalments only.

The simple interest rate 7% is also called the **flat rate**.

Example 28

A washing machine is priced at \$450. It may be bought on the following hire purchase terms: a deposit of 15%, simple interest of $10\frac{2}{3}\%$ per year over 2 years; repayment to be paid monthly.

Find *(a) the monthly instalment;*
(b) the total hire purchase price of the washing machine;
(c) the percentage of money saved if a housewife buys the washing machine by paying \$450 immediately.

Solution

The deposit = 15% of \$450 = $\frac{15}{100} \times \$450 =$ **\$67.50**

The amount remaining = \$450 – \$67.50 = \$382.50

The interest on \$382.50 for 2 years = $\$382.50 \times \frac{32}{3} \times \frac{1}{100} \times 2 = \$81.60$

Additional amount to pay in 24 monthly instalments = \$382.50 + \$81.60 = **\$464.10**

(a) The monthly instalment = \$464.10 ÷ 24 = \$19.34 (correct to the nearest cent.)

(b) The total hire purchase price = the deposit + additional amount
= \$67.50 + \$464.10 = \$531.60

(c) On hire purchase terms, the additional amount the housewife would have to pay = interest on \$382.50 for 2 years = \$81.60 (or \$531.60 – \$450)

Percentage of money she could save on cash terms = $\frac{81.60}{450} \times 100\% = 18\frac{2}{15}\%$

Exercise 12h

1. For each of the following **(i)** find the additional amount you have to pay by hire purchase and **(ii)** express the additional amount obtained in (i) as a percentage of the cash price:

	Cash price	*Hire purchase terms*		
		Down payment	*Monthly instalment of*	*Number of instalments*
(a)	\$360	\$50	\$40	10
(b)	\$900	\$150	\$75	12
(c)	\$25 000	\$10 000	\$500	36

2. Peter buys a window air-conditioner at \$900. He pays 20% deposit and the outstanding balance plus interest in 48 months. Interest on the balance is charged at 10%. Find
(a) the cost of his monthly instalment;
(b) the amount he saves by paying cash.

3. On each of the following **(i)** find the hire purchase price of the goods and **(ii)** express the amount saved by paying cash as a percentage of the cash price:

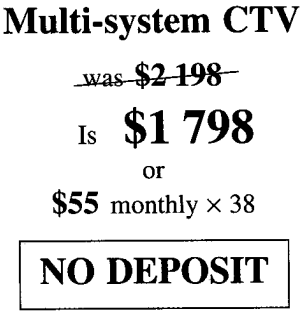
Item	*Cash Price*	*Deposit*	*Number of Instalments*	*Monthly Instalment*
(a) VCD player	\$200	10%	24	\$9
(b) Printer	\$450	15%	18	\$25
(c) 3-seater sofa	\$1 600	25%	30	\$52

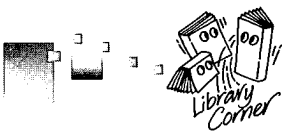
4. For each of the following, find **(i)** the monthly instalment and **(ii)** the difference in the hire purchase price and the cash price as a percentage of the cash price:

	Cash price	*Hire purchase terms*
(a)	\$800	\$100 deposit; balance 8%; 1 year
(b)	\$8 000	\$3 200 deposit; balance 10%; $2\frac{1}{2}$ years
(c)	\$1 200	\$200 deposit; balance 15%; $1\frac{1}{3}$ years

5. The cash price of a computer package deal was \$3 200. Mary paid a 15% down payment and the outstanding balance plus interest over 24 months. Interest on the balance was charged at 9.5%.
   (a) Find the cost of the package deal if it is bought on hire purchase.
   (b) Find the difference between the hire purchase price and the cash price.
   (c) Express the difference obtained in (b) as a percentage of the cash price.

6. Answer the following questions by referring to the advertisement on the right.
   (a) Find the percentage discount for payment in cash compared to the original price, giving your answer correct to one decimal place.
   (b) What is the difference between the hire purchase price and the original price?
   (c) What is the rate of simple interest charged for hire purchase? (Give your answer correct to three significant figures.)





Money Exchange

Every day, major banks will display the exchange rates of the various currencies. These rates change or fluctuate every day and are determined by the demand and supply of the various currencies.

Different countries use different forms of currency and their units of money are called by various names. The United Kingdom uses the sterling pound (£), the United States of America uses the American dollar (US\$), Thailand uses the baht (B)), Malaysia uses the ringgit (M\$), Indonesia uses the rupiah (R), the Philippines uses the peso (P) and Singapore uses the Singapore dollar (S\$).

We can buy or sell foreign currencies at any bank or through a money changer.

The table below shows the exchange rates of the various currencies displayed by a major bank on July 1999.

	Currency	*Buying*	*Selling*
Singapore dollar to one unit of foreign currency	Australian dollar (A$)	1.1140	1.1300
	Canadian dollar (C$)	1.1350	1.1580
	New Zealand dollar (NZ$)	0.8820	0.9010
	Sterling pound (£)	2.6450	2.6730
	US dollar (US$)	1.6930	1.7040
Singapore dollar to 100 units of foreign currency	Chinese renminbi	19.10	19.60
	Deutschemark (DM)	88.10	89.10
	French franc (FR)	26.20	26.60
	Hongkong dollar (HK$)	21.70	22.00
	Indian rupee	3.750	4.150
	Indonesian rupiah (Rp)	0.02400	0.02700
	Japanese yen (¥)	1.3900	1.4200
	Philippines peso (P)	4.400	4.550
	Thai baht (B)	4.530	4.660

Table 12.1

The table shows that the bank will buy a currency at a lower rate than it will sell that currency. For example, we need to pay S$113 to buy A$100 from the bank but we will get only S$111.40 for selling A$100 to the bank. The difference is the profit the bank makes.

Example 29

Convert the following foreign currencies to Singapore dollars using Table 12.1. Give your answers correct to the nearest cent.
(a) US$185 *(b) 5 000 Thai baht*

Solution

(a) The bank will buy US dollars at US$1 = S$1.693.
US$185 = 185 × S$1.693 = S$313.21 (correct to the nearest cent)

(b) The bank will buy Thai baht at 100 Thai baht = S$4.530.
5 000 Thai baht = $\frac{5\,000}{100}$ × S$4.530 = S$226.50.

Example 30

Convert (a) S$250 to sterling pound; (b) S$5 000 to Japanese yen.
Give your answers correct to the nearest unit of the foreign currency.

Solution

(a) The bank will sell sterling pounds at £1 = S$2.673 or S$1 = £$\frac{1}{2.6730}$.
S$250 = 250 × £$\frac{1}{2.6730}$ = £94 (correct to the nearest £)

(b) The bank will sell Japanese yen at ¥100 = S$1.4200 or S$1 = ¥$\frac{100}{1.4200}$.
S$5 000 = 5 000 × ¥$\frac{100}{1.4200}$ = ¥352 113 (correct to the nearest ¥)

Exercise 12i

1. Convert the following foreign currencies to Singapore dollars using Table 12.1. (Buying rate) Give your answers correct to the nearest cent where necessary.
   (a) US$450 **(b)** £3 000
   (c) 8 000 Renminbi **(d)** 2 200 peso
   (e) C$720 **(f)** 875 000 rupiah
   (g) 25 000 Thai baht **(h)** 980 French franc
   (i) ¥2 000 000 **(j)** NZ$640
   (k) A$8 540 **(l)** 4 200 rupee

2. Convert the following amounts in Singapore currency to the stated foreign currency using Table 12.1. (Selling rate) Give your answers correct to the nearest unit of the foreign currency.
   (a) S$7 500 to US$
   (b) S$875 to £
   (c) S$5 250 to NZ$
   (d) S$275 to Thai baht
   (e) S$5 000 to peso
   (f) S$225 to rupiah
   (g) S$420 to ¥
   (h) S$850 to French franc
   (i) S$370 to C$
   (j) S$1 250 to A$
   (k) S$2 200 to HK$
   (l) S$1 500 to Deutschemark

3. A money changer exchanged Thai baht (B) and US dollars (US$) at a rate of 7.70 B = US$1.
   (a) Calculate, in Thai baht, the amount received for US$150.
   (b) Calculate, in US$, the amount received for 1 617 B.

4. The exchange rate between the English pound (£) and the German mark (M) during a particular day was £1 to 3.10 M.
   (a) How many German marks would be equivalent to £320?
   (b) How many pounds would be equivalent to 868 M?

5. Roger put S$8 500 in a 1-year US$ fixed deposit account with a bank at 4% simple interest per annum when the exchange rate was at US$1 = S$1.70.
   (a) How much did he invest in US dollars?
   (b) At the end of one year he withdrew all his money when the exchange rate was at US$1 = S$1.65. Calculate the number of Singapore dollars he made from this investment.

Taxation

The government imposes various forms of taxes which include direct taxes and indirect taxes on its residents to finance public spending on national defence, education, etc. Direct taxes include income taxes, property taxes and profit taxes whereas indirect taxes include duties, motor vehicle taxes, goods and services taxes (GST), value-added taxes, etc.

Property Tax

A **property tax** is charged on the owner of land, houses, flats or buildings at a standard rate of 16%, as of 1998, on the **annual value** of the property.

Example 31

The annual value of a flat is \$1 020. Find the half year tax payable at a rate of 16%.

Solution

Half year tax payable = $\$1\,020 \times 16\% \times \frac{1}{2}$

$= \$1\,020 \times 16 \times \frac{1}{100} \times \frac{1}{2} = \$81.60$

Value-added Tax and GST

In some countries, when you buy an article, you have to pay a certain amount of tax known as the value-added tax in addition to the price of the article. The tax payable is usually given as a certain percentage of the selling price. In Singapore, a goods and services tax (GST) of 3% is imposed on goods bought and services rendered which came into effect on 1 April 1994.

Example 32

An article has a value-added tax of 15% imposed on it. If the marked price of the article is \$64, calculate the total amount a man has to pay if he wants to buy it.

Solution

Value-added tax payable = 15% of \$64 = $\frac{15}{100} \times \$64 = \$9.60$

$\therefore$ the total amount the man has to pay = \$64 + \$9.60 = \$73.60.

Example 33

An ink jet printer is advertised as \$618 inclusive of a 3% GST. What is the original price of the printer?

Solution

Original price : \$618 = 100 : 103

$\therefore$ original price = $\frac{100}{103} \times \$618 = \$600$

Income Tax

Income tax is charged on all incomes derived from Singapore or received in Singapore from sources outside Singapore during the year started 1 January and ended 31 December. The income tax payable is calculated based on the chargeable income.

Chargeable income = Total income – Reliefs

The reliefs include personal relief, wife relief, child relief, life insurance premiums, contributions to the Central Provident Fund (CPF) and gifts to charitable organisations in the form of cash, etc. The amount of income tax payable is calculated according to the tax rates. Below is an extract of the tax rates from Explanatory Notes on how to prepare the returns sent to taxpayers in 1998 by the Inland Revenue Department:

	Chargeable Income ($)	*Rate (%)*	*Gross Tax Payable ($)*
On the first	7 500	2	150.00
On the next	12 500	5	625.00
On the first	20 000		775.00
On the next	15 000	8	1 200.00
On the first	35 000		1 975.00
On the next	15 000	12	1 800.00
On the first	50 000		3 775.00
On the next	25 000	16	4 000.00

Table 12.2

Example 34

Calculate the tax payable for a chargeable income of $8 450 according to the tax rates given in Table 12.2.

Solution

Chargeable income			Tax
On the first	$7 500		$150.00
On the next	$ 950	at 5%	$ 47.50
	$8 450		$197.50

∴ the tax payable = $197.50

Example 35

In 1998, Richard earned a gross annual income of $48 000. Calculate his income tax if he was entitled to the following reliefs: personal, $3 000; wife, $1 500; 3 children, $1 500 each; handicapped brother, $2 500; life insurance premiums and CPF contributions, $5 000; and gifts to charitable organizations, $200.

Solution

Total reliefs = $3 000 + $1 500 + 3 × $1 500 + $2 500 + $5 000 + $200 = $16 700

Chargeable income = $48 000 – $16 700 = $31 300

Chargeable income			Tax
On the first	$20 000		$ 775.00
On the next	$11 300	at 8%	$ 904.00
	$31 300		$1 679.00

∴ Richard's income tax was $1 679.00

Exercise 12j

1. The annual value of a property is $12 300. Find the property tax payable for a period of 8 months at the rate of 16%.
2. Calculate the total amount of money a man has to pay for an article marked at $85 with a 14% value-added tax imposed on it.
3. How much has a customer to pay for an article costing $240 with a 3% GST imposed on it?
4. An article is advertised for $169.95 inclusive of a 3% GST. Find the original price.

(For questions 5 and 6, refer to Table 12.2)

5. Find the income tax of a tax payer if his chargeable income is
   (a) $3 000, **(b)** $6 000,
   (c) $12 000, **(d)** $18 000.
6. For parts (a) and (b), use the following information:

   Reliefs: personal, $3 000; wife, $1 500; children, $1 500 each; handicapped children, $2 500 each; parent, $2 500.

   (a) A married man with three children, one of whom is handicapped, earned $24 400 last year. Given that his wife is not working and that he contributed $3 660 towards CPF as well as paid $720 on his life insurance policy, find the income tax he has to pay.

   ***(b)** Mr Huang earned $18 000 in 1998. He has two children, a wife and a mother to support. He contributed $2 700 towards CPF and $120 to a charitable organisation. He also paid $640 for his life insurance premium. Calculate the amount of income tax Mr Huang has to pay.

Problems Involving the Use of Tables and Charts

A lot of information is often organized and put together in the form of tables and charts. The following in-class activity gives you some practice at using and interpreting tables and charts.

In-Class Activity

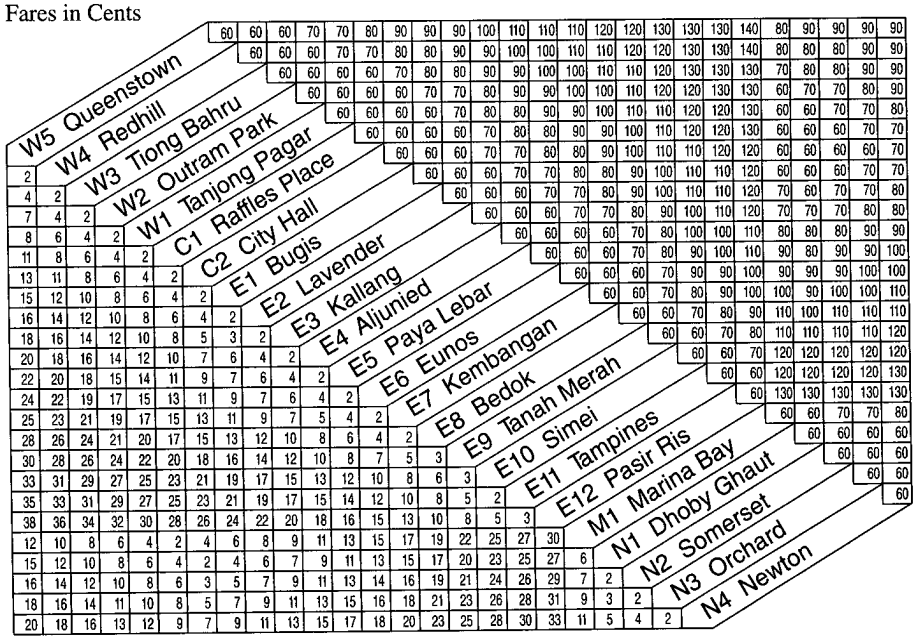
You can carry out these activities on your own.

MRT Travel Times and Fares

Table 12.3 shows travel times in minutes between some MRT stations and station-to-station fares in cents. Travel times include station stop but exclude transfer and waiting times. For example, you have to pay 60 cents to travel from City Hall to Somerset and the travelling time is 3 minutes.

1. Use Table 12.3 to find the fare payable and the travel time from
   (a) Tampines to Raffles Place; **(b)** City Hall to Tiong Bahru;
   (c) Dhoby Ghaut to City Hall; **(d)** Pasir Ris to Redhill.

Fares in Cents



Travel Times in Minutes

Table 12.3 (as of 1999)

2. A man left his home at 08 45 and arrived at Simei station 8 minutes later. Three minutes later, he boarded an MRT train and alighted at Orchard station. After half an hour, he boarded another MRT train with his friends at Orchard station to visit someone in Tiong Bahru. Two and one-quarter hours later, he boarded a train at Tiong Bahru station to return home. Using Table 12.3, calculate
   (a) the man and his friends' arrival time at Tiong Bahru station;
   (b) the man's arrival time at Simei station on his return journey;
   (c) the man's total MRT train fare.

Postal Charges

The table below shows the local mail postage rates for letters, postcards, small packets and printed papers as well as their respective surface mail rates to different places of the world in July 1999:

Service	*Weight Step Not Over*	*Singapore*	*Malaysia and Brunei Darussalam*		*Other foreign countries*
Letters	20 g 50 g 100 g 250 g 500 g	22¢ 30¢ 50¢ 80¢ $1.50	20 g 50 g 100 g every additional 100 g	35¢ 50¢ 80¢ 70¢	40¢ 70¢ $1.00 90¢
Postcards		22¢	30¢		30¢
Small packets & Printed papers	20 g 50 g 100 g 200 g 300 g 400 g 500 g	22¢ 30¢ 40¢ 50¢ 60¢ 70¢ 80¢	20 g 50 g 100 g every additional 100 g	30¢ 40¢ 70¢ 60¢	40¢ 60¢ 90¢ 80¢

Table 12.4

For example, it will cost

(i) 35¢ to send a letter weighing 20 g to Malaysia;
(ii) 90¢ + 80¢ or \$1.70 to send a packet weighing 200 g to the United Kingdom;
(iii) 70¢ + 7 × 60¢ or \$4.90 to send some magazines weighing 790 g to Brunei.

3. Use Table 12.4 to find the cost for mailing the following articles:

(a) 2 postcards to Malaysia, 1 postcard to Brunei and 4 postcards to Hong Kong.
(b) 8 magazines to the same address in Thailand, each weighing 120 g.
(c) a shirt weighing 110 g and a pair of trousers weighing 660 g to Indonesia.
(d) a packet of curry powder weighing 990 g to New Zealand.
(e) a letter with photographs enclosed weighing 220 g to USA.
(f) 6 letters each weighing 49 g: 2 to Tokyo, 3 to Kuala Lumpur and 1 to Manila.
(g) 4 letters to a Singapore address each weighing 18 g and 3 letters to a Singapore address each weighing 150 g.

ABC Photocopying Services

Table 12.5 provides the operators of the photocopying services a quick reference to the charges (in cents) of photocopying work. For example, the charge for 37 copies of the photocopying work is 185 cents or \$1.85.

<table>
<tr><td>1</td><td>10</td><td>11</td><td>77</td><td>21</td><td>105</td><td>31</td><td>155</td><td>41</td><td>205</td><td rowspan="11">51 to 100 copies
$3\frac{1}{2}$ cents each</td><td rowspan="11">101 to 150 copies
3 cents each.

More than 150 copies
3 cents each plus 10% off the first 100 copies.

For more than 100 copies 3% GST is chargeable on the amount payable.</td></tr>
<tr><td>2</td><td>20</td><td>12</td><td>84</td><td>22</td><td>110</td><td>32</td><td>160</td><td>42</td><td>210</td></tr>
<tr><td>3</td><td>30</td><td>13</td><td>91</td><td>23</td><td>115</td><td>33</td><td>165</td><td>43</td><td>215</td></tr>
<tr><td>4</td><td>40</td><td>14</td><td>98</td><td>24</td><td>120</td><td>34</td><td>170</td><td>44</td><td>220</td></tr>
<tr><td>5</td><td>50</td><td>15</td><td>105</td><td>25</td><td>125</td><td>35</td><td>175</td><td>45</td><td>225</td></tr>
<tr><td>6</td><td>60</td><td>16</td><td>112</td><td>26</td><td>130</td><td>36</td><td>180</td><td>46</td><td>230</td></tr>
<tr><td>7</td><td>70</td><td>17</td><td>119</td><td>27</td><td>135</td><td>37</td><td>185</td><td>47</td><td>235</td></tr>
<tr><td>8</td><td>80</td><td>18</td><td>126</td><td>28</td><td>140</td><td>38</td><td>190</td><td>48</td><td>240</td></tr>
<tr><td>9</td><td>90</td><td>19</td><td>133</td><td>29</td><td>145</td><td>39</td><td>195</td><td>49</td><td>245</td></tr>
<tr><td>10</td><td>100</td><td>20</td><td>140</td><td>30</td><td>150</td><td>40</td><td>200</td><td>50</td><td>250</td></tr>
</table>

Table 12.5

4. **(a)** From the above table, what is the charge per copy if the number of copies of the photocopying work is
(i) from 11 to 20 inclusive, **(ii)** from 21 to 50 inclusive?

(b) Use the table to find the cost of the following photocopying work:
(i) 7 copies; **(ii)** 19 copies; **(iii)** 42 copies;
(iv) 68 copies; **(v)** 124 copies; **(vi)** 220 copies.

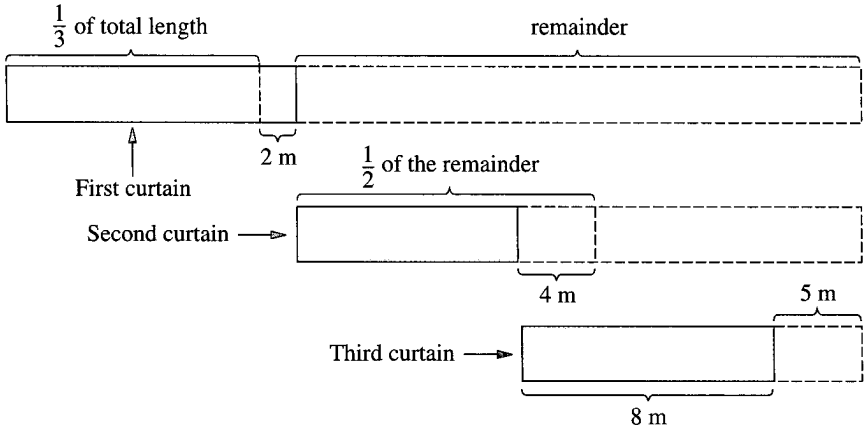
(c) **(i)** Do you notice that the charge for 10 copies is enough to pay for 14 copies when the number of copies of photocopying work exceed 10?
(ii) Similarly, the charge for 50 copies is enough to pay up to how many copies of photocopying work when the number of copies exceed 50?

Example 36

A housewife bought a piece of curtain material. She used $\frac{1}{3}$ of the material and 2 m more to make her first curtain. For her second curtain, she used up $\frac{1}{2}$ of the remaining material less 4 m. She used 8 m for her third curtain and had 5 m of the material left. Find the length of the curtain material she bought.

Solution

Strategy 1: Use a diagram and work backwards



From the diagram (starting from the bottom):

(1) She had 8 + 5 = 13 m of the material remaining after making the second curtain.

(2) $\frac{1}{2}$ of the remaining = 13 − 4 = 9 m

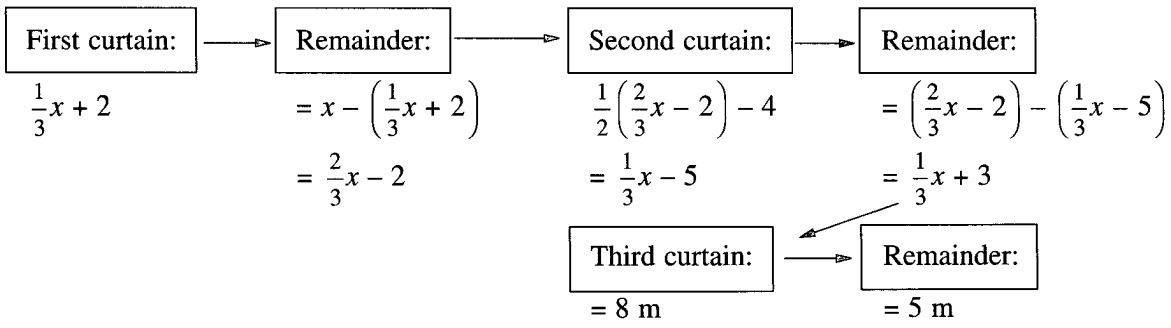
$\therefore$ after making her first curtain, she had $2 \times 9 = 18$ m of the material left.

(3) $\frac{2}{3}$ of the total length of the material = 18 + 2 = 20 m.

$\therefore$ the length of curtain material the housewife bought = $20 \times \frac{3}{2} = 30$ m.

Strategy 2: Use an equation

Let x be the total length of the curtain material.



Hence, the remainder from the second curtain = length of third curtain + remainder of 5 m.

$$\therefore \quad \frac{1}{3}x + 3 = 8 + 5, \quad \frac{1}{3}x = 10 \quad \text{and} \quad x = 30 \text{ m}$$

Example 37

Su Mei, Li Li and Alvin each receives some sweets. Su Mei gives part of her sweets to Li Li and Alvin so that their respective number of sweets is doubled; Li Li gives part of her sweets to Alvin and Su Mei so that their respective number of sweets is doubled and Alvin also gives part of his sweets to Su Mei and Li Li so that their respective number of sweets is doubled. If all three of them eventually end up with 240 sweets, how many sweets did each of them receive originally?

Solution

Sometimes, a combination of several heuristics is used to solve a problem. Example 37 makes use of three heuristics.

Strategy: Use tabulation, before-and-after comparison and work backwards

	Su Mei (S)	*Li Li* (L)	*Alvin* (A)
Original number of sweets	390	210	120
S → L, A	60	420	240
L → A, S	120	120	480
A → S, L	240	240	240

Note: A → S, L indicates that Alvin gives part of his sweets to Su Mei and Li Li.

We can fill the table working from the bottom of the table.

(1) After Alvin has given part of his sweets to Su Mei and Li Li, each of them has 240 sweets (last line of the table).

(2) Before receiving sweets from Alvin,

Su Mei and Li Li each has $\frac{1}{2} \times 240 = 120$ sweets

Before giving sweets to Su Mei and Li Li,

Alvin has 240 + 120 + 120 = 480 sweets (second line from the bottom).

(3) Before receiving sweets from Li Li,

Su Mei has $\frac{1}{2} \times 120 = 60$ sweets and Alvin has $\frac{1}{2} \times 480 = 240$ sweets.

Before giving sweets to Su Mei and Alvin,

Li Li has $120 + 60 + 240 = 420$ sweets (third line from the bottom).

(4) Before receiving sweets from Su Mei,

Li Li has $\frac{1}{2} \times 420 = 210$ sweets and Alvin has $\frac{1}{2} \times 240 = 120$ sweets

Before giving sweets to Li Li and Alvin,

Su Mei has $60 + 210 + 120 = 390$ sweets.

Thus originally, Su Mei's number of sweets = 390, Li Li's number of sweets = 210, and Alvin's number of sweets = 120.

Exercise 12k

1. Wei Lin was trying a number trick on Edmond. She told him to choose a number, multiply it by 4, subtract 7 from the product, then add 11 and double the result. Edmond's final answer was 64. What number did he start with?

2. Mrs Yong bought some apples. She used 1 more than half of them for a pudding. She then used 1 more than half of the remainder for a pie. She gave 1 more than half of those that were left to her children and had 1 remaining. If she paid \$4.40 for the apples, find the cost of 1 apple.

3. Forty eight marbles are divided into 3 groups. Then, some marbles are removed from the first group and put in the second group so that the number of marbles in the second group is doubled. Some marbles are removed from the second group and put in the third group so that the number of marbles in the third group is doubled and some marbles are removed from the third group and put in the first group so that the number of marbles in the first group is doubled. As a result, there are equal number of marbles in each group. Find the number of marbles in each group originally.

4. On a bus, 99 40-cent tickets and 80-cent tickets costing a total of \$56 were issued. Find the difference between the number of 40-cent tickets and the number of 80-cent tickets issued.

5. In an examination consisting of 15 questions, 8 marks are awarded for a correct answer and 4 marks are deducted for a wrong answer. How many correct answers must one get to score 72 marks in this examination?

6. A durian seller sells half of his durians plus half a durian to his first customer. He then sells half of the remainder plus half a durian to his second customer, and half of the remainder plus half a durian to his third customer. He repeats the same process for the next customer and so on. After he has served the seventh customer, he finds that he has sold all his durians. How many durians had he originally?

Summary

1. A percentage is a fraction whose denominator is 100 and we use % to represent percent. A percentage can be converted to a fraction by dividing it by 100.

2. Profit = Selling price – Cost price, Loss = Cost price – Selling price

3. Discount = Original Selling price – Sale price

4. If the simple interest on $\$P$ for T years at $R\%$ per annum is $\$I$, then $I = \frac{PRT}{100}$.

5. Chargeable income = Total income – Reliefs

Review Questions 12

1. A school is given a 15% discount for buying textbooks in bulk. How much does the school have to pay for 110 books costing \$10 each before the discount?

2. The profit made on a certain camera is 30% of the cost price. If the profit is \$270, find
   (a) the cost price and
   (b) the selling price of the camera.

3. Yiwei receives \$28 per week in pocket money. If she decides to save 20% of it, find how much she will
   (a) save and
   (b) spend in a year.

4. Calculate the simple interest on \$5 640 invested for 7 months at 6% per annum.

5. Arrange 1.74, $1\frac{2}{3}$, 1.56, 173% and $1\frac{3}{4}$ in descending order.

6. Given that $0.7 : \frac{7}{8} = 7\% : x$, find x.

7. Given that a is 30% of b, find the value of $\frac{a}{4b}$, expressing your answer as a fraction in its lowest terms.

8. A bank offers two schemes of investment. Scheme A pays tax-free interest of 4%. Scheme B pays interest of 6% on which a tax of 20% has to be paid. A man has \$5 000 to invest. Calculate his earnings under the two different schemes for 1 year.

9. A bank exchanges British currency for Singapore currency at the rate of S\$2.30 to £1.
   (a) Calculate the amount of Singapore dollars that can be exchanged for £120.
   (b) Calculate, in £, the amount exchanged for S\$1 600 by a customer who also had to pay an extra 3% commission for this transaction.

10. A fruit seller has 120 oranges. Given that he has 20% more apples than oranges and 40% less oranges than pears, find the number of apples and the number of pears the fruit seller has.

11. A shopkeeper sold two articles for \$48 each. He made a 25% profit on one article and a loss of 20% on the other. What was his net gain or loss on the sale of the two articles?

12. Mr Chen and Miss Wang decided to buy a new car costing \$60 000.

(a) Mr Chen paid for his new car in cash and was given a discount. Given that he paid \$57 000 for his new car, calculate the percentage discount he received.

(b) Miss Wang agreed to pay 60% of the price of the car as a deposit and the balance at $3\frac{1}{2}\%$ simple interest per annum over a period of 3 years. Calculate the amount of each monthly instalment.

13. (a) After a company had paid $42\frac{1}{2}\%$ of the profit it had made as tax, \$41 400 000 remained. Calculate the amount of the profit.

(b) Given that 26.9% of the profit remained after tax was set aside for further investment, calculate this amount set aside, correct to the nearest \$100 000.

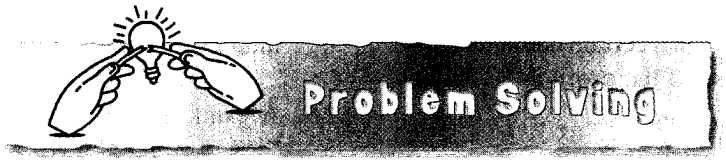
(c) If \$26 496 000 was paid to shareholders, find what percentage this was of the \$41 400 000 available.

***14.** (a) A retailer bought a compact disc from a manufacturer for \$20. In addition to that, he paid a 15% value-added tax. If he sold the disc to a customer for \$26, calculate the cash profit he made.

(b) The manufacturer later increased the price of the compact disc by 20%. At the same time, the value-added tax was increased to 25%.

(i) If the retailer made the same cash profit as before, calculate the price a customer had to pay for a disc.

(ii) Find the cash profit the retailer made on each disc if he sold each one at a price which was 30% more than the cost price.



1. John accepted a reduction of 15% in his salary when his company was not doing well. Now his company's financial position has improved and his boss wants to restore his original salary. By what percentage must this reduced salary be increased?

2. The radius of a cylinder is increased by 15% and its height is decreased by 20%. Find the percentage change in the volume of the cylinder.

3. A man bought some articles at a discount of 25% of the list price. He set the marked price of each article such that after giving a discount of 20% of the marked price he still made a profit of $33\frac{1}{3}\%$ of the selling price. What percentage of the list price was his marked price?

4. Mr Sim instructed his assistant to place an order for 5 pairs of leather shoes and a number of pairs of canvas shoes. A pair of leather shoes costs two and a half times as much as a pair of canvas shoes. His assistant made a mistake in the order and the number of pairs of the two types of shoes had been interchanged. This increased the bill by $33\frac{1}{3}\%$. Find the ratio of the number of pairs of canvas shoes to the number of pairs of leather shoes in the order that Mr Sim instructed his assistant to place.

5. Kelvin read 60 pages of a book on the first day. This was 20% more than the number of pages he read on the second day. Given that he read $\frac{1}{6}$ of the book on the second day, find the number of pages in the book he had read for both days.

Revision Exercise III No. 1

1. A shopkeeper bought a radio from a wholesaler for \$25. In addition, he paid a value-added tax of 15% on the cost price. He then sold the radio for \$31.50. Calculate the cash profit made by the shopkeeper.

2. A man walks at a rate of 1.25 m/s. Find the time he takes to walk 3.75 km.

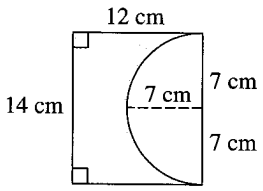
3. Find the simple interest you will obtain if you deposit \$600 in a bank for 9 months at $4\frac{1}{2}\%$ per annum.

4. Find the ratio of the weight of a Japanese car weighing 4 200 kg to that of a German car weighing 7 200 kg.

5. What is the difference in average wages between employing 18 men at a wage of \$380 each and 33 women at a wage of \$208 each?

6. A sum of money is distributed among 3 boys, A, B and C, in the ratio 2 : 4 : 14. If B gets \$1.20 more than A, how much money does C get?

7. The figure shows a semicircle of radius 7 cm being enclosed in a rectangle of sides 14 cm by 12 cm. Calculate the area of the shaded region. $\left(\text{Take } \pi = \frac{22}{7}\right)$



8. It was projected that Singapore will need 1.2 million cubic metres of water daily at the turn of the 20th century. The Singapore Power (SP) planned to invest \$900 million to build a desalination plant capable of producing 30 million gallons of water per day. If 1 gallon is equivalent to 4.5 litres,
   (a) how many m^3 of water can the desalination plant produce per day?
   (b) how many desalination plants, correct to the nearest whole number, must be built if there are no other sources of water supply?

9. It took two and half years and 2.85 million m^3 of earth to fill the disused Sin Seng quarry in Rifle Range Road. If each truck can carry 6.25 m^3 of earth per trip, how many trips are needed to fill the quarry? If the cost of transport and material for each truck load is \$45, how much would it cost for the transport and material?

10. The height of water inside a rectangular cuboid of length 8 cm and width 6 cm is $2\frac{1}{2}$ cm. This water is poured into an empty cylindrical tank of diameter 14 cm. Find the height of water in the cylinder.
    $\left(\text{Take } \pi = \frac{22}{7}\right)$

Revision Exercise III No. 2

1. A man earned an annual income of \$24 500 in 1998. He was allowed a deduction of \$1 500 relief for each of his three children and a personal relief of \$3 000. If he was charged a tax rate of 4% on the first \$5 000 and 6% on his remaining income, calculate the amount of tax he had to pay.

2. A bookseller bought 4 dozen books at \$15.50 each. At what price was each book sold if his total profit was \$240?

3. How many books costing \$8.40 each can be bought with \$200? How much money will then be left?

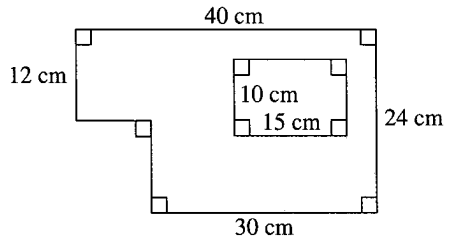
4. A man bought a car for \$33 000. He made a first payment of \$12 000 and borrowed the rest from a bank at 10% per annum simple interest. At the end of the first year, he repaid a certain sum to the bank after which he still owed the bank \$9 000. Calculate the sum he repaid.

5. A car uses $27\frac{1}{2}$ litres of petrol for a journey of 220 km. How much petrol will it need to cover a distance of 680 km?

6. The volume of a rectangular box is 96 cm^3 and its height is 4 cm. What is the area of its base? If the length of the base is 8 cm, find the width.

7. Find the total surface area and volume of a solid cylindrical block of diameter 28 cm and height 12 cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

8. Find the area of the shaded part in the figure below. If the figure represents the cross-section of a solid whose height is 7.5 cm, find the volume.



9. A pipe discharges 48 litres of water per minute into a rectangular tank of length 3.2 m and width 2.5 m. After 30 minutes, find the height of the water in the tank.

10. Carol deposits \$800 in a bank that offers interest of 3% per annum. If the money and its interest are not withdrawn but allowed to compound annually, how much will she have at the end of 3 years? (Give your answer correct to the nearest cent.)

Revision Exercise III No. 3

1. **(a)** A shopkeeper sells an article for \$250, thus making a profit of 25%. Find the cost price of the article.
   (b) A new car cost \$64 000. After one year its value depreciated by 20%. Find its value at the end of the first year.
   (c) A man saves 20% of his income. What is his income if he spends \$1 360 a month?

2. Two men, A and B, can paint a house in 8 days. A alone can paint it in 12 days. How long will B take to paint the house by himself?

3. Calculate the simple interest on \$250 invested for 4 years at 8% per annum.

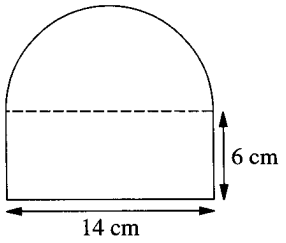
4. 784 marbles are shared among x, y and z in the ratio 3 : 5 : 8. How many marbles does each get?

5. Convert £53.35 to Malaysian ringgit if the exchange rate is M\$5.95 to £1, giving your answer correct to the nearest ten cents.

6. Find the value of the following:
   (a) 75% of \$12;
   (b) 3% of 40 kg;
   (c) $8\frac{1}{2}$% of 200 m^3;
   (d) 45% of 500 km.

7. **(a)** A rectangular tank $1\frac{1}{2}$ m long and 88 cm wide contains water to a depth of 65 cm. The water is transferred to an empty rectangular tank 2 m long and 1 m wide. Find the depth of the water in centimetres.
   (b) Find the total surface area and volume of a solid cylinder of diameter 14.6 cm and height 16.8 cm. If the density of the solid is 3.8 g/cm^3, find its weight.
   $\left(\text{Take } \pi = \frac{22}{7}\right)$

8. The figure shows a rectangle with a semi-circle on one of its sides. Calculate the perimeter and the area of the figure.
   $\left(\text{Take } \pi = \frac{22}{7}\right)$



9. Find the volume of a solid of uniform cross-section with an area of 6.5 cm^2 and height, 24 cm. (Take $\pi = 3.142$)

10. **(a)** The perimeter of a rectangle is 24 cm and its length is twice its breadth. Find the area of the rectangle.

(b) A swimming pool is 50 m long and 20 m wide and its depth varies uniformly from 1 m to 3.5 m. Find the volume of the pool in cubic metres.

Revision Exercise III No. 4

1. A man buys 5 kg of beef at \$12.50 per kg. In addition, for every kilogram of beef purchased, he has to pay a consumption tax of 6% on the selling price. Calculate the total amount of money that he has to pay.

2. The cash price of a television set is \$940. However, it will cost more if it is bought by hire-purchase where a down payment of \$160 and 24 equal monthly payments of \$40 each have to be made. Find

(a) the extra cost involved if a man decides to buy it by hire-purchase;

(b) the difference between the hire-purchase price and the cash price as a percentage of the cash price.

3. There are 31 sweets to be shared among 3 boys, *A*, *B* and *C*. *A* has 2 sweets more than *B* and *B* receives 1 more than *C*. How many sweets does each receive?

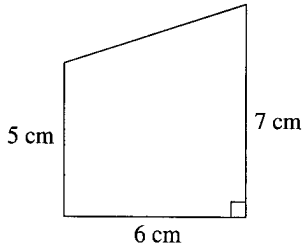
4. The driver of a car 3 m long took 5 seconds to drive through a tunnel 44 m long. Find the average speed of the car in km per hour.

5. A man bought 400 dozen pencils at \$0.80 a dozen. He sold half of them at \$1.05 a dozen and the rest at \$0.75 a dozen. Find his profit.

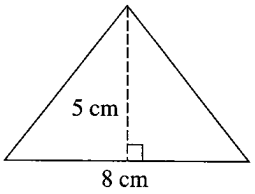
6. Mr Rajoo puts \$4 500 in an insurance trust that offers 5.5% interest. If he does not make any withdrawals and the money and interest in the trust is allowed to compound annually, how much will he have at the end of 3 years? Give your answer correct to the nearest cent.

7. Find the area of the figures below.

(a)

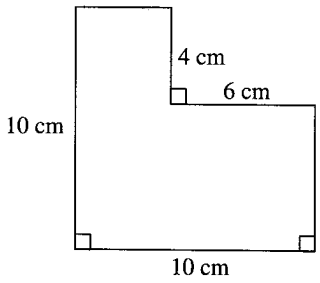


(b)



8. The height of water in a cylinder of radius 12 cm is 28 cm. This water is transferred into a rectangular cuboid of length 24 cm, width 18 cm and height 40 cm. Find the height of water level in the rectangular cuboid. $\left(\text{Take } \pi = \frac{22}{7}\right)$

9. Find the area of the cross-section of the metal plate shown in the figure. If its thickness is 1.5 cm, find the volume.



10. A rectangular tank measured internally is 2.5 m long, 1.5 m wide and 0.8 m high. How many litres of water are needed to fill the tank? The water in the tank is transferred into small cylindrical tanks each of diameter 70 cm and height 40 cm. Find the number of complete cylindrical tanks that can be filled. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Revision Exercise III No. 5

1. **(a)** Given that $p = \frac{1}{4}$ and $q = \frac{2}{7}$, express $\frac{p}{q}$ as a percentage.

(b) A school has 750 pupils and 32% of them wear spectacles. How many pupils do not wear spectacles?

(c) If a man sells an article for \$15, he would make a loss of 25%. How much must he sell it in order to make a profit of 5%?

2. **(a)** The average speed of a car is 108 km/h. How many metres can the car travel in 3 minutes?

(b) At a book fair, a book was reduced in price from \$7.50 to \$6.00. If the first price gives a 50% profit, find the percentage profit of the book sold at the reduced price.

3. A shopkeeper buys a flower vase for \$60 and prices it at \$85. Find the cash price a customer has to pay if the shopkeeper gives him a 5% discount when he pays cash. Find also the percentage profit made by the shopkeeper.

4. A, B and C share a sum of money in the ratio 3 : 4 : 9. If B has \$2.80 more than A, how much is C's share?

5. **(a)** If I sold my car for \$8 400, I would lose 25%. How much did I pay for my car?

(b) A man bought a house for \$327 500 and sold it at a gain of 12%. Find the gain.

6. The volume of a rectangular block is 720 cm^3. Find its height if the area of the base is 60 cm^2.

7. Find the total surface area of a cube of side 6 cm.

8. If the area of a parallelogram is 32.8 cm^2 and the height between the two parallel sides is 16.4 cm, calculate the length of the base.

9. Mrs Li has 2 m of cake frill. She uses it for trimming a square cake of side 20 cm and the circumference of a round cake of diameter 28 cm. How much of the cake trimming is left over? $\left(\text{Take } \pi = \frac{22}{7}\right)$

10. 2.8 litres of paint of density 1.8 g/cm^3 is mixed with 2.2 litres of turpentine of density 0.9 g/cm^3. Find the mass and density of the mixture.

CHAPTER 13

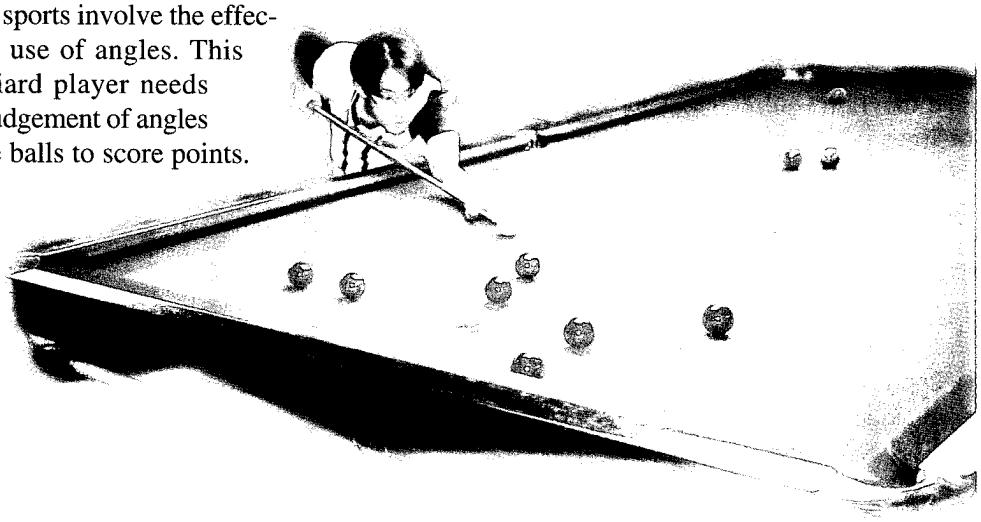
Basic Geometrical Concepts and Properties

In this chapter, you will learn how to

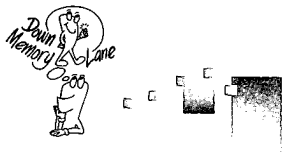
- identify various plane polygons and some simple solid figures;
- calculate unknown angles involving adjacent angles on a straight line, vertically opposite angles, angles at a point, alternate angles, corresponding angles and interior angles between parallel lines;
- draw parallel and perpendicular lines;
- construct angle bisectors and perpendicular bisectors.

Preliminary Problem

Many sports involve the effective use of angles. This billiard player needs to have a good judgement of angles when hitting the balls to score points.



Points



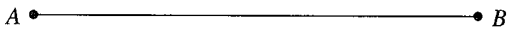
The word "geometry" is derived from the Greek words "ge" (earth) and "metrein" (to measure). Euclid's masterpiece, "The Elements", survived as the basic textbook for over 2 000 years. The geometry that we are to study in this book is sometimes referred to as Euclidean geometry.

The basic geometric figure is a **point**. All other geometric figures are made up of a collection of points. The smallest dot you can mark on your paper with a sharp pencil will give you an idea of what is meant by a geometric point. A point is only an idea in our mind; it is not a physical object and we regard it as having a position but not size or shape. We use a dot or sometimes a cross to mark the position of a point. We normally use capital letters to name points. Thus we speak of point A, point B, point C, etc.

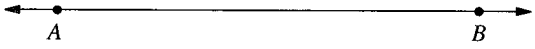
A B C

Lines

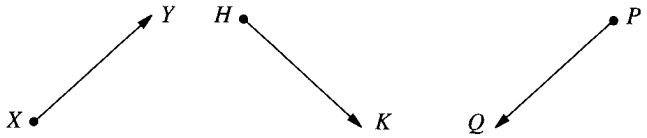
A **line** is the path described by a moving point. A straight **line segment** is formed when we use a ruler to join two points, say, A and B.



We call the line segment AB or BA. A and B are called the end-points. If we extend the line segment AB in each of the two directions indefinitely, we get a **line**. This is represented by the following diagram.

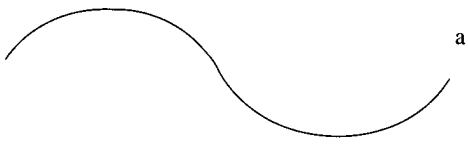


The diagrams on the right show parts of lines with only one end-point and extending in only one direction. We call them **rays**.



Thus we have ray XY, ray HK and ray PQ.

A line is either straight or curved. The diagram shows a curved line. A curved line is also called a **curve**.



a curved line

In this book, when we speak of a line, it refers to a straight line whereas a curve refers to a curved line. Also, we shall simply use AB to denote **line AB, line segment AB, ray AB** and the **length of AB**.

Planes

A **plane** is a flat surface in which any two points are joined by a straight line lying entirely on the surface. The floor of a classroom is an example of a **horizontal plane** and the wall of a classroom is an example of a **vertical plane**.

Solids

A **solid** is a three-dimensional shape or object. The box shown in Fig. 13.1 has six flat surfaces of equal size and each surface is part of a plane. We call this solid figure a **cube**. The box shown in Fig. 13.2 has six flat surfaces of three different sizes. We call this solid figure a **cuboid**.

This is a cube where $AB = BC = CG = 10$ cm.

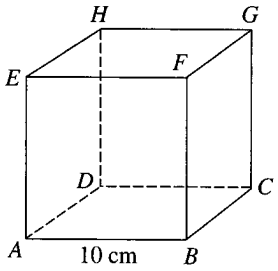


Fig. 13.1

This is a cuboid where $AB = 12$ cm, $BC = 6$ cm and $CG = 8$ cm.

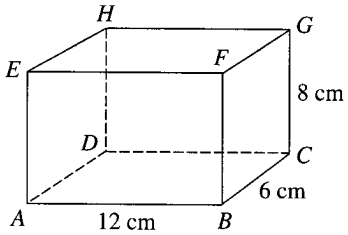


Fig. 13.2

Fig. 13.3 shows a **pyramid** which has five flat surfaces. Each surface is part of a plane. Fig. 13.4 shows a prism with five flat surfaces. Each surface forms part of a plane. Fig. 13.5 is an example of a prism with seven flat surfaces.

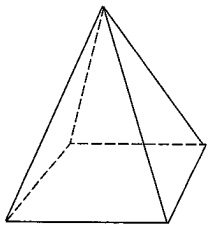


Fig. 13.3

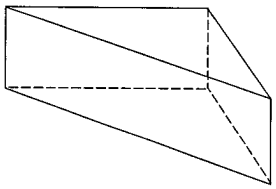


Fig. 13.4

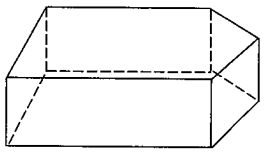


Fig. 13.5

Can you show a net of each of the above solids?

Curved Surfaces

A surface which is not flat does not form part of a plane. Such a surface is called a **curved surface**. For example, the surface of a basketball is a curved surface. The basketball is an example of a **sphere** (Fig. 13.6).

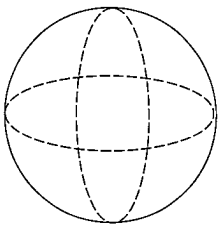


Fig. 13.6

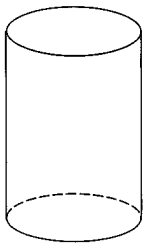


Fig. 13.7

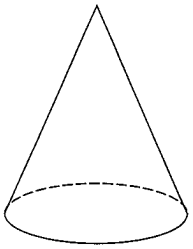


Fig. 13.8

Fig. 13.7 shows a solid figure which has two flat surfaces and a curved surface. It is called a cylindrical solid figure, or a **cylinder**.

A **cone** has only one flat surface and a curved surface (Fig. 13.8).

Intersecting Lines

Fig. 13.9 shows two lines, AB and PQ, on the same plane having a common point X. We say that the two lines **intersect** at X. Point X is called the **point of intersection**.

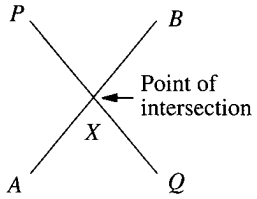
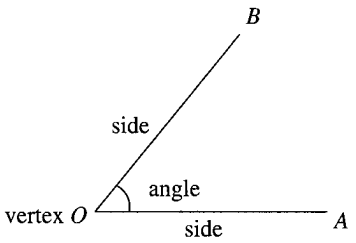


Fig. 13.9

Angles

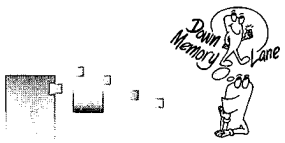
When two rays OA and OB meet at a point O, an **angle** is formed. O is known as the vertex of the angle and OA and OB are the sides or arms of the angles.

The angle is called angle AOB or angle BOA and is written as $A\hat{O}B$ or $B\hat{O}A$. Another way of writing this angle is $\angle AOB$ or $\angle BOA$. We may also call it angle O and write $\hat{O}$ or $\angle O$ when it is clear which angle we are referring to.



The Protractor and Angle Measure

The standard unit for measuring angles is one degree (written as 1°). It is defined as $\frac{1}{360}$ of a complete revolution. By definition, one complete rotation about a point has an angle of 360°.



The system of naming angles was first used by the Babylonians (3 000–2 000 BC). The idea of 360 parts was suggested to be a consequence of the Babylonians estimation of a year having 360 days.

How many different angles are there in the figure?

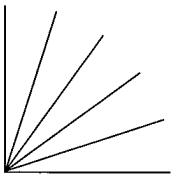


Fig. 13.10 shows a protractor which is used to measure angles. To measure an angle, place the protractor so that its centre A is at the vertex of the angle and its base AB along one side of the angle. Note under which graduation mark the other side passes. Thus, in Fig. 13.11(a), the angle x is 60°. Fig. 13.11(b) shows another way of measuring the angle.

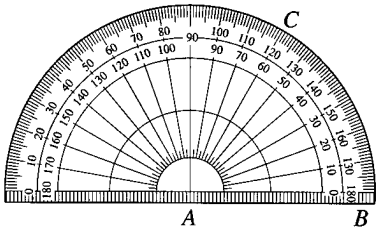


Fig. 13.10

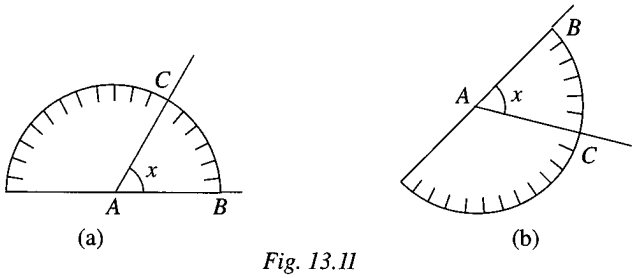
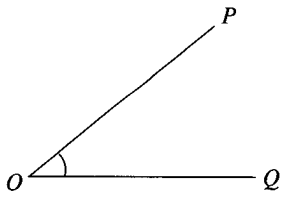


Fig. 13.11

Notice that the graduation marks on the protractor are marked with two sets of numbers, one greater than 90° and the other less than 90°. Hence, when using the protractor, use your common sense to choose the correct set of numbers. For example, if one arm of the angle to be measured lies along AB, the set of numbers to be used is the one in which the numbers increase as you read the graduations from AB towards AC.

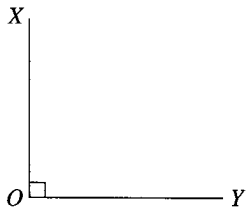
Different Kinds of Angles

An **acute angle** is less than 90°.



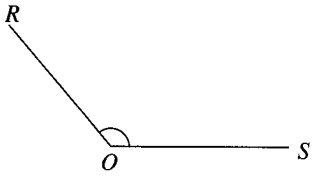
$P\hat{O}Q$ is an acute angle.

A **right angle** is equal to 90°.



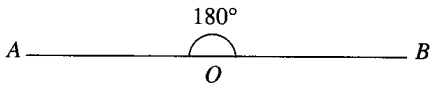
$X\hat{O}Y$ is a right angle.

An **obtuse angle** is larger than 90° but less than 180°.



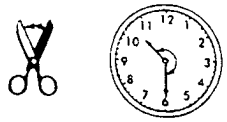
$R\hat{O}S$ is an obtuse angle.

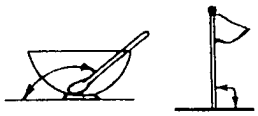
A **straight angle** is equal to 180°.



$A\hat{O}B$ is a straight angle.

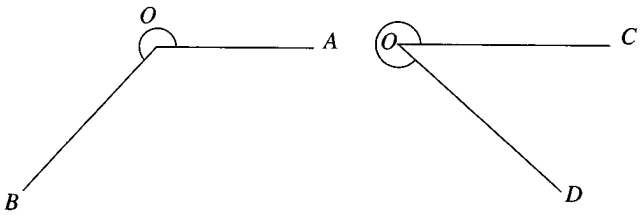
Can you name the angles formed by each of the following objects found in everyday life?



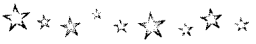


Look around your surroundings to see other examples of angles formed by everyday objects and instruments.

A **reflex angle** is larger than 180° but less than 360°. Both $A\hat{O}B$ and $C\hat{O}D$ are reflex angles.

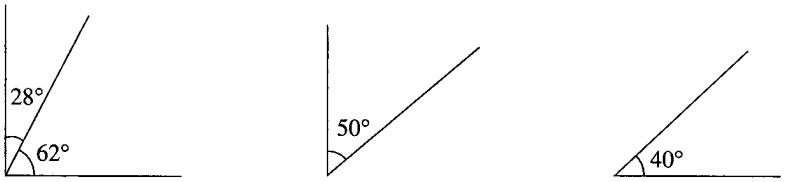


An old man uses a magnifying glass which can enlarge things exactly three times. How many degrees will the angle of 2° appear to him?



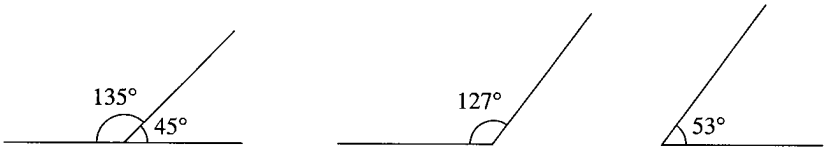
Complementary Angles

Two angles are called **complementary angles** if their **sum** is **90°**. The angles 28° and 62° are said to be complementary and so are the angles 40° and 50°, 30° and 60°, etc.



Supplementary Angles

Two angles are called **supplementary angles** if their **sum** is **180°**. The angles 45° and 135° are supplementary angles and so are 53° and 127°, 105° and 75°, etc.



Adjacent Angles on a Line

In Fig. 13.12, $X\hat{O}Z$ and $Y\hat{O}Z$ are called **adjacent angles** because

(a) they have a common vertex O,
(b) they have a common side OZ and
(c) they lie on opposite sides of the common arm.

The sum of adjacent angles on a line is equal to 180°.
(Abbreviation for reference: **adj. ∠s on a line.**)

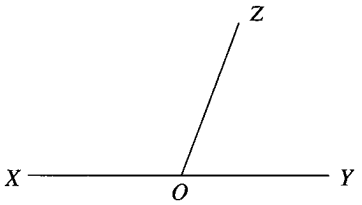


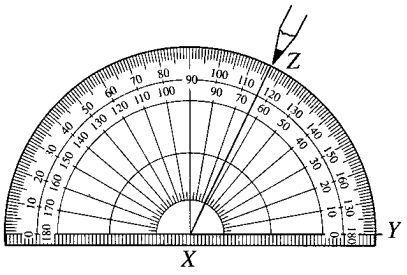
Fig. 13.12

Using the same method construct angles of sizes 35°, 83°, 110° and 165°.

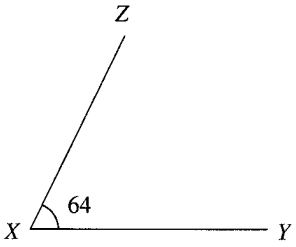
To Construct an Angle Using a Protractor

Suppose you have a line XY and wish to construct angle YXZ of 64°. Place a protractor such that its centre is at X and its base is along XY as shown in Fig. 13.13(a). Make the 64° graduation with your sharp pencil and label it Z. Join Z to X and we have $X\hat{Y}Z = 64°$ (Fig. 13.13(b)).

Method: Use a protractor



(a)



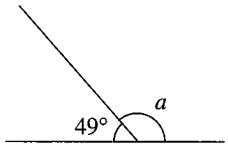
(b)

Fig. 13.13

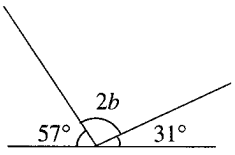
Example 7

Find the values of a, b and c in the following diagrams:

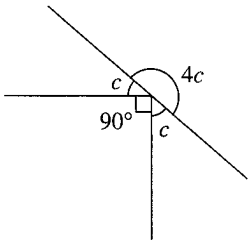
(a)



(b)



(c)



Solution

(a) $49° + a = 180°$ (adj. ∠s on a line)

$\therefore \quad a = 180° - 49° = 131°$

(b) $57° + 2b + 31° = 180°$ (adj. ∠s on a line)

$2b = 180° - 57° - 31° = 92°$

$\therefore \quad b = 46°$

(c) $90° + c + c + 4c = 360°$ (angles at a point)

$6c = 270°$

$\therefore \quad c = 45°$

Vertically Opposite Angles

Vertically opposite angles are formed when two straight lines intersect each other. The angles BOC and AOD in Fig. 13.14 are said to be vertically opposite. The angles AOC and BOD are also vertically opposite each other. We shall prove that vertically opposite angles are equal.

With the notation in Fig. 13.14,

$a + b = 180°$ (adj. $\angle$s on a line)
$b + c = 180°$ (adj. $\angle$s on a line)
$a + b = b + c$
$a = c$

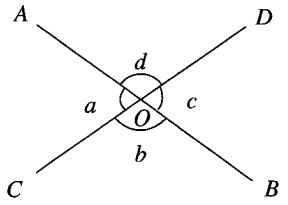
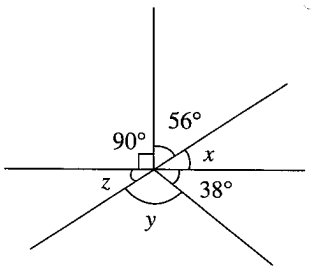


Fig. 13.14

Similarly, we can also show that $b = d$. Hence, **vertically opposite angles are equal**. (Abbreviation for reference: **vert. opp. $\angle$s**.)

Example 2

Find the values of x, y and z in the given diagram.



Solution

$90° + 56° + x = 180°$ (adj. $\angle$s on a line)
$\therefore \quad x = 180° - 90° - 56° = 34°$
$z = x = 34°$ (vert. opp. $\angle$s)
$\therefore \quad z = 34°$

$z + y + 38° = 180°$ (adj. $\angle$s on a line)
$34° + y + 38° = 180°$
$\therefore \quad y = 180° - 38° - 34°$
$= 108°$

Exercise 13a

1. Give two examples of each of the following simple solids:

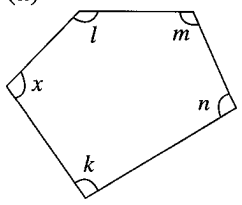
(a) cube **(b)** cuboid **(c)** prism
(d) cylinder **(e)** pyramid **(f)** cone
(g) sphere

2. What shape is each of the following solids? Make a sketch of each:

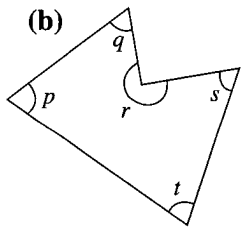
(a) a tennis ball **(b)** a can of milk
(c) a compact disc **(d)** a tent
(e) a heap of sand
(f) the end of a sharpened pencil

3. Measure the angles marked in the following diagrams to the nearest degree:

(a)

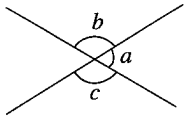


(b)

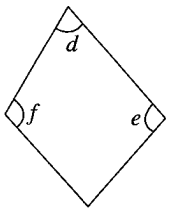


4. Name and measure each of the marked angles:

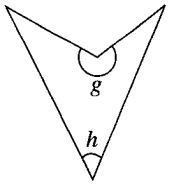
(a)



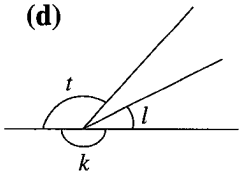
(b)



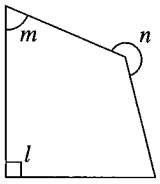
(c)



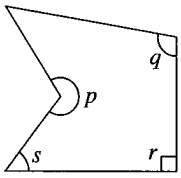
(d)



(e)



(f)



5. Find the measure of the complementary angle of each of the following angles:

(a) 18° **(b)** 46° **(c)** 53°
(d) 64° **(e)** 7°

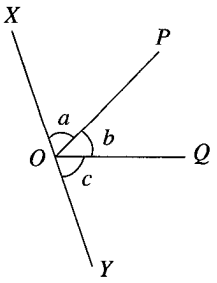
6. Find the measure of the supplementary angle of each of the following angles:

(a) 36° **(b)** 12° **(c)** 102°
(d) 171° **(e)** 88°

7. Use a protractor to draw the following angles:

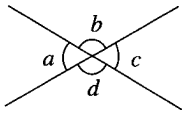
(a) 20° **(b)** 157° **(c)** 197°
(d) 242° **(e)** 320° **(f)** 285°

8. Refer to the figure on the right in which XOY is a straight line, and answer the following questions:



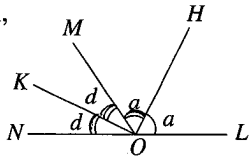
(a) Find a, given that $b = 45°$ and $c = 86°$.
(b) Find a, given that $b = 2a$ and $c = 3a$.
(c) Find b, given that $a + c = b$.
(d) Find c, given that $a = b = c$.

9. Using the figure on the right, find y in each of the following cases:



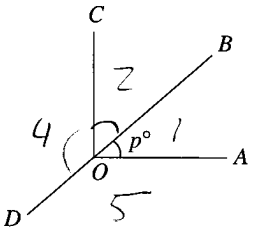
(a) If $a = 2y°$ and $c = (y + 30)°$.
(b) If $a = (3y + 40)°$ and $c = (y + 60°)$.
(c) If $a = (10y - 20)°$ and $c = (6y + 16)°$.
(d) If $a = y°$ and $b = 2y°$.
(e) If $b = (y + 30)°$ and $d = (4y + 16)°$.

10. In the diagram given,



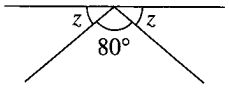
(a) write an equation involving a and d;
(b) find the value of $H\hat{O}K$;
(c) if $d = 25°$, find a.

11. In the diagram below, $A\hat{O}B = p°$. If $B\hat{O}C$ is two times $A\hat{O}B$, $C\hat{O}D$ is four times $A\hat{O}B$ and $D\hat{O}A$ is five times $A\hat{O}B$, find the values of all the four angles.

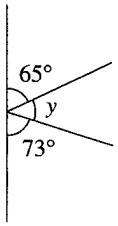


12. Calculate the unknown(s) in each of the following:

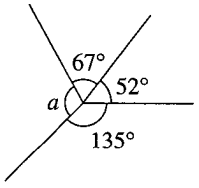
(a)



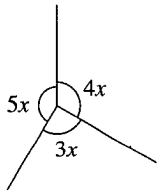
(b)



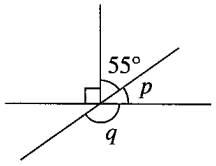
(c)



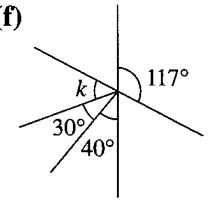
(d)



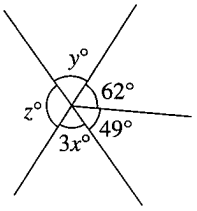
(e)



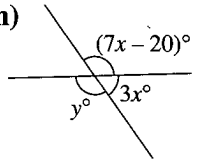
(f)



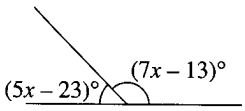
(g)



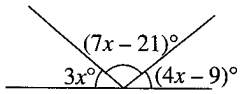
(h)



(i)



(j)



Parallel Lines

Parallel lines are lines which extend in the same direction and remain the same distance apart. We can take parallel lines as two points moving in the same direction. In geometry, a pair of parallel lines is represented by either a pair of single or double arrows (Fig. 13.15).

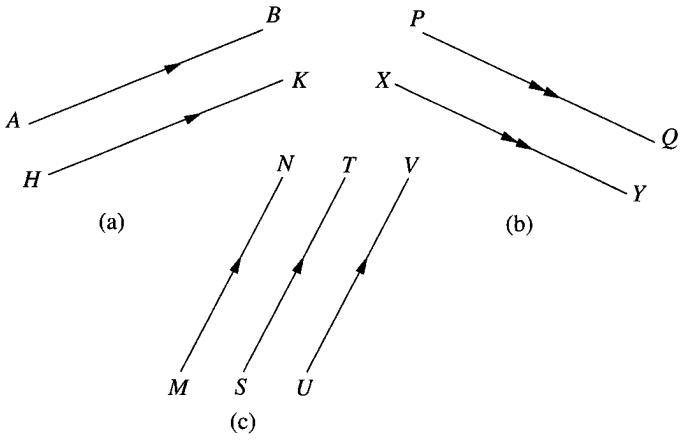


Fig. 13.15

The ruled lines in your exercise books are examples of parallel lines and so are railway lines. Can you identify other examples of parallel lines around you?

We use the symbol "//" to represent "is parallel to". Thus, in Fig. 13.15(a), *AB//HK* means that *AB* is parallel to *HK* and in Fig. 13.15(b), *XY//PQ*. In Fig. 13.15(c), *MN//ST* and *ST//UV*. This also implies that *MN//UV*.

In Fig. 13.16, the line *PQ* is called a **transversal**. The angles k_1 and k_2 are called **corresponding angles (corr. ∠s)**. Similarly, l_1 and l_2 are also called corresponding angles. Use your protractor to measure the angles k_1, k_2, l_1 and l_2. What do you notice about the size of angles k_1 and k_2, and l_1 and l_2? Can you name two other pairs of corresponding angles in Fig. 13.16?

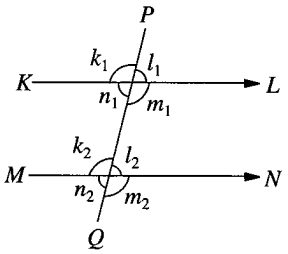


Fig. 13.16

Fig. 13.17 shows two parallel lines *AB* and *CD* cut by a transversal *PQ*. The angles a_1 and a_2 are called **alternate angles (alt. ∠s)**. Similarly, b_1 and b_2 are also called alternate angles. Use a protractor to measure the angles a_1, a_2, b_1 and b_2. What do you notice about the size of angles a_1 and a_2, and b_1 and b_2?

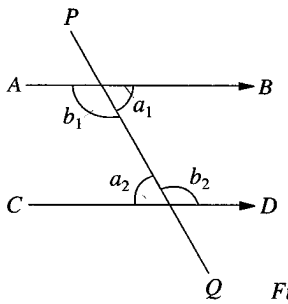


Fig. 13.17

Fig. 13.18 shows two parallel lines AB and XY cut by a transversal PQ. The angles a and b are called **interior angles**. The angles x and y are also interior angles. Use your protractor to measure the angles a, b, x and y. Find the value of $(a + b)$ and of $(x + y)$. Are the interior angles supplementary?

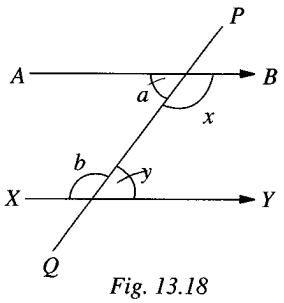


Fig. 13.18

We can conclude from the above discussion that when two parallel lines are cut by a transversal,

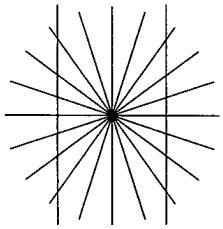
1. **the corresponding angles are equal;**
2. **the alternate angles are equal;**
3. **the interior angles are supplementary.**

The converse statements for the above are also true. That is, when two straight lines are cut by a transversal, and

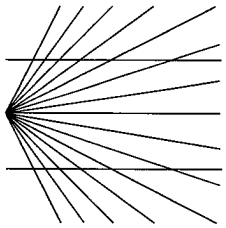
1. **if the corresponding angles are equal, then the two lines are parallel;**
2. **if the alternate angles are equal, then the two lines are parallel;**
3. **if the interior angles are supplementary, then the two lines are parallel.**

Which of the following pairs of lines are parallel?

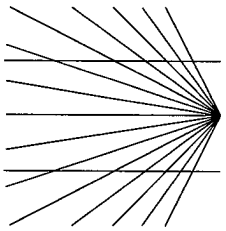
(a)



(b)



(c)



Example 3

Calculate the unknowns in Fig. 13.19.

Solution

$a = 48°$ (corr. $\angle$s, $AB//CD$)
$b = 61°$ (vert. opp. $\angle$s)
$d = b = 61°$ (corr. $\angle$s, $AB//CD$)
$b + c = 180°$ (interior $\angle$s)
$c = 180° - 61° = 119°$

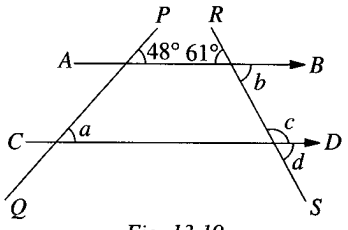


Fig. 13.19

Example 4

Find the unknowns in Fig. 13.20.

Solution

$i + 30° = 69°$ (corr. $\angle$s, $OA//BC$)
$i = 69° - 30° = 39°$
$j = i$ (corr. $\angle$s, $OA//BC$)
$j = 39°$

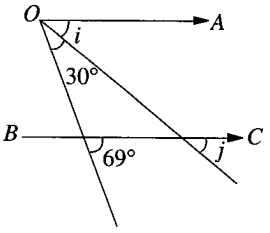


Fig. 13.20

Example 5

Find the values of f, g and h in Fig. 13.21.

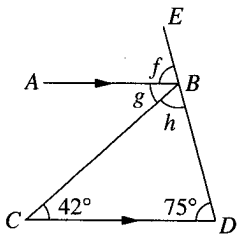


Fig. 13.21

Solution

$g = 42°$ (alt. ∠s, *AB*//*CD*)
$f = 75°$ (corr. ∠s, *AB*//*CD*)
$f + g + h = 180°$ (adj. ∠s on a line)
$h = 180° - 42° - 75°$
$= 63°$

Example 6

Find the value of x in Fig. 13.22.

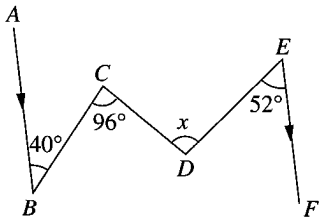


Fig. 13.22

Solution

At the points *C* and *D*, two lines both parallel to *AB* and *EF* are drawn as shown in Fig. 13.23.

$y_1 = 40°$ (alt. ∠s, *AB*//*PQ*)
$\therefore\ y_2 = 96° - 40°$
$= 56°$
$y_2 = y_3 = 56°$ (alt. ∠s, *PQ*//*RS*)
$y_4 = 52°$ (alt. ∠s, *RS*//*EF*)
$\therefore\ x = y_3 + y_4$
$= 56° + 52°$
$= 108°$

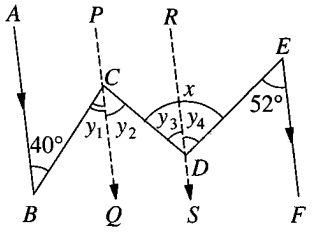


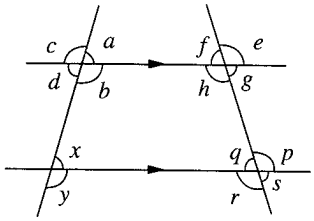
Fig. 13.23

Exercise 13b

1. From the diagram below, list

(a) three pairs of corresponding angles;

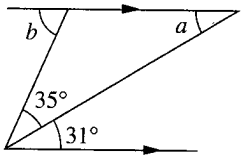
(b) three pairs of alternate angles.



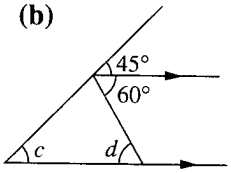
2. Refer to the diagram in Question 1. If $a = 73°$ and $e = 106°$, find the values of p, q, r, s, x and y.

3. Calculate the unknown(s) in the following diagrams:

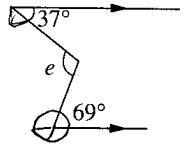
(a)



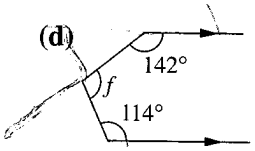
(b)



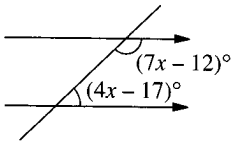
(c)



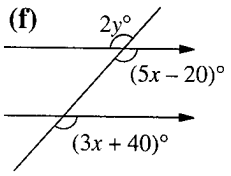
(d)



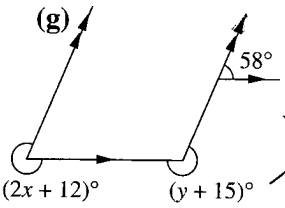
(e)



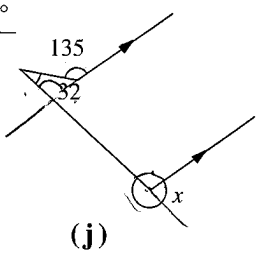
(f)



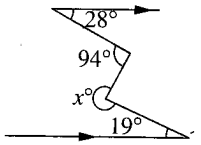
(g)



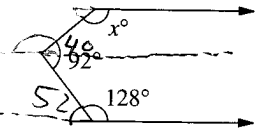
(h)



(i)



(j)



Drawing Parallel Lines Using a Set-square and a Ruler

Given: A straight line AB.

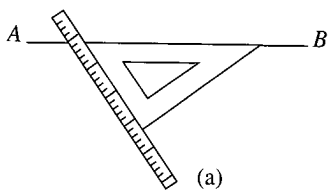
To construct: A line parallel to AB.

Construction steps:

(1) Place the set square on the line AB and place the ruler as shown in Fig. 13.24(a).

(2) By moving the set square along the length of the ruler, we can draw lines parallel to AB.

For example, by moving the set square down against the ruler, we can draw a line PQ parallel to AB (see Fig. 13.24(b)).



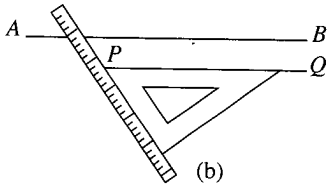
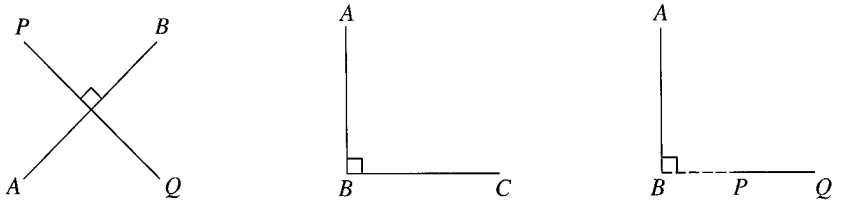


Fig. 13.24

Perpendicular Lines

Geometers Sketch Pad (GSP) is a powerful tool to do geometrical constructions. Find out how you can draw (a) perpendicular lines, (b) parallel lines, (c) circles, (d) angle bisectors and (e) perpendicular bisectors using GSP.

Two lines which are at right angles to each other are said to be **perpendicular** to each other. We use the symbol "$AB \perp PQ$" to denote "AB is perpendicular to PQ".



(a) $AB \perp PQ$ (b) $AB \perp BC$ (c) $AB \perp QP$ (produced)

Fig. 13.25

Drawing Perpendicular Lines Using a Set-square and a Ruler

(a) **Given:** A point K on a line AB.

To construct: A line through K perpendicular to AB.

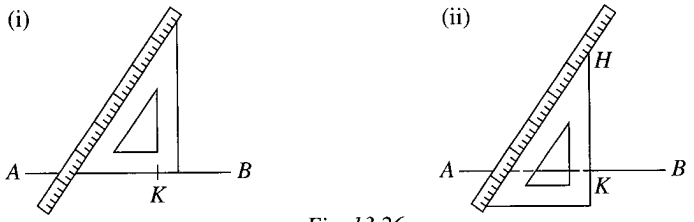


Fig. 13.26

Construction steps:

(1) Place the set-square on the line AB and place the ruler as shown in Fig. 13.26(i).

(2) Slide the set square along the length of the ruler until the other edge of the set square passes through K. Draw HK as shown in Fig. 13.26(ii). We write $AB \perp HK$.

(b) **Given:** A point K which is not on the line AB.

To construct: A line through K perpendicular to AB.

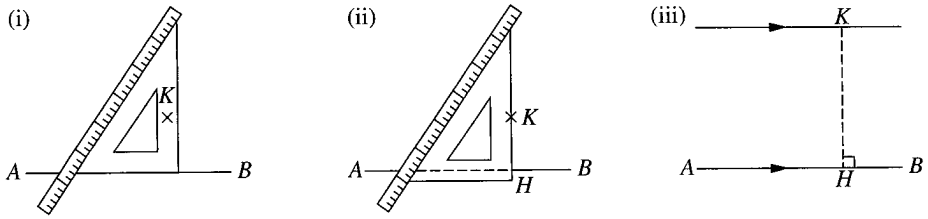


Fig. 13.27

Construction steps:

(1) Place one edge of the set square on AB as shown in Fig. 13.27(i).

(2) Slide the set square along the length of the ruler until the other edge passes through K. Draw HK as shown in Fig. 13.27(ii). We write $HK \perp AB$.

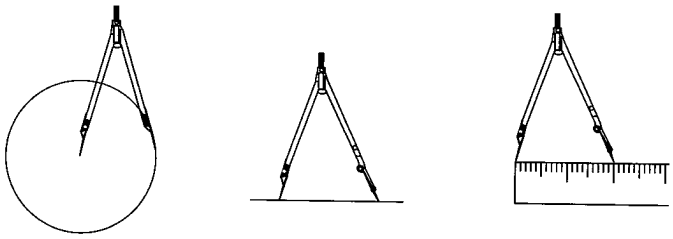
Note: The length of KH gives the distance of the point K to the line AB. In addition, if we draw a line passing through K and parallel to AB, KH also gives the distance between the two parallel lines.

1. *Any point on the perpendicular bisector of PQ is equidistant from P and from Q.*

2. *The line segment drawn from any vertex of a triangle perpendicular to the opposite side is called the altitude or height of the triangle.*

Use of Compasses

A pair of **compasses** is a mathematical instrument often used for drawing circles, marking off lengths and measuring the length of a line segment.

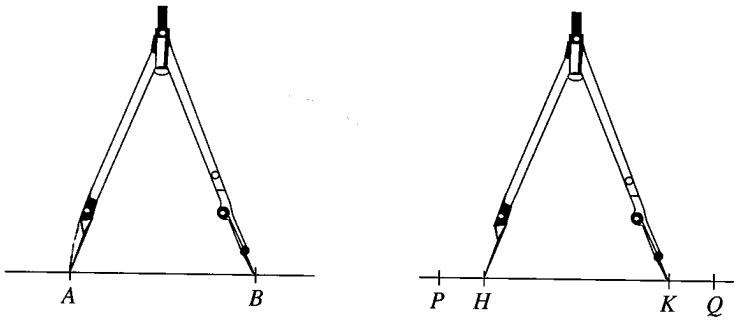


drawing a circle

measuring a line segment

The following explains how to use a compass.

To mark off a length equal to AB on the line PQ, i.e., HK:

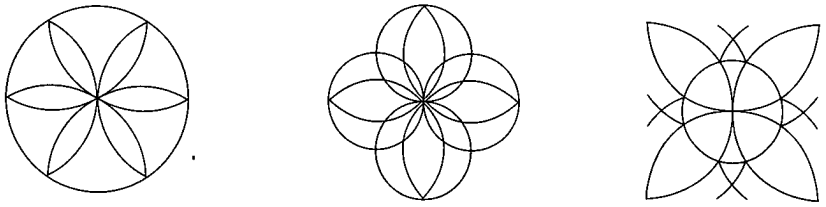


Construction steps:

(1) Adjust the arms of the compasses until they touch AB.
(2) Mark a point H on another line, PQ.
(3) With H as centre and radius AB, draw an arc to cut PQ at K. Hence, $AB = HK$.

In-Class Activity

You can create designs with your pair of compasses. Some of these designs are shown below. Try to create other designs of your own.

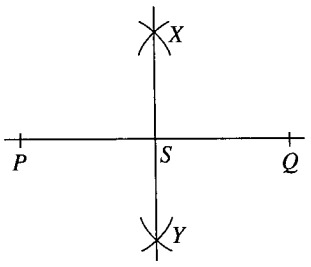


Perpendicular Bisector and Angle Bisector

Given: A straight line PQ.

To construct: A perpendicular bisector of a given line segment.

Construction steps:



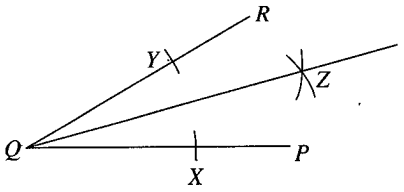
(1) Taking P and Q as centres and a radius greater than half of PQ, draw arcs above and below PQ such that the arcs cut each other at X and Y.

(2) Join XY to cut PQ at S. XY is called the **perpendicular bisector** of PQ. If you measure PS and SQ, you will find that $PS = SQ$. Thus, S is called the **midpoint** of PQ.

Given: An angle PQR.

To construct: The angle bisector of a given angle.

Construction steps:



(1) Taking Q as centre and a fixed radius, cut QP at X and QR at Y.

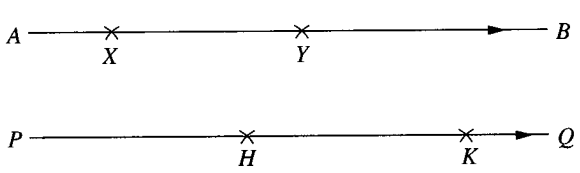
(2) Taking X and Y as centres and the same radius, draw arcs to cut each other at Z.

(3) Join QZ.

You will find $P\hat{Q}Z = R\hat{Q}Z$ and hence QZ is the angle bisector of $P\hat{Q}R$.

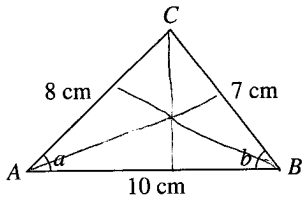
Exercise 13c

1. In the figure, AB and PQ are parallel lines. Draw two perpendicular lines from points X and Y, on AB, to PQ. Similarly, draw two perpendicular lines from H and K to AB.



Measure the lengths of these perpendicular lines. What do you notice about their lengths?

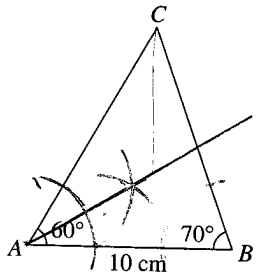
2. Draw the following triangle ABC with the given dimensions accurately and measure the angles marked a and b.



Using a set square, draw a line from C touching AB and perpendicular to AB. Also, draw a line from B touching AC and perpendicular to AC. Measure the length of these perpendicular lines. Using a set square, draw the line through A touching BC and perpendicular to BC. Do these three perpendicular lines meet at the same point?

3. Draw the triangle ABC with the given dimensions accurately. Measure AC and BC.

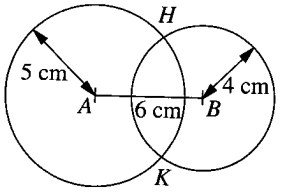
Using a pair of compasses, construct the angle bisectors of $B\hat{A}C$, $A\hat{B}C$ and $A\hat{C}B$. Do the angle bisectors meet at the same point?



4. Draw a line PQ of length 8 cm. Construct the perpendicular bisector of PQ.

5. Draw an angle ABC of 78°. Construct the bisector of $A\hat{B}C$.

6. Draw the diagram shown accurately. Measure HK.



Summary

1. The **sum of all the angles at a point** is 360°.

2. The **sum of adjacent angles** on a straight line is 180°.

3. Fig. 13.28 shows two parallel lines, AB and CD, with a transversal PQ.
   (a) x and y are **vertically opposite angles** and are equal in size.
   (b) x and q are **corresponding angles** and are equal in size.
   (c) e and b are **alternate angles** and are equal in size.
   (d) e and y are **interior angles**. Their sum is equal to 180°.

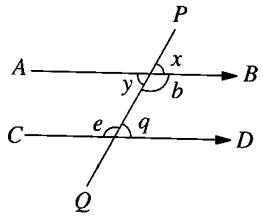
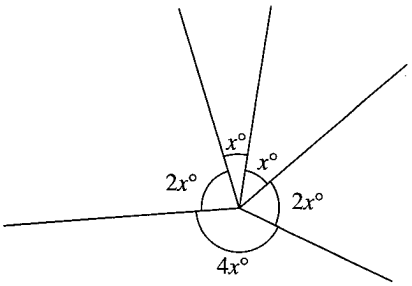


Fig. 13.28

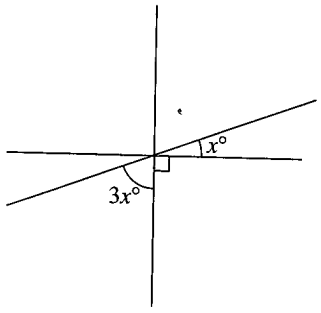
Review Questions 13

1. Find the value of the unknown(s) in each of the following:

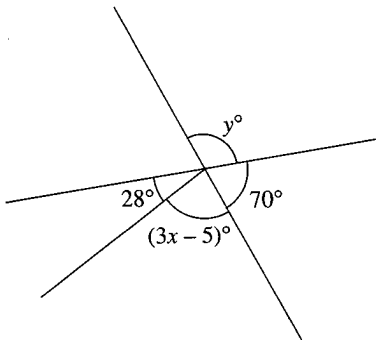
(a)



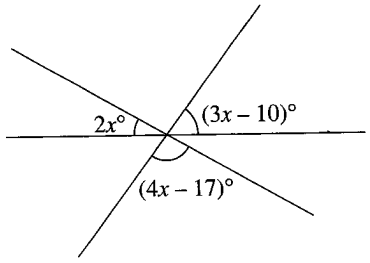
(b)



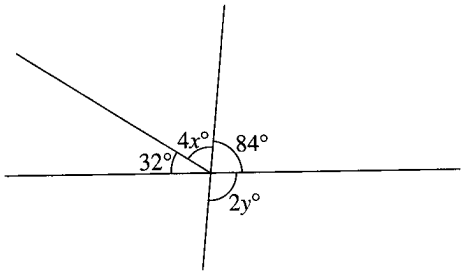
(c)



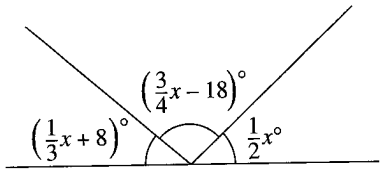
(d)



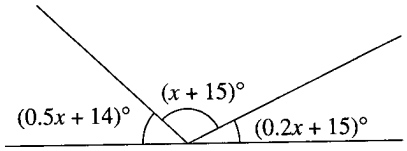
(e)



(f)



(g)



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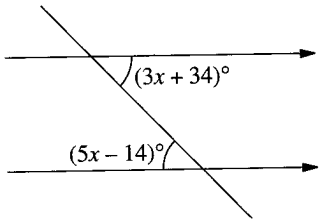
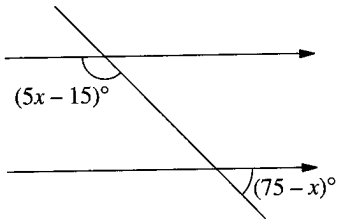


(i)



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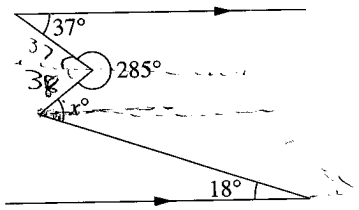
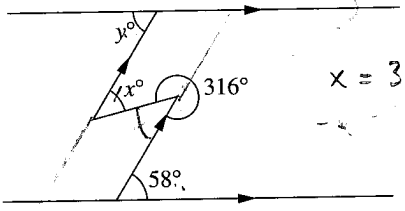
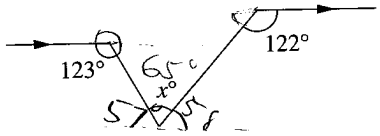


(k)

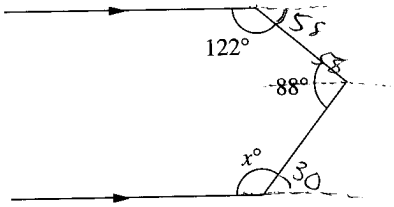


$x = 360 - 316°$

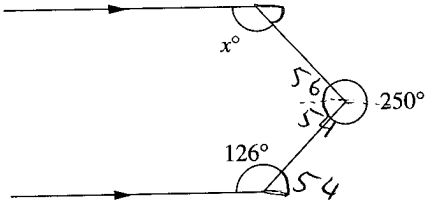
(l)

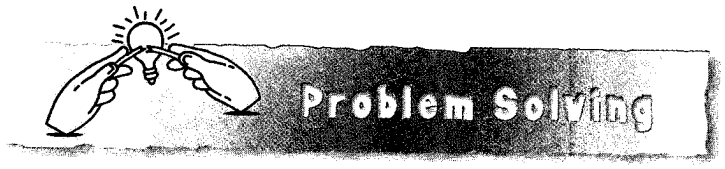


(m)

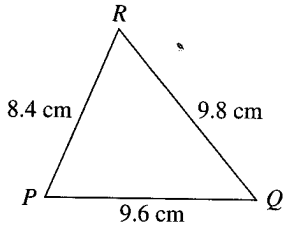


(n)



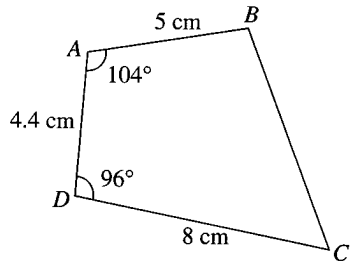


1. Draw the triangle PQR accurately. Measure $P\hat{Q}R$ and $R\hat{P}Q$. Using a pair of compasses, construct the angle bisectors of $P\hat{Q}R$ and $R\hat{P}Q$. Let the two lines meet at X. Measure the length of PX and RX. What is the distance of X from PQ? With X as centre, construct a circle to touch PQ, PR and QR.

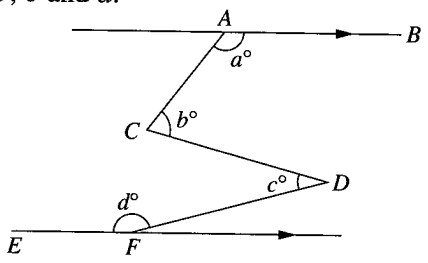


2. Construct triangle ABC such that AB = 12 cm, BC = 11 cm and AC = 9.6 cm. Construct the perpendicular bisectors of AB and BC. Let the two bisectors meet at K. With K as centre, construct a circle to pass through A, B and C. Find out the name of this circle from your library.

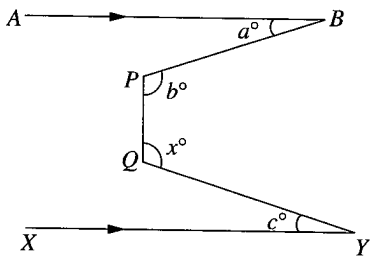
3. Draw the given quadrilateral $ABCD$ accurately. Construct a line parallel to AB and passing through D. Produce this line to cut BC at X. Measure DX and BX.



4. In the figure, $B\hat{A}C = a°$, $A\hat{C}D = b°$, $C\hat{D}F = c°$ and $D\hat{F}E = d°$. Form an equation connecting a, b, c and d.



5. In the figure, AB is parallel to XY, $A\hat{B}P = a°$, $B\hat{P}Q = b°$, $P\hat{Q}Y = x°$ and $X\hat{Y}Q = c°$. Express x in terms of a, b and c.



CHAPTER 14

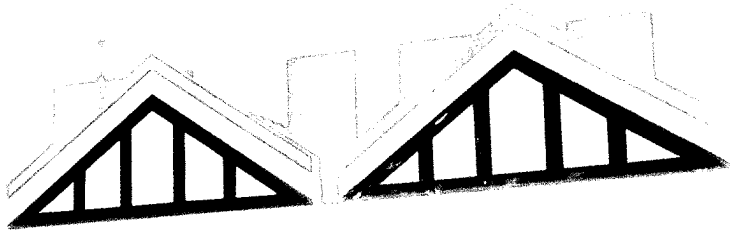
Angle Properties of Triangles and Quadrilaterals

In this chapter, you will learn how to

- △ calculate the unknown angles involving triangles and quadrilaterals using the angle properties of these figures;
- △ construct simple geometrical figures from given information.

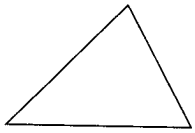
Preliminary Problem

Triangles and quadrilaterals are common designs well-liked by people in the design of their houses and exteriors. Can you see triangles and quadrilaterals in this picture?

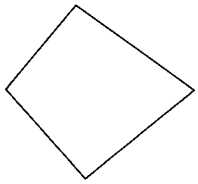


Polygons

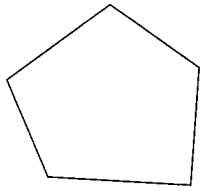
A plane figure with three or more straight edges as its sides is called a **polygon**. Each polygon is named after the number of sides it contains. The following names are given to some common polygons.



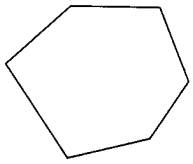
Triangle
(3-sided)



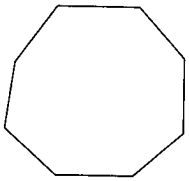
Quadrilateral
(4-sided)



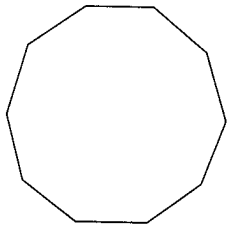
Pentagon
(5-sided)



Hexagon
(6-sided)



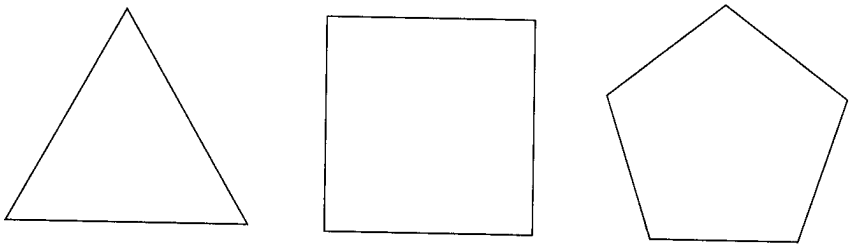
Octagon
(8-sided)



Decagon
(10-sided)

Types of Polygons
Polygons have different names, depending on the number of corners or sides they have.

In general, a polygon with n sides is called an n-gon. Thus a polygon with 12 sides is called a 12-gon and a polygon with 25 sides is called a 25-gon. A **regular polygon** is one in which all its sides and all its angles are equal. The following figures are some examples of regular polygons.



In this chapter, you will learn about properties of triangles and quadrilaterals.

Triangles

A plane figure formed by having three straight edges as its sides is called a **triangle**.

Fig. 14.1 shows a triangle ABC, which can be denoted by $\triangle ABC$, formed by the three sides AB, BC and CA. The points A, B and C are called the vertices (singular: vertex) of the triangle. $\angle ABC$, $\angle BAC$ and $\angle BCA$ are called the interior angles of the triangle, or simply the angles of $\triangle ABC$.

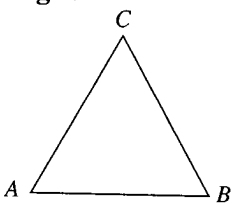


Fig. 14.1

Triangles can be classified according to:

(a) The number of equal sides

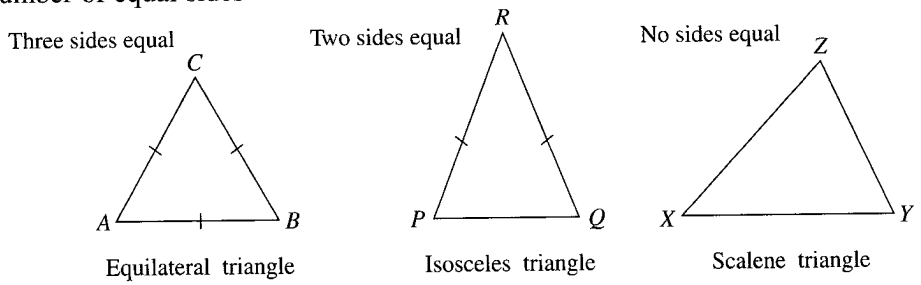


Fig. 14.2

(b) The types of angles

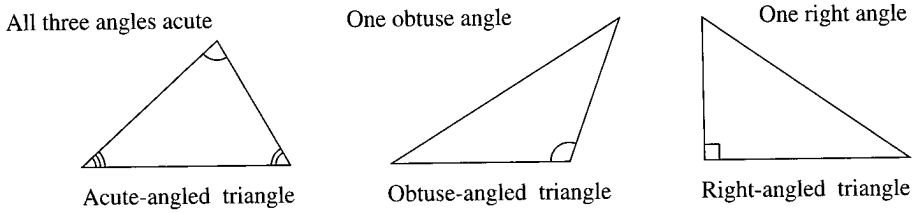


Fig. 14.3

Notes: (1) All the three angles in an equilateral triangle are equal in size. Each angle is 60°.
(2) The two base angles of an isosceles triangle are equal, i.e., $\angle RPQ = \angle RQP$.
(3) All the three angles in a scalene triangle are different in size.

Construction of Triangles

To construct a triangle accurately, we use a pair of compasses, a ruler and a protractor. However, not all three instruments are often used at the same time.

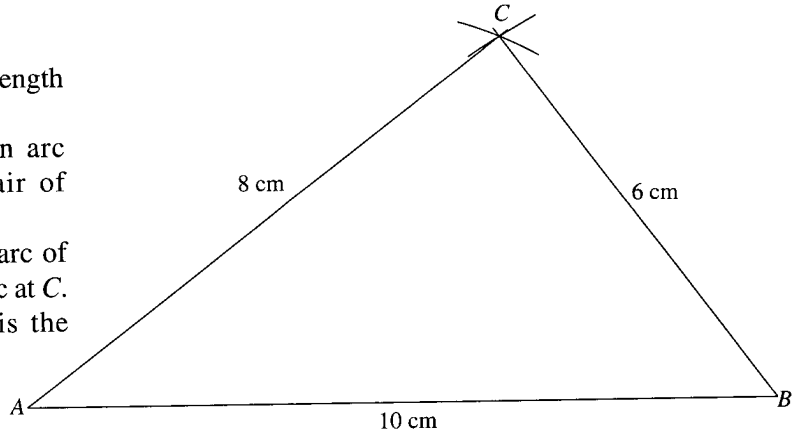
Example 1

Construct $\triangle ABC$ in which $AB = 10$ cm, $BC = 6$ cm and $AC = 8$ cm. Measure $\angle ABC$, $\angle BCA$ and $\angle BAC$. What is the sum of the three angles? What type of triangle is this?

Solution

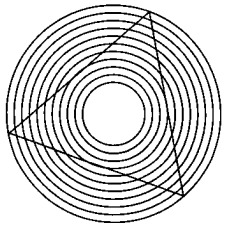
Construction steps:

(1) Draw a line segment AB of length 10 cm.
(2) With A as centre, draw an arc of radius 8 cm with a pair of compasses.
(3) With B as centre, draw an arc of radius 6 cm to cut the first arc at C.
(4) Join AC and BC. $\triangle ABC$ is the required triangle.



By measurement with a protractor, $\angle ABC \approx 53°$, $\angle BAC \approx 37°$ and $\angle ACB \approx 90°$. The sum of the three angles $\approx 53° + 37° + 90° = 180°$. $\triangle ABC$ is a right-angled triangle since $\angle ACB = 90°$. It can also be called a scalene triangle since all the angles are different.

Are the sides of the triangle straight?



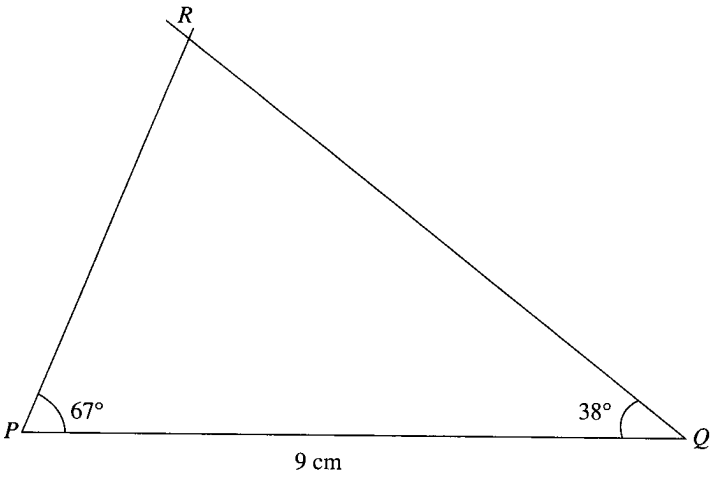
Example 2

Construct $\triangle PQR$ in which $PQ = 9$ cm, $\angle PQR = 38°$ and $\angle QPR = 67°$. Measure the angle PRQ and the length of the sides PR and RQ. What is the sum of the three interior angles of the triangle. What type of triangle is this? What is the length of the side facing the smallest angle? And what is the length of the side facing the largest angle?

Solution

Construction steps:

(1) Draw a line segment PQ 9 cm long.
(2) Using a protractor at points P and Q, draw an angle of 67° and one of 38° respectively.
(3) Produce the other arms of angles P and Q to meet at R. $\triangle PQR$ is the required triangle.



By measurement, $\angle PRQ = 75°$, $PR = 5.6$ cm and $QR = 8.5$ cm. The sum of the three interior angles $= 38° + 67° + 75° = 180°$. $\triangle PQR$ is an acute-angled triangle. It can also be called a scalene triangle. The length of the side facing the smallest angle is 5.6 cm and that of the side facing the largest angle is 9 cm.

The result of using a protractor to find the measure of an angle is not likely to be accurate.

Example 3

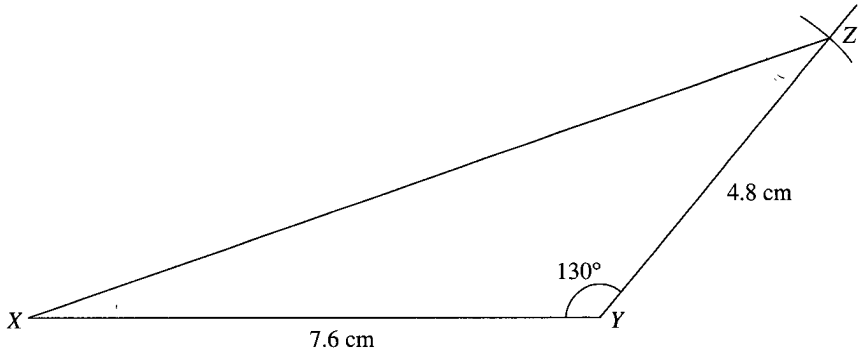
Construct $\triangle XYZ$ in which $XY = 7.6$ cm, $\angle XYZ = 130°$ and $YZ = 4.8$ cm. Measure the length of XZ and the angles ZXY and YZX. What is the sum of the three interior angles of $\triangle XYZ$? What type of triangle is this? What is the size of the angle facing the shortest side?

Solution

Construction steps:

(1) Draw a line segment XY of length 7.6 cm.
(2) Use a protractor to construct an angle of 130° at Y.

(3) Use a pair of compasses, set to a radius of 4.8 cm, to draw an arc to cut the produced arm of angle Y at Z. $\triangle XYZ$ is the required triangle.



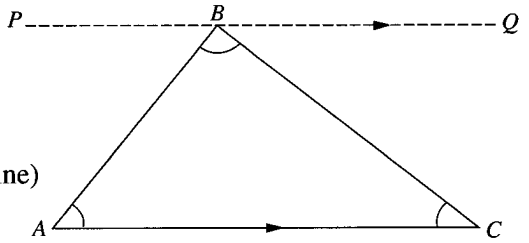
By measurement, $XZ = 11.3$ cm, $\hat{YZX} = 31°$ and $\hat{YXZ} = 19°$. The sum of the three interior angles = $130° + 31° + 19° = 180°$, $\triangle XYZ$ is an obtuse-angled triangle. It is also an example of a scalene triangle. The size of the angle facing the shortest side is 19°.

Angle Properties of Triangles

Use the Geometers' Sketch Pad to verify that the sum of the interior angles of a triangle is equal to 180°.

From the above three examples, we find that the sum of the interior angles of each triangle always adds up to 180°, or 2 right angles. Do you also notice that the length of one side varies according to the size of the angle opposite it? In other words, the longest side is opposite the largest angle and the shortest side is opposite the smallest angle.

Draw PQ parallel to AC and passing through B.



We have $B\hat{A}C = P\hat{B}A$ (alt. $\angle$s, $PQ//AC$)

and $B\hat{C}A = Q\hat{B}C$ (alt. $\angle$s, $PQ//AC$)

Now, $P\hat{B}A + A\hat{B}C + Q\hat{B}C = 180°$ (adj. $\angle$s on a str. line)

$\therefore\ B\hat{A}C + A\hat{B}C + B\hat{C}A = 180°$

(**Abbreviation:** $\angle$ sum of $\triangle$)

Exterior and Interior Opposite Angles

Fig. 14.4 shows $\triangle ABC$ with BA produced to P, BC produced to Q and CB produced to R. p, q and r are called the exterior angles of $\triangle ABC$. In particular, a and b are referred to as the interior opposite angles with reference to q. Similarly, a and c are the interior opposite angles with reference to r, etc.

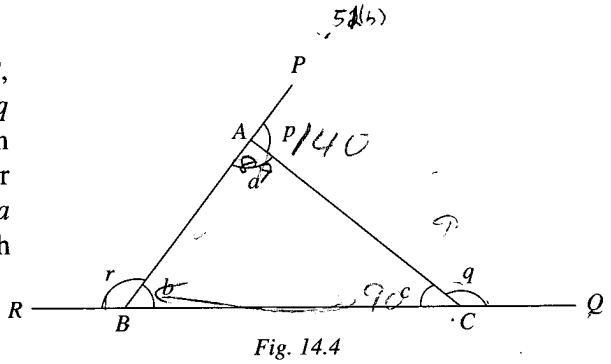


Fig. 14.4

Use Geometers' Sketch Pad to explore the relationship between the sum of the interior opposite angles and their corresponding exterior angles.

Using a protractor, measure the angles a, b, c, p, q and r. Find the values of $(a + b)$, $(b + c)$ and $(a + c)$. What do you notice about these values? What can you say about the sum of the interior opposite angles?

We can establish the fact that the sum of the interior opposite angles of a triangle is equal to its exterior angle by the following proof.

Refer to triangle ABC with notations as given in Fig. 14.5

We have $a + b + c = 180°$ ($\angle$ sum of $\triangle$)

and $q + c = 180°$ (adj. $\angle$s on a str. line)

$\therefore \quad a + b = q$

(**Abbreviation:** ext. $\angle$ = sum of int. opp. $\angle$s)

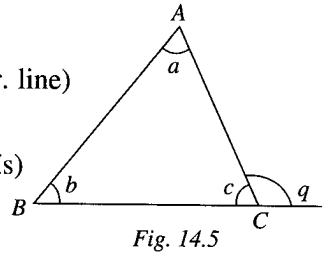
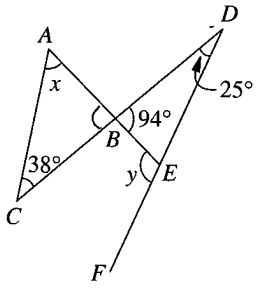


Fig. 14.5

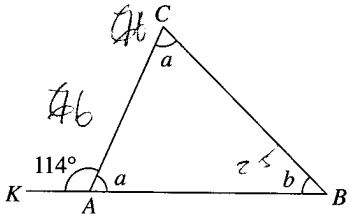
Example 4

Find the unknown angles marked in each of the following diagrams:

(a)



(b)



Solution

(a) $A\hat{B}C = 94°$ (vert. opp. $\angle$s)

$x + 94° + 38° = 180°$ ($\angle$ sum of $\triangle$)

$\therefore \quad x = 180° - 94° - 38° = 48°$

$y = 94° + 25°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$\therefore \quad y = 119°$

(b) $114° + a = 180°$ (adj. $\angle$s on a str. line)

$\therefore \quad a = 180° - 114° = 66°$

$114° = 66° + b$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$\therefore \quad b = 114° - 66°$

$= 48°$

Exercise 14a

1. Sketch, in each case, a triangle ABC with the size of $\hat{A}$ and $\hat{B}$ given below. In each case find $\hat{C}$ and classify each triangle (i) by its sides (ii) by its angles:

(a) $\hat{A} = 20°$, $\hat{B} = 60°$ **(b)** $\hat{A} = 70°$, $\hat{B} = 40°$ **(c)** $\hat{A} = 60°$, $\hat{B} = 60°$

(d) $\hat{A} = 42°$, $\hat{B} = 48°$ **(e)** $\hat{A} = 65°$, $\hat{B} = 50°$ **(f)** $\hat{A} = 25°$, $\hat{B} = 112°$

2. The following are base angles of isosceles triangles. In each case, find the third angle of the isosceles triangle:

(a) 42° **(b)** 82° **(c)** 18° **(d)** 64°

3. Construct a triangle ABC such that $AB = 85$ mm, $BC = 110$ mm and $AC = 95$ mm. Name and measure the size of the smallest angle of the triangle.

4. Construct an equilateral triangle with sides 9.5 cm each.

5. Construct a right-angled triangle such that the two shorter sides are each equal to 6 cm. Measure the length of the longest side and the size of the smallest angle.

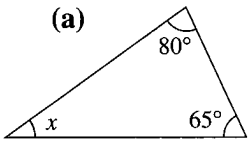
6. Construct a triangle ABC such that $\hat{A} = 75°$, $AB = 6$ cm and $AC = 7$ cm. Measure BC and $\hat{B}$. What type of triangle is this?

7. Construct a triangle PQR such that $PQ = 8$ cm, $\hat{P} = 48°$ and $\hat{Q} = 56°$. Measure PR, QR and $\hat{R}$. What type of triangle is this?

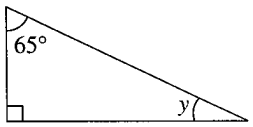
8. Construct a triangle LMN such that $\hat{L} = 90°$, $LM = 5$ cm and $MN = 9$ cm. Measure LN, $\hat{M}$ and $\hat{N}$. What is the name of this triangle?

9. Calculate the values of the unknown in each of the following diagrams:

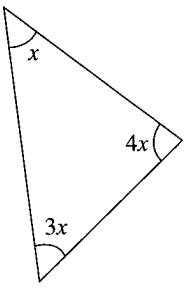
(a)



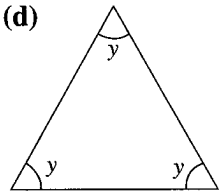
(b)



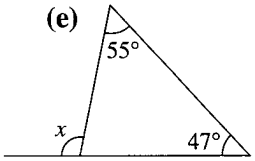
(c)



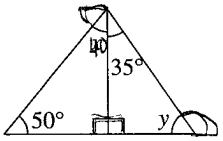
(d)



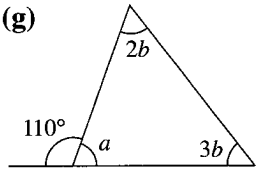
(e)



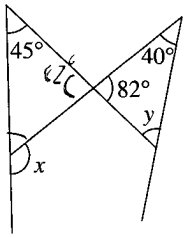
(f)



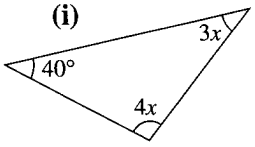
(g)



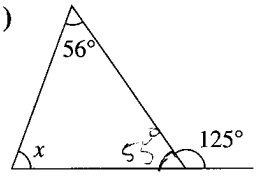
(h)



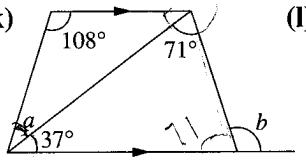
(i)



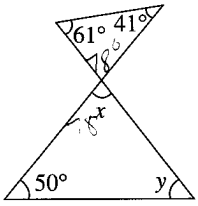
(j)



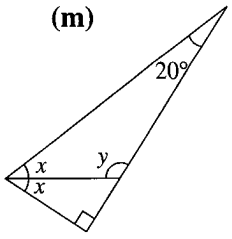
(k)



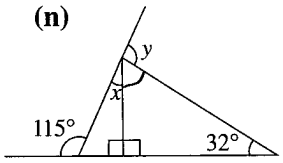
(l)



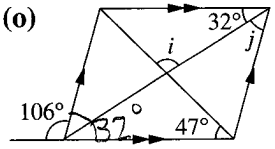
(m)



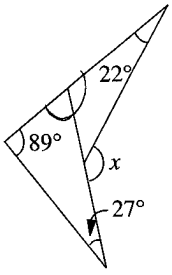
(n)



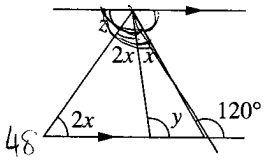
(o)



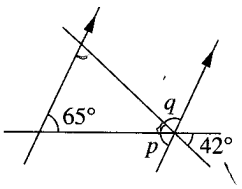
(p)



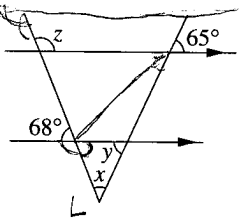
(q)



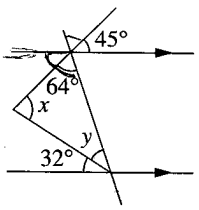
(r)



(s)



(t)



***10.** The angles of a triangle are $(x-35)°$, $(x-25)°$ and $\left(\frac{1}{2}x - 10\right)°$. Form an equation in x and hence find x.

***11.** If the sizes of the angles of a triangle are $3x°$, $5x°$ and $4x°$, find the smallest angle of the triangle.

***12.** In the triangle ABC, $\hat{A} = 50°$, $\hat{C} = 26°$ and AB is produced to D. Find $A\hat{B}C$ and $C\hat{B}D$.

Quadrilaterals

A quadrilateral is a plane figure with four sides and four angles. Fig. 14.6 shows a quadrilateral $ABCD$. It is named by taking the vertices in order either in a clockwise or anticlockwise direction. Thus, $ABCD$, $BCDA$, $DCBA$ and $CBAD$ are correct ways of naming the quadrilateral but $ABDC$, $ACBD$ and $CDBA$ are not. The line segments AC and BD that join the opposite vertices are called **diagonals**. The sum of the four angles of a quadrilateral equals 360°.

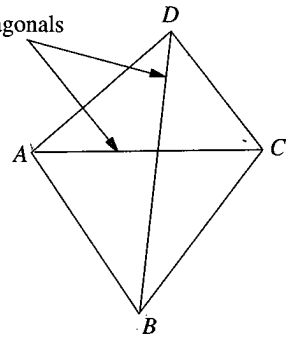
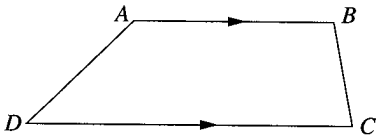


Fig. 14.6

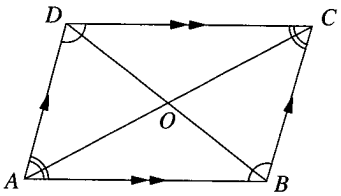
The following are some quadrilaterals and their properties:

1. Trapezium

A trapezium is a quadrilateral with exactly one pair of opposite sides parallel, i.e., $AB//DC$.



2. Parallelogram



(a) A parallelogram is a quadrilateral with two pairs of opposite sides parallel, i.e. $AB//DC$ and $AD//BC$.

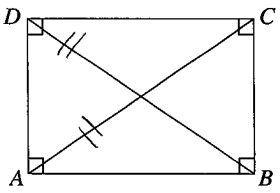
(b) The opposite sides of a parallelogram are equal in length, i.e., $AB = DC$ and $AD = BC$.

(c) The diagonals of a parallelogram bisect each other, i.e., $AO = OC$ and $BO = OD$.

(d) The opposite angles of a parallelogram are equal, i.e., $\hat{A} = \hat{C}$ and $\hat{B} = \hat{D}$. Furthermore $\hat{A} + \hat{D} = 180°$ and $\hat{B} + \hat{C} = 180°$.

Use the open tool Geometers' Sketch Pad to explore the many properties of quadrilaterals.

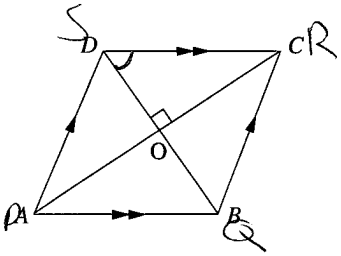
3. Rectangle



(a) A rectangle is a parallelogram where all its angles are right angles. The properties of a parallelogram are also applicable to a rectangle.

(b) The diagonals of a rectangle are equal in length, i.e., $AC = BD$.

4. Rhombus

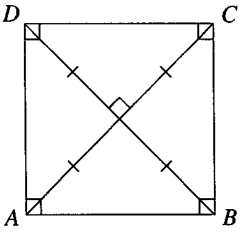


(a) A rhombus is a parallelogram with all its sides equal in length, i.e., $AB = BC = CD = AD$.

(b) The diagonals of a rhombus bisect each other at right angles, i.e., $A\hat{O}B = C\hat{O}D = B\hat{O}C = A\hat{O}D = 90°$ and $AO = OC$, $DO = OB$.

(c) The diagonals of a rhombus bisect the angles, i.e., $D\hat{A}O = B\hat{A}O$, $A\hat{B}O = C\hat{B}O$, $B\hat{C}O = D\hat{C}O$ and $A\hat{D}O = C\hat{D}O$.

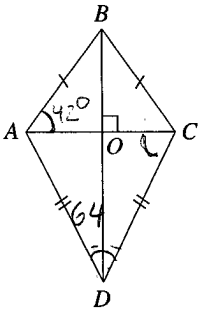
5. Square



(a) A square is a rhombus where all its angles are right angles, i.e., $\hat{A} = \hat{B} = \hat{C} = \hat{D} = 90°$.

(b) The diagonals of a square are equal in length, i.e., $AC = BD$.
The diagonals of a square bisect each other at right angles.

6. Kite

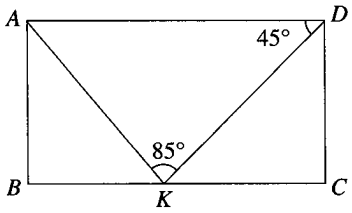


(a) A kite is a quadrilateral with 2 pairs of equal adjacent sides, i.e., $AB = BC$ and $AD = DC$.

(b) The longer diagonal bisects the other diagonal at right angles, i.e., $AO = OC$ and $A\hat{O}B = B\hat{O}C = A\hat{O}D = C\hat{O}D = 90°$.

Example 5

ABCD is a rectangle in which $A\hat{K}D = 85°$ and $A\hat{D}K = 45°$. Calculate (a) $B\hat{A}K$; (b) $B\hat{K}A$.

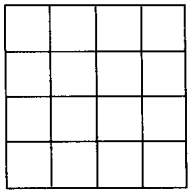


Solution

(a) $K\hat{A}D + 85° + 45° = 180°$ ($\angle$ sum of $\triangle$)
$K\hat{A}D = 180° - 85° - 45° = 50°$
$B\hat{A}K + 50° = 90°$ ($B\hat{A}D$ is a right angle)
$\therefore\ B\hat{A}K = 90° - 50° = 40°$

(b) $B\hat{K}A + 40° + 90° = 180°$ ($\angle$ sum of $\triangle$, $A\hat{B}K = 90°$)
$\therefore\ B\hat{K}A = 180° - 40° - 90° = 50°$

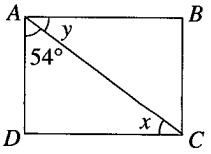
How many squares are there in the given figure?



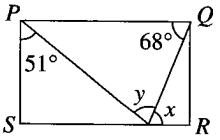
Exercise 14b

1. For each of the rectangles given, calculate the unknown angles marked x and y:

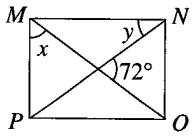
(a)



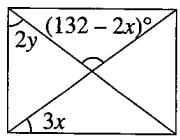
(b)



(c)

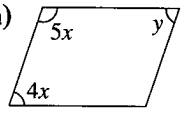


(d)

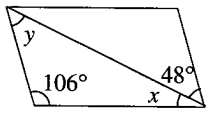


2. For each of the parallelograms given, calculate the values of x and y:

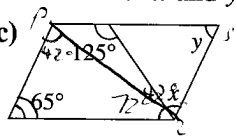
(a)



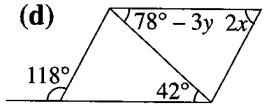
(b)



(c)

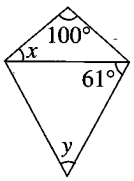


(d)

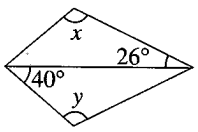


3. For each of the kites given, calculate the values of x and y:

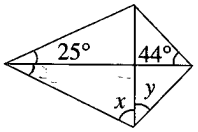
(a)



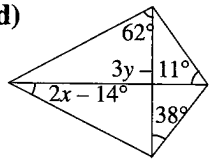
(b)



(c)

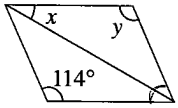


(d)

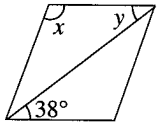


4. For each of the rhombuses given, calculate the unknown angles marked x and y:

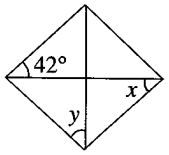
(a)



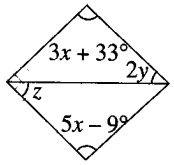
(b)



(c)

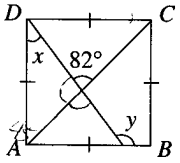


(d)



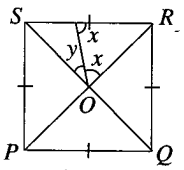
***5.** For each of the figures given, calculate the unknown angles marked x and y:

(a)



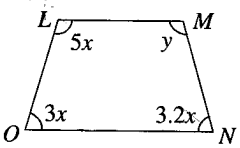
ABCD is a square.

(b)



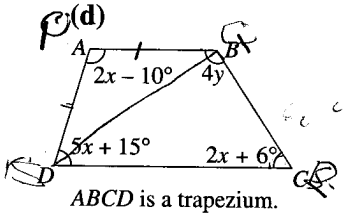
PQRS is a square.

(c)



LMNO is a trapezium.

(d)



ABCD is a trapezium.

6. In a rectangle $ABCD$, X is the mid-point of AB and $C\hat{X}D = 118°$, calculate

(a) $A\hat{D}X$ **(b)** $X\hat{C}D$

7. In a parallelogram $PQRS$, $Q\hat{P}R = 42°$ and $Q\hat{R}S = 70°$, calculate

(a) $P\hat{Q}R$ **(b)** $P\hat{R}Q$

8. In a rhombus $PQRS$, $P\hat{Q}R = 108°$, calculate

(a) $Q\hat{R}S$ **(b)** $Q\hat{P}R$ **(c)** $Q\hat{S}R$

9. In a kite $ABCD$, $AB = BC$, $AD = CD$, $A\hat{D}C = 64°$ and $B\hat{A}C = 42°$, calculate

(a) $A\hat{C}D$ **(b)** $A\hat{B}C$

10. In a trapezium $PQRS$, PQ is parallel to SR, $PQ = PS$, $P\hat{S}R = 62°$ and $Q\hat{R}S = 52°$, calculate

(a) $P\hat{Q}S$ **(b)** $S\hat{Q}R$

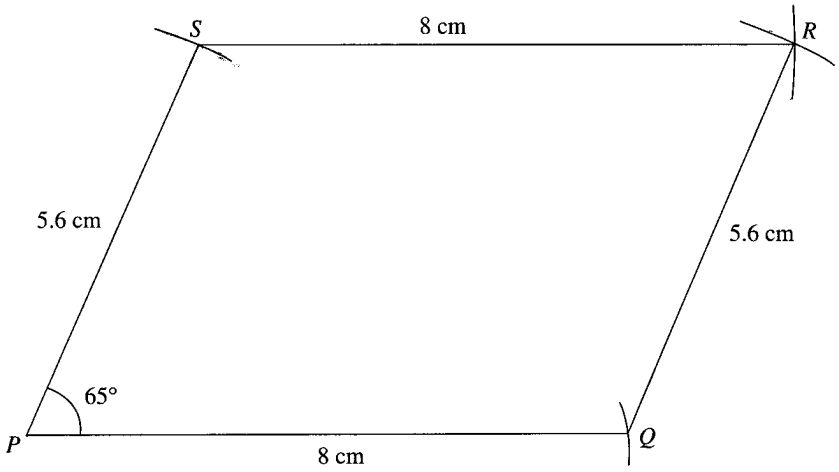
Construction of Quadrilaterals

The following examples illustrate the steps for the construction of quadrilaterals.

Example 6

Construct a parallelogram PQRS where PQ = 8 cm, PS = 5.6 cm and $S\hat{P}Q = 65°$. Measure the length of the diagonals PR and QS.

Solution



Construction steps:

(1) Draw a line segment PQ of length 8 cm.
(2) Using a protractor, construct an angle of 65° at P, so that PQ is one side of the angle, and produce the other arm of the angle.
(3) With P as centre and radius 5.6 cm, cut the arm of P at S.
(4) With Q as centre and radius 5.6 cm, draw an arc.
(5) With S as centre and radius 8 cm, draw an arc to cut the arc in (4) at R. $PQRS$ is the required parallelogram.

By measurement, PR = 11.5 cm and QS = 7.6 cm.

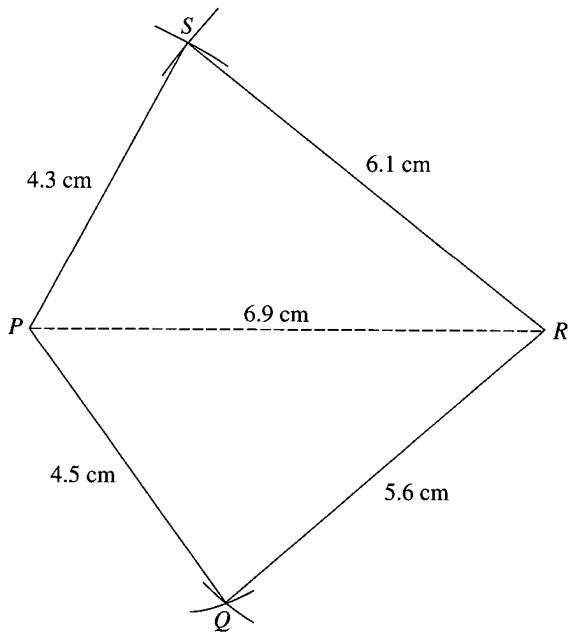
Example 7

Construct a quadrilateral PQRS where PQ = 4.5 cm, QR = 5.6 cm. RS = 6.1 cm, PS = 4.3 cm and diagonal PR = 6.9 cm. Measure $\hat{Q}$, $\hat{R}$ and $\hat{S}$.

Solution

Construction steps:

(1) Draw a line segment PR of length 6.9 cm.
(2) With P and R as centres and radii 4.5 cm and 5.6 cm respectively, draw two arcs to cut at Q.
(3) With P and R as centres and radii 4.3 cm and 6.1 cm respectively, draw two arcs to cut at S.
(4) Join PQ, QR, RS and PS. $PQRS$ is the required quadrilateral.



By measurement, $\hat{Q} = 86°$, $\hat{R} = 78°$ and $\hat{S} = 81°$.

Draw a rectangle of any size. Use your ruler to locate the midpoints of the sides carefully. Join these midpoints to form a new quadrilateral. What is the name of the quadrilateral you have obtained? Repeat the above by drawing

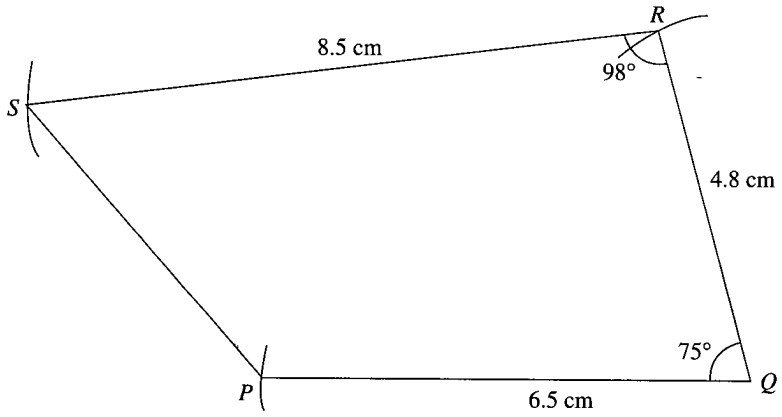
(a) a trapezium;
(b) a parallelogram;
(c) a kite;
(d) a rhombus;
(e) a quadrilateral of 4 unequal lengths.

What conclusion can you draw?

Example 8

Construct a quadrilateral PQRS where PQ = 6.5 cm, QR = 4.8 cm, RS = 8.5 cm, $\hat{Q} = 75°$ and $\hat{R} = 98°$. Measure the length of PS and the angles P and S.

Solution



Construction steps:

(1) Draw a line segment PQ of length 6.5 cm.
(2) Using a protractor, construct $\hat{Q} = 75°$ and produce the arm of Q.
(3) With Q as centre and radius 4.8 cm, draw an arc to cut the produced arm of $\hat{Q}$ at R.
(4) Using a protractor, construct $\hat{R} = 98°$ and produce the arm of $\hat{R}$.
(5) With R as centre and radius 8.5 cm, draw an arc to cut the produced arm of $\hat{R}$ at S.
(6) Join PS and $PQRS$ is the required quadrilateral.

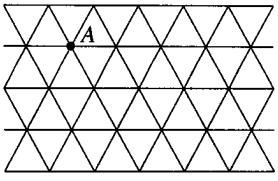
By measurement, $PS = 4.8$ cm, $\hat{P} = 131°$ and $\hat{S} = 56°$.

Exercise 14c

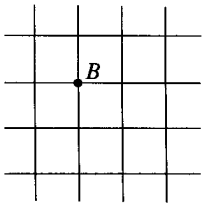
1. Construct a parallelogram $ABCD$ with $AB = 10$ cm, $BC = 12$ cm and $\hat{B} = 80°$. Measure BD.
2. Construct a trapezium $ABCD$ where $AB = 5.6$ cm, $BC = 11.2$ cm, $\hat{B} = 80°$ and $\hat{C} = 70°$. Measure AC.
3. Construct a rhombus $ABCD$ where $AB = 7.5$ cm and $AC = 12$ cm. Measure the length of the other diagonal.
4. Construct a rhombus $PQRS$ where $PQ = 6$ cm and $\hat{Q} = 115°$. Measure the length of the diagonals.
5. Construct a rectangle of sides 84 mm and 96 mm. Measure the length of the diagonals and the acute angle made by these diagonals.
6. Construct a quadrilateral $ABCD$ given that $AB = 65$ mm, $BC = 46$ mm, $AD = 58$ mm, $\hat{A} = 105°$ and $\hat{B} = 120°$. Measure AC and BD.
7. Construct a quadrilateral $ABCD$ given that $AB = 5.3$ cm, $BC = 6.3$ cm. $CD = 6.7$ cm. $\hat{B} = 75°$ and $\hat{C} = 60°$. Measure AD.
8. Construct a quadrilateral $PQRS$ where $PQ = 5.6$ cm, $\hat{Q} = 80°$, $\hat{R} = 95°$, $QR = 6.2$ cm and $RS = 9.2$ cm. Measure PS.
9. Construct a quadrilateral $ABCD$ where $BC = 60$ mm, $CD = 90$ mm, $AD = 60$ mm, $AB = 45$ mm and $BD = 90$ mm. Measure $A\hat{D}C$.
10. Construct a quadrilateral $PQRS$ given that $PQ = PR = PS = 90$ mm, $RS = 75$ mm and $QR = 120$ mm. Measure $Q\hat{P}S$.
11. Construct a quadrilateral $PQRS$ where $PS = 6$ cm, $RS = QR = 9$ cm and $P\hat{S}R = Q\hat{R}S = 110°$. Measure PQ.

Tessellation of Regular Polygons

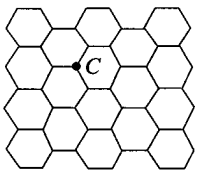
If you look at the floors of houses and shopping centres, you will most probably notice that many of them are tiled. The tiles used are usually in the shape of a square or a rectangle. A pattern formed by fitting together regular figures which completely cover a plane surface is called **tessellation**. The following diagrams show how planes may be tessellated by equilateral triangles, squares and regular hexagons.



A tessellation formed by equilateral triangles.



A tessellation formed by squares.



A tessellation formed by regular hexagons.

What other tessellations are made up of combinations of other regular polygons? Can you design one on your own? What is the sum of the corner angles at each of the points *A*, *B* and *C*?

Find out whether rhombuses, regular octagons and regular decagons tessellate.

However, not all regular polygons tessellate. *For example*, regular pentagons do not tessellate. When they are put together as shown in Fig. 14.7, they leave a gap in between.

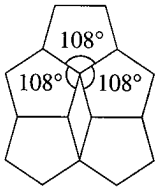


Fig. 14.7

Sometimes a tessellation may be make up of two or more regular polygons. Fig. 14.8 shows a tessellation formed by squares and regular octagons.

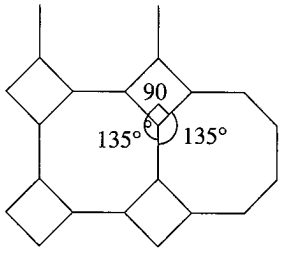
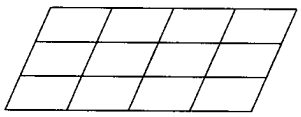


Fig. 14.8

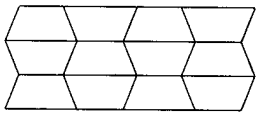
Tessellation formed by squares and regular octagons.

The honeycomb illustrates tessellation found in nature.

We can also tessellate irregular polygons.



Tessellation formed by parallelograms

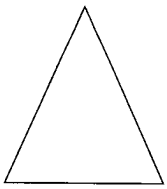


Tessellation formed by isosceles trapeziums (Two sides equal)

In-Class Activity

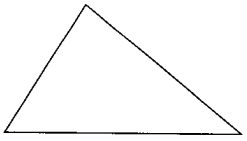
1. Which of the following figures will tessellate?

(a)



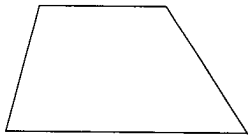
Isosceles triangle

(b)



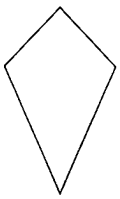
Scalene triangle

(c)



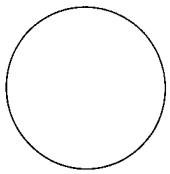
Scalene trapezium (No sides equal)

(d)



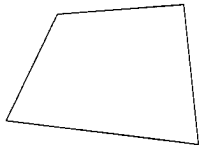
Kite

(e)



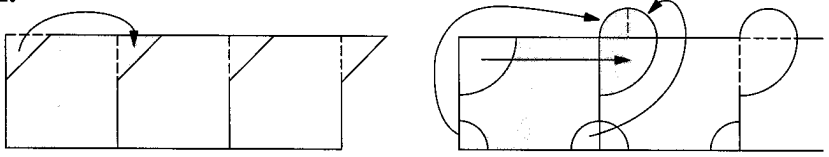
Circle

(f)



Quadrilateral

2.



We can design figures of different shapes which tessellate. In each of the diagrams above, we start with a simple design. Then, we remove a piece from a corner and add it onto the opposite side and we will have a new figure which tessellates. Create a few new tessellating patterns on your own in this way.

The Dutch artist, Maurits Escher, produced many tessellations of fishes, birds, reptiles and human beings. find out more on the works produced by Escher in your library.

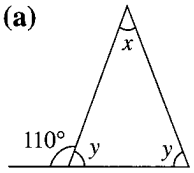
Summary

1. A **scalene triangle** is a triangle with no two sides being equal.
   An **isosceles triangle** is a triangle with two sides equal in length. The base angles of an isosceles triangle are equal in size.
   An **equilateral triangle** is a triangle with all three sides equal in length. The sizes of the three angles are also equal.
2. An **acute-angled triangle** is one where all the three angles are acute, i.e., less than 90°.
   An **obtuse-angled triangle** is a triangle with one of its angles obtuse, i.e., more than 90°.
   A **right-angled triangle** is a triangle with one of its angles equal to 90°.
3. The **sum of the angles of a triangle** is 180°.
   The **exterior angle** of a triangle is equal to the **sum of the interior opposite angles**.
4. A quadrilateral is a 4-sided plane figure. The sum of the angles of a quadrilateral is equal to 360°.

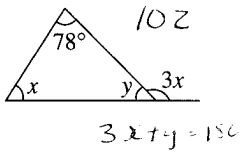
Review Questions 14

✶**1.** For each of the following figures, calculate the values of the unknowns:

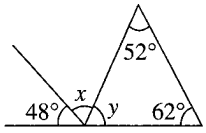
(a)



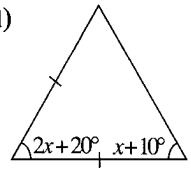
(b)



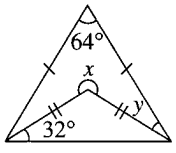
(c)



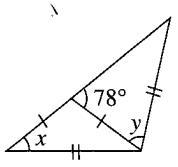
(d)



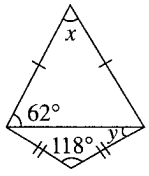
(e)



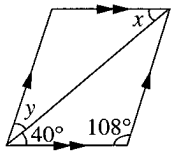
(f)



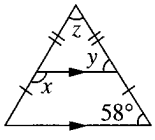
(g)



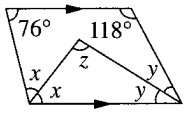
(h)



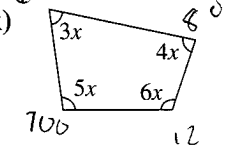
(i)



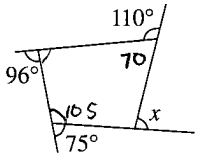
(j)

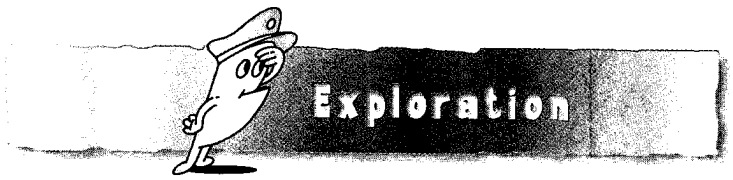


(k)



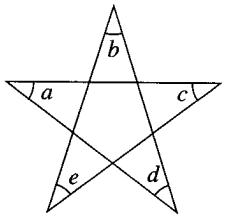
(l)



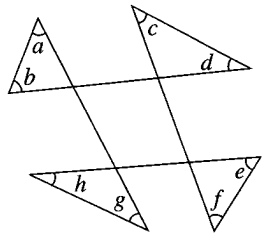


1. Find the sum of the marked angles in each of the following diagrams:

(a)

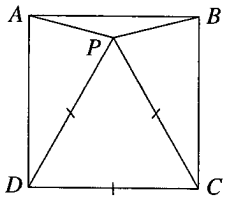


(b)



2. Construct a trapezium $PQRS$ where PQ = 8.4 cm, QR = 4.8 cm, RS = 4.8 cm and $\hat{Q} = 70°$. Measure PS, $\hat{R}$, $\hat{S}$ and $\hat{P}$.

3. $ABCD$ is a square and $\triangle DPC$ is an equilateral triangle. Find $A\hat{P}B$.



CHAPTER 15

Statistics

In this chapter, you will learn how to

- collect, classify and tabulate data;
- read and interpret tables and statistical diagrams;
- construct bar graphs, pie charts, pictograms, stem and leaf diagram, line graphs and histograms with equal intervals.

Preliminary Problem

The government collects numerical information about all aspects of life, including education. Each year, the government has increased the level of spending on education in order to create a first-rate education system. Students including those in the picture benefit from the all-round Singaporean eduation system.

Introduction to Numerical Data

We live in a world of figures. Consider the following statements:

(1) The enrolment of students in a particular school is 1 000.
(2) There are altogether 152 secondary schools in Singapore.
(3) The literacy rate among local residents aged 15 years and above was about 95% in 1999.
(4) Deaths resulting from smoking was about 56 000 in a city in 1998.
(5) Eight out of 10 film stars use a certain brand of perfume.

We can continue to give figures like those shown above. Have you ever wondered how such figures are collected, summarised and finally presented so that a reader can easily understand them? Numerical data can be obtained in many ways. The data collected can be summarised and presented by means of tables and charts, or graphs.

Statistics is a science of collecting, organising, interpreting and analysing data.

Misleading data

Statistical data can be misleading. Therefore, it is often useful to consider some of the following points before we draw our conclusions from given statistical information:

1. *the source and authority of the statistical information,*
2. *the size of the sample involved,*
3. *deliberate omissions of information,*
4. *any bias involved.*

Collection and Organisation of Data

We collect and analyse data to answer questions as well as solve problems.

Consider the following example:

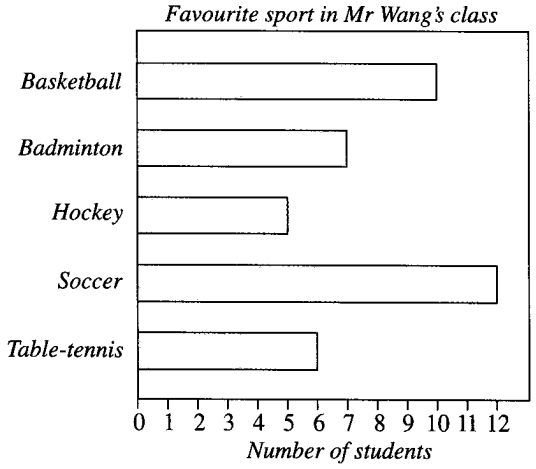
Suppose Mr Wang of ABC Boys' School wants to know the type of sports his 40 students enjoy.

1. To **collect** the information, he gives each student a survey form containing the names of sports. Each student is to select only one favourite sport.

2. When the counting is done, he **organises** the results (information or data collected) in the form of a table as shown below.

Favourite sport	Basketball	Badminton	Hockey	Soccer	Table-tennis	**Total**
Number of students	10	7	5	12	6	40

3. He **presents** the information found in the table using a **diagram**.



4. He **interprets** the information and draws the following conclusions:

(a) The most popular sport among his students is soccer.
(b) More than half of the class prefer either soccer or basketball.
(c) Hockey is the least popular sport.

Are the following statements true?

1. *Students who smoke perform worse academically than non-smokers. Thus, smoking will make one stupid.*
2. *It is more dangerous to drive today than it was in 1950 because statistics have shown that there are more road accidents today than there were in 1950.*
3. *Of all car accidents, 20% are caused by people who drink alcohol and drive, whereas 80% are caused by those who are sober. Therefore, there is no cause for alarm when people drink alcohol and drive.*
4. *Singaporeans eat 1.2 hamburgers on the average per year.*

The example in the previous page shows a very simple way of collecting and organising data. In practice, data can be collected in many ways, depending on the amount of data available and the purpose of the survey.

Collection of Data Using a Questionnaire

Suppose the principal of ABC Boys' School has to plan the Sports Budget. To make a better decision, he wants to know the relative popularity of the sports played in the school. He asks Mr Wang to conduct a survey by giving each student a copy of the questionnaire shown below:

QUESTIONNAIRE
on
Relative popularity of Sports played at ABC Boys' School

Which is your favourite sport?

Check the box next to the sport you like best.

Choose only ONE sport. Badminton ☐ Soccer ☐
Basketball ☐ Table-tennis ☐
Hockey ☐

Mr Wang records the results of the survey in a table as shown below.

Favourite sport	*Tally*	*No. of students*
Badminton	𝍸 /	136
Basketball	𝍸 𝍸	180
Hockey	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ////	84
Soccer	𝍸 //	212
Table-tennis	𝍸 ///	108
	Total	720

He does this in an organised manner by using the method of **tallying**. A tally is put in the corresponding space in the table, matching the checked box in the questionnaire. Tallies are grouped in fives (𝍸) with the fifth tally crossing the first four for counting convenience.

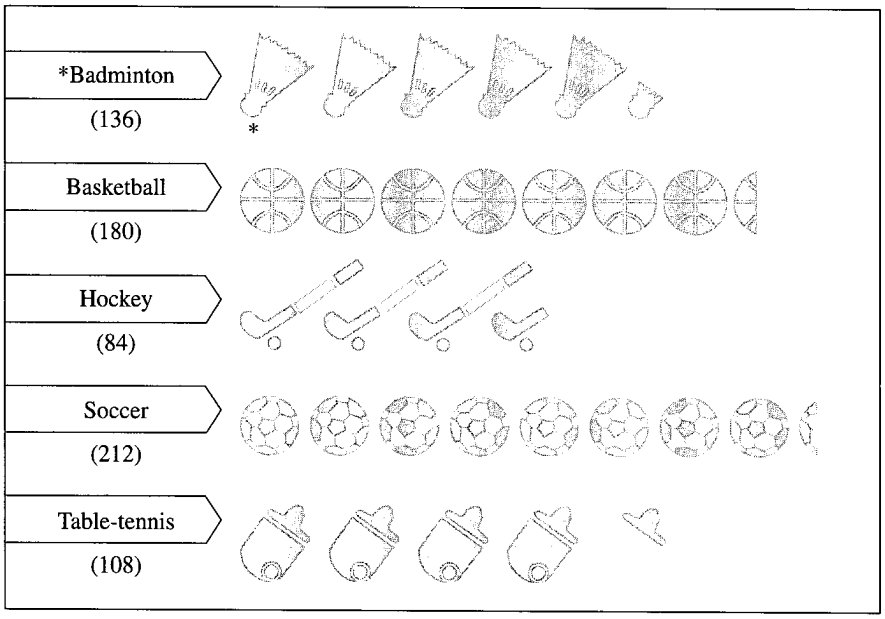
From the table in the previous page, can you tell the relative popularity of the various types of sports in the school? You might have difficulty understanding and interpreting the information displayed in the table. You will probably be able to make better comparisons of the data if you present the information in a diagram as shown in the following sections.

Pictograms

*A **pictogram** uses pictures to represent statistics.*

Mr Wang can draw different diagrams to display the results of the survey, so as to give a clearer picture of the relative popularity of the various sports. One such diagram is a **pictogram**, shown in Fig. 15.1 below.

Popularity of sports played at ABC Boys' School



Each figure represents 25 pupils.

Fig. 15.1

Each figure in the pictogram above represents 25 pupils and a fraction of a figure means a corresponding fraction of 25. It should be noted that this is not a very accurate method of showing the **exact** number of pupils. It merely gives us a quick comparison of the relative popularity of the sports played in the school.

Bar Graphs

Mr Wang may also represent the information in a bar graph as shown in Fig. 15.2.

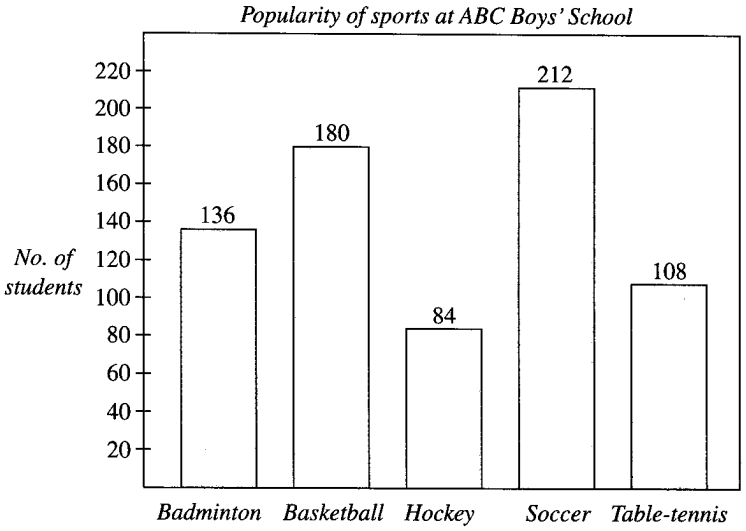


Fig. 15.2

The vertical axis at the side shows the number of students in accordance to the different categories of their favourite sports, with these categories being labelled along the horizontal axis. A bar is constructed representing each category, with the length of the bar proportional to the number of students in that category. The bars should be of the same width. Note that there is a space between each category on the horizontal axis to distinguish clearly between the categories.

One can see at a glance that soccer is the most popular sport as it has the longest bar.

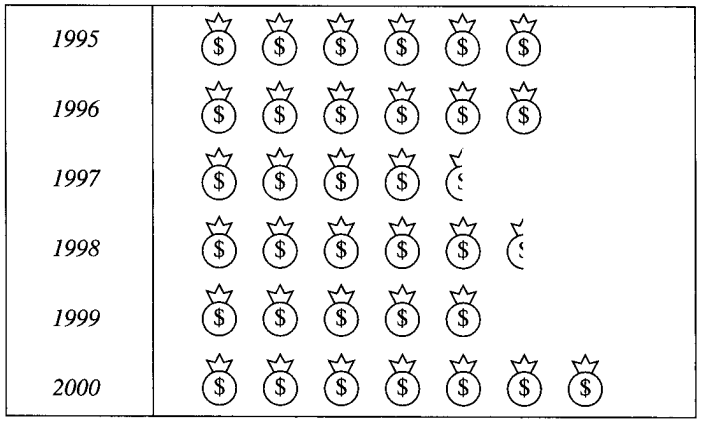
Can you tell which are the second, third and fourth most popular sports? Which is the least popular sport?

Example 1

The table below shows the profits, after taxation, of a company from 1995 to 2000.

(a) What was the profit in
(i) 1998?
(ii) 2000?

(b) In which year was the profit smallest? By how much had the profit decreased that year from the previous year?



Each (\$) *represents 1 million dollars.*

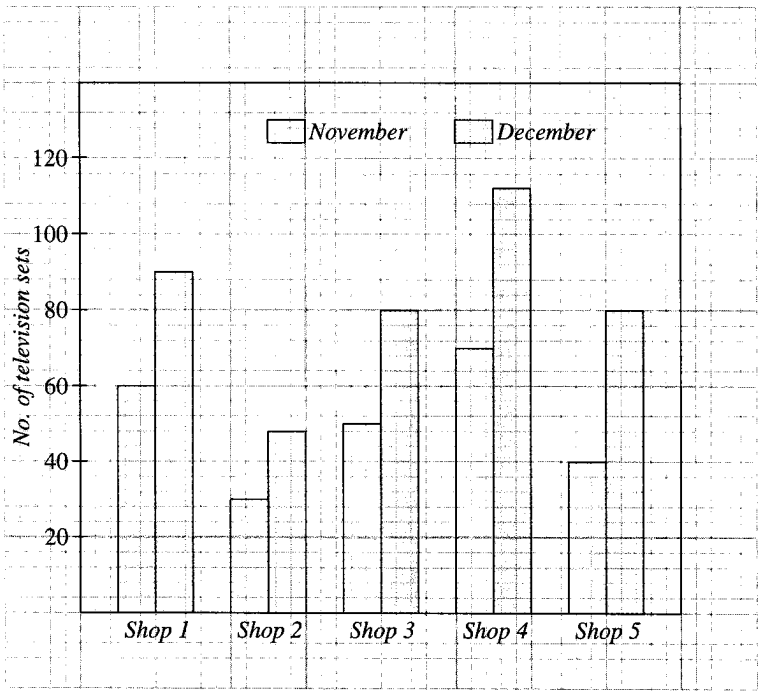
Solution

(a) The profit in 1998 was about $5\frac{1}{2}$ million dollars and in 2000 it was 7 million dollars.

(b) In 1997 the profit was the smallest and it had decreased from 1996 by about $1\frac{1}{2}$ million dollars.

Example 2

A company owns five electrical shops. The bar graph at the side shows the number of television sets sold in the five shops in November and December in a certain year.



Study the graph above and answer the following questions:

(a) Find the total number of television sets sold in the five shops in
(i) November;
(ii) December.

(b) Express the total number of television sets sold in December as a percentage of the total number of television sets sold in November and December.

(c) (i) Which shop enjoyed the greatest increase in the sales of television sets?
(ii) Express the increase as a percentage of the number of television sets sold in November.

Solution

(a) (i) The total number of television sets sold in the five shops in November
$= 60 + 30 + 50 + 70 + 40$
$= 250$

(ii) The total number of television sets sold in the five shops in December
$= 90 + 48 + 80 + 112 + 80$
$= 410$

(b) Percentage of the total number of television sets sold in December

$$= \frac{410}{250 + 410} \times 100\%$$

$= 62\%$ (correct to the nearest whole number)

(c) (i) Shop 4 enjoyed the greatest increase in sales.

(ii) Percentage increase in sales $= \frac{112 - 70}{70} \times 100\%$

$= 60\%$

Exercise 15a

1. The following pictogram shows the average weekly pocket money that students from each of the five secondary two classes receive.

Class	*Average weekly pocket money for five secondary two classes*
Sec 2A	□ □ □ □
Sec 2B	□ □ □
Sec 2C	□ □ □ □ □ □ □
Sec 2D	□ □ □ □ □ □
Sec 2E	□ □ □ □ □

Each □ *represents \$10.*

(a) What is the ratio of Sec 2D's average weekly pocket money to Sec 2B's average weekly pocket money?

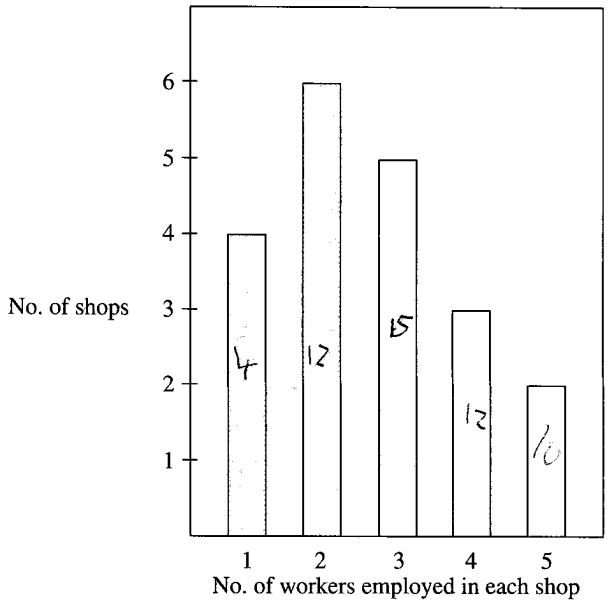
(b) Express Sec 2A's average weekly pocket money as a percentage of Sec 2D's average weekly pocket money.

2. The bar graph illustrates the results of a survey carried out in the shops of a certain housing estate.

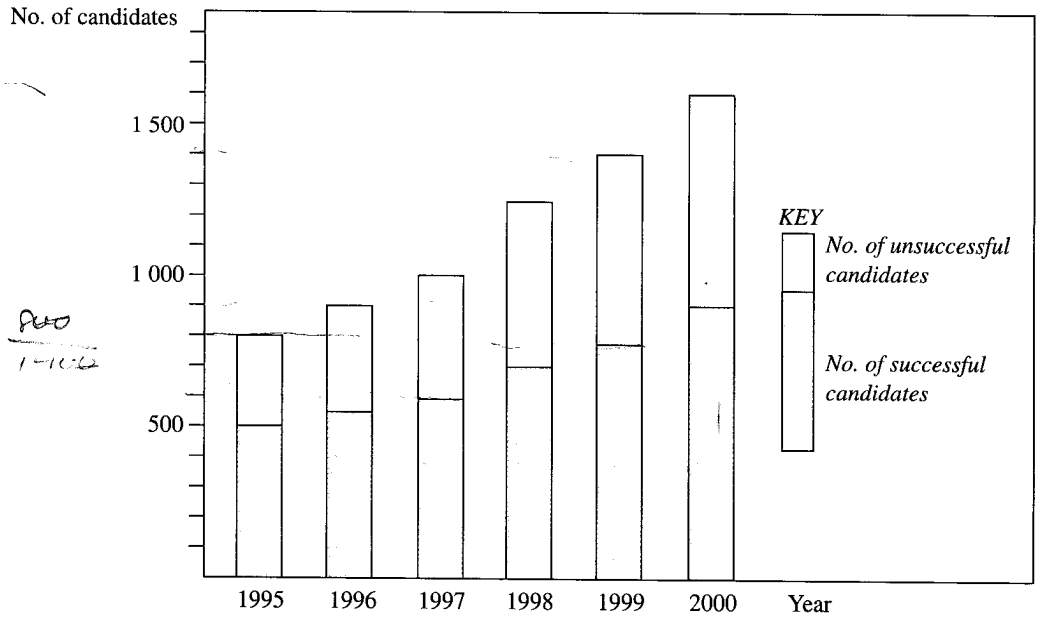
Calculate

(a) the total number of workers;

(b) the percentage of shops hiring 3 or more workers.

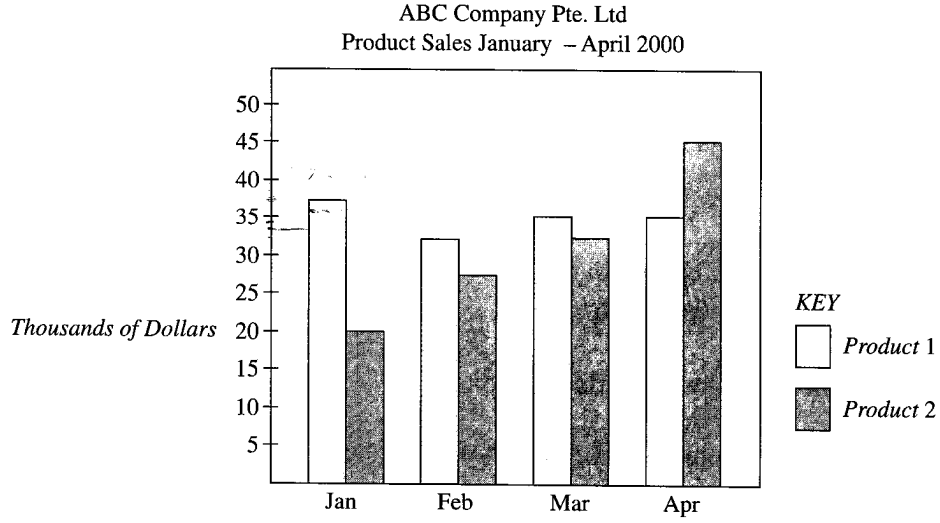


3. The graph below shows the number of candidates who sat for an examination and the number who were successful.



(a) How many candidates sat for the examination in 1997?
(b) How many passed in 2000?
(c) What fraction of the number who took the examination in 1999 passed?

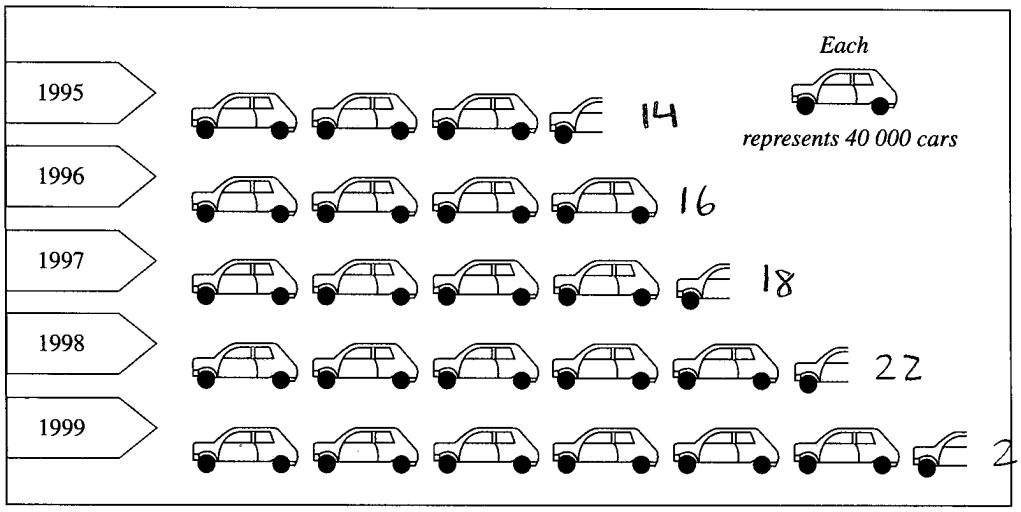
4. The graph below shows the product sales of ABC Company in the first four months of 2000.



(a) Which product has almost the same amount of sales over the four-month period?
(b) Which month showed the greatest increase in the sales of product 2?
(c) How much were the sales of product 1 in January?
(d) How much were the sales of product 2 in April?

(e) What were the total sales of product 1?
(f) What were the total sales of product 2?
(g) What fraction of the total product sales of the company did the sales of product 1 make up during the first four months?

5. The pictogram below illustrates the number of private cars registered each year from 1995 to 1999 in a certain city.



(a) In which year was the greatest number of vehicles registered? Estimate the number of vehicles registered in that year.
(b) Estimate the number of cars registered in each of the other years.
(c) If, in 1997, the registration fee for each car was $500, estimate the total amount the Registry of Vehicles collected for that year.
(d) Give an estimate of the percentage increase in the number of vehicles registered from 1998 to 1999.

6. The circulation of a certain newspaper from 1995 to 1999 is given as follows:

Year	1995	1996	1997	1998	1999
No. of copies (in thousands)	250	275	290	315	280

Use a bar graph to illustrate this information.

7. In a certain city, the average daily number of traffic accidents in a week is given as follows:

Day	Sun	Mon	Tues	Wed	Thur	Fri	Sat
No. of accidents	70	40	30	35	50	55	80

Illustrate this information with a bar graph.

8. The table below shows the number of students who play squash, tennis and badminton. Illustrate the data using a pictogram.

Sport	Squash	Tennis	Badminton
No. of students	40	60	50

(a) Find the ratio of the number of squash players to the number of badminton players.
(b) Calculate the percentage of pupils who play badminton.

Collection of Data Through Observation

In-Class Activity

(a) Carry out a traffic survey by watching and noting the types of vehicles which pass by your school during a 15-minute period.

(b) Copy and complete the survey sheet below.

Name : _____ Date : _____

Location of survey : _____ Time : From _____ To _____

Types of vehicles	*Tally*	*No. of vehicles*
Bus		
Taxi		
Private car		
Motorcycle		
Lorry		
Pick-up or Van		
Total		

Note: You may change the list of vehicles to suit your situation. Students in the class should work in small groups and choose different times and locations to conduct the survey.

(c) Display the information you collected in the form of a pictogram or a bar chart. Remember to give a short title for your diagram.

(d) With the help of your diagram, interpret the information and draw some conclusions.

Pie Charts

*A **pie chart** represents relative quantities by areas of sectors of a circle.*

If the principal of ABC Boys' School wants to know what fraction of the school chose a particular sport, then the diagram in Fig. 15.3, a **pie chart**, would be useful to him. The pie chart displays clearly the fraction of the school choosing a particular type of sport.

Fig. 15.3 is constructed by dividing a circle into different sectors. Each sector corresponds to the percentage of a category of students who like a particular sport. The angle of each sector is proportional to the number the sector represents.

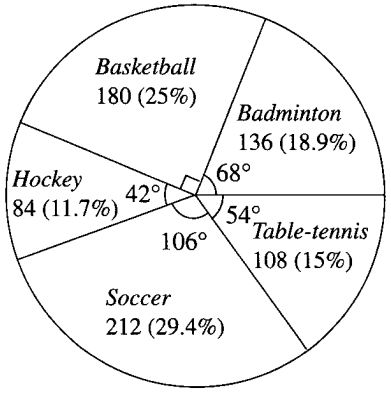


Fig. 15.3

The angle of each sector can be found easily, e.g. if 212 out of 720 students prefer soccer, then the angle of the sector for soccer is $\frac{212}{720}$ of $360° = \frac{212}{720} \times 360° = 106°$.

These days, we usually use software programmes on the computer to help us construct bar graphs, pie charts, etc.

The table below shows the percentage of each category of students and the angle of the sector for each category.

Favourite Sport	*No. of students*	*Percentage*	*Angle of sector*
Badminton	136	$\frac{136}{720} \times 100\% = 18.9\%$	$\frac{136}{720} \times 360° = 68°$
Basketball	180	$\frac{180}{720} \times 100\% = 25\%$	$\frac{180}{720} \times 360° = 90°$
Hockey	84	$\frac{84}{720} \times 100\% = 11.7\%$	$\frac{84}{720} \times 360° = 42°$
Soccer	212	$\frac{212}{720} \times 100\% = 29.4\%$	$\frac{212}{720} \times 360° = 106°$
Table-tennis	108	$\frac{108}{720} \times 100\% = 15\%$	$\frac{108}{720} \times 360° = 54°$

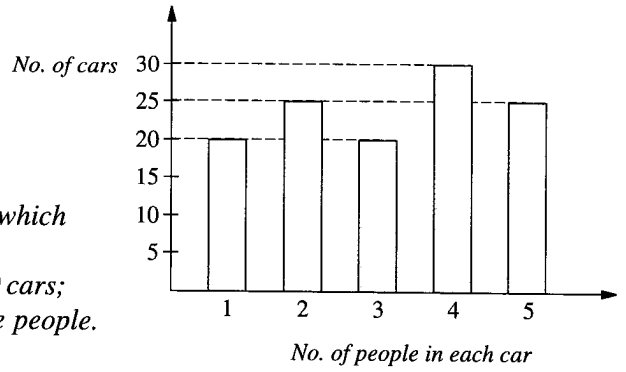
Although pie charts are particularly useful where the proportions of a whole are more important than the actual numerical values, they have some disadvantages. For instance, it is difficult to interpret a pie chart representing data which involves too many categories. In constructing pie charts, long calculations are often needed and actual measurements of angles using protractors are necessary to produce an accurate diagram.

Example 3

The bar graph illustrates the results of a survey conducted to find the number of people in each car in a random survey of 120 cars at a traffic junction.

Calculate

(a) the angle in a pie chart of the sector which represents cars with 2 people;
(b) the total number of people in these 120 cars;
(c) the percentage of cars with 4 or more people.



Solution

(a) No. of cars with 2 people = 25
$\therefore$ the angle of the sector in a pie chart representing this is $\frac{25}{120} \times 360° = 75°$

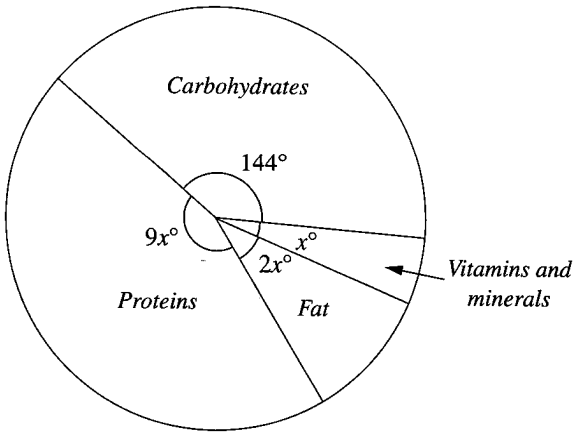
(b) Total no. of people in these 120 cars = $(1 \times 20) + (2 \times 25) + (3 \times 20) + (4 \times 30) + (5 \times 25)$
$= 375$

(c) No. of cars with 4 or more people = 30 + 25 = 55
$\therefore$ the percentage of cars with 4 or more people = $\frac{55}{120} \times 100\% = 45.8\%$

Example 4

The pie chart shows the nutritional composition of a fast-food product.

(a) Calculate the value of x.
(b) What is the percentage of fat in the fast-food product?
(c) Given that one such fast-food product contains 120 grams of carbohydrates, calculate the total weight of the fast-food product.



Solution

(a) $144° + 9x° + 2x° + x° = 360°$
$12x = 216$
$\therefore \quad x = 18$

(b) The angle representing fat in the product $= 2 \times 18°$
$= 36°$

$\therefore$ the percentage of fat in the fast-food product $= \frac{36°}{360°} \times 100\%$
$= 10\%$

(c) The angle representing carbohydrates is 144° and this constitutes 120 g.

$\therefore$ the total weight of the product $= \left(\frac{360}{144} \times 120\right)$ g
$= 300$ g

Collection of Data Through Interviews

In-Class Activity

(a) Conduct interviews with students in your class to find out their daily means of transport to school.
(b) Decide on the number of students you want to interview.
(c) Think of where and when to interview the students, e.g. in the canteen during recess time, or in the morning before students go to their classes for lessons. Do not forget to thank the students you have interviewed for their participation.
(d) Design a form for recording the results.
(e) Use a diagram, preferably a pie chart, to display your data.
(f) Interpret the information from your diagram and draw your conclusions.

Exercise 15b

1. The main products of a manufacturing company are corn oil (50%), margarine (30%), peanut oil (15%) and others (5%). Display the above data using a pie chart.

2. 72 students of a certain school were asked to indicate a place of interest in Singapore they would like to visit during their holidays. The table below shows the results.

Place of interest	Bird Park	Chinese Gardens	Science Centre	Sentosa Island	Zoo
No. of students	9	5	19	24	15

Use a bar graph to illustrate the results.

3. 100 people took part in a survey on their favourite fruit. The results were tabulated as follows:

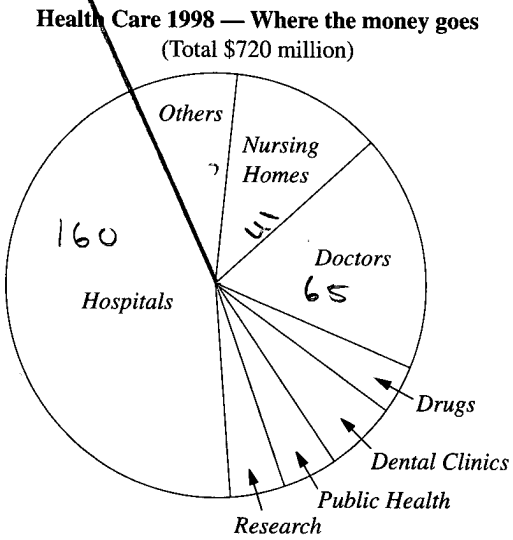
Local Fruit	Banana	Durian	Mangosteen	Papaya	Rambutan
No. of people	30	25	10	15	20

(a) Illustrate the results using a
(i) bar graph; **(ii)** pie chart.

(b) Which diagram shows more clearly that one quarter of the people like durian most?

(c) Which diagram shows more clearly that banana is more popular than durian?

4. The pie chart below shows the health care expenditure for the year 1998:



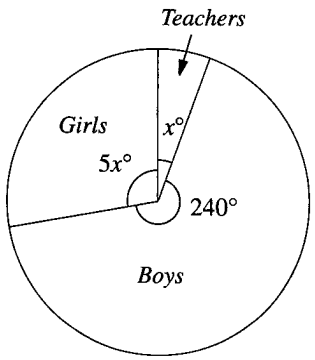
(a) Measure the angle of each sector using a protractor and calculate the amount of money allocated to each category.

(b) Express the expenditure of each category as a percentage of the total expenditure on Health Care.

5. The pie chart shows the number of pupils and teachers in a certain school.

(a) Calculate the value of x.

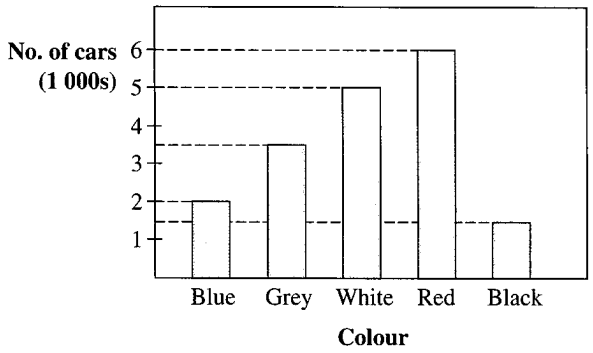
(b) If there are 45 teachers in the school, how many
(i) boys are there in the school?
(ii) girls are there in the school?



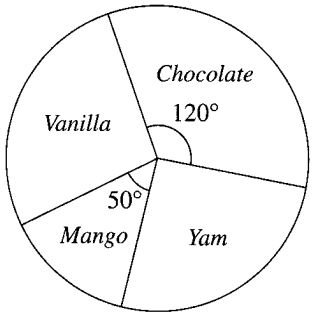
6. The bar graph shows the number of cars of different colours sold in one year in a certain city.

(a) Which colour is the most popular?

(b) Illustrate the information using a clearly labelled pie chart.



***7.** Pupils in a class were asked to indicate which one of the four ice-cream flavours – vanilla, chocolate, yam or mango – they preferred. The following pie chart shows the results.



(a) If one-quarter of the class preferred yam flavour, state the angle in the yam sector.

(b) Calculate the angle in the vanilla sector.

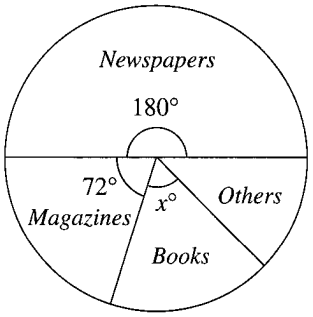
(c) If 5 students indicated a preference for mango flavour, calculate the number of students in the class.

(d) Calculate the percentage of students in the class who preferred vanilla flavour.

***8.** The daily output of two products, X and Y, in a factory are 6 tonnes and 14 tonnes respectively. If the output is represented by a pie chart, calculate the angle of the sector representing the output of product Y.

***9.** A factory produces three products, A, B and C, in the ratio of $1 : x : 5$. When the output is illustrated by a pie chart, the angle of the sector representing the output of C is 120°. Find x.

***10.** The pie chart shows the sales of a publishing company.



(a) What percentage of the total sales does each of the following make up?

(i) newspapers **(ii)** magazines

(b) Given that books make up $17\frac{1}{2}\%$ of the total sales, find x.

The table below shows the government recurrent expenditure on university education in thousand dollars.

Year	87/88	88/89	89/90	90/91	91/92	92/93
Expenditure ('000)	293 443	259 014	288 571	351 476	841 154	412 524

Year	93/94	94/95	95/96	96/97	97/98
Expenditure ('000)	431 554	502 999	520 289	546 120	669 004

By plotting the points corresponding to the data and then joining the points by line segments, we obtain the line graph as shown in Fig. 15.4.

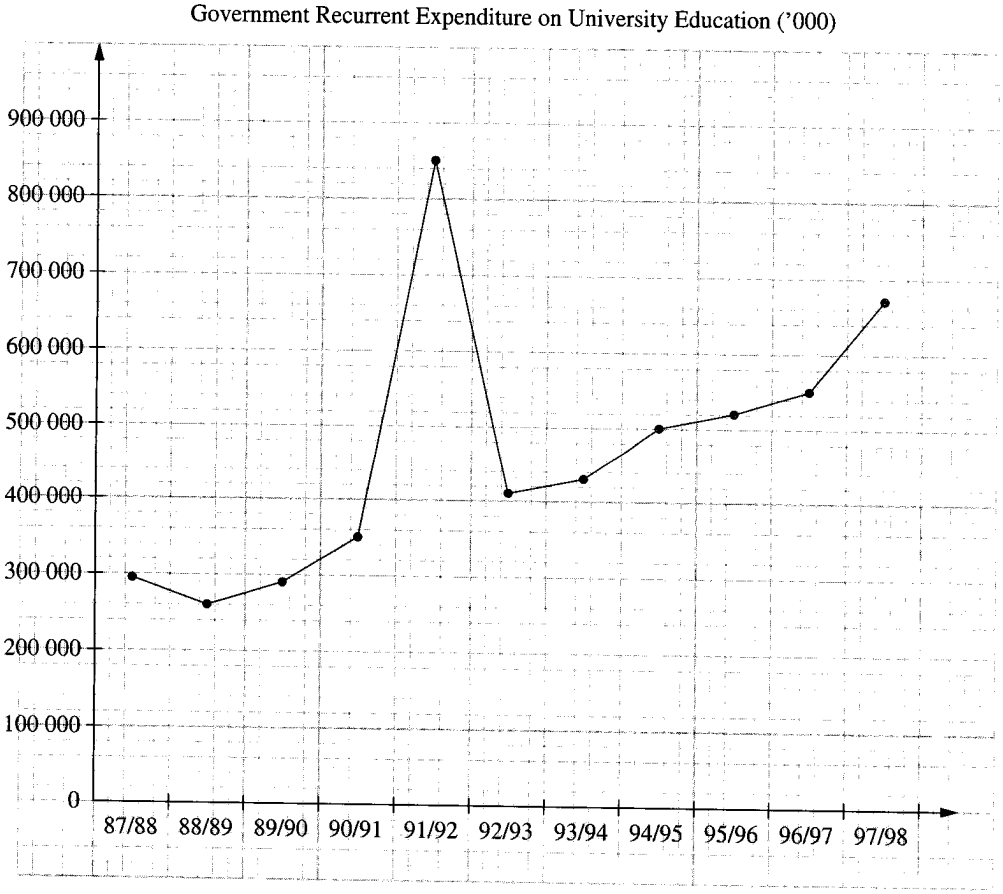


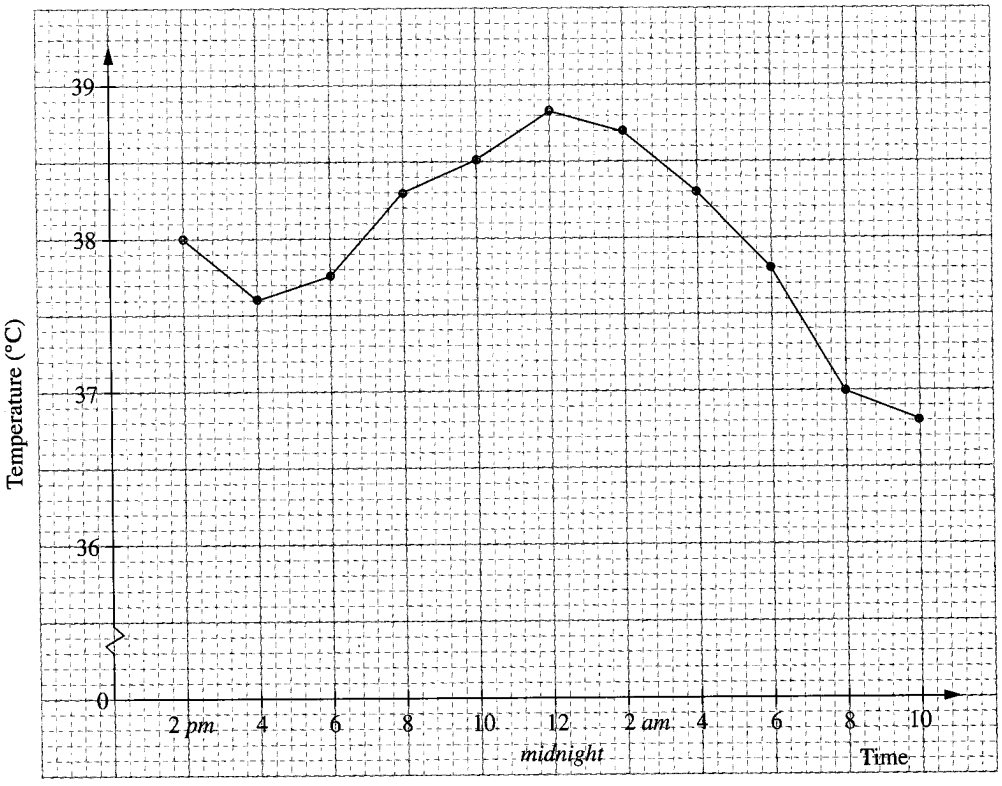
Fig. 15.4

A line graph is a suitable graph to construct when we wish to show a rising or a falling trend in a set of data over a period of time. The graph in Fig. 15.4 shows that the government recurrent expenditure on university education reached its lowest point in the 88/89 fiscal year, after which it increased gradually for the next two years. The expenditure hit the highest point in the fiscal year 91/92 following a big jump from the previous year. The expenditure then took a dip in the year 92/93 after which it rose gradually for the next five years.

NB: Although adjacent points are joined by a line segment, the intermediate values, other than the values recorded, have no meaning.

Example 5

The following graph shows the temperatures of a child who developed fever. His temperature was taken every two hours starting from 2 p.m.



(a) At what time was his temperature the highest on the graph?
(b) What was his temperature at 4 p.m., 9 p.m. and 3 a.m.?
(c) When was the child's temperature 37°C, 37.4°C and 38°C?

Solution

(a) His temperature was the highest at 12 midnight.

(b) His temperature at 4 p.m. was 37.6°C. His temperatures at 9 p.m. and 3 a.m. can be estimated using intermediate values because it is reasonable to assume that the temperature changes gradually within a relatively short period of 2 hours.

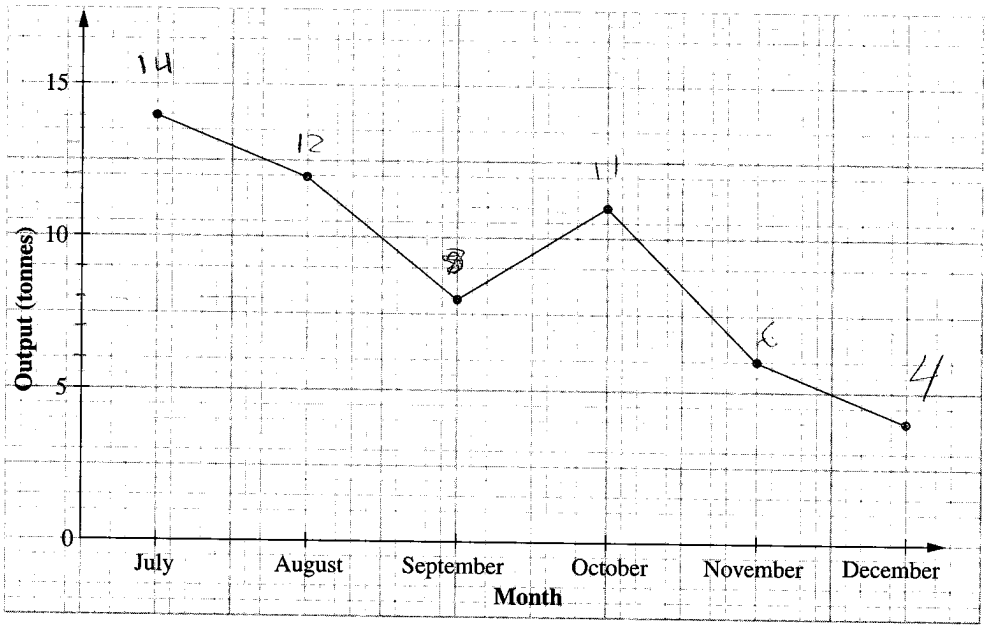
From the graph,
his temperature at 9 p.m. was about 38.4°C;
his temperature at 3 a.m. was about 38.5°C.

(c) The child had a temperature of 37°C at 8 a.m., 37.4°C at 7 a.m., 38°C at 2 p.m., at about 6.50 p.m. and at about 5.10 a.m.

Exercise 15c

1. The line graph below shows the monthly tin output of a tin mine from July to December in 1999. Study the graph and estimate

(a) the largest monthly output;
(b) the smallest monthly output;
(c) the total output for the six months.

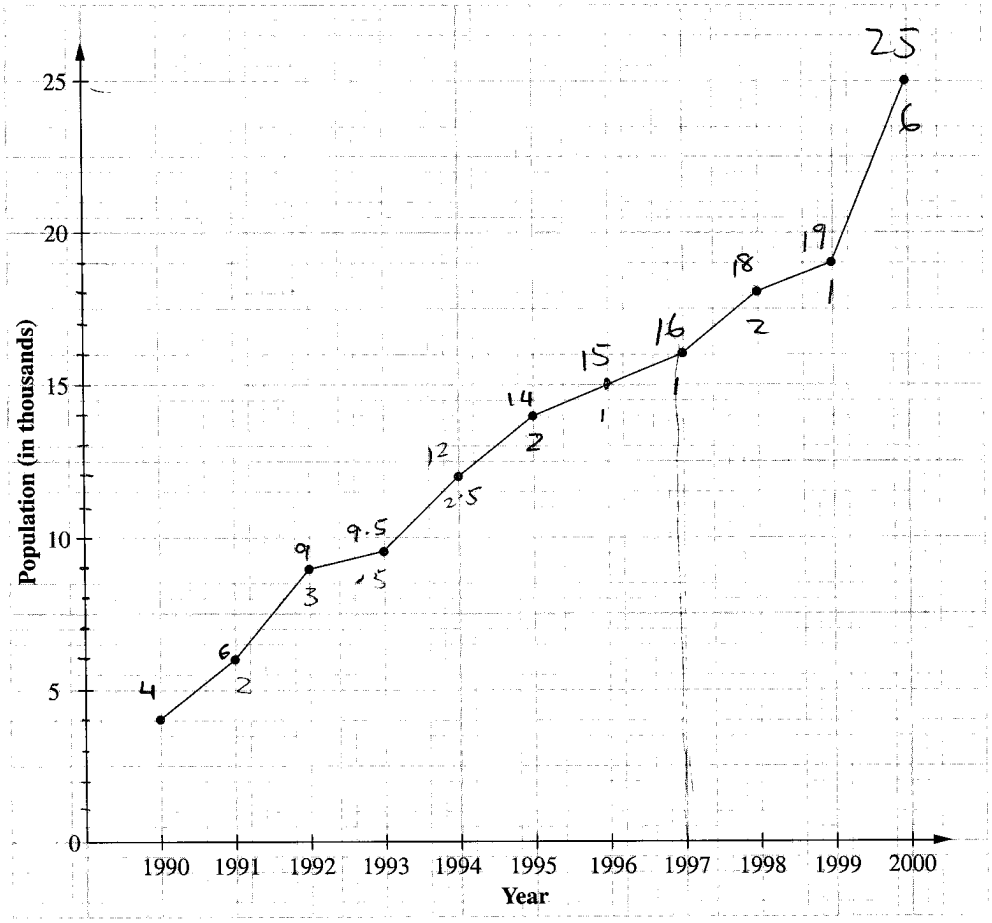


2. The temperature of a patient, taken every 3 hours, was recorded as shown in the table below:

Time	3 p.m.	6 p.m.	9 p.m.	12 midnight	3 a.m.	6 a.m.	9 a.m.
Temperature (°C)	39	39	39.5	37.5	39	38	37

(a) Display the data using a line graph.
(b) From the graph, estimate the patient's temperatures at 5 p.m. and at 1 a.m.

3. The line graph below shows the population of a town from 1990 to 2000.



(a) Between which two years did the town have the greatest number increase in population?
(b) Find the percentage increase in the population from 1997 to 2000.

Dot Diagram

A dot diagram, or a dot plot, provides an easy way to organise data. A dot diagram consists of a horizontal number line and dots placed above the number line. The dots represent the values in a set of data.

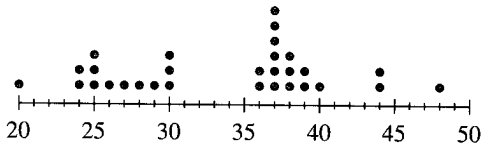
Consider the marks scored by thirty students in a test.

48	20	27	26	24	39	40	30	37	37
29	44	37	38	25	44	25	30	28	37
38	24	30	36	39	37	25	38	37	36

Rearrange the marks in numerical order from the smallest to the biggest.

20, 24, 24, 25, 25, 25, 26, 27, 28, 29, 30, 30, 30, 36, 36, 37, 37, 37, 37, 37, 37, 38, 38, 38, 39, 39, 40, 44, 44, 48

Now create the dot diagram as shown below.

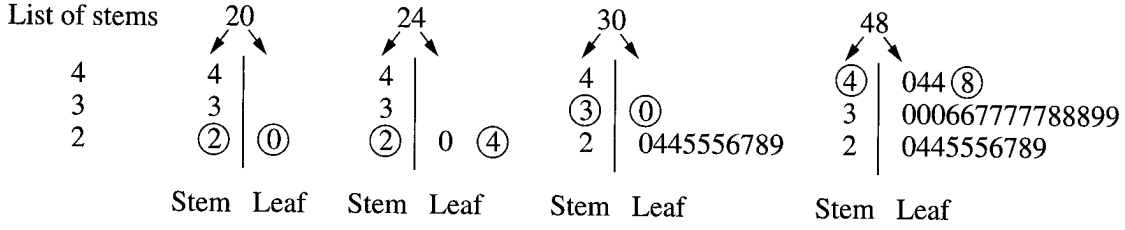


The diagram shows all the marks scored by the thirty students. It shows that the lowest score is 20, and that the highest score is 48. Most students score more than 35 marks. The most common score is 37.

Stem and Leaf Diagram

The stem and leaf diagram is closely related to the dot diagram. In the stem and leaf diagram, however, numerical digits are used to present the data, instead of using number lines and dots. In constructing the stem and leaf diagram, two parts, a stem and a leaf, are extracted from each value.

Consider the set of same marks scored by the thirty students mentioned above. The first step in constructing the stem and leaf diagram is to arrange the marks in numerical order, the same way as that when constructing the dot diagram. Next, separate the numbers into their stem and leaf parts. For example, the first number is 20. We split it into its stem digit '2' and its leaf digit '0', and the leaf digit is written to the right of the corresponding stem as shown below. The same process is then repeated for each of the remaining numbers.



Alternatively, we may arrange the numbers as shown below first.

40 44 44 48
30 30 30 36 36 36 37 37 37 37 37 37 38 38 38 39
20 24 24 25 25 25 26 27 28 29

Next, split each number in the first column into its stem and leaf parts.

S	**L**
4	**0** 44 44 48
3	**0** 30 30 36 36 36 37 37 37 37 37 37 38 38 38 39
2	**0** 24 24 25 25 25 26 27 28 29

This is followed by removing the stem digit of each of the other numbers.

S	L														
4	**0**	4	4	8											
3	**0**	0	0	6	6	6	7	7	7	7	7	7	8	8	8	9
2	**0**	4	4	5	5	5	6	7	8	9						

The final step is to simply bring the leaf digits closer together.

S	L
4	**0**448
3	**0**006667777778889
2	**0**445556789

To get a better picture of the distribution of the marks, we may choose to display a stem twice, i.e. one stem for the leaves 0 – 4 and the other for the leaves 5 – 9, as shown below.

4	8
4	044
3	667777778889
3	000
2	5556789
2	0444

From this stem and leaf diagram, the quick impression we have of the test is that most students score below 40 marks. No student scores below 20 marks and one student scores a high mark of 48, assuming that the full mark is 50. The most common score is 37. The test appears to be an easy one that discriminates the students quite well. The bulk of the students' score from 25 marks to 39 marks, with a few good students scoring high marks of 44 and above, and a few weak students scoring comparatively low marks of 24 and below.

Exercise 15d

1. The following data represent the travel times, in minutes, from home to office of 30 company executives:

68	37	18	48	25	12	64	35	40	43
34	28	54	57	43	31	38	43	50	39
44	41	26	17	19	12	35	53	60	48

(a) Represent this data set in a dot diagram.
(b) Represent this data set in a stem and leaf diagram.
(c) What is the most common travel time?
(d) What is the percentage of executives who take less than half an hour to reach the office?

2. The following are the weight, in kg, of 40 boxes.

80	83	84												
86	86	86	87	88	88	89	89							
90	90	90	90	90	90	90	91	91	92	92	93	93	93	94
95	96	96	97	97	99									
100	101	102	104	104										
106	108	108												

(a) Represent this data set in a dot diagram.
(b) Copy and complete the stem and leaf diagram below.

8	0		
8	6	6	
9	0	0	0
9	5	6	
10	0	1	
10	6		

(c) Write down the most common mass.
(d) 50% of the boxes have a mass of below a kg each. Find the value of a.

3. The following data represents the attention span, in minutes, of 30 preschool-age children.

~~5.4~~	~~2.0~~	~~6.3~~	~~5.5~~	~~3.0~~	~~5.4~~	~~3.6~~	~~5.4~~	~~3.3~~	~~4.6~~
~~7.3~~	4.4	~~6.6~~	~~5.4~~	~~5.2~~	~~7.0~~	~~3.8~~	~~5.8~~	~~5.7~~	~~4.5~~
~~5.7~~	~~4.9~~	~~6.5~~	~~2.7~~	~~6.1~~	~~7.2~~	~~2.2~~	~~6.7~~	~~4.1~~	~~5.6~~

(a) Draw a dot diagram to represent the data.
(b) Copy and complete the stem and leaf diagram below.

7	0		
6	1	3	
5	2	4	4
4	4	5	
3	0	3	
2	0		

NB: The leaf unit = 0.1

(c) What is the most common attention span?
(d) What is the percentage of children with attention spans falling below 6 minutes?

4. The following diagram represents the scores of the students in two different schools for a common examination. In each school, 29 students took the examination.

ABC School								XYZ School							
Leaves								Stem	Leaves						
						4	0	5	2	6	8	9			
					9	9	6	6	2	5	8	8	9	9	
			9	8	5	3	2	0	7	4	6	7	8	8	9
		9	9	7	7	6	6	4	2	8	0	3	4	4	6	7	7
	9	8	8	7	6	6	3	2	0	9	0	2	7	8			
										10	0	0					

(a) Which school had the "high scorer"?
(b) Which school had the "low scorer"?
(c) Which school did better in the examination?
(d) Combining the scores of the two schools, draw a dot diagram and also a stem and leaf diagram representing all 56 values.

Frequency Tables

Consider the marks scored by 40 students in a Science test marked out of a total of 10.

8	6	4	3	5	5	2	9	2	7
9	3	3	7	7	5	8	3	7	3
4	8	7	8	2	4	6	2	4	1
7	7	6	2	6	4	4	6	10	6

The teacher would like to know the overall performance of the students after the test to get a feedback on the students' progress. He may be interested in:

(a) the average score
(b) the middle score
(c) the most common score
(d) the highest score
(e) the lowest score
(f) the range of scores
(g) whether the test was too difficult or too easy.

He can find what he wants from the list of marks. But he will have some difficulty in doing so because the marks are not arranged in some convenient order. Hence, as discussed earlier, a set of unsummarised data, which we call raw data, has to be simplified and then arranged in an orderly fashion first, so that we will be able to understand and use them later.

We notice from the list of marks that some marks appear more than once. We arrange the marks according to the number of times each mark appears on the list.

Mark	*Tally*	*Frequency*
0		0
1	/	1
2	𝍸	5
3	𝍸	5
4	𝍸 /	6
5	///	3
6	𝍸 /	6
7	𝍸 //	7
8	////	4
9	//	2
10	/	1
Total frequency		40

The marks are arranged in order of magnitude. We go through the list of marks and keep a tally as shown in the table above. The number of times each mark appears is called its **frequency**. The table which gives the frequency of each score is called a **frequency table**.

Histograms

A diagram can be constructed to illustrate the information given in the frequency table. Such a diagram is called a **histogram**. The histogram is easier to understand than the frequency table.

Fig. 15.5 below shows the histogram representing the frequency of the marks obtained in the Science test taken by 40 students.

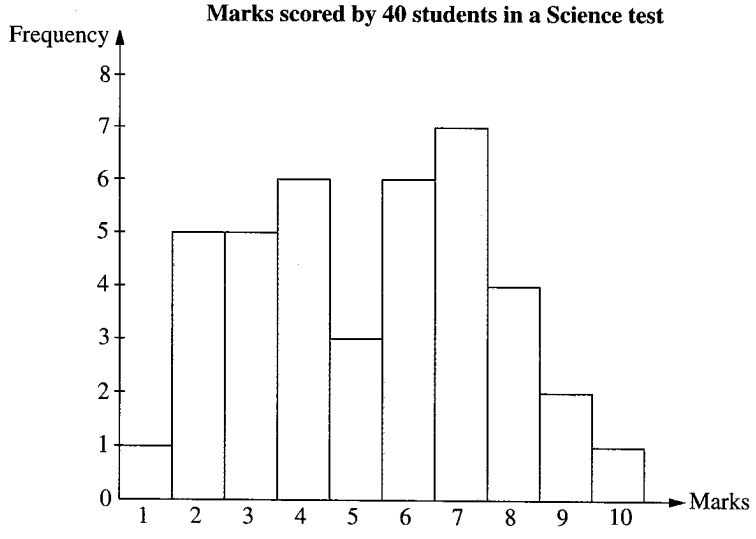


Fig. 15.5

A histogram is actually a vertical bar graph with no space in between the bars. However, the areas of the bars or the rectangles, and not the heights, are proportional to the numbers they represent. In our example, the bases of the rectangles are equal. So the height of each rectangle will be proportional to the frequency.

In-Class Activity

Using the histogram in Fig. 15.5, try answering questions like:

(1) What was the most common score?
(2) What was the middle score, that is, the score with half of the pupils scoring less than it?
(3) What was the highest score?
(4) What was the lowest score?
(5) What was the range, that is, the difference between the highest and the lowest scores?

Collection of Data by Measuring

This method of collecting data is useful in controlling the quality of products from a production line. For example, in a factory producing electrical light bulbs, the quality control process may involve measuring the lifespan of a certain number of bulbs selected at random from the production line. The results of the actual calculations involving this number of bulbs are then analysed to determine whether the electrical light bulbs produced overall are up to the acceptable standard.

In-Class Activity

(a) Measure the lengths, correct to the nearest cm, of the shoes worn by each of your classmates.
(b) Record the results and arrange them in the form of a frequency table.
(c) Display your information using a histogram.
(d) Comment on your results.

Exercise 15e

1. In a spelling test, the number of mistakes incurred by each of the 30 pupils in a primary one class is given below:

3 4 6 0 2 2 4 3 5 3
4 2 2 3 1 5 3 0 4 5
4 3 4 0 3 2 6 3 1 0

(a) Construct a frequency table for the number of spelling mistakes.
(b) Draw a histogram to illustrate the results.
(c) What is the most common number of mistakes?
(d) What is the highest number of mistakes?

2. The teachers of a certain school were asked to indicate the average number of hours they spend on marking students' assignments each day. The following set of data was obtained.

6	4	3	1	2	2	3	1	4
1	2	5	3	4	5	2	2	3
3	1	2	2	3	1	4	2	

(a) Construct a frequency table and draw a histogram illustrating the results.
(b) How many teachers responded to the survey?
(c) What is the longest number of hours spent?
(d) What is the most common number of hours spent?

***3.** 100 crates of oranges imported from Country *A* were inspected and the number of rotten oranges recorded is shown below:

9 24 84 88 75 30 14 16 9

Rotten oranges	0	1	2	3	4	5	6	7	8	9
No. of crates	4	9	12	28	22	15	5	2	2	1

Another 100 crates of oranges imported from Country *B* were also inspected and the number of rotten oranges recorded was:

30 16 12 4 10 17 7 8

Rotten oranges	0	1	2	3	4	5	6	7	8
No. of crates	51	30	8	4	1	2	2	1	1

(a) Draw a histogram to represent each frequency table.
(b) What is the largest number of rotten oranges for each exporter?
(c) Find the total number of rotten oranges for each exporter.

Summary

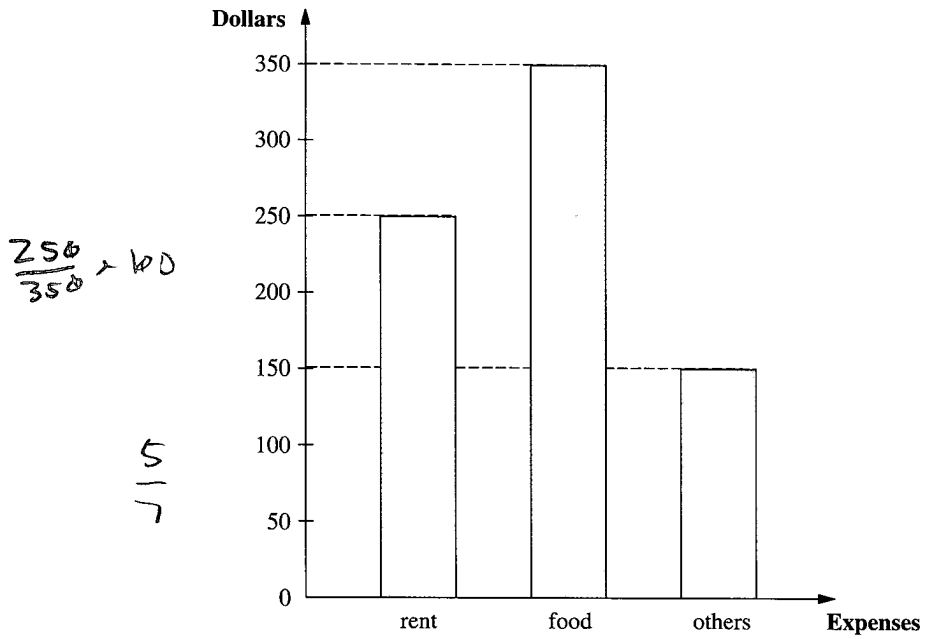
1. **(a)** Numerical data can be obtained in many ways.
(b) The data collected can be summarised in a systematic way by tabulation.
(c) The tabulated data is usually presented in a graphical form.
(d) Some common statistical graphs are **pictograms, bar graphs, pie charts** and **line graphs**.

2. Dot diagrams as well as stem and leaf diagrams provide an easy way of organising data.
In a dot diagram, values are presented by dots above a horizontal number line.
In a stem and leaf diagram, a value is split into two parts, namely a stem and a leaf.

3. **(a)** A set of data, or raw data, can be arranged in an orderly way in the form of a **frequency table**.
(b) A frequency table can be represented graphically by a **histogram**.
(c) A **histogram** is a vertical bar graph with no space in **between** the bars.
(d) The **area** of each bar is proportional to the frequency it represents.

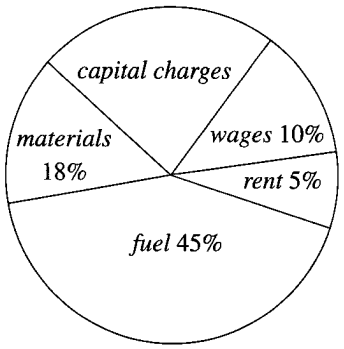
Review Questions 15

1. The bar graph below illustrates the monthly expenditure of a family.



Draw a pie chart to represent this information, marking out the size of the angle in each sector clearly.

2. For the period of 1998–1999, the gross income of a company was \$63 million. The profit of the company before tax was \$8 million after the costs of running the company were deducted from the gross income.



(a) Find the running costs for the period of 1998–1999.

(b) The running costs are represented in the following pie chart:

(i) What percentage of the running costs belonged to capital charges?

(ii) What is the measurement of the angle used to represent the materials in the pie chart?

(iii) Find the actual amount of money spent on fuel.

3. The table shows the number of goals scored by each team in a soccer tournament.

No. of goals	0	1	2	3	4	5
No. of teams	15	19	8	7	1	0

(a) How many teams scored more than two goals?

(b) Draw a histogram to represent the data.

***4.** During a one-month period, the number of sick leave days of 100 workers in a factory was recorded as shown in the table below:

No. of sick leave days	0	1	2	3	4
No. of workers	45	32	14	6	3

(a) Represent the data using a histogram.
(b) What is the most common number of days of sick leave?

***5.** The following data represent the scores of 30 students in a quiz:

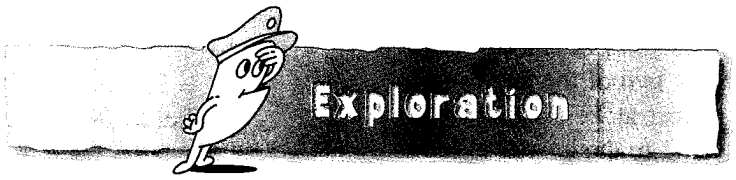
110	84	107	83	83	112	87	80	117	91
104	113	110	124	118	79	116	116	94	113
93	94	110	95	93	104	76	115	91	90

(a) Draw a dot diagram to represent the data.
(b) Represent the data in a stem and leaf diagram.
(c) What is the most common score?
(d) There is an exceptionally high score. Identify this score.

6. The following data represent the weights, in grams, of 50 pencil sharpeners:

7.2	7.7	9.0	5.2	7.7	7.0	7.2	7.4	9.5	7.2
8.4	6.9	7.4	7.2	8.5	7.1	8.0	10.5	7.6	10.2
8.6	9.3	9.1	7.5	7.2	9.8	11.9	8.3	9.4	7.9
6.8	9.2	8.6	8.6	8.4	7.2	7.4	8.5	7.7	7.0
7.0	7.3	7.6	6.9	9.2	10.4	7.3	9.4	9.1	8.1

(a) Draw a **(i)** dot diagram to represent the data;
(ii) stem and leaf diagram to represent the data.
(b) Find the most common weight.
(c) Calculate the percentage of sharpeners having weights greater than 9.0 grams?



1. The following table shows the amounts, in dollars, spent on food, rent, clothing, fuel and other items by two families in a week.

	Food	*Rent*	*Clothing*	*Fuel*	*Others*	**Total**
Family *A*	160	56	48	32	24	320
Family *B*	180	x	63	45	42	y

(a) Draw a pie chart of radius 4 cm to represent the amounts spent by Family *A*.
(b) In the pie chart for Family *B*, the angle of the sector representing the amount spent on rent is the same as that for Family *A*. Calculate the values of x and y.

Revision Exercise IV No. 1

1. The Political and Economic Risk Consultancy (PERC) ranked the following countries according to the level of corruption in the country based on a scale of 0 to 10: zero being the 'cleanest' and most transparent and 10 being the most corrupt. The following table show the results of the survey of some Asian countries in 1999. Represent the figures in the form of a bar chart.

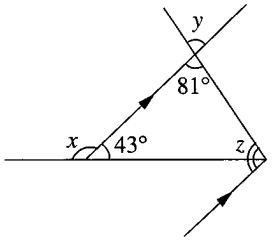
Country	*Corruption Score*
China	9.00
India	9.17
Japan	4.25
Malaysia	7.50
Indonesia	9.91
Singapore	1.55
South Korea	8.20
Thailand	7.57
Vietnam	8.50
the Philippines	6.71

2. A triangle has sides 6 cm, 8 cm and 9 cm. Find the length of the shortest altitude of the triangle by means of an accurate drawing.

3. Draw a triangle with sides 12 cm, 8 cm and 10.5 cm. Bisect any two of the angles and let the bisectors meet at X. Construct the perpendicular from X to the longest side of the triangle. Measure this perpendicular.

4. A bus is supposed to start at 12 25 and to reach its destination at 13 50. It starts 3 minutes late and arrives 10 minutes late. How long did it take to reach its destination?

5. In the given diagram, find the values of x, y and z.



6. **(a)** A square piece of metal with side 14 cm costs \$4.90. How much will a similar piece of metal in the shape of a rectangle with sides 20 cm by 12 cm cost? (Assume that the cost of the metal is proportional to the area.)

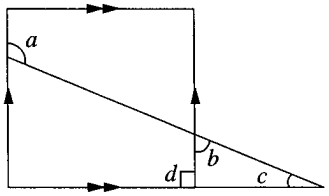
(b) A trader buys a typewriter for \$120 and sells it for \$138. Find the profit percent.

7. **(a)** How long will it take a principal sum of \$2 400 to amount to \$2 880 at a simple interest rate of 6% per annum?

(b) The driver of a train averaging 32 km/h takes 3 hours for a journey. What is the length of the journey? How long would the driver of an express train averaging 48 km/h take for the same journey?

8. In the figure, $d = 90°$, show that

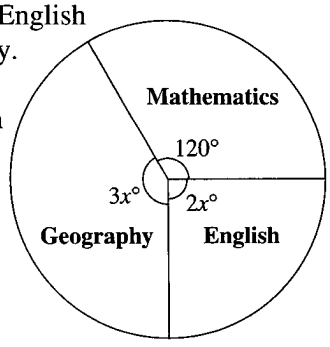
(a) $a - c = 90°$, **(b)** $a + b = 180°$.



9. **(a)** Draw $\triangle PQR$ such that $PQ = 90$ mm, $QR = 100$ mm and $RP = 67.5$ mm. A point T lies on PQ such that $QT = 30$ mm. Draw a line through T parallel to QR to cut RP at S. Measure the length of RS.

(b) Construct $\triangle ABC$ such that $BC = 7.5$ cm, $AC = 6$ cm and $A\hat{C}B = 60°$. Also construct the angle bisector of $A\hat{B}C$. Measure the length of AB.

10. Each student in a group of 240 was asked to choose his/her favourite subject from Mathematics, English and Geography. The result is represented on the pie chart given on the right.



Calculate
(a) the value of x;
(b) the number of pupils who chose English;
(c) the fraction of pupils who chose Geography.

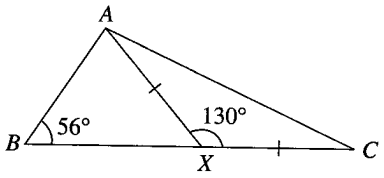
Revision Exercise IV No. 2

1. Draw $\triangle ABC$ with sides each of length 10.8 cm. Mark a point P on AB such that $PA = 2.4$ cm. On BC, mark a point Q such that $QC = 8.6$ cm. Measure PQ.

2. Construct a rhombus of side 6 cm and one of its diagonals 8 cm. Measure the length of the other diagonal.

3. In $\triangle ABC$, $A\hat{B}C = 64°$, $AB = AC$ and BC is produced to D.
Calculate
(a) $B\hat{A}C$; **(b)** $A\hat{C}D$.

4. In the given figure $AX = XC$, $A\hat{X}C = 130°$ and $A\hat{B}C = 56°$. Calculate the value of
(a) $C\hat{A}X$; **(b)** $B\hat{A}X$.



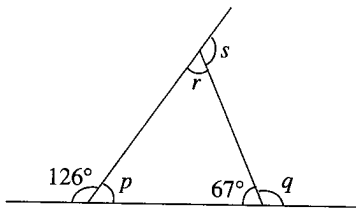
5. Two trains 245 m and 315 m long are travelling towards each other at 90 km/h and 54 km/h respectively on parallel lines. How long do the trains take to pass one another from the time they meet each other?

$\left(\textbf{Note: } 1 \text{ km/h} = \frac{5}{18} \text{ m/s}\right)$

6. Construct a parallelogram with diagonals 7.5 cm and 10.2 cm, and with the shorter sides 3.6 cm long. Measure the length of the longer sides.

7. **(a)** If $x\%$ of 300 is equal to 15% of 220, find x.
(b) Express 5.4 km as a percentage of 6.4 km
(i) exactly;
(ii) in decimal form correct to 3 significant figures.

8. Find the angles marked p, q, r and s in the figure.



9. Construct $\triangle ABC$ where $AB = 8$ cm, $BC = 7$ cm and $CA = 10$ cm. Construct a point X inside the triangle such that it is lying on the bisector of $A\hat{B}C$ and is 3 cm from AC.

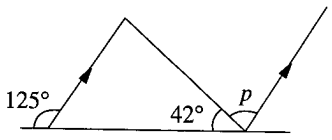
10. In a school, the pupils are divided into three groups, A, B and C. $\frac{1}{3}$ of the pupils are in group A, $\frac{5}{12}$ in group B and the rest in group C. If there are 150 pupils in group C, find
(a) the total number of pupils in the school;
(b) the ratio of the number of pupils in group A to the number of pupils in group B.

Revision Exercise IV No. 3

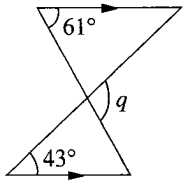
1. Construct a parallelogram with one of its diagonals 100 mm and two of its sides 48 mm and 109 mm. Measure the length of the other diagonal.

2. Construct a trapezium $PQRS$ in which PQ is parallel to SR, $P\hat{S}R = 90°$, $QP = 3.6$ cm, $PS = 7.2$ cm and $SR = 12.6$ cm. Measure QR and $P\hat{Q}R$.

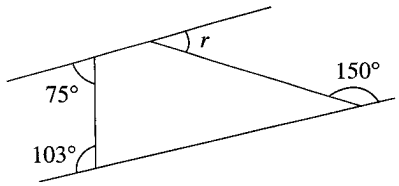
3. Find the angles marked p, q and r in the following figures.
(a)



(b)



(c)



4. (a) The capacity of a tank is 60 000 litres. It is being filled at a rate of 1 250 litres per hour. How long will it take to fill $\frac{5}{8}$ of the tank?

(b) A clock gains 12 min 15 s in one week. How many days will it take to gain 3 h 30 min?

5. Construct a quadrilateral $ABCD$ where $AB = CD = 8$ cm, $BC = 6$ cm, $A\hat{B}C = 90°$ and $B\hat{C}D = 130°$. Measure the length of AD.

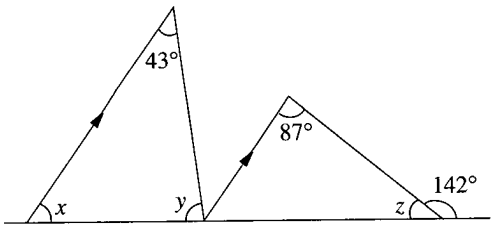
6. If \$1 560 amounts to \$1 833 after 2 years and 4 months, find the rate of simple interest.

7. (a) If 80% of a number is 400, find the number.

(b) What percentage of \$55.00 is \$13.20?

(c) A bag was sold for \$23.80 at a loss of $12\frac{1}{2}$%. Find the cost price of the bag.

8. Find the unknown angles x, y and z in the figure below.



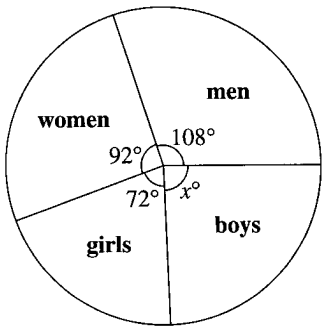
9. Given that $7x = 2y$, find the ratio $x : y$.

10. The pie chart below shows the number of people taking part in the big walk organised by the Singapore Sports Council. If 1 656 girls took part in the walk, find

(a) x;

(b) the percentage of participants who are men;

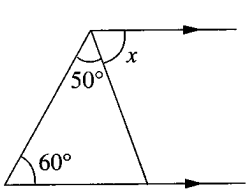
(c) the number of participants who are boys.

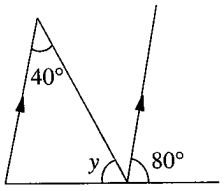


Revision Exercise IV No. 4

1. A Malaysian tourist exchanges M\$480 for Singapore dollars at a rate of M\$100 = S\$45.50. Find the amount of Singapore dollars he can get.

2. Find, in the figure, the angles marked x and y.



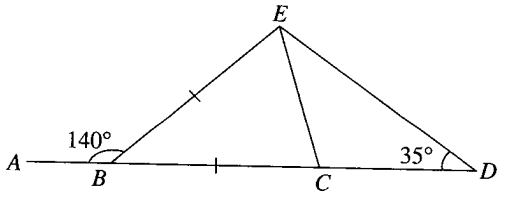


3. 12 men takes 5 days to build a road 200 m long. How many days will 20 men take to build a road 400 m long?

4. A bookseller bought 50 books for \$225 and sold them for \$5.40 each. Find his percentage profit.

5. Construct a parallelogram with sides 7.5 cm and 9 cm and with one of its interior angles as 62°. Measure its diagonals.

6. In the figure, $ABCD$ is a straight line, $BE = BC$, $A\hat{D}E = 35°$ and $A\hat{B}E = 140°$. Show that $CE = CD$.



7. Construct $\triangle PQR$ such that $PQ = 10$ cm, $QR = 9$ cm and $RP = 8$ cm. A point S on PR is 2 cm from R. Draw a line through S parallel to RQ to cut PQ at T. Measure PT.

8. In an isosceles triangle the base angle is 12° greater than the vertical angle. Find the size of the vertical angle.

9. Each pupil in a class of 40 was asked to state the length of time he/she spends on private tuition in a week. The results are shown in the following table.

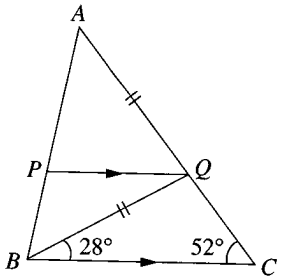
(t hours) *Length of time spent*	*No. of pupils*
$t = 0$	8
$0 < t \leqslant 2$	4
$2 < t \leqslant 4$	12
$4 < t \leqslant 6$	10
$6 < t \leqslant 8$	6

Illustrate the above information by using a pie chart. What is the angle representing pupils who have 2 to 4 hours of private tuition per week?

10. To ensure the long term availability of fresh water for its citizens, the Singapore Government decided to build a desalination plant scheduled for completion in the year 2004. Due to the Asian economic crisis in 1998, the cost of building the plant has dropped from an initial projected cost of S$1.2 billion to about $912 million. Find the percentage savings for building the plant.

Revision Exercise IV No. 5

1. In the figure, $AQ = BQ$ and PQ is parallel to BC. If $Q\hat{B}C = 28°$ and $Q\hat{C}B = 52°$, find $A\hat{P}Q$.

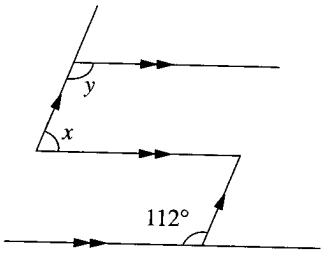


2. Construct a parallelogram $ABCD$ in which $AB = 7.5$ cm, $BC = 6$ cm and $A\hat{B}C = 50°$. Draw the perpendiculars from C to AB and AD. Measure the length of these perpendiculars.

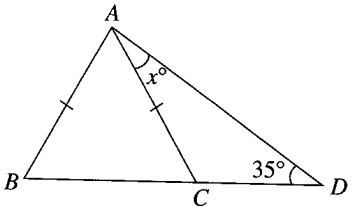
3. From a square wooden disc of side 10 cm, a circular disc of radius 5 cm is cut out. What percentage of wood remains? (Take $\pi = 3.14$)

4. **(a)** If $5\frac{1}{2}\%$ of $2M$ is 110, find M.
   (b) A man bought a pen for $12.50. He sold it to a customer at 24% profit. Find the selling price.
   (c) A man bought an HDB flat for $150 000 and sold it later for $162 000. Find his percentage gain.

5. Find the angles marked x and y in the figure.

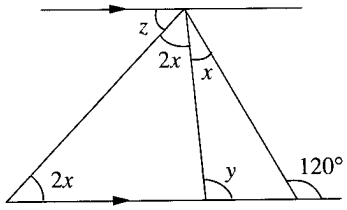


6. In the figure, BCD is a straight line and $\triangle ABC$ is an equilateral triangle. Given that $A\hat{D}C = 35°$ and $D\hat{A}C = x°$, find x.



7. A man uses a ruler to measure a rectangle 45 cm by 36 cm. His result is 44.6 cm by 35.8 cm. Find his error per cent in the value he obtains for the perimeter.

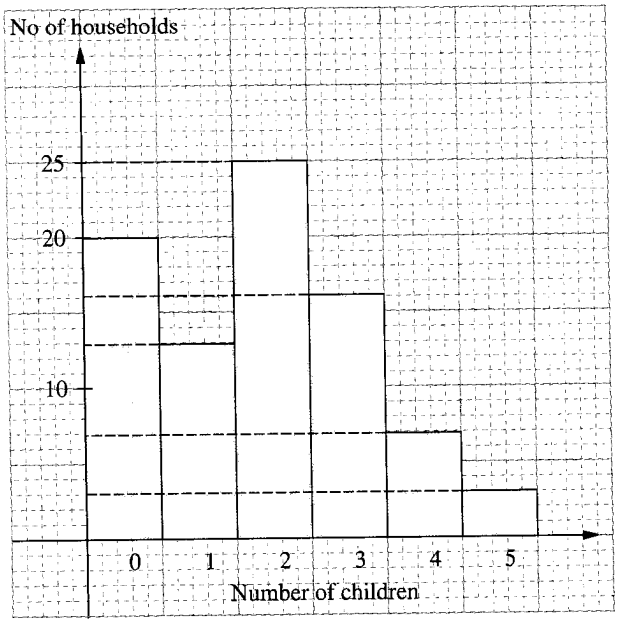
8. Find x, y and z in the given figure.



9. **(a)** ABC is a triangle in which $B = C = 70°$. AC is produced to P and the bisector of BCP cuts AB produced at Q. Find AQC.

(b) Construct a rhombus of side 7 cm and one of the diagonals 11 cm. Measure the other diagonal. Measure also one of the acute angles.

10. The histogram displays the results of a survey on the number of children, the residents of a newly completed condominium have.



Find

(a) the number of households;

(b) the total number of children in the survey;

(c) the percentage of households having exactly 2 children.

End-of-Year Examination Speciman Paper 1

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. Simplify **(a)** $4 - 1\frac{1}{2} \times 1\frac{3}{4}$, [2]

(b) $3\frac{3}{8} \div \frac{3}{4} + \frac{5}{16}$. [2]

2. A ball-point pen weighs 12.4 g. Find the total mass of 2 560 such ball-point pens, giving your answer in kilograms. [3]

3. The perimeter of a rectangle is 28 cm and its width is 6 cm. Find its area. [3]

4. Evaluate each of the following, giving your answer correct to 2 decimal places:

(a) $9.264\,6 \div 0.6$ [2]

(b) 3.156×27.2 [2]

5. A, B and C share \$345 in the ratio 1 : 5 : 17. How much does each receive? [4]

6. A customer paid \$117 for a watch after a 35% discount. What was the original price of the watch? [3]

7. A school librarian bought 56 books, some at \$3.50 each and others at \$4.50 each. If the total cost of these books was \$240, how many books that cost \$3.50 each were bought? [4]

8. Solve the following equations:

(a) $4 = \frac{1}{5}(2x - 3)$ [2]

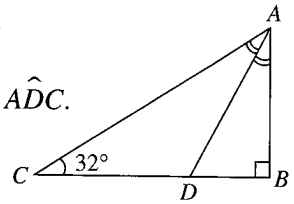
(b) $3x - 4 = 7 - 4(x - 5)$ [2]

9. Simplify each of the following:

(a) $12x - 2[2x - 3(x - 5)]$ [2]

(b) $\frac{x-2}{5} - \frac{2x-3}{4} + \frac{5-x}{10}$ [3]

10. In the figure, ABC is a right-angled triangle with $A\hat{B}C = 90°$, $A\hat{C}B = 32°$ and $D\hat{A}B = D\hat{A}C$.
Calculate
(a) $D\hat{A}B$, **(b)** $A\hat{D}C$.



[4]

11. Consider the number pattern below:

$$1 = 1$$
$$1 + 2 + 1 = 4$$
$$1 + 2 + 3 + 2 + 1 = 9$$
$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$$
$$\vdots \quad \vdots$$

(a) Write down the 5th line of the sequence. [1]

(b) Find the value of $1 + 2 + 3 + \ldots + 8 + 9 + 8 + 7 + \ldots + 3 + 2 + 1$. [1]

(c) If $1 + 2 + 3 + \ldots + (x - 1) + x + (x - 1) + \ldots + 3 + 2 + 1 = 169$, find the value of x. [1]

12. **(a)** A solid copper cylinder, 8 cm long and 4 cm in diameter, is melted and recast into a length of wire 2 mm in diameter. How long is the wire? [3]

(b) Mr Lee puts \$5 000 in a bank that pays a compound interest of 4% per year. If he leaves the principal and the interest in the bank for another year, how much will he have at the end of the second year? [2]

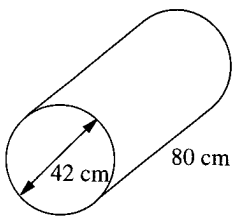
13. Construct a triangle ABC with $AB = 7.8$ cm. $A\hat{B}C = 55°$ and $BC = 6.8$ cm. Measure the length of AC. [4]

Part II (50 marks) **Time: 1 h 15 min**

*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. A lawn roller is 80 cm wide with a diameter of 42 cm. Find the area it covers in 40 revolutions. Give your answer in square metres and take π to be $\frac{22}{7}$. [4]



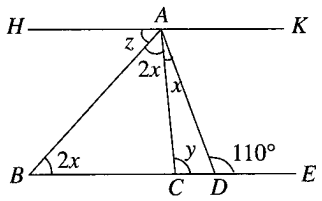
2. An open cylindrical tank with diameter 28 cm contains water to a depth of 30 cm. Find the volume of the water inside the tank, giving your answer in litres. Find also the total surface area of the tank that is in contact with the water. [6]

3. A rectangular field has sides $5x$ metres by $4x$ metres. Calculate the value of x in each of the following cases:

(a) The perimeter of the field is 450 metres. [3]

(b) The area of the field is 2 000 square metres. [3]

4. In the figure, HAK is parallel to BDE, $A\hat{D}E = 110°$, $A\hat{B}C = B\hat{A}C = 2x$, $C\hat{A}D = x$, $A\hat{C}D = y$ and $B\hat{A}H = z$. Calculate the values of x, y and z. [6]



5. An open rectangular box 24 cm long, 18 cm wide and 10 cm deep internally is made of wood 2 cm thick.

(a) What volume of wood is used to make the box? [4]

(b) What is its capacity in litres? [3]

Section B (28 marks)

6. (a) Find the mass, in kg, of a cylindrical metal bar 3 m long and 7 cm in diameter if the density of the metal is 4.2 g/cm^3.

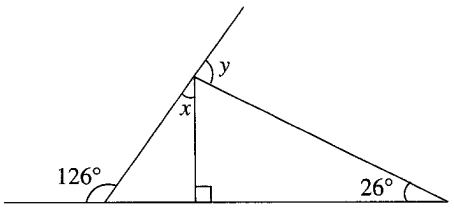
$\left(\text{Take } \pi = \frac{22}{7}\right)$ [4]

(b) Evaluate each of the following, giving your answer correct to 4 significant figures:

(i) $\sqrt{432.9} \div 7.6$ [2]

(ii) $\frac{5}{\sqrt[3]{965}} + \frac{3.2^2}{1.85^5}$ [2]

7. (a) Calculate the values of x and y in the diagram. [4]



(b) The amount of pocket money Alvin received is just enough for him to buy 10 plates of chicken rice or 15 plates of fried noodles. If he wishes to buy equal number of plates of chicken rice and fried noodles, how many of each type can he buy with the money? [3]

8. (a) Four people contributed sums of money to the Community Chest of Singapore in the ratio 2 : 3 : 5 : 8. If the largest amount contributed is \$24, calculate the total amount contributed by the four people. [3]

(b) A tourist exchanged M\$900 for Singapore dollars (S\$) at M\$2.25 to S\$1. He spent S\$245 in Singapore and exchanged the remainder for Malaysian ringgit at the same rate. How many Malaysian ringgit did the tourist receive? Give your answer correct to the nearest ringgit. [3]

End-of-Year Examination Specimen Paper 2

Part 1 (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. Simplify **(a)** $7\frac{1}{2} - 3\frac{3}{4} \div \frac{3}{8}$, [2]

(b) $4\frac{2}{3} \times 3\frac{3}{8} - 9\frac{7}{8}$. [2]

2. (a) Express 0.035 as a fraction in its lowest terms. [1]

(b) Express $1\frac{9}{40}$ as a decimal. [1]

(c) Express 2.004 56 correct to two decimal places. [1]

(d) Express 0.025 47 correct to two significant figures. [1]

3. (a) Find the LCM of 25, 80 and 120. [2]

(b) Simplify $3(5x - 7) - 2(3x - 5y) + 2(3y - 7x) - (x - 2y)$. [2]

4. Solve the equation $x - \frac{x-1}{4} = \frac{x+2}{5}$. [3]

5. The area of a trapezium is 1 500 cm^2. Find the height between the parallel sides if their lengths are 14 cm and 36 cm. [3]

6. How many litres of water are required to fill a rectangular storage tank 12 m long, 6 m wide and 2 m deep? [3]
(1 litre = 1 000 cm^3)

7. A solid concrete cylinder of diameter 70 cm is 120 cm high. What is its volume in cubic metres? [3]

8. At a sale, a sofa set selling at a discount of 25% fetches \$3 570. Find the original price of the sofa set. [3]

9. A trader buys x pencils at \$$y$ each and sells them at 3 for \$$z$. Find an expression for his total profit. [3]

10. A bicycle wheel has a diameter of 70 cm. Find

(a) the circumference of the wheel; [2]

(b) the distance the bicycle travels after the wheel has made 400 revolutions.

$\left(\text{Take } \pi = 3\frac{1}{7}\right)$ [2]

11. A Singaporean obtained US\$2 500 at a rate of S\$1.70 to US\$1. He spent US\$1 850 and exchanged the remaining US dollars for Singapore dollars at a rate of S\$1.68 to US\$1. How many equivalent Singapore dollars had he used up? [4]

12. Construct a parallelogram $ABCD$ in which $AB = 9$ cm, $AD = 6$ cm and $B\hat{C}D = 55°$. On the same diagram, construct a perpendicular from D to AB and measure its length. [4]

13. Consider the following number pattern:

$2 \times 2 = 1 \times 3 + 1$
$3 \times 3 = 2 \times 4 + 1$
$4 \times 4 = 3 \times 5 + 1$
$5 \times 5 = 4 \times 6 + 1$
$\vdots \qquad \vdots$
$n \times n = 13 \times 15 + 1$

(a) Write down the 10th line in the pattern. [1]

(b) Find the value of n. [1]

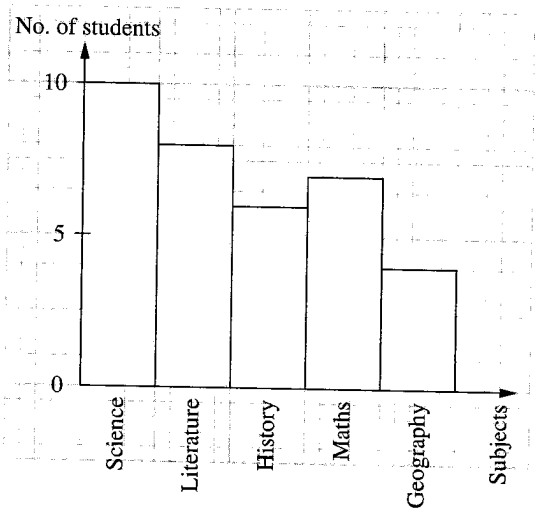
(c) Use the above pattern to calculate the value of 199^2. [2]

14. The students of a Secondary 1 class were asked to name their favourite subject. The bar graph illustrates the result. Find

(a) the total number of students in the class; [1]

(b) the fraction of students who like Maths; [1]

(c) the fraction of students who like either History or Geography. [1]



Part II (50 marks) Time: 1 h 15 min

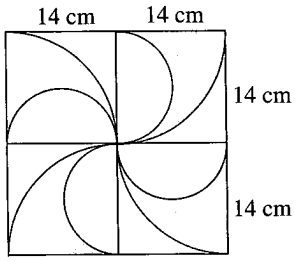
*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. The density of a piece of metal is 5.8 g/cm^3. Find the mass of a piece of the metal with a volume of 25 cm^3. [3]

2. A father is four times as old as his son. Five years ago, the sum of their ages was 70. Find their present ages. [4]

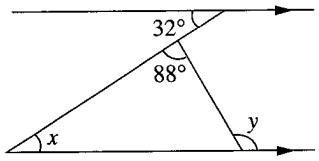
3. **(a)** The figure shows a pattern made up of straight lines, semicircles and arcs of quadrants of circles. Calculate the total area of the shaded regions. [6]



(b) A motorcycle wheel has a diameter of 42 cm. Find the number of complete revolutions it makes in moving a distance of 1.25 km. [3]

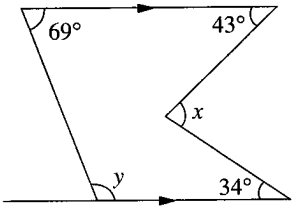
4. **(a)** Find the size of the unknown angles marked x and y on the following diagrams:

(i)



[3]

(ii)



[3]

Section B (28 marks)

5. **(a)** A man left $\frac{1}{8}$ of his money to a school, $\frac{1}{4}$ to charity and the remainder to his family. If the family received \$15 000, how much money did the man have originally? [3]

(b) A sum of money is distributed among three people, A, B and C, in the ratio 5 : 7 : 13. If B has \$5 200 more than A, how much does C receive? [4]

6. **(a)** A man travelled 200 km at 60 km/h and then at 48 km/h for the next 160 km of his journey. Find his average speed for the whole journey. [4]

(b) A man earns a taxable income of \$24 000 a year and pays \$900 in taxes. What percentage of his income is taxed? [3]

7. **(a)** Evaluate each of the following, giving your answer correct to 4 significant figures:

(i) $84^2 \div 3.56^3$ **(ii)** $\frac{\sqrt{84.6}}{\sqrt[3]{0.014}} \times 0.6^3$ [4]

(b) Find the principal amount that will earn a simple interest of \$1 638 in 3 years 3 months at 6% per annum. [3]

8. A rectangular pond 10 m by 8 m is surrounded by a concrete path 20 cm wide and 25 cm thick. Calculate the volume of concrete used in making the path, giving your answer in m^3. If the density of the concrete is 2.8 g/cm^3, calculate the weight of the concrete. [7]

End-of-Year Examination Specimen Paper 3

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. Simplify

(a) $1\frac{1}{4} - \frac{1}{8} \times \left(2\frac{1}{3} + 1\frac{2}{9}\right)$; [2]

(b) $4\frac{2}{3} + 3\frac{1}{4} \div \left(3\frac{2}{5} + 3\frac{1}{10}\right)$. [2]

2. Simplify the following:

(a) $4a \times 5 + 3a$ [1]

(b) $18y - 5y \times 3$ [1]

(c) $2(a + b + c) - 4(2a - b - 3c)$ [2]

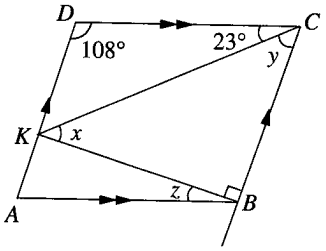
3. Solve the following equations:

(a) $10 - 3y = 21 - 5y$ [2]

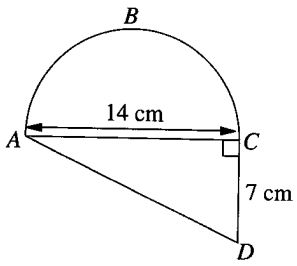
(b) $11x + 10 - 6x = 23 + 4x - 9$ [2]

(c) $\frac{4x-2}{3} + \frac{2x+17}{6} = \frac{7x-3}{2}$ [3]

4. $ABCD$ is a parallelogram in which $A\hat{D}C = 108°$. Given that KB is perpendicular to BC and $K\hat{C}D = 23°$, find the values of x, y and z. [4]



5. In the figure, ABC is a semicircle and ACD is a triangle. Find the area of the whole figure with the dimensions given. [4]



6. $\$2x$ is shared among 3 persons, A, B and C, in the ratio 3 : 2 : 5. If B gets \$45, find the value of x. [3]

7. The cost of material, labour and administration for an advertising campaign is in the ratio 8 : 5 : 2. If the total cost of the campaign is \$3 525, find the cost of labour. [3]

8. A school field measures 120 m by 60 m. A plan of the field is drawn to a scale of 1 : 500. Find the area of the field on the plan in cm^2. [4]

9. The cost of a bottle of milk is NZ\$0.65 in New Zealand. Find the cost of 20 bottles of milk in Singapore dollars. [3]
(Take NZ\$1 = S\$0.92)

10. A shopkeeper buys \$143 worth of pens at \$6.50 each. If he sells them at \$7.80 each, find his profit percentage. [4]

11. How long will it take to earn a simple interest of \$337.50 from a principal sum of \$2 500 at 6% per annum? [3]

12. A driver covers 180 km in 2 h 30 min. If he is driving at a constant speed, find the distance he covers in 5 minutes. [3]

13. Construct $\triangle PQR$ where PQ = 6.4 cm, QR = 9.6 cm and PR = 8 cm. Find, by measurement, the distance from P to the midpoint of QR. [4]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. When 8 is added to $\frac{4}{5}$ of a number x, the result is equal to x. Find x. [3]

2. Find the volume of water that falls onto a flat roof 12.4 m long and 8.2 m wide during a day when 12 mm of rain is recorded. This water is later transferred into cylindrical containers each of radius 28 cm and height 60 cm. How many containers can be completely filled? [6]

3. (a) The sum of three consecutive odd numbers is 63. Find the numbers. [3]

(b) A woman is 7 times as old as her daughter. If in 5 years' time she is 4 times as old as her daughter, what are their present ages? [3]

4. (a) A housewife buys 11 m of cloth at \$8.20 per metre. How much change will be given to her if she pays with 2 fifty-dollar notes? [3]

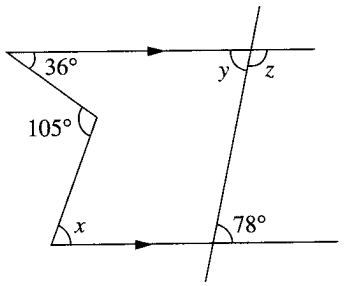
(b) Find the cost of painting the outer surface of a closed rectangular tank 3.6 m long, 2.5 m wide and 2 m deep with paint at 75 cents per square metre. [4]

Section B (28 marks)

5. **(a)** A man cycles at v km/h. Find the distance he travels in t hours. How long will it take him to travel s km? [4]

(b) A car uses 1 litre of petrol to travel 9 km. If 1 litre of petrol costs \$1.14, find the cost needed for the car to travel a distance of 135 km. [3]

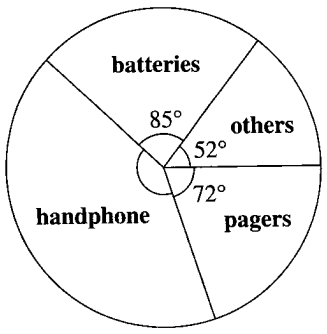
6. **(a)** Find the angles marked x, y and z on the figure. [4]



(b) Construct a parallelogram of sides 6.9 cm and 11.1 cm, and whose longer diagonal is 14.4 cm. Measure the length of the other diagonal. [4]

7. The inner dimensions of an open wooden rectangular box are 10 cm by 8 cm for the base and 5 cm for the height. If the thickness of the wood is 1 cm, calculate the volume of the material used in making the box. Find the weight of the box if the density of the wood is 0.7 g/cm^3. [6]

8. **(a)** The pie chart below shows the sales of a shop dealing with telecommunications in a week.



(i) What percentage of the sales were from pagers? [1]

(ii) If the total sales of the week amounted to \$72 000, calculate the amount of sales for handphones. [3]

(b) 12 men can renovate a house in 15 days if they work 7 hours a day. If 14 men were asked to renovate a similar house in 10 days, how many hours a day must the men work? [3]

End-of-Year Examination Specimen Paper 4

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. Evaluate

(a) $(2 + 5)^2 - (17 - 15)^4 \div 2\frac{2}{3}$, [2]

(b) $1 - 0.04 + \frac{1}{4}$. [2]

2. **(a)** Express 0.086 479 as a decimal

(i) correct to 2 decimal places; [1]

(ii) correct to 2 significant figures. [1]

(b) Express 99 225 in prime factors and hence, find the value of $\sqrt{99\,225}$. [2]

3. Solve the following equations:

(a) $5(3 - x) = 7(2x - 5)$ [2]

(b) $\frac{x}{3} - \frac{2x - 5}{5} = \frac{3}{15}$ [3]

4. Two towns, A and B, are 180 km apart. Motorist P sets off from A for B at 12 20 and travels at a speed of 50 km/h. Motorist Q sets off from A for B at 12 45 at a speed of 60 km/h. Who will arrive at B first and at what time? [3]

5. Copy and complete the following:

(a) 3 657 m = _____ km [1]

(b) 2.5 m^2 = _____ cm^2 [1]

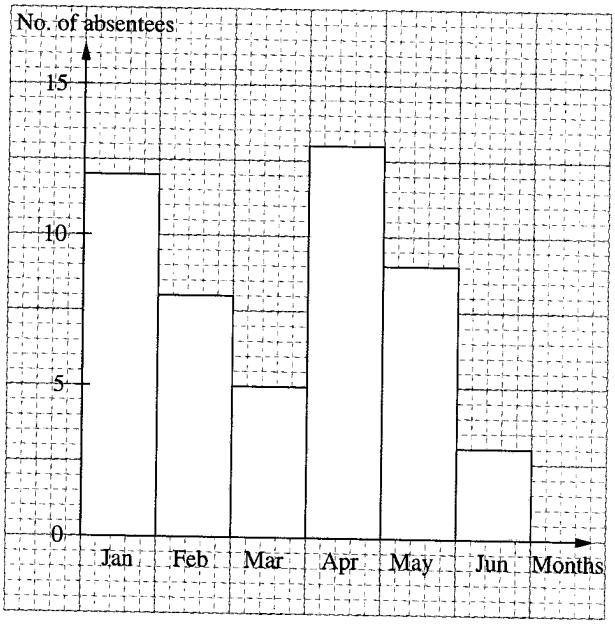
(c) 1.864 l = _____ cm^3 [1]

(d) 0.84 g = _____ kg [1]

6. A sum of money is divided among A, B and C in the ratio 15 : 8 : 7. If B has \$4.50 more than C, find the original sum of money. [3]

7. A shopkeeper bought 650 eggs for \$70, 62 eggs were broken and the shopkeeper sold the rest at \$2 for 14 eggs. What was his gain or loss per cent? [3]

8. The histogram shows the number of absentees of a class during the first six months in a school in Singapore.



(a) Find the total number of absentees. [1]

(b) Suggest a simple reason why there were fewer absentees in the months of March and June. [2]

9. (a) If $2 : x = 5 : 9$, find x. [2]

(b) If $p = \frac{3}{4}$ and $q = 1\frac{5}{7}$, express $\frac{p}{q}$ as a percentage. [2]

10. Find two consecutive odd numbers such that the greater number added to 3 times the smaller number makes a total of 86. [4]

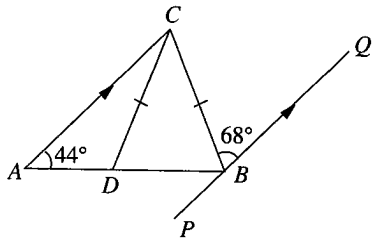
11. Simplify

(a) $2(5x - y) - 3(6x - 5y) + (x - 7y)$; [2]

(b) $\frac{x+2}{3} - \frac{x-5}{5} + \frac{2-x}{4}$. [3]

12. In the diagram, AC is parallel to PQ, $CD = CB$, $B\hat{A}C = 44°$ and $C\hat{B}Q = 68°$. Calculate

(a) $A\hat{B}C$, **(b)** $A\hat{C}D$. [4]



13. A motorist travels x km in t hours. If he travels at the same speed, how far will he travel in

(a) 3 hours; [1]

(b) y hours? [2]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. Arrange the following numbers in descending order:

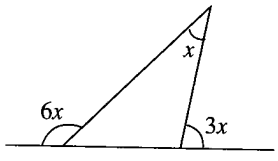
$\frac{22}{7}$, 3.142, $\frac{47}{15}$, $3\frac{3}{20}$ [2]

2. Calculate the simple interest on \$7 800 for $3\frac{1}{2}$ years at 6% per annum. [3]

3. Construct a rhombus of side 8 cm with one of its interior angles measuring 70°. Measure the length of the diagonals. [4]

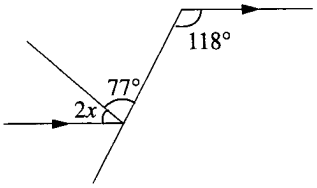
4. Find the value of x in each of the following diagrams:

(a)



[3]

(b)



[3]

5. Construct the following in a single diagram.

(a) $\triangle ABC$ in which $AB = 8$ cm, $A\hat{B}C = 65°$ and $B\hat{A}C = 42°$. [2]

(b) The perpendicular from C to AB. [2]

(c) The point E such that CE is parallel to AB and $C\hat{B}E = 58°$. [3]

Section B (28 marks)

6. A worker was paid \$2.25 an hour and if he worked overtime, he was paid \$3.50 an hour. If the worker received \$127 for 52 hours of work, how many hours of overtime work did he do? [5]

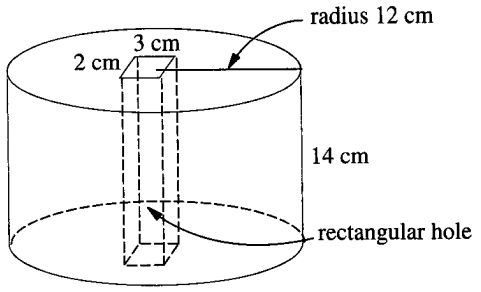
7. A rectangular flower bed has a perimeter of 96 m. Given that its length is twice its breadth, calculate its length and its area. [4]

The flower bed is surrounded by a concrete path of width 40 cm and height 15 cm. Calculate the volume of concrete used for the path, giving your answer in m^3. [4]

8. A cylindrical water container of diameter 28 cm and height 35 cm is $\frac{7}{10}$ full of water. How many complete glasses of water, each of volume 186 cm^3, can be filled? What is the volume of water left over then? [7]

$\left(\text{Take } \pi = \frac{22}{7}\right)$

9. A rectangular hole measuring 2 cm by 3 cm is cut out from a solid cylinder of radius 12 cm and height 14 cm as shown in the diagram.



(a) Find the volume of the remaining solid. [5]

(b) If the density of the material used to make the solid is 1.5 g/cm^3, find the weight of the remaining solid. [3]

End-of-Year Examination Specimen Paper 5

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. Simplify the following:

(a) $2\frac{3}{4} \div 1\frac{2}{3} - 1\frac{1}{3}$ [2]

(b) $3\frac{2}{3} + 1\frac{1}{2} \times \frac{3}{4}$ [2]

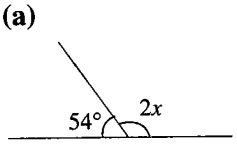
2. Simplify the following:

(a) $2(3x - 5) - 3(5x - 3)$ [2]

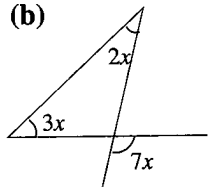
(b) $2(a + 3b) + 7(2b - a)$ [2]

3. Find the value of the unknown in each of the following diagrams:

(a)



(b)



[4]

4. Estimate each of the following, giving your answer correct to 1 significant figure:

(a) $\sqrt{905}$ **(b)** $\frac{586}{291}$

(c) $8.01^2 - 0.48$ **(d)** $\sqrt[3]{8\,024}$ [4]

5. If $3(x - 1) - 5(x - 3) = 3$, find the value of $7x - 5$. [3]

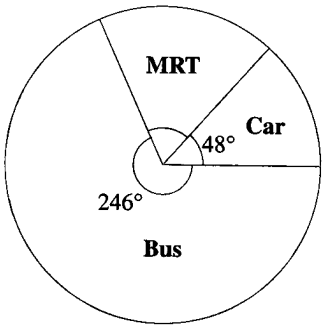
6. Express **(a)** c cents in dollars, [1]

(b) m millimetres in metres, [1]

(c) k kilograms in grams. [1]

7. A tourist from England wishes to exchange sterling pounds for Singapore dollars. How many complete sterling pounds does he need to exchange for S$1 800 if the exchange rate is £1 to S$2.72? [3]

8. The results of a survey of the mode of transport used by the students of a school is illustrated by the pie chart. If the number of students travelling by car is 160, how many students travel by MRT? [3]



9. **(a)** The perimeter of a square is 36 cm. Find its area. [2]

(b) The area of a triangle ABC is 36 cm^2 and $AB = 18$ cm. Find the perpendicular height from C to AB. [2]

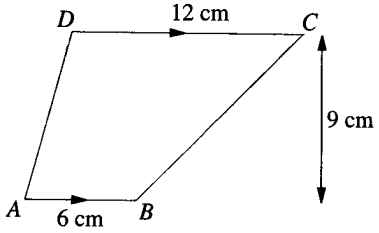
10. A farmer uses $\frac{1}{3}$ of his land for rubber, $\frac{3}{8}$ for palm oil, $\frac{1}{6}$ for durians and the remaining 23 hectares for cocoa. Find the total area of the land. [3]

11. Meiling's mother is 8 times as old as Meiling. In 10 years' time, Meiling's mother will be only 3 times as old as Meiling. Find their present ages. [4]

12. The road distance between two towns, A and B, is 550 km. A car leaves A for B at an average speed of 72 km/h and a lorry leaves B for A, travelling along the same road as the car, at an average speed of 38 km/h at the same time. How long will it take before the two vehicles meet? [4]

13. Construct a parallelogram with sides 54 mm and 108 mm, and one of its angles, 64°. Measure the lengths of the two diagonals. [4]

14. The diagram shows a trapezium $ABCD$ with AB parallel to DC. Given that $AB = 6$ cm, $DC = 12$ cm and the height between the parallel sides equals 9 cm, calculate the area of $ABCD$. [3]



Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. A man cycles at v km/h.

(a) Find the distance he travels in 5 hours. [1]

(b) Find the time needed to travel s km. [2]

2. A cylinder P has radius r cm and height $2h$ cm. A second cylinder Q has radius $3r$ cm and height h cm. Find the ratio of $\frac{\text{volume of } P}{\text{volume of } Q}$. [4]

3. **(a)** A motorist takes $5\frac{3}{4}$ hours to travel from P to Q at V km/h. If he increases his speed by 5 km/h, the time taken will be reduced by half an hour. Find V. [5]

(b) The sum of 3 consecutive even numbers is 108. Find the numbers. [3]

4. Construct, in a single diagram,

(a) $\triangle ABC$ where $AB = 9$ cm, $BC = 8$ cm and $AC = 7$ cm; [2]

(b) a point D such that CD is parallel to AB and $CD = 5$ cm; [2]

(c) the perpendicular from C to AB. [3]

Section B (28 marks)

5. **(a)** Solve the equation

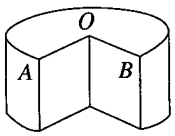
$$\frac{2}{3}(4x - 1) - \frac{5}{6}(2x + 1) = \frac{1}{2}. \quad [3]$$

(b) If water is running into a tank at the rate of 3 metres per second along a trough of rectangular cross-section 18 cm wide and 15 cm deep, find the amount of water that runs into the tank in 1 minute. [4]

6. **(a)** Given that 12.5% of A is 42, find A.

(b) Calculate the time needed for \$4 800 to earn a simple interest of \$420 at 7% per annum. [3]

7. A cylindrical piece of cake of radius 14 cm and thickness 8 cm stands on a horizontal table. $\frac{1}{4}$ of the cake is removed by cutting vertically downwards through the radii OA and OB as shown in the diagram.



Find **(a)** the volume of the remaining cake. [3]

(b) the total surface area of the remaining cake. [5]

$\left(\text{Take } \pi = \frac{22}{7}\right)$

8. A shopkeeper bought 1 840 apples for \$350. 52 apples were bad. He repacked the rest in boxes of 6 each and sold all of them at \$1.80 per box. Calculate

(a) the number of boxes sold; [3]

(b) the total profit he made if the cost of an empty box was 6 cents each. Express this profit as a percentage of the total cost of apples and boxes, giving your answer correct to 1 decimal place. [4]

Answers

Exercise 1a *(Pg 5)*

1. **(a)** $a > 80\,000$
(b) $b > 90$ $c > 80$
(c) $83 \leqslant d \leqslant 95$
2. **(a)** 1, 2, 3, 4, 5, 6, 7
(b) 26, 28, 30, 32, 34
(c) 41, 43, 45, 47, 49, 51
(d) 64, 65, 66, 67, 68, 69
(e) 24, 27, 30, 33, 36, 39
(f) 86, 88, 90, 92, 94, 96, 98
(g) 55, 57, 59, 61, 63
(h) 74, 76, 78, 80, 82, 84, 86, 88, 90

Exercise 1b *(Pg 6)*

1. **(a)** 1 735 **(b)** 15 455
2. **(a)** 58 257 **(b)** 2 368
3. **(a)**
$$\begin{array}{r} 249 \\ +\ 586 \\ \hline 835 \end{array}$$
(b)
$$\begin{array}{r} 284 \\ -\ 196 \\ \hline 88 \end{array}$$
(c)
$$\begin{array}{r} 663 \\ 792 \\ +\ 587 \\ \hline 2\,042 \end{array}$$
4. **(a)** $x = 2, y = 2, z = 3$
(b) $x = 6, y = 8, z = 7$
(c) $x = 1, y = 7, z = 3$
(d) $x = 2, y = 6, z = 9$
5. **(a)** 10 234, 98 765, 88 531
(b) **(i)** 97 531, 86 420
(ii) 10 468, 23 579
(c) 97 531, 86 420

Exercise 1c *(Pg 8)*

1. **(a)** 15 **(b)** 854 **(c)** 18
(d) 23
2. **(a)** 29 **(b)** 44 **(c)** 56
(d) 68 **(e)** 152 **(f)** 266
(g) 150 **(h)** 300 **(i)** 120
(j) 130
3. **(a)** 143 **(b)** 128 **(c)** 280
(d) 200 **(e)** 200 **(f)** 1 150

Exercise 1d *(Pg 10)*

1. **(a)** 6 194 **(b)** 85 920
(c) 273 097
2. **(a)** 124 **(b)** 716 **(c)** 4 938
3. **(a)** 32 **(b)** 342 **(c)** 629

Exercise 1e *(Pg 12)*

1. **(a)** ×, × **(b)** +, + **(c)** +, +
(d) ×, × **(e)** ×, × **(f)** +, +
2. **(a)** 7 **(b)** 4, 5 **(c)** 12
(d) 6 **(e)** 13 **(f)** 6, 9

Exercise 1f *(Pg 13)*

1. **(a)** 242 424 **(b)** 242 424
2. 1 450 548
3. **(a)** 450 **(b)** 620
(c) 590 **(d)** 770
(e) 1 500 **(f)** 6 100
(g) 12 910 **(h)** 81 600
(i) 199 900 **(j)** 650 500
(k) 44 400 **(l)** 222 200
(m) 1 111 000 **(n)** 8 100
(o) 7 200 **(p)** 7 700
(q) 7 992 **(r)** 9 995

Exercise 1g *(Pg 15)*

1. **(a)** = **(b)** > **(c)** <
(d) > **(e)** > **(f)** =
(g) < **(h)** > **(i)** <
(j) <
2. **(a)** $12 - (7 - 2) = 7$
(b) $3 \times (5 + 7) = 36$
(c) $3 \times (5 + 2 \times 4) = 39$
(d) $3 \times (5 + 2) \times 4 = 84$
(e) $(3 \times 5 + 2) \times 4 = 68$
(f) $4 \times (6 - 3) \times 5 = 60$
3. **(a)** 72 **(b)** 112 **(c)** 1
(d) 62 **(e)** 40 **(f)** 5
(g) 440 **(h)** 45 **(i)** 15
(j) 8 **(k)** 282 **(l)** 492
(m) 209
4. 4, 12, \$6
5. 8, 72, 4, 20, \$98
6. 8
7. 5

Exercise 1h *(Pg 18)*

1. **(a)** **(i)** 590 **(ii)** 600
(b) **(i)** 6 840 **(ii)** 6 800
(iii) 7 000
2. **(a)** **(i)** 2 200 **(ii)** 2 000
(b) **(i)** 10 080
(ii) 10 100
(iii) 10 000
(iv) 10 000
3. **(a)** 82 650 **(b)** 82 600
(c) 83 000 **(d)** 80 000
4. **(a)** **(i)** 55 730
(ii) 55 700
(iii) 56 000
(b) **(i)** 380 010
(ii) 38 000
(iii) 38 000
(c) **(i)** 9 969 970
(ii) 9 970 000
(iii) 9 970 000
(d) **(i)** 76 636 790
(ii) 76 636 800
(iii) 76 637 000
5. **(a)** $14\,500 \leqslant a < 15\,500$ or $14\,500 \leqslant a \leqslant 15\,499$
(b) $245\,000 \leqslant b < 255\,000$ or $245\,000 \leqslant b \leqslant 254\,999$

Exercise 1i *(Pg 20)*

1. **(a)** 8 400, 8 652
(b) 50, 52
(c) 80 000, 82 110
(d) 13, 13
(e) 3 400 000, 3 413 160
2. **(a)** (ii) **(b)** (i) **(c)** (ii)
(d) (i) **(e)** (i)
3. **(a)** 52 796 190
(b) 26 011 001
(c) 64 512 270
(d) 1 091
(e) 475 742 484
(f) 25
(g) 55 013
(h) 13 805
(i) 35 074 576
(j) 6 991 771
(k) 8 022 509
(l) 30 838
(m) 5 899
(n) 53 935 970
(o) 3 430 818
(p) 138 479 550
(q) 91 214 514

Review Questions 1 *(Pg 22)*

1. **(c)** 355 **(d)** 3 000
(e) 1 600 **(f)** 8 684
(g) 1 107 **(h)** 12 242
2. **(a)** 1 510 **(b)** 1 350
(c) 720 **(d)** 7 700
(e) 28 **(f)** 40 000
(g) 998 **(h)** 2 024
(i) 38 021
3. **(a)** 80 **(b)** 1 660
(c) 37 **(d)** 1
(e) 27 **(f)** 30
(g) 0 **(h)** 1 150
(i) 130 **(j)** 0
(k) 19 **(l)** 220
4. **(a)** 97 000 **(b)** 3 648 000
(c) 4 000 000 **(d)** 190
(e) 70 **(f)** 200
(g) 68 **(h)** 230
(i) 200 **(j)** 20 000

Exercise 2a *(Pg 27)*

1. **(a)** 1, 2, 4, 8, 16
(b) 1, 2, 4, 7, 14, 28
(c) 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
(d) 1, 2, 4, 5, 10, 20, 25, 50, 100
(e) 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
(f) 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210
2. **(a)** 4, 8, 12, 16, 20, 24
(b) 7, 14, 21, 28, 35, 42
(c) 9, 18, 27, 36, 45, 54
(d) 12, 24, 36, 48, 60, 72
(e) 17, 34, 51, 68, 85, 102
(f) 21, 42, 63, 84, 105, 126
3. 54, 126, 196
4. 1, 2, 3, 4, 8, 9, 12, 16, 48, 144
5. 24, 32, 56, 72, 64, 40, 96, 120
6. 4, 14, 28, 32, 56
7. **(a)** 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40, 48, 60, 80, 96, 120, 160, 240, 480
(b) 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 25, 30, 40, 50, 60, 75, 100, 120, 150, 200, 300, 600
(c) 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40, 48, 60, 64, 80, 96, 120, 160, 192, 240, 320, 480, 960
(d) 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 26, 36, 39, 52, 78, 104, 117, 156, 234, 312, 468, 936
(e) 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 45, 54, 60, 72, 90, 108, 120, 135, 180, 216, 270, 360, 540, 1 080
(f) 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 25, 30, 40, 48, 50, 60, 75, 80, 100, 120, 150, 200, 240, 300, 400, 600, 1 200
8. **(a)** 2 bags of 24 gums, 3 bags of 16 gums, 4 bags of 12 gums, 6 bags of 8 gums, 8 bags of 6 gums, 12 bags of 4 gums, 16 bags of 3 gums, 24 bags of 2 gums
(b) 3 bags of 15 gums, 5 bags of 9 gums, 9 bags of 5 gums, 15 bags of 3 gums
(c) 3 bags each contains 16 orange-flavoured gums and 15 lime-flavoured gums
9. Prime numbers: (a), (c), (f); Composite numbers: (b), (d), (e)
10. 31, 37, 41, 43, 47
11. **(b)** Prime numbers: (i), (ii), (iv), (v)
12. 2, 3; Yes
13. **(a)** Yes **(b)** Yes **(c)** No
14. 13, 31 and 17, 71

Exercise 2b *(Pg 29)*

1. **(a)** 2, 5 **(b)** 2, 4
(c) 2, 4, 5 **(d)** 2, 4
(e) 5 **(f)** none
(g) 2, 4, 5 **(h)** 5
(i) 2, 4 **(j)** 2, 4
(k) 2, 4 **(l)** 2, 5
2. **(a)** 3, 9 **(b)** 3, 9 **(c)** 3, 9
(d) 3, 9 **(e)** 11 **(f)** 3, 11
(g) 3, 11 **(h)** 3, 11 **(i)** 3, 11
3. **(a)** 6, 12 **(b)** 10 **(c)** 6, 12
(d) 15 **(e)** 6, 10, 12, 15
(f) 6, 10, 15
4. Unit digit 0, sum of digits divisible by 3. 660, 540 divisible by 30
5. No, Yes, Yes, No
6. Yes, Yes
7. No

Exercise 2c *(Pg 31)*

1 **(a)** 7^2 **(b)** $2^2 \times 5^2$
(c) 3×7^3 **(d)** $5^2 \times 11^3$
(e) $2^3 \times 13^2 \times 31$
(f) $5^3 \times 19^2 \times 23 \times 29^2$
2. **(a)** $2^2 \times 7$ **(b)** $2^4 \times 3$
(c) 2×3^3 **(d)** $2^3 \times 11$
(e) $2^2 \times 3^3$ **(f)** $2^4 \times 3^2$
(g) $2^6 \times 3$ **(h)** 2^8
3. **(a)** $2 \times 2 \times 2$
(b) $2 \times 2 \times 3 \times 3$
(c) $2 \times 3 \times 5$
(d) $2 \times 2 \times 2 \times 3 \times 3$
4. **(a)** 2^4 **(b)** $2^3 \times 5$
(c) $3^2 \times 5$ **(d)** $2^3 \times 7$
(e) $2^2 \times 3 \times 5$ **(f)** $2^2 \times 3 \times 7$
(g) $2 \times 3 \times 19$ **(h)** $2^3 \times 3 \times 5$
5. **(a)** $2^2 \times 5^2$ **(b)** 5^3
(c) 3×7^2 **(d)** $2^3 \times 3^3$
(e) $3^2 \times 5^2$ **(f)** $2^3 \times 3^2 \times 5$
(g) $3^4 \times 7$ **(h)** $2^3 \times 3^4$

Exercise 2d *(Pg 33)*

1. **(a)** 3 **(b)** 2, 4
(c) 3 **(d)** 7
(e) 3, 9 **(f)** 3, 5, 15
(g) 2, 3, 4, 6, 12
(h) 3, 5, 15
2. **(a)** 6 **(b)** 6 **(c)** 14
(d) 15 **(e)** 8 **(f)** 16
(g) 5 **(h)** 7 **(i)** 8
(j) 12 **(k)** 28 **(l)** 9
(m) 3 **(n)** 18 **(o)** 33
(p) 4
3. **(a)** 9 **(b)** 21 **(c)** 16
(d) 12 **(e)** 16 **(f)** 15
(g) 12 **(h)** 180 **(i)** 350
4. **(a)** 30 cm **(b)** 12
5. **(a)** 14 cm **(b)** 37

Exercise 2e *(Pg 36)*

1. **(a)** 21 **(b)** 65
(c) 18 **(d)** 30
(e) 21 **(f)** 24
(g) 36 **(h)** 75
(i) 72 **(j)** 150
(k) 1 755 **(l)** 162
(m) 300 **(n)** 1 080
(o) 144 **(p)** 1 575
(q) 250 **(r)** 400
(s) 4 410

2. **(a)** 90 **(b)** 48
(c) 72 **(d)** 126
(e) 5 236 **(f)** 61 425
(g) 72 **(h)** 72
3. **(a)** 1 417 500 **(b)** 16 200
(c) 138 600
4. **(a)** 6, 126 **(b)** 7, 84
(c) 13, 78 **(d)** 70, 420
(e) 15, 450 **(f)** 112, 672
5. 8.01 p.m.
6. 5 p.m.

Exercise 2f *(Pg 38)*

1. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144
2. 16, 25, 49, 81, 100, 1, 144, 169, 225, 400
3. 121, 144, 169, 196, 225, 256, 289, 324, 361, 400
4. **(a)** 8 **(b)** 12 **(c)** 17
(d) 60
5. **(a)** 6 **(b)** 9 **(c)** 12
(d) 14 **(e)** 16 **(f)** 18
(g) 21 **(h)** 22
6. **(a)** 34 **(b)** 36 **(c)** 42
(d) 99 **(e)** 105 **(f)** 186
7. 27, 64, 343, 512, 729, 1 000
8. **(a)** 11 **(b)** 19 **(c)** 13
(d) 12 **(e)** 180
9. **(a)** $3^3 \times 5^3$; 15 **(b)** 2^{12}, 16
(c) $2^9 \times 3^3$; 24 **(d)** $2^6 \times 7^3$; 28
(e) $2^6 \times 3^6$; 36 **(f)** $3^6 \times 5^3$; 45
(g) 2^{18}; 64 **(h)** $2^9 \times 3^6$; 72
10. 3 136 cm^2
11. 48 cm
12. 1331 cm^3
13. 14 cm

Exercise 2g *(Pg 42)*

1. **(a)** 1 600 **(b)** 3 600
(c) 12 100 **(d)** 27 000
(e) 64 000 **(f)** 1 000 000
(g) 40 000 **(h)** 27 000 000
2. **(a)** 6 **(b)** 3 **(c)** 8
(d) 5 **(e)** 9 **(f)** 12
(g) 10 **(h)** 30
3. **(a)** 676 **(b)** 1 369
(c) 6 084 **(d)** 9 801
(e) 15 129 **(f)** 2 197
(g) 24 389 **(h)** 39 304
(i) 300 763 **(j)** 1 295 029
(k) 31 **(l)** 47
(m) 59 **(n)** 106
(o) 263 **(p)** 16
(q) 41 **(r)** 68
(s) 91 **(t)** 124
(u) 1 650 **(v)** 18 717
(w) 15 704 **(x)** 426
(y) 103

Review Questions 2 *(Pg 43)*

1. **(a)** F **(b)** F **(c)** T
(d) F **(e)** F **(f)** F
(g) T **(h)** F
2. **(a)** 18 **(b)** 6 **(c)** 13
(d) 5
3. **(a)** 60 **(b)** 144 **(c)** 630
(d) 2 640
4. **(a)** 6 **(b)** 4 **(c)** 8
5. **(a)** 45 **(b)** 120
(c) 7 689 **(d)** 8 165
(e) 1 183
7. 30 m
8. 8 cm
9. (4, 120), (12, 40), (20, 24), (8, 60)
10. (8, 0), (4, 4), (0, 8), (9, 8)
11. 532 $(19 \times 7 \times 2 \times 2)$

Exercise 3a *(Pg 47)*

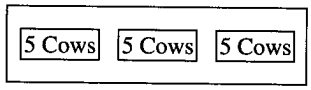
1. **(a)** 14, 17, 20 **(b)** 40, 50, 60
(c) 80, 87, 94 **(d)** 48, 40, 32
(e) 46, 73, 82 **(f)** 55, 43, 39
2. **(a)** multiply the preceding term by 3; 81, 243, 729
(b) multiply the preceding term by 2; 96, 192, 384
(c) divide the preceding term by 2; 100, 50, 25
(d) multiply the preceding term by 3; 324, 972, 2 916
3. **(a)** add 5 to preceding term; 34, 39
(b) add 11 to preceding term; 72, 83
(c) subtract 6 from preceding term; 49, 43
(d) subtract 9 from preceding term; 63, 54
(e) multiply preceding term by 2; 240, 480
(f) divide preceding term by 3; 27, 9
4. **(a)** 15, 21, 28 **(b)** 37, 42, 47
(c) 33, 32, 27 **(d)** 10, 8, 9
(e) 29, 58, 61 **(f)** 21, 34, 55

Exercise 3b *(Pg 50)*

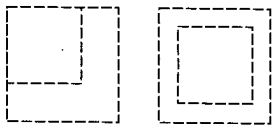
1. **(a)** $1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2 = (5 + 1)^2$
$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7^2 = (6 + 1)^2$
(b) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 144 = 12^2 = (11 + 1)^2$
(c) $a = 25$, $c = 13$, $d = 12$
2. **(a)** $2 + 6^2 = 38$ **(b)** 8
3. **(a)** $10^2 - 9^2 = 19 = 10 + 9$
(b) 1 195
(c) $m = 41$, $n = 40$
4. **(a)** $\frac{11 \times 12}{2} + (11 - 1)^2 = 166$
(b) $p = 10$, $q = 136$
5. **(a)** 9, 12 **(b)** $t = 3n$
(c) **(i)** 150 **(ii)** 29
6. **(a)** 16, 25
(b) $T = (N + 1)^2$
(c) **(i)** 100 **(ii)** 10

Exercise 3c *(Pg 57)*

1. **(a)** $3 + 1 = 4$, $4 + 1 = 5$, $5 + 1 = 6$, $6 + 1 = 7$
(b) 50 **(c)** 100
2. **(a)** **(i)** $\frac{8 - 2}{2} = 3$, $\frac{10 - 2}{2} = 4$, $\frac{12 - 2}{2} = 5$, $\frac{14 - 2}{2} = 6$
(ii) $2(3) + 2 = 8$,
$2(4) + 2 = 10$,
$2(5) + 2 = 12$,
$2(6) + 2 = 14$
(b) **(i)** 9 **(ii)** 14
(c) **(i)** 46 **(ii)** 74
3. **(a)** $\frac{4 \times (4 - 1)}{2} = 6$,
$\frac{5 \times (5 - 1)}{2} = 10$,
$\frac{6 \times (6 - 1)}{2} = 15$,
$\frac{7 \times (7 - 1)}{2} = 21$
(b) 190
4. 120 **5.** 39 min
6. 5 games, 26 games
7. 15 minutes
8.



9.



Review Questions 3 *(Pg 60)*

1. **(a)** 25, 31, 37 **(b)** 42, 57, 75
(c) 16, 26, 42 **(d)** 85, 81, 77
(e) 21, 25, 29 **(f)** 27, 38, 51

2. **(a)** 9 **(b)** 6 **(c)** 16
(d) 42 **(e)** 28 **(f)** 26
(g) 95

3. **(a)** $1\,111\,111\,111 - 22\,222 = 33\,333^2$
(b) $x = 1\,111\,111\,111\,111\,111$
$y = 22\,222\,222$

4. **(a)** **(i)** $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28 = \frac{7 \times (7 + 1)}{2}$
(ii) $k = 9$
(b) **(i)** $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = 1 + 8 + 27 + 64 + 125 + 216 = 441 = 21^2$
(ii) $x = 8^3$, $y = 512$, $z = 1\,296$

5. **(a)** 108
(b) 630, 2 460

Exercise 4a *(Pg 64)*

1. **(a)** $\frac{1}{6}$ **(b)** $\frac{2}{9}$ **(c)** $\frac{5}{8}$
(d) $\frac{6}{13}$ **(e)** $\frac{5}{12}$ **(f)** $\frac{11}{100}$

2. **(a)** one-ninth
(b) two-sevenths
(c) five-twentieths
(d) thirty five-hundredths

3. **(a)** $\frac{5}{7}$ **(b)** $\frac{4}{9}$

Exercise 4b *(Pg 66)*

1. **(a)** 12 **(b)** 10 **(c)** 52
(d) 3 **(e)** 135 **(f)** 187

2. $\frac{1}{4} = \frac{4}{16} = \frac{16}{64}$

4. **(a)** 12, 18, 30
(b) 6, 32, 28

5. **(a)** $\frac{1}{2}$ **(b)** $\frac{8}{11}$ **(c)** $\frac{6}{13}$
(d) $\frac{5}{6}$ **(e)** $\frac{5}{8}$ **(f)** $\frac{7}{13}$

6. (a), (b), (c)

7. **(a)** $\frac{7}{10}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{6}{13}$
(d) $\frac{1}{2}$ **(e)** $\frac{2}{3}$ **(f)** $\frac{7}{25}$
(g) $\frac{3}{7}$ **(h)** $\frac{5}{11}$

Exercise 4c *(Pg 69)*

1. **(a)** (i) **(b)** (ii) **(c)** (i)
(d) (iii) **(e)** (i) **(f)** (iii)
(g) (ii) **(h)** (iii)

2. **(a)** $\frac{7}{3}$ **(b)** $\frac{14}{11}$
(c) $\frac{68}{9}$ **(d)** $\frac{23}{5}$
(e) $\frac{23}{4}$ **(f)** $\frac{23}{6}$
(g) $\frac{31}{13}$ **(h)** $\frac{156}{11}$

3. **(a)** $3\frac{1}{7}$ **(b)** 3
(c) $5\frac{5}{6}$ **(d)** $4\frac{2}{3}$
(e) $16\frac{4}{5}$ **(f)** $15\frac{1}{2}$
(g) $10\frac{1}{10}$ **(h)** $11\frac{2}{13}$

4. **(a)** $\frac{4}{10}$ **(b)** $\frac{1}{16}$
(c) $\frac{1}{3}$ **(d)** $\frac{5}{9}$

5. **(a)** $\frac{6}{7}$ **(b)** $\frac{1}{2}$
(c) $\frac{7}{8}$ **(d)** $\frac{2}{3}$

6. Peter

7. Susan

8. **(a)** $\frac{5}{8}, \frac{3}{4}, \frac{11}{12}$
(b) $\frac{4}{9}, \frac{2}{3}, \frac{5}{6}$
(c) $\frac{1}{3}, \frac{1}{2}, \frac{4}{7}$
(d) $\frac{7}{11}, \frac{2}{3}, \frac{5}{6}$

9. **(a)** $\frac{5}{6}, \frac{3}{4}, \frac{7}{12}, \frac{5}{9}$
(b) $\frac{11}{12}, \frac{4}{5}, \frac{3}{4}, \frac{7}{10}$
(c) $\frac{2}{3}, \frac{5}{8}, \frac{1}{2}, \frac{5}{12}$
(d) $\frac{5}{6}, \frac{7}{9}, \frac{13}{18}, \frac{2}{3}$

Exercise 4d *(Pg 73)*

1. **(a)** $\frac{3}{4}$ **(b)** $1\frac{1}{20}$
(c) $1\frac{1}{2}$ **(d)** $1\frac{9}{14}$

2. **(a)** $\frac{1}{6}$ **(b)** $\frac{1}{4}$
(c) $\frac{1}{6}$ **(d)** $\frac{2}{7}$

3. **(a)** $\frac{11}{14}$ **(b)** $1\frac{3}{70}$
(c) $\frac{1}{10}$ **(d)** $\frac{11}{60}$

4. **(a)** $3\frac{4}{5}$ **(b)** $5\frac{13}{18}$
(c) $5\frac{43}{56}$ **(d)** $9\frac{27}{200}$
(e) $3\frac{29}{200}$ **(f)** $3\frac{3}{8}$
(g) $3\frac{9}{10}$ **(h)** $2\frac{5}{12}$
(i) $4\frac{1}{15}$ **(j)** $3\frac{5}{24}$

5. **(a)** $1\frac{1}{14}$ **(b)** $1\frac{7}{8}$
(c) $4\frac{13}{28}$ **(d)** $5\frac{1}{60}$
(e) $5\frac{11}{15}$ **(f)** $4\frac{3}{8}$
(g) $\frac{29}{45}$ **(h)** $3\frac{5}{9}$

6. $4\frac{4}{15}$ km

7. $4\frac{3}{4}$ litres

8. **(a)** $2\frac{1}{3}$ hours or 2 hours and 20 minutes
(b) first classroom
(c) $\frac{1}{6}$ hour or 10 minutes

9. **(a)** $4\frac{3}{4}$ cups
(b) 2 cups

10. $4\frac{1}{2}$ hours

Exercise 4e *(Pg 78)*

1. **(a)** 16 **(b)** $6\frac{2}{3}$
(c) $\frac{10}{21}$ **(d)** 1
(e) 15 **(f)** $\frac{3}{8}$
(g) 9 **(h)** $2\frac{34}{273}$

2. **(a)** 6 **(b)** $2\frac{1}{12}$

(c) $1\frac{7}{128}$ (d) $5\frac{1}{3}$

(e) $\frac{1}{4}$

3. (a) 4 pupils (b) 9 oranges
(c) 32 km (d) 15 hours
(e) 8 kg

4. (a) $\frac{9}{14}$ (b) $10\frac{1}{2}$ kg

(c) $3\frac{3}{4}$ kg

(d) (i) $5\frac{5}{8}$ kg

(ii) $1\frac{1}{8}$ kg

5. (a) $4\frac{2}{3}$ hours or 4 hours and 40 minutes

(b) (i) $9\frac{4}{5}$ hours or 9 hours and 48 minutes

(ii) $1\frac{19}{30}$ hours or 1 hour and 38 minutes

Exercise 4f *(Pg 80)*

1. (a) $\frac{7}{36}$ (b) $\frac{1}{6}$

(c) $3\frac{3}{4}$ (d) $1\frac{7}{10}$

(e) $6\frac{1}{6}$ (f) $1\frac{4}{5}$

2. (a) $\frac{13}{90}$ (b) $\frac{2}{9}$

(c) 10 (d) $1\frac{13}{36}$

(e) 0 (f) $18\frac{1}{24}$

Exercise 4g *(Pg 82)*

1. 216 hectres
2. \$39
3. (a) 24 boys (b) 12 girls
4. 25
5. 85
6. 35

Exercise 4h *(Pg 85)*

1. (a) 0.43 (b) 0.057
(c) 0.000 8 (d) 1.35
(e) 8.25 (f) 2.3
(g) 15.096 (h) 7.000 5
(i) 85.12 (j) 2.62
(k) 19.024 (l) 101.000 11

2. (a) $\frac{3}{4}$ (b) $\frac{9}{25}$

(c) $\frac{1}{40}$ (d) $\frac{3}{500}$

(e) $\frac{21}{200}$ (f) $3\frac{3}{4}$

(g) $\frac{1}{80}$ (h) $15\frac{1}{4}$

(i) $84\frac{5}{8}$

3. (a) 1.625 (b) 0.837 5
(c) 2.687 5 (d) 0.875
(e) 0.593 75 (f) 1.468 75

Exercise 4i *(Pg 86)*

1. (a) $0.\dot{6}$ (b) $0.\dot{4}\dot{5}$
(c) $0.\dot{8}\dot{7}$ (d) $0.2\dot{2}\dot{7}$
(e) $0.\dot{5}71\,428\dot{}$
(f) $0.486\,\dot{1}$

2. (a) $0.\dot{4}$, R
(b) $2.\dot{3}$, R
(c) $0.6\dot{7}$, R
(d) $0.\dot{5}\dot{9}$, R
(e) 0.776, N.R.
(f) $0.29\dot{5}\,\dot{4}$, R

Exercise 4j *(Pg 88)*

2. 1.04, 1.10, 1.115, 1.145, 1.18
3. (a) 2.1, 2.4, 2.7
(b) 3.03, 3.12, 3.16
(c) 0.02, 0.035, 0.065, 0.08
(d) 1.22, 1.25, 1.28, 1.31, 1.345, 1.38
4. (a) < (b) > (c) <
(d) > (e) >, > (f) <, <
5. (a) 0.8, 0.4, 0.3

(b) $1\frac{3}{4}$, 1.54, $\frac{5}{4}$

(c) 1.9, 1.88, 1.13

(d) $0.\dot{6}\dot{5}$, $0.6\dot{5}$, $\frac{13}{20}$, $0.\dot{6}0\dot{5}$

(e) $3.1\dot{4}$, $\frac{22}{7}$, $3.\dot{1}\dot{4}$, 3.14

(f) 2.201, 2.102, $2.0\dot{2}$, 2.012

Exercise 4k *(Pg 89)*

1. (a) 1.358 (b) 336.728
(c) 916.462 (d) 267.203
2. (a) 597.45 (b) 271.91
(c) 0.061
3. (a) 18.97 (b) 297.102
(c) 127.18 (d) 32.365
4. (a) 2.47 (b) 13.28
(c) 4.849 (d) 9.634
(e) 8 (f) 306.826
5. (a) 88.578 (b) 215.23
(c) 11.219

Exercise 4l *(Pg 92)*

1. (a) 0.3 (b) 2.523
(c) 0.024 6 (d) 6.57
(e) 0.007 2 (f) 0.745 6
2. (a) 0.028 5 (b) 591.25
(c) 25.248 (d) 1 599.36
(e) 76.8 (f) 1.538 1
(g) 0.090 62 (h) 0.014
3. (a) 7.36 (b) 1 851.7
(c) 1 502.9 (d) 17 900
(e) 6.6 (f) 124
4. (a) 75.38 (b) 0.002 9
(c) 6.24 (d) 0.000 066
(e) 4 (f) 0.086 5
5. (a) 10.6 (b) 5.7
(c) 0.11 (d) 8 000
(e) 11.1 (f) 30.588
6. (a) 4 (b) 9
(c) 12.8 (d) 0.099
(e) 1 152 (f) 0.078 125

Exercise 4m *(Pg 93)*

1. 60
2. 40 cents
3. \$12.05
4. \$29.33; \$20.67
5. packs of 5.

Exercise 4n *(Pg 96)*

1. (a) (i) 5 (ii) 5.42
(b) (i) 16 (ii) 15.82
(c) (i) 8 (ii) 7.86
(d) (i) 131 (ii) 130.83
2. (a) 712.893 (b) 0.003
(c) 0.827 (d) 7.024
3. 0.286
4. (a) 0.44 (b) 0.64
(c) 0.64 (d) 0.73

Exercise 4o *(Pg 97)*

1. **(a)** 467.622 **(b)** 35.8
(c) 154.92 **(d)** 26.156
(e) 15.846 **(f)** 51.791

2. **(a)** $\frac{19}{20}$, 0.95

(b) $1\frac{5}{6}$, 1.83

(c) $4\frac{1}{6}$, 4.17

(d) $53\frac{19}{27}$, 53.70

(e) $3\frac{1}{3}$, 3.33

(f) $8\frac{63}{64}$, 8.98

3. **(a)** 70, 67.71
(b) 1 000, 1 012.82
(c) 3 101, 3 131.36
(d) 290, 291.08
(e) 20, 19.79

Review Questions 4 *(Pg 99)*

1. **(a)** $\frac{7}{10}, \frac{3}{4}, \frac{4}{5}$

(b) $\frac{7}{12}, \frac{5}{8}, \frac{2}{3}$

(c) $\frac{4}{7}, \frac{7}{12}, \frac{25}{42}, \frac{9}{14}$

2. **(a)** $\frac{5}{8}, \frac{7}{12}, \frac{5}{9}, \frac{13}{24}$

(b) $\frac{2}{7}, \frac{4}{15}, \frac{5}{21}, \frac{6}{35}$

(c) $\frac{6}{11}, \frac{13}{25}, \frac{7}{15}, \frac{9}{20}, \frac{2}{5}$

3. Xinyu

4. **(a)** $3\frac{3}{4}$ **(b)** $2\frac{10}{21}$
(c) $\frac{13}{24}$ **(d)** $1\frac{3}{10}$
(e) $4\frac{1}{9}$ **(f)** $3\frac{29}{72}$

5. **(a)** 3 **(b)** $1\frac{1}{2}$
(c) $6\frac{2}{9}$ **(d)** $\frac{3}{10}$
(e) $\frac{1}{2}$ **(f)** $\frac{3}{5}$
(g) $5\frac{31}{42}$ **(h)** $22\frac{1}{2}$

6. **(a)** 31 **(b)** 9.26
(c) 0.156 52 **(d)** 0.12
(e) 120 **(f)** 413

7. **(a)** $\frac{9}{16}$ **(b)** 3.12
(c) $\frac{2\,000}{16\,947}$ **(d)** 1 051.48
(e) $\frac{361}{432}$

8. 25, yes.

Revision Exercise I No 1 *(Pg 101)*

1. **(a)** 34 **(b)** 160
(c) 800 **(d)** 4 224

2. **(a)** 900 **(b)** 30 **(c)** 2

3. 3, 180

4. **(a)** 3 **(b)** $\frac{1}{2}$ **(c)** $\frac{3}{4}$

5. **(a)** 21, 25 **(b)** $\frac{5}{6}, \frac{6}{7}$
(c) $5 \times 7, 6 \times 8$

6. **(a)** 7.91 **(b)** 61.56
(c) 29.63 **(d)** 10.89

7. **(a)** 3 **(b)** 2
(c) $\frac{3}{40}$ **(d)** 2

8. **(a)** 2 900 **(b)** 0.67
(c) $\frac{17}{40}$ **(d)** 2.84

9. **(a)** \$89 **(b)** \$11

10. **(a)** 5, 85
(b) 3, 5, 7; 51, 85, 119

Revision Exercise I No 2 *(Pg 101)*

1. **(a)** 69, 61, 52 **(b)** 8, 7, 10
(c) 0, 12, 24

2. **(a)** 5.476 **(b)** 4

3. **(a)** 180 **(b)** 52

4. **(a)** **(i)** 504 **(ii)** 280
(iii) 3 465
(b) **(i)** 8 **(ii)** 6

5. **(a)** 20 **(b)** 57 **(c)** 10

6. **(a)** 0.3 125 **(b)** $\frac{9}{40}$
(c) **(i)** 9.91 **(ii)** 9.9

7. 500 g pack, 10¢ per kg

8. **(a)** 0.98 **(b)** 12.23
(c) 2.04 **(d)** 2

9. **(a)** $6^2 - 2 \times 6 = 24$
(b) 9

10. **(a)** $4\frac{1}{10}$ **(b)** $3\frac{5}{8}$
(c) $\frac{1}{3}$

Revision Exercise I No 3 *(Pg 102)*

1. 10.5

2. **(a)** 907 **(b)** 22.7 022
(c) 50

3. **(a)** $\frac{14}{25}$ **(b)** 0.29

4. **(a)** $\frac{13}{20}$ **(b)** $\frac{3}{13}$
(c) $\frac{3}{4}$ **(d)** $\frac{57}{68}$

5. $x = 4$, $y = 1$, $z = 2$

6. **(a)** **(i)** 3×2^4 **(ii)** $2^2 \times 3^4$
(b) **(i)** 7 **(ii)** 3
(c) **(i)** 504 **(ii)** 210

7. **(a)** 3, 6 **(b)** 77, 88
(c) 65, 78

8. 17, 19, 23, 29, 31, 37, 41, 43

9. \$100

10. **(a)** $\frac{3}{4}, \frac{5}{6}, \frac{8}{9}, \frac{11}{12}$
(b) $\frac{2}{5}, \frac{3}{8}, \frac{1}{3}, \frac{1}{4}$

Revision Exercise I No 4 *(Pg 103)*

1. **(a)** 3 **(b)** 47.7
(c) 17.1 **(d)** −1.725

2. **(a)** $\frac{77}{96}$ **(b)** $\frac{11}{48}$

3. **(a)** No **(b)** No **(c)** No
(d) Yes **(e)** No **(f)** No

4. **(a)** **(i)** 0.175 **(ii)** 0.4 125
(iii) 0.36 **(iv)** 1.3 125
(b) **(i)** $\frac{33}{500}$ **(ii)** $\frac{23}{40}$
(iii) $\frac{7}{8}$ **(iv)** $\frac{7}{16}$

5. 5 h 36 min

6. 9

7. \$48.09; \$1.91

8. **(a)** 96, 4 **(b)** 840, 4

9. **(a)** 125, 216
(b) 1 024, 4 096
(c) 16, 22

10. **(a)** $\frac{1}{2}$ **(b)** $\frac{2}{9}$

Revision Exercise I No 5 *(Pg 103)*

1. **(a)** $\frac{5}{8}$ **(b)** $1\frac{6}{7}$
(c) $5\frac{29}{48}$ **(d)** $\frac{17}{24}$

2. 720 m

3. **(a)** 104 **(b)** 11

4. **(a)** 80 **(b)** $1\frac{1}{4}$
(c) $1\frac{1}{3}$ **(d)** $1\frac{2}{5}$
5. **(a)** 8.03 **(b)** 1.67
(c) 36.04
6. **(a)** 2, 4, 6, 8, 10, 12, 14
(b) 1, 3, 5, 7, 9, 11
(c) 2, 3, 5, 7, 11, 13, 17, 19, 23
(d) 1, 2, 4, 8, 16, 32
7. **(a)** 16, 192 **(b)** 3, 756
(c) 12, 360 **(d)** 4, 1 344
8. $15 580
9. **(a)** $\frac{3}{56}$ **(b)** $\frac{2}{5}$
(c) $0.57\dot{1}\,\dot{4}$, $\frac{4}{7}$, $0.571\,\dot{4}$, $0.5\dot{7}1\,\dot{4}$
10. **(a)** 26, 37, 50
(b) 0.005, 0.000 6, 0.000 07
(c) 2, 0, –2
(d) 23, 37, 60
(e) $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$

Exercise 5a *(Pg 107)*

1. **(a)** negative
(b) **(i)** +20 **(ii)** +90
(iii) –9
2. **(a)** 30 m above sea-level
(b) –35
(c) rotating 30° anti-clockwise
(d) speed of 45 km/h of a car moving towards west
3. **(a)** –1 **(b)** +2 **(c)** –4
4. **(a)** < **(b)** > **(c)** <
(d) < **(e)** > **(f)** <
5. **(a)** –5° < 12°
(b) $200 > –$120
(c) –40 m < –25 m
6. **(a)** –90, –60, –6, –2, 0
(b) –1 500, –1, 1, 2
8. **(a)** > **(b)** < **(c)** >
(d) > **(e)** < **(f)** >

Exercise 5b *(Pg 109)*

1. **(a)** –5 **(b)** –5
(c) –7 **(d)** –6
(e) 8 **(f)** –9
2. **(a)** –11 **(b)** –16
(c) –14 **(d)** 20
(e) –21 **(f)** –20
(g) –30

Exercise 5c *(Pg 111)*

1. **(a)** 3 **(b)** –1
(c) –1 **(d)** –4
(e) 4 **(f)** –1
2. **(a)** 8 **(b)** 8
(c) 7 **(d)** 11
(e) –15 **(f)** –26
(g) –41 **(h)** –15
(i) –99
3. **(a)** –9 **(b)** –14
(c) 25 **(d)** 13
(e) –35 **(f)** 56
(g) 26 **(h)** –153
4. 9°C
5. **(a)** $ (6 000 – 2 000 – 5 500)
(b) $1 500 loss

Exercise 5d *(Pg 113)*

1. **(a)** –5 **(b)** –4
(c) –4 **(d)** –6
(e) 4 **(f)** 0
(g) 6 **(h)** –4
2. **(a)** 5 **(b)** –7
(c) –11 **(d)** –17
(e) 15 **(f)** 14
(g) 1 **(h)** 11
(i) –65 **(j)** 70
(k) –270 **(l)** –38
3. **(a)** 0 **(b)** –1
(c) –4 **(d)** 8
(e) 1 **(f)** 8
(g) 17 **(h)** –13
(i) –50
4. **(a)** –24 **(b)** –41
(c) –21 **(d)** –368
(e) –7 **(f)** 96
(g) –29 **(h)** –152
(i) –50

Exercise 5e *(Pg 115)*

(a) –21 **(b)** 6 **(c)** –32
(d) 0 **(e)** 280 **(f)** 0
(g) –2 **(h)** 61 **(i)** –6
(j) –55 **(k)** 0 **(l)** 8
(m) 0 **(n)** –16 **(o)** 192
(p) 0 **(q)** 0 **(r)** 28

Exercise 5f *(Pg 116)*

1. **(a)** ×, × **(b)** +, +
(c) +, + **(d)** ×, ×
2. **(a)** –28, 60 **(b)** 12, –120
(c) –14, –50 **(d)** 8, 9, 11
(e) 10, –20 **(f)** –15, –9, 90
3. **(a)** 13 **(b)** –80
(c) 4 **(d)** 125
(e) 40 **(f)** –18
(g) 28 **(h)** 0
(i) –4 **(j)** –9
(k) –17

Exercise 5g *(Pg 119)*

1. **(a)** $\frac{3}{4}$ **(b)** –1
(c) $-1\frac{1}{10}$ **(d)** $-3\frac{9}{10}$
(e) $-\frac{11}{12}$ **(f)** $-\frac{13}{20}$
(g) $\frac{3}{20}$ **(h)** $-7\frac{13}{28}$
(i) $\frac{7}{12}$ **(j)** $2\frac{7}{16}$
2. **(a)** $4\frac{1}{2}$ **(b)** $-\frac{2}{15}$
(c) $1\frac{11}{16}$ **(d)** $-\frac{1}{18}$
(e) 6 **(f)** $\frac{3}{13}$
(g) $\frac{112}{115}$

Exercise 5h *(Pg 121)*

1. **(a)** R **(b)** R **(c)** R
(d) I **(e)** I **(f)** R
(g) R **(h)** I **(i)** R
(j) R **(k)** R **(l)** R
(m) I **(n)** R **(o)** I
2. **(a)** T **(b)** F **(c)** T
(d) T **(e)** T **(f)** F
3. **(a)** 6 095 **(b)** 367 770
(c) –423 **(d)** –33 973
(e) $-\frac{65}{903}$ **(f)** $-3\frac{122}{539}$
(g) $2\frac{159}{184}$ **(h)** $-\frac{2\,821}{34\,542}$
(i) $\frac{598}{1\,225}$
4. **(a)** 11.36 **(b)** 4.33
(c) –2.47 **(d)** 2.37
(e) 0.50 **(f)** 23.60
(g) 19 940.66
(h) 113 269.73
(i) –0.07
(j) 0.79

Review Questions 5 *(Pg 122)*

1. **(a)** 67 **(b)** −28
(c) −22 **(d)** −100
(e) −36 **(f)** 390
(g) 1 130 **(h)** −279
2. **(a)** −120 **(b)** −12
(c) 56 **(d)** 3
(e) 10 **(f)** −24
(g) 6 **(h)** 54
(i) 87
3. **(a)** 28 **(b)** −88 **(c)** −26
(d) −4 **(e)** 3 **(f)** −102
4. **(a)** $-\frac{19}{30}$ **(b)** $-3\frac{5}{12}$ **(c)** $-\frac{4}{9}$
(d) $-3\frac{5}{24}$ **(e)** $1\frac{5}{18}$ **(f)** 35
5. $-\frac{11}{13}$
6. **(a)** $1.428,\ 1.4\dot{2}8\dot{,}\ 1\frac{3}{7},\ 1.428\dot{}$
(b) $-3.6\dot{,}\ -3\frac{7}{11},\ -3.6\dot{3},\ -3.63$
(c) $-1\frac{4}{9},\ -1.43\dot{5},\ -1.4\dot{3}\dot{5},$ $-1.4\dot{3}\dot{5}$

Exercise 6a *(Pg 127)*

1. 4 kg **2.** No. **3.** B
4. **(a)** (iii) **(b)** (iii) **(c)** (ii)
(d) (i) **(e)** (i)
5. **(a)** (iii) **(b)** (iii) **(c)** (ii)
(d) (iii) **(e)** (iv)

Exercise 6b *(Pg 132)*

1. **(a)** 460 g **(b)** 700 g
(c) 3 cm **(d)** 123.5 cm
(e) 18 **(f)** 30 m
(g) 12.4 cm
(h) 4 300 pupils
(i) 850 km **(j)** 22.6 mm
2. **(a)** 3 **(b)** 3 **(c)** 5
(d) 3 **(e)** 2 **(f)** 2
(g) 4 **(h)** 4
(i) 1, 2, 3, 4, 5, 6 or 7
(j) 3 **(k)** 2 **(l)** 2
3. **(a)** 3.1 **(b)** 1.484
(c) 0.003 **(d)** 20
(e) 3.1 **(f)** 0.57
(g) 0.056 8 **(h)** 217.01
(i) 3.60 **(j)** 15.70
(k) 0.003 6 **(l)** 6.0
(m) 0.035 **(n)** 18.0
(o) 8.4 **(p)** 120.41
(q) 4.83 **(r)** 12
(s) 0.050 **(t)** 0.080
(u) 0.010 1 **(v)** 0.103
4. **(a)** 4
(b) 400 000 000
5. 0.05
6. **(a)** 0.010 5 **(b)** 0.442
(c) 31.2 **(d)** 11.1
7. 28, 0
8. 0.211, 3
9. 0.008 3, 2
10. 117.96, 5

Review Questions 6 *(Pg 133)*

1. **(a)** (iii) **(b)** (ii)
(c) (ii) **(d)** (ii)
2. **(a)** (i) **(b)** (ii)
(c) (ii) **(d)** (i)
3. **(a)** 0.086 **(b)** 0.085 7
(c) 5.10 **(d)** 730 000
(e) 0.06 **(f)** 0.06
(g) 0.006 14 **(h)** 0.004
4. 20
5. **(a)** 50 **(b)** 2 000

Exercise 7a *(Pg 137)*

1. **(a)** $\square \times 7 = 91$
(b) $(\square - 5) \times 4 = 28$
(c) $\square \times 5 + 4 = 19$
(d) $(\square - 2) \times 3 = 12$
(e) $(\square + 15) \times \triangle = 84$
(f) $2\square = \$32$
(g) $\square + 2\triangle = \$8.40$
(h) $2\square + 5\triangle = 47$
(i) $5\square + 3\triangle = \$98$
(j) $320\square + 450\triangle = \$128$
(k) $2 \times \frac{3}{4}\square + 4\square = 6$ days
(l) $\square + 2\triangle \leqslant \$3\,000$
(m) $3\square + 2\triangle = 2\frac{2}{3}$ h

Exercise 7b *(Pg 141)*

1. **(a)** $-5x^3 + 7x^2 + 3x + 4$
(b) $7x^4 + 7x^3 - 5x^2 - 4x$
(c) $5x^3 + 4x^2 - 7x + 4$
(d) $5x^5 - 6x^3 + 7x^2 + 7$
2. **(a)** $4 + 7x - 3x^2 + 4x^3$
(b) $-4x - 9x^2 + 8x^3 + 4x^5$
(c) $-4a + 2a^2 + 3a^5 - 4a^6$
(d) $-3b - 4b^2 + 4b^3 + 7b^5$
3. **(a)** $2x + 14$ **(b)** $5a - 14$
(c) $8k$ **(d)** $\frac{x}{3y}$
(e) $2x + 6y$ **(f)** $\frac{y}{2} - 5x$
4. **(a)** $2x + y$ **(b)** $7k$
(c) $2t - 15$ **(d)** $3u - 4$
(e) $\frac{v}{2} + 8$ **(f)** $\$\left(\frac{h}{2} + 2k\right)$
(g) $\$(1.82x + 2.22y)$
5. **(a)** −36 **(b)** 24
(c) 2 **(d)** −6
(e) −55 **(f)** 164
6. **(a)** $\$1.10x$ **(b)** $\frac{3x}{3+k}$
(c) $5(h + 3)$ **(d)** $\frac{1}{4}(m - 4)$
(e) kn eggs **(f)** xz km
7. **(a)** $3(x + 1)$ **(b)** $5(x - 6)$
(c) $4x + 8$ **(d)** $\frac{x - 14}{3}$
8. **(a)** $-1\frac{5}{6}$ **(b)** $-3\frac{1}{4}$
(c) $-1\frac{1}{2}$ **(d)** $-\frac{1}{5}$
(e) $-\frac{5}{6}$ **(f)** $-\frac{16}{21}$
(g) $-\frac{8}{11}$ **(h)** $-\frac{7}{36}$

Exercise 7c *(Pg 143)*

1. **(a)** $14x$ **(b)** 0
(c) $a - 6b$ **(d)** $2c - d$
(e) $7x + 6y$ **(f)** $4pq$
(g) $2ef$ **(h)** $13de - bc$
(i) $2p^2$ **(j)** $4a^2$
(k) $-q^3$ **(l)** $t^3 + t$
2. **(a)** $60n$ **(b)** $-12a$
(c) $8k$ **(d)** $5a$
(e) $\frac{3}{2}m$ **(f)** $-4b$
(g) $-8n$ **(h)** $2a$
(i) $3u$ **(j)** $-\frac{c}{11}$
(k) $9v$ **(l)** $\frac{3}{a}$
(m) $-\frac{3}{d}$ **(n)** $20m$
3. **(a)** $-14k^2$ **(b)** $32b^2$
(c) $-9x^2$ **(d)** $\frac{2}{3}y^2$
(e) $15ab$ **(f)** $-14mn$
(g) $12uv$ **(h)** $\frac{6b}{a}$

(i) $\frac{4m}{3n}$ (j) $-70a^2b$

(k) d^2e (l) $\frac{10x^3}{3y}$

(m) $\frac{5c}{d}$ (n) $6d^3e^2f$

(o) $5c^3d^2$

Exercise 7d *(Pg 145)*

1. (a) $5a + 7b$ (b) $15v - 2u$
(c) $3b - 5a$ (d) $4x - 8y$
(e) $11m - 8n$ (f) $7k - 18h$
(g) $5ax - 32bx + 27cx$
(h) $13x - 4y$ (i) $27b - 9a$
(j) $9p - 14q$
(k) $7a - 9b + 10c$
(l) $-6b - 4k$ (m) $9y - 5x - 6$
(n) $6q - 5p + 20$

2. (a) $2x - 3$ (b) $4p - 2$
(c) $a - b + c + d$
(d) $28 - 4x$ (e) $a - 2$
(f) $4p - 2q$ (g) $-6a - 16$
(h) $25c + 5d$ (i) $17x - 2y$
(j) $a - 2b$
(k) $9w - 11r - 6t$
(l) $3y - 6x$ (m) $-24x - 28y$
(n) $200y$ (o) $12 - 13a$
(p) $10b - 7a$ (q) $4a - 4b$
(r) $-9x - 10y$

3. (a) $\frac{17x - 20}{35}$ (b) $\frac{28x + 26}{15}$

(c) $\frac{23x - 3}{10}$ (d) $\frac{10x - 25}{28}$

(e) $\frac{18 - 17x}{20}$ (f) $\frac{23x - 6}{20}$

Exercise 7e *(Pg 147)*

1. (a) $4x^2 - x + 8$
(b) $x^3 + 9x^2 - 3x - 8$
(c) $2a^2 + 4a + 13$
(d) $2a + c$
(e) $6p - 2q + 3r$
(f) $x + 2y - 4z$
(g) $x^3 + 5x^2 + 4x + 15$
(h) $3p + 5q + 3r - 5s$
(i) $2xz$
(j) $2x^4 + 3x^3 - 4x^2 - 10x + 5$
(k) $4x^4 + 8x^2 + 3x - 14$
(l) $10x^2y + 10xy + xy^2$

2. (a) $x^2 + 4x - 2$
(b) $-x^2 + x - 5$
(c) $2a + 5b - 10c$
(d) $p - 6q - 3r + s$
(e) $x^3 - 6x^2 - 7x + 10$
(f) $-a^3 - 3a^2 + 2a - 2$
(g) $2a + 2b - 9c - 4d$
(h) $5a^5 + 7a^4 - 9a^3 + 3a + 8$
(i) $5a^5 + a^4 - 3a^3 - 2a^2 - 16$
(j) $2a^6 - 5a^5 - 10a^4 + 11a^3 - 12a^2 + 3a - 8$

3. (a) $5a^2 - 2a$ (b) $-3a + 3b$
(c) $-8a + 26b - 62ab$
(d) $13a - 8b - 2c$
(e) $3a + 6b - 10c$
(f) $-37a + 12b - 37c$
(g) $7a - 7b - 12c$
(h) $-12a - 53b + 54c$
(i) $61a - 68b + 7c$
(j) $18a + 60b - 52c$

Review Questions 7 *(Pg 148)*

1. (a) 27 (b) 8 (c) -34
(d) -18 (e) 3 (f) 81

2. $-\frac{1}{2}$

3. 6

4. (a) $60f - 68e$ (b) $2m - 9$
(c) $4 - k$ (d) $7 - 7x$
(e) 17 (f) $-10x - 2$
(g) $6x + 4$
(h) $a - b + c - d - e$
(i) $126 - 45a$ (j) $3x + 7y$
(k) $13a - 18$ (l) $30y - 18x$
(m) $\frac{34a - 5b - 21c}{30}$
(n) $\frac{26a + 2b}{3a}$
(o) $\frac{10x - 3y + 19}{15}$
(p) $\frac{3x + 35y - 2z - 120}{42}$

5. $10p^2 + 7pq + 4q^2$

6. $-x^2 - x - 15$

7. $9a^2 - 10ab + b^2$

Exercise 8a *(Pg 152)*

1. (a) 2 (b) 47 (c) 4
(d) 8 (e) 8 (f) 15
(g) 12 (h) 12 (i) 9
(j) 2 (k) 10 (l) 5

2. (a) 2 or -2
(b) 3 or -3
(c) not possible
(d) 25 (e) 27 (f) -8
(g) 60 (h) 24

Exercise 8b *(Pg 154)*

1. (a) 5 (b) 6 (c) 0
(d) -5 (e) -3 (f) 0
(g) 7 (h) -11 (i) $\frac{1}{3}$

2. (a) 2 (b) 3 (c) -2
(d) -3 (e) 1 (f) 7
(g) $1\frac{1}{2}$ (h) 7

3. (a) identity (b) identity
(c) -1 (d) $-16\frac{1}{2}$
(e) identity (f) $4\frac{1}{6}$
(g) identity

Exercise 8c *(Pg 156)*

1. (a) 10 (b) $7\frac{1}{2}$
(c) -21 (d) $1\frac{19}{21}$
(e) $2\frac{2}{5}$ (f) 12
(g) 36 (h) -60
(i) $-2\frac{1}{2}$ (j) 15
(k) $25\frac{5}{7}$ (l) 13
(m) $2\frac{2}{3}$ (n) $\frac{-1}{3}$
(o) $-4\frac{3}{4}$ (p) $-1\frac{1}{3}$
(q) -2 (r) 22

2. (a) -10 (b) 2
(c) 0.15 (d) 4
(e) -0.64 (f) 1.21
(g) 34 (h) 2.10
(i) 2.92 (j) 0.907

Exercise 8d *(Pg 157)*

1. 86 **2.** 86 **3.** 1 386
4. 22 **5.** 5 **6.** 32
7. $1\frac{5}{6}$ **8.** $2\frac{1}{2}$ **9.** 15
10. 3 **11.** $4\frac{1}{8}$ **12.** 2
13. -123 **14.** -30 **15.** $6\frac{1}{3}$
16. $5\frac{3}{10}$

Exercise 8e *(Pg 158)*

(a) $S = a + b + c$ **(b)** $P = xy$
(c) $D = a - e$ or $e - a$
(d) $A = \frac{1}{2}\pi r^2$ **(e)** $C = \frac{3m}{25}$
(f) $A = \frac{1}{4}(m + n + p + q)$ years old
(g) $x = 180 - 2y$
(h) $T = 60a + b$
(i) $T = dp + cq$
(j) **(i)** $r = p + q$
(ii) $\frac{xy(p+q)}{py+qx}$ kmh^{-1}

Exercise 8f *(Pg 160)*

1. 5 000 g; 1 000x g
2. \$24; \$4p; \$$pq$ **3.** $\frac{1}{4}m$ kg
4. $2b$ km
5. 60 m minute
6. $\frac{y}{7}$ weeks **7.** $20x$
8. $\frac{1}{2}b$ **9.** $\frac{21}{v}$ hours
10. \$$(a + b)$
11. $6b$ years; $(7b + 2y)$ years
12. $(2x + 4)$ years
13. $n - 1$, n, $n + 1$
14. $(5u + 3v)$ km

Exercise 8g *(Pg 161)*

1. **(a)** $x + 7 = 18$
(b) $(x - 2) \times 3 = 24$
(c) $(x - 5) \times 7 = 63$
(d) $(24 - x) \div 5 = 4$
(e) $x + (x + 1) + (x + 2) = 63$
(f) $x + (x + 3) = 43$
(g) $6x = 2x + 16$
(h) $2[x + (x + 5)] = 32$
(i) $2(x + 2x) = 54$
2. \$70
3. 15
4. $3x + x + 2x = 4\,800$
5. $3x + 2(2x) = 450$

Exercise 8h *(Pg 163)*

1. 11, 12, 13, 14 **2.** 14
3. 4 **4.** 16, 64
5. 19, 15 and 13 **6.** \$20, \$10
7. 8 700 kg **8.** 27
9. 9, 36 **10.** \$3.50
11. \$32, \$128, \$96
12. \$152 **13.** 12

Review Questions 8 *(Pg 164)*

1. **(a)** $2\frac{9}{11}$ **(b)** $-4\frac{1}{4}$ **(c)** 30
(d) $\frac{1}{4}$ **(e)** -6 **(f)** $\frac{4}{}$
(g) $\frac{17}{19}$ **(h)** $6\frac{3}{5}$
2. **(a)** identity **(b)** $-1\frac{3}{5}$
(c) -29 **(d)** identity
(e) identity **(f)** $2\frac{35}{37}$
3. 45
4. 120 kg, 240 kg, 120 kg
5. 120 **6.** 16 hours
7. 9 **8.** 24 days
9. 8 **10.** $3\frac{3}{5}$ km
11. 8, 40 **12.** 21, 24
13. 36 km/h **14.** 25, 13
15. 25 cents

Revision Exercise II No 1 *(Pg 167)*

1. $404x$
2. $20y - 8x$
3. **(a)** $3a$ **(b)** $28c^2$
(c) $5ab$ **(d)** $10a + c$
(e) $14a^2$ **(f)** 0
(g) $2xy$ **(h)** 0
(i) $20a$ **(j)** $8x$
4. **(a)** 6 **(b)** $5\frac{2}{11}$
(c) $12\frac{3}{7}$ **(d)** 11
5. **(a)** 138 **(b)** 10
6. 24 yrs
7. **(a)** -39 **(b)** -156
(c) 830
8. **(a)** $-7\frac{9}{20}$ **(b)** $1\frac{29}{72}$
9. **(a)** $>$ **(b)** $<$
(c) $>$ **(d)** $=$
10. **(a)** -450 **(b)** 26
(c) 16 **(d)** 12

Revision Exercise II No 2 *(Pg 167)*

1. $2b - 60a$
2. **(a)** -1 **(b)** 3
(c) 14 **(d)** -18
3. **(a)** $\frac{1}{6}$ **(b)** $-\frac{4}{7}$
4. 42 **5.** 9 mths
6. **(a)** 0 **(b)** -72
(c) 1 **(d)** -75
7. **(a)** $2\frac{1}{22}$ **(b)** $6\frac{1}{6}$ **(c)** $-5\frac{7}{8}$
8. **(a)** 40 **(b)** 20 **(c)** 4
9. $\frac{5x}{18t}$
10. **(a)** 49.735 **(b)** 14.442
(c) 111.595

Revision Exercise II No 3 *(Pg 168)*

1. $3r - 2q - s$
2. **(a)** 20 **(b)** -15
(c) 32 **(d)** 36
3. **(a)** 9 **(b)** 0.15
(c) 15 **(d)** $3\frac{14}{15}$
4. 135
5. 45 yrs
6. **(a)** -7 **(b)** -44 **(c)** 98
7. **(a)** 490.63 **(b)** 27.07
(c) 0.27
8. **(a)** $>$ **(b)** $<$ **(c)** $<$
(d) $>$ **(e)** $>$ **(f)** $<$
9. **(a)** 13.875 6 **(b)** 583.5
10. **(a)** $\frac{11}{13}$ **(b)** $-5\frac{1}{18}$

Revision Exercise II No 4 *(Pg 168)*

1. **(a)** 5.34 **(b)** 0.090
(c) 5 000 **(d)** 12 100
2. **(a)** $2x^2 - x - 2 - 6y^2$
(b) $6z - 2y$
3. $-\frac{1}{30}x - 1\frac{1}{6}y$
4. 6
5. **(a)** $1\frac{2}{3}$ **(b)** 8 **(c)** 14
(d) 4 **(e)** 2 **(f)** 5
6. $7\frac{1}{5}xy$ km
7. **(a)** 2 000 000 **(b)** 900 000
(c) 5 000 **(d)** 4 000
8. **(a)** 0.003 **(b)** 2 000
(c) 10 **(d)** 3
9. **(a)** 10 **(b)** -18
(c) $1\frac{3}{7}$ **(d)** $-\frac{4}{13}$
10. **(a)** 7 yrs **(b)** 56 yrs

Revision Exercise II No 5 *(Pg 169)*

1. **(a)** 5 **(b)** 6
(c) -1 **(d)** 12
2. $\frac{4x+6}{3}$

3. 5

4. (a) 6 (b) $2\frac{2}{3}$ (c) 11
(d) 5 (e) 8 (f) $-1\frac{1}{4}$

5. (a) $12mn$ (b) $3xy$
(c) $28a$ (d) $8.5a$

6. (a) 6.2 (b) 2.595
(c) 10

7. (a) 4 (b) $-\frac{1}{4}$
(c) 27 (d) $-\frac{3}{10}$

8. (a) 13 (b) -280
(c) $-1\frac{11}{20}$ (d) $\frac{1}{4}$

9. (a) 25 yrs (b) 18 yrs

10. (a) 8.840 (b) 212.3
(c) 2 094 (d) 1.895

Mid-Year Examination Specimen Paper 1 *(Pg 170)*

Part I

1. (a) False (b) True
(c) False (d) True

2. (a) 12 (b) 1 950

3. (a) $4\frac{5}{18}$ (b) $\frac{2}{9}$

4. (a) 24 (b) -4
(c) -20 (d) 0

5. (a) $\frac{7}{8}$ (b) 1.387 5

6. (a) (i) 32.75 (ii) 32.7
(b) 87.5

7. $2^4 \times 17^2$, 68

8. (a) $-7x^4 - 8x^2 + 10x + 3$
(b) $2x^3 + 2x^2 + 8x + 10$

9. $\frac{1}{2}$

10. 251

11. -6

12. $(20x + 12y)¢$

13. $13\frac{1}{5}$

14. (a) 1.9, 2.5 (b) 94, 143

Part II

1. (a) $10 - 5x$ (b) 8

2. 64 kg

3. (a) $\frac{1}{8 \times 9} = \frac{1}{8} - \frac{1}{9}$
(b) $\frac{1}{2\,550}$ (c) 18, 19

4. (a) $6\frac{1}{5}$ (b) $-\frac{3}{11}$

5. (a) $3\frac{24}{25}$ (b) 2 780
(c) 1.474 (d) -6.514

6. (a) 30
(b) (i) 0.150 8
(ii) 0.002 32

7. (a) $6k - h + 18m$
(b) $\frac{4x + 27y}{15}$

8. (a) $6\frac{3}{5}$ (b) 7.2
(c) $-14 + 11x - 8x^2 + 2x^3$

Mid-Year Examination Specimen Paper 2 *(Pg 171)*

Part I

1. (a) 2 772 (b) 243

2. (a) $22\frac{1}{2}$ (b) 22.702 2

3. (a) $6a$ (b) $-y$

4. (a) $-2\frac{1}{3}$ (b) 3.2

5. 9 yrs

6. \$11.25

7. (a) $\frac{15}{16}$ (b) $2\frac{13}{21}$

8. (a) $0.2\dot{1}$ (b) 0.212 1

9. (a) 60 (b) 6 000

10. $155y$ sec

11. (a) 69, 133 (b) $\frac{23}{29}, \frac{33}{40}$

12. (a) 38°C (b) -3°C

13. $\frac{-38x - 31}{15}$

14. (a) -16 (b) 126

Part II

1. (a) 22 (b) 43
(c) 1 086

2. (a) 33.98 (b) 4.829

3. (a) 5 (b) 10

4. (a) 14, 21 (b) $\frac{14a - 19}{20}$

5. \$11

6. (a) $\frac{-45}{368}$ (b) 26, 10 cm

7. (a) (–2, 6), (–1, 5), (1, 3)
(b) (–2, –1), (1, 2)
(c) (–1, 5, 6), (1, 3, 6), (1, 4, 5), (2, 3, 5)

8. (a) $10x^2 - 5x - 2$
(b) $6x^3 + 2x^2 - 2x + 1$

9. (a) $1 + 3 + 5 + \ldots + 17 + 19 = 100 = 10^2$
(b) 225 (c) 13

Mid-Year Examination Specimen Paper 3 *(Pg 173)*

Part I

1. 294

2. 15, 1 800

3. $2^6 \times 7^2$, 56

4. $2\frac{1}{2}$

5. (a) $<$ (b) $<$
(c) $>$ (d) $<$

6. 7

7. 0.000 35

8. 336, 11 048

9. (a) $3\frac{1}{2}$ (b) $3\frac{3}{16}$

10. (a) 30, 38 (b) $7\frac{2}{3}$, 10

11. (a) 0.625 (b) $\frac{43}{50}$
(c) 0.002 6 (d) 2.4

12. (a) $12x - 7y$ (b) $7z - 11x$

13. (a) 8 (b) 21

14. \$23.50

Part II

1. (a) 3.629 (b) 65.99

2. (a) 85 (b) -3

3. 9 yrs, 36 yrs

4. (a) $(12a + 7b)¢$
(b) (i) $5 < 7 < 10$
(ii) $c < a < b$ or $b < a < c$

5. (a) 29, 37 (b) 50, 65

6. (a) $\frac{5}{24}$
(b) $-4x^5 + 4x^4 + x^2 + 13x - 7$

7. (a) 0.016 26 (b) 37 560

8. (a) $x - 6$, 42 (b) 875 124

9. (a) $\frac{46}{3x}$ (b) $16\frac{2}{5}$

Mid-Year Examination Specimen Paper 4 *(Pg 174)*

Part I

1. $0.1\dot{7}$, 0.177, $0.\dot{1}7$, 0.178

2. 540, 45

3. $2^3 \times 3^6$, 18

4. 30

5. (a) $1\frac{2}{3}$ (b) $\frac{1}{4}$

6. $2\frac{3}{5}$

7. $\frac{4x + 7}{15}$

8. (a) 28 (b) 8
9. $4x^3 - 4x + 13$
10. $-\frac{1}{2}$
11. 15 yrs, 45 yrs
12. (a) $70x$ km (b) $\frac{50x}{11}$
(c) $1\,000k$
13. $28xy - 15x^2 - 4x - 26y$
14. $\frac{56}{135}$ **15.** $\frac{xy}{15}$

Part II
1. 1.17 **2.** 3
3. $11\frac{1}{6}$
4. $\frac{169}{273}, \frac{34}{51}, \frac{88}{121}, \frac{76}{95}$
5. $5\frac{2}{45}$ **6.** \$360
7. (a) (i) 66 000
(ii) 168.7
(b) $\frac{3x - y}{14}$
8. (a) (i) $10^2 - 9^2 = 19$
(ii) 197
(iii) $x = 79$, $y = 78$
(b) $4x^2y^3 - 1 - 3x$
9. (a) $-24\frac{1}{2}$
(b) (i) 441 (ii) 105
10. (a) 96, 145 (b) 81

Mid-Year Examination Specimen Paper 5 *(Pg 176)*

Part I
1. (a) 143 (b) $208\frac{3}{4}$
2. (a) $2\frac{1}{60}$ (b) $\frac{5}{6}$
3. 5, 3 780
4. (a) 2 (b) –1
(c) $\frac{1}{4}$ (d) 7
5. (a) $a - 4b$ (b) $-5m - 4n$
6. –75
7. (a) $1\frac{3}{35}$ (b) $\frac{8}{15}$
8. $1\frac{2}{3}$ **9.** $2^8 \times 3^2$, 48
10. $\frac{8}{11}, \frac{5}{7}, \frac{2}{3}, \frac{4}{9}$
11. $2x^3 - 8x^2 + 8x - 5$
12. (a) 56 133 (b) 518
13. (a) $k = 5$ (b) 9
(c) 1 200
14. (a) 5, 6, 7, 8 (b) –9

Part II
1. (a) 0.058 92 (b) –4.500
2. 3.5 **3.** \$502 **4.** $\frac{6}{17}$
5. (a) $5\frac{7}{24}$
(b) $2x^3 - 12x^2 - 16x + 12$
6. (a) 49 (b) 49
7. (a) 1 (b) 32, 34
8. (a) 80¢ (b) \$565
(c) –6°C
9. (a) 18 (b) 25 yrs

Exercise 9a *(Pg 181)*

1. (a) 27 cm (b) 32 cm
(c) 28 cm
2. (a) 41 cm (b) 60 cm
(c) 42 cm
3. 6
4. 1.65 km
5. 17 m
6. $(200a + 2b)$ cm

Exercise 9b *(Pg 184)*

1. (a) 0.000 85 (b) 0.025
(c) 63 000 (d) 4 060
(e) 4 440 (f) 31 000
(g) 0.000 053 7
(h) 280 000 (i) 0.053 2
(j) 6.945 (k) 0.034
(l) 0.046 2
2. (a) 20 m, 24 m²
(b) 6 m, 28 m
(c) 4 m, 12.4 m
(d) 7 m, 31.5 m²
(e) 23 mm, 598 mm²
3. 972 **4.** 2 460 cm²
5. 272 m² **6.** 810 cm²
7. \$2 152.50 **8.** 275 m²
9. 81 cm² **10.** 55.2 cm²
11. 5.65 ha
12. 18 cm, 180 cm²

Exercise 9c *(Pg 190)*

1. (a) 35 cm² (b) 39 cm²
(c) $13\frac{1}{2}$ cm²
2. 17 cm
3. (a) 102 cm² (b) $17\frac{1}{2}$ cm²
(c) 21 cm
4. (a) 84 (b) 7 (c) 5.5
5. 157.5 cm²
6. (a) 54 (b) 10 (c) 13
7. \$14 080
8. (a) 8 (b) 23
(c) 12 (d) 21
9. (a) 60 cm² (b) 762 cm²
(c) 76 cm² (d) 168 cm²
(e) 659 cm² (f) 63 cm²
(g) 810 m² (h) 1 040 m²

Exercise 9d *(Pg 195)*

1. (a) 20 m, $62\frac{6}{7}$ m, $314\frac{2}{7}$ m²
(b) 28 mm, 56 mm, 2 464 mm²
(c) 14 m, 28 m, 88 cm
(d) 1.8 m, $11\frac{11}{35}$ m, $10\frac{32}{175}$ m²
2. (a) 220 mm, 3 850 mm²
(b) 88 cm, 616 cm²
(c) 110 cm, $962\frac{1}{2}$ cm²
(d) $14\frac{2}{3}$ cm, $17\frac{1}{9}$ cm²
3. (a) 21.98 cm, 38.47 cm²
(b) 86.66 m, 597.98 m²
(c) 2.32 cm, 0.43 cm²
(d) 32.97 cm, 86.55 cm²
4. (a) 72 cm, 308 cm²
(b) 56 cm, 217 cm²
(c) 96 cm, $661\frac{1}{2}$ cm²
(d) 22.4 cm, 35 cm²
(e) $28\frac{4}{7}$ cm, $50\frac{3}{7}$ cm²
(f) 396 cm, 10 234 cm²
5. 20 cm
6. 3 684 cm²
7. 11.73 cm/min
8. $35\frac{1}{5}$ m/s
9. 301
10. 45.4 cm

Review Questions 9 *(Pg 197)*

1. 42 cm² **2.** 18 cm²
3. 128 cm² **4.** 13 cm, 117 cm²
5. 864
6. 12 cm; 8 cm, 4 cm
7. 14
8. 44 m
9. 88 cm, 154 cm²

Exercise 10a *(Pg 204)*

1. **(a)** 480 cm^3, 376 cm^2
(b) 420 cm^3, 358 cm^2
(c) 115 200 mm^3, 27 360 mm^2
(d) $7\frac{1}{2}$ cm^3, $41\frac{1}{2}$ cm^2
(e) $\frac{21}{64}$ cm^3, $3\frac{43}{160}$ cm^2
(f) 4.095 cm^3, 19.26 cm^2
2. **(a)** 2 160 mm^3, 1 284 mm^2
(b) 8 cm, 158 cm^2
(c) 2.5 cm, 89.5 cm^2
(d) 8 m, 432 m^2
(e) $3\frac{1}{4}$ cm, $102\frac{5}{8}$ cm^2
(f) 5 cm, 540 cm^3
3. **(a)** 69 300 *l* **(b)** 33 345 *l*
(c) 6.84 *l* **(d)** 35.568 *l*
4. 96 cm^2
5. 702
6. 16 cm, 1 420 cm^2
7. 2.9 m
8. 31.35 cm
9. **(a)** 456 000
(b) $25.08 million, $836
10. 6 703.2 m^3

Exercise 10b *(Pg 207)*

1. $3\frac{1}{8}$ g/cm^3 **2.** 2.41 g/cm^3
3. 1.3 g/cm^3 **4.** 72.8 g
5. 125 cm^3 **6.** 250 cm^3
7. **(a)** 672 cm^3 **(b)** 1881.6 g
8. **(a)** 1 232 cm^3 **(b)** 7.63 g/cm^3

Exercise 10c *(Pg 210)*

1. **(a)** 18 450 cm^3
(b) 600 cm^3
(c) 1 404 cm^3
(d) 369 840 cm^3
(e) 16 644 cm^3
(f) 960 cm^3
(g) 1 332 cm^3
(h) 770 cm^3
(i) 4 725 cm^3
2. **(a)** 6 cm^2, 42 cm^3
(b) 14 cm, 693 cm^3
(c) 32 cm, 240 cm^2
(d) 400 cm, 95.94 cm^2
3. 102 480 m^3
4. 153 cm^3, 250 cm^2
5. 5 625 m^3, 1 937.5 m^2
6. 4 500 cm^3, 1 650 cm^2
7. $31\frac{1}{4}$ cm
8. 3.66 m^2
9. **(a)** 180 cm^3 **(b)** 264 cm^2
10. 1 728 *l*, 0.872 m^3
11. 49 500 cm^3
12. 25 cm
13. 138.6 *l*

Exercise 10d *(Pg 216)*

1. **(a)** 1 848 cm^3, 836 cm^2
(b) 4.526 m^3, 17.35 m^2
(c) 44 550 mm^3, 7 354.3 mm^2
2. **(a)** 8 cm **(b)** 28 cm
3. **(a)** 42 cm **(b)** 21 cm
4. 400
5. 7.425 *l*
6. 1 056 cm^3
7. 19 250 cm^2
8. 20.1 m^2
9. 11 550 m^3, 154
10. 123.75 *l*, 206
11. **(a)** 4 400 cm^3 **(b)** 37.84 kg
12. **(a)** 3 080 cm^3
(b) 0.844 g/cm^3
13. 0.9856 cm^3, 5.32 g, 10 348.8 m^3, 55 881 tonnes

Review Questions 10 *(Pg 218)*

1. **(a)** 108 cm^3, 168 cm^2
(b) 88 cm^3, 152 cm^2
(c) 1 217.86 cm^3, 1 233.57 cm^2
2. **(a)** rectangular cuboid
(b) cylinder
(c) rectangular cuboid
(d) triangular pyramid
(e) triangular prism
(f) square pyramid
3. 2.8 m
4. 3 000 **5.** 64
6. 1 800 **7.** 1 350 *l*
8. 104.16 kg **9.** 172.8 kg
10. 1 690 kg **11.** 0.81 m^3
12. 105 cm **13.** 1.216 kg
14. 1 h 55 min 30 sec
15. 291 000 m^3, 366 660 kg
16. **(a)** 495 652 trips;
(b) 543 truck loads

Exercise 11a *(Pg 223)*

1. **(a)** 6 **(b)** 3
(c) 12 **(d)** 7
2. **(a)** 3 : 5 **(b)** 11 : 2
(c) 4 : 5 **(d)** 1 : 50
(e) 3 : 4 **(f)** 4 : 5 : 3
(g) 8 : 4 : 1 **(h)** 3 : 5 : 7
(i) 21 : 6 : 5 **(j)** 2 : 3 : 5
3. **(a)** **(i)** 5:16 **(ii)** $\frac{5}{16}$
(b) **(i)** 7 : 12 **(ii)** $\frac{7}{12}$
(c) **(i)** 5 : 2 **(ii)** $\frac{5}{2}$
(d) **(i)** 2 : 3 **(ii)** $\frac{2}{3}$
(e) **(i)** 5 : 12 **(ii)** $\frac{5}{12}$
(f) **(i)** 7 : 120 **(ii)** $\frac{7}{120}$
4. 1 : 4
5. 4 : 21
6. **(a)** 8 : 3 **(b)** 5 : 8
7. 13 : 12 : 7
8. 2 : 3 : 6 : 7
9. **(a)** 4 : 9 **(b)** 2 : 3
10. 3 : 7 : 24 : 5

Exercise 11b *(Pg 225)*

1. 168
2. 64
3. **(a)** 25 kg **(b)** 64 m
(c) 87.5 ha **(d)** 1 cm^2
4. **(a)** 7 : 5 **(b)** 4 : 3
5. **(a)** 5 : 7 **(b)** 3 : 4
6. 19 : 24
7. $104
8. 7.7 cm by 12.6 cm
9. $7.20
10. $35 200

Exercise 11c *(Pg 227)*

1. **(a)** 15 **(b)** 200
(c) 19 **(d)** 5
2. $150 **3.** 272 km
4. 240 **5.** 221 g
6. $172.50 **7.** $5 000
8. $35.10 **9.** 500 g
10. **(a)** 25 g **(b)** 350 g
(c) 18 m^2
11. 15
12. **(a)** **(i)** $9.90 **(ii)** $16.60
(b) **(i)** $8.00 **(ii)** $13.50

Exercise 11d *(Pg 230)*

1. **(a)** 08 00 **(b)** 14 00
(c) 17 30 **(d)** 21 42

(e) 12 00 (f) 00 45
(g) 00 00 (h) 02 42

2. (a) 3.30 am (b) 3 pm
(c) 11.12 pm (d) 7.15 pm
(e) 9.23 am (f) 12 pm
(g) 12.05 am (h) 12.00 am

3. (a) 02 30 (b) 17 55 (c) 06 45

4. (a) 20 45 (b) 03 35 (c) 18 30
(d) 06 35 the next day
(e) 2 h 5 min
(f) 3 h 25 min
(g) 12 h 28 min
(h) 10 h 40 min
(i) 22 35
(j) 19 00 (Thurs)

5. 6 h 37 min

6. 07 13 the next day

7. 10 51

8. 7 h 10 min, 05 30

9. 50 min

10. (a) 4 h 55 min
(b) 7 h 20 min
(c) 6 h 50 min
(d) 6 h 25 min
(e) 13 h 15 min

Exercise 11e *(Pg 234)*

1. (b) 8 m/s (c) $6\frac{2}{3}$ m/s
(d) 440 km (e) 500 m
(f) 5 s

2. (a) 5 m/s (b) 20 m/s
(c) 25 m/s

3. (a) 36 km/h (b) 126 km/h
(c) 1 800 km/h

4. 22 s

5. 10 53

6. (a) 13 42 (b) 112 km/h

7. 80 m

8. (a) 19 km (b) 110 km/h

9. 10 km/h

10. 48 km/h

11. (a) 5 m/s (b) 4 s
(c) 7.5 m/s

12. (a) 60 m (b) 4 s
(c) 16 m/s

Exercise 11f *(Pg 237)*

1. (a) $2\frac{6}{7}$ (b) $3\frac{8}{9}$ (c) 33
(d) 18 (e) 8 (f) 8

2. (a) 70 (b) uy

3. (a) 7 : 5 (b) 3 : 8
(c) 9 : 5 (d) 55 : 24

4. 2

5. (a) \$64 (b) \$60 (c) $\$\frac{ac}{b}$

6. $3\frac{3}{5}$ kg

7. 7.5 m; 32 books

8. (a) 72 books (b) $41\frac{2}{3}$ kg

Exercise 11g *(Pg 240)*

1. (b), (c), (d)

2. 40 min **3.** 5 books

4. 20 days **5.** $17\frac{1}{2}$ h

6. (a) 840 cattle (b) 40 days

7. 28 **8.** 2 days

9. 36 hours

Exercise 11h *(Pg 242)*

1. \$1.50, \$6, \$7.50

2. 50 cm, 150 cm, 200 cm

3. 25 g, 40 g, 45 g

4. 56 cm, 84 cm, 168 cm

5. \$3 920

6. \$4 000

7. \$70, \$154

8. 120 kg, 240 kg, 120 kg

9. (a) 20 m (b) 50 cm

10. (a) 40 m (b) 60 cm

11. (a) 1 cm : 250 m
(b) 750 m (c) 32 cm

12. (a) 10 kg, 30 kg (b) \$3

13. (a) 54 *l*, 9 *l*, 6 *l* (b) 46 *l*

14. (a) 9 kg, 15 kg, 21 kg
(b) \$10.80

15. (a) 150 g, 650 g
(b) 40 g, 480 g
(c) 19 : 93 : 48
(d) 95 g

Exercise 11i *(Pg 245)*

1. 9; 6, 7, 8, 9, 10, 11, 12, 13, 14

2. 280 m

3. 37 800 m

4. 37

Review Questions 11 *(Pg 246)*

1. 3 : 4

2. 240 cm by 288 cm

3. 333 000

4. 9 : 10 : 12

5. (a) 45 (b) 15

6. (a) 06 47 (b) 1.2 km

7. (a) 09 51 (b) 88 km/h

8. (a) \$18.54 (b) 5 kg

9. (a) (i) 2 h 40 min
(ii) 6 h 25 min
(b) (i) 53 min
(ii) 1 219 words

10. (a) 63 m (b) 7 m

11. $53\frac{1}{3}$ km/h

12. (a) \$1 050, \$750, \$300
(b) \$1 225, \$875, \$350, \$175, \$175

13. 1 : 60

14. (a) 12 30
(b) 3 h 20 min; 58.5 km/h

15. (a) 7 h 15 min
(b) \$11.90

Exercise 12a *(Pg 253)*

1. (a) 0.06 (b) 0.11
(c) 0.22 (d) 0.63
(e) 1.79 (f) 0.002 7
(g) 0.287 (h) 1.346
(i) 0.035 (j) 0.052 5
(k) 0.000 74 (l) 0.543 7
(m) 0.000 063 (n) 0.011 25
(o) 0.008 75 (p) 0.507 5

2. (a) 17% (b) 57.5%
(c) 83% (d) 236%
(e) 9% (f) 2.5%
(g) 0.8% (h) 256.4%
(i) 0.05% (j) 120%
(k) 400% (l) 625%

3. (a) 75% (b) 90%
(c) 85% (d) $4\frac{4}{5}$%
(e) 120% (f) 48%
(g) 124% (h) 240%

4. (a) 66.6% (b) 57.1%
(c) 22.2% (d) 58.3%
(e) 133.3% (f) 83.3%
(g) 242.9% (h) 90.9%

5. 60%, 0.6; $\frac{11}{100}$, 0.11; 17.5%, $\frac{7}{40}$; 9.5%, $\frac{19}{200}$; $\frac{157}{200}$, 0.785

6. $\frac{1}{24}$

7. $4\frac{11}{16}$%

8. (a) $4\frac{1}{2}$%, $\frac{12}{32}$, 0.39

(b) 22%, 0.222, $\frac{2}{9}$

(c) 0.6, 64%, $\frac{2}{3}$

Exercise 12b *(Pg 255)*

1. **(a)** 30% **(b)** 4.5% **(c)** 125%

2. **(a)** 75% **(b)** $33\frac{1}{3}\%$

(c) 67% **(d)** $16\frac{2}{3}\%$

(e) 1.5% **(f)** 30%

3. **(a)** **(i)** $\frac{5}{6}$ **(ii)** $83\frac{1}{3}\%$

(b) **(i)** $\frac{13}{16}$ **(ii)** $81\frac{1}{4}\%$

4. 30%

5. 6%

6. 20%, 30%, 45%, 5%

7. **(a)** \$99.20 **(b)** 325

(c) 3.60 m **(d)** 62.5 cm

(e) 91.8 litres **(f)** 4.84 hours

8. 7 225 people

9. 1 190 tires

10. **(a)** 47.4 kg **(b)** 237 000 kg

11. **(a)** 400 000

(b) **(i)** 800 000

(ii) 1 200 000

(iii) 1 600 000

(iv) 2 000 000

Exercise 12c *(Pg 259)*

1. 63 **2.** 135

3. 140 **4.** 240

5. \$228.80 **6.** \$122 400

7. \$1 800 **8.** \$65

9. 355 houses **10.** 5.4 m

11. **(a)** \$55.80

(b) **(i)** \$9.18 **(ii)** \$367.20

12. 15% **13.** \$86 400

14. 20% **15.** 16.64%

16. **(a)** $11\frac{1}{9}\%$ **(b)** 40 minutes

(c) 180 seconds

(d) 192 seconds

(e) James

Exercise 12d *(Pg 262)*

1. **(a)** 12.5% **(b)** 7.14%

(c) 25% **(d)** 13.8%

2. \$135 **3.** \$18.75

4. 16.7% **5.** \$17 296

6. \$180

7. **(a)** \$800 **(b)** \$1 080

8. \$600 **9.** 24%

10. 32% **11.** \$240

12. \$180 **13.** 99¢

Exercise 12e *(Pg 264)*

1. **(a)** 12% **(b)** 20%

2. **(a)** **(i)** \$4.50 **(ii)** \$40.50

(b) **(i)** \$11 **(ii)** \$99

3. **(a)** \$700 **(b)** \$810

4. **(a)** \$87.50 **(b)** \$164

5. \$500

6. **(a)** \$72 **(b)** \$72.20

7. \$28

8. \$700

9. \$1 600, \$1 120

10. **(a)** 38cents **(b)** $15\frac{5}{9}\%$

11. \$1 310 **12.** \$18 000

13. \$90

14. \$36, \$48, \$60

Exercise 12f *(Pg 267)*

1. **(a)** \$6 720, \$18 720

(b) 4 yrs, \$720

(c) \$300, \$408

(d) 4%, \$4 200

(e) \$3 600, 5%

(f) 7%, \$1 989

(g) 6%, 540

(h) \$1 200, $1\frac{1}{2}$ yrs

2. \$7 084 **3.** \$1 100

4. \$360 **5.** \$132

6. 10 years **7.** $4\frac{1}{4}\%$

8. \$1 019.5 **9.** \$16

10. \$6 100 **11.** \$20 000

Exercise 12g *(Pg 270)*

1. **(a)** \$94.50 **(b)** \$257.34

(c) \$1 244.03 **(d)** \$149.84

(e) \$1 556.25

2. \$5 829.57 **3.** \$103.13

4. \$9 001.46

Exercise 12h *(Pg 271)*

1. **(a)** **(i)** \$90 **(ii)** 25%

(b) **(i)** \$150 **(ii)** $16\frac{2}{3}\%$

(c) **(i)** \$3 000

(ii) 12%

2. **(a)** \$21 **(b)** \$288

3. **(a)** **(i)** \$236 **(ii)** 18%

(b) **(i)** \$517.50 **(ii)** 15%

(c) **(i)** \$1 960 **(ii)** 22.5%

4. **(a)** **(i)** \$63 **(ii)** 7%

(b) **(i)** \$200 **(ii)** 15%

(c) **(i)** \$75 **(ii)** $16\frac{2}{3}\%$

5. **(a)** \$3 716.80 **(b)** \$516.80

(c) 16.15%

6. **(a)** 18.2% **(b)** \$108

(c) 5.13%

Exercise 12i *(Pg 274)*

1. **(a)** S\$761.85 **(b)** S\$7 935

(c) S\$1 528 **(d)** S\$96.80

(e) S\$817.20 **(f)** S\$210

(g) S\$1 132.5 **(h)** S\$256.76

(i) S\$27 800 **(j)** S\$564.48

(k) S\$9 573.56 **(l)** S\$157.50

2. **(a)** US\$4 401 **(b)** £327

(c) NZ\$5 827

(d) 5 901 Thai baht

(e) 109 890 peso

(f) 833 333 rupiah

(g) ¥29 577

(h) 3 195 French franc

(i) C\$320

(j) A\$1 106

(k) HK\$10 000

(l) 1 684 Deutschemark

3. **(a)** 1 155 Thai baht

(b) US\$210

4. **(a)** 992 M **(b)** £280

5. **(a)** US\$5 000 **(b)** S\$330

Exercise 12j *(Pg 277)*

1. \$1 312 **2.** \$96.90

3. \$247.20 **4.** \$165

5. **(a)** \$60 **(b)** \$120

(c) \$375 **(d)** \$675

6. **(a)** \$276 **(b)** \$90.80

Exercise 12k *(Pg 282)*

1. 7 **2.** 20 cents

3. 22, 14, 12 **4.** 17

5. 11 **6.** 127

Review Questions 12 *(Pg 283)*

1. \$935

2. **(a)** \$900 **(b)** \$1 170

3. **(a)** \$291.20 **(b)** \$1 164.80

4. \$197.40

5. $1\frac{3}{4}$, 1.74, 173%, $1\frac{2}{3}$, 1.56

6. $\frac{7}{80}$

7. $\frac{3}{40}$

8. \$200, \$240

9. **(a)** S\$276 **(b)** £674.78

10. 144, 168

11. loss \$2.40

12. **(a)** 5% **(b)** \$736.67

13. **(a)** \$72 000 000
(b) \$11 137 000
(c) 64%

14. **(a)** \$3
(b) **(i)** \$33 **(ii)** \$9.00

Revision Exercise III No 1 *(Pg 285)*

1. \$2.75 **2.** 50 min
3. \$20.25 **4.** 7 : 12
5. \$24 **6.** \$8.40
7. 91 cm^2
8. **(a)** 135 000 m^3 **(b)** 9
9. 456 000 trips, \$20 520 000
10. $\frac{60}{77}$ cm

Revision Exercise III No 2 *(Pg 285)*

1. \$920 **2.** \$20.50
3. 23, \$6.80 **4.** \$14 100
5. 85 *l*
6. 24 cm^2, 3 cm
7. 2 288 cm^2, 7 392 cm^3
8. 690 cm^2, 5 175 cm^3
9. 18 cm
10. \$874.18

Revision Exercise III No 3 *(Pg 286)*

1. **(a)** \$200 **(b)** \$51 200
(c) \$1 700
2. 24 days
3. \$80
4. 147, 245, 392
5. M\$317.43
6. **(a)** \$9 **(b)** 1.2 kg
(c) 17 m^3 **(d)** 225 km
7. **(a)** 42.9 cm
(b) 1 106 cm^2, 2 814 cm^3, 10.69 kg
8. 48 cm, 161 cm^2
9. 156 cm^3
10. **(a)** 32 cm^2 **(b)** 2 250 m^3

Revision Exercise III No 4 *(Pg 287)*

1. \$66.25
2. **(a)** \$180 **(b)** 19.15%
3. $A = 12$, $B = 10$, $C = 9$
4. 33.84 km/h
5. \$40
6. \$5 284.09
7. **(a)** 36 cm^2 **(b)** 20 cm^2
8. $29\frac{1}{3}$ cm
9. 76 cm^3, 114 cm^3
10. 3 000 *l*, 19

Revision Exercise III No 5 *(Pg 288)*

1. **(a)** 87.5% **(b)** 510
(c) \$21
2. **(a)** 5 400 m **(b)** 20%
3. \$80.75, 34.58%
4. \$25.20
5. **(a)** \$11 200 **(b)** \$39 300
6. 12 cm
7. 216 cm^2
8. 2 cm
9. 32 cm
10. 7.02 kg, 1.404 g/cm^3

Exercise 13a *(Pg 296)*

2. **(a)** spherical **(b)** cylindrical
(c) cylindrical **(d)** prism
(e) conical **(f)** conical
3. **(a)** $x = 100°$, $k = 94°$, $l = 135°$, $m = 115°$, $n = 96°$
(b) $p = 70°$, $q = 65°$, $r = 270°$, $s = 62°$, $t = 73°$
4. **(a)** $a = 62°$, $b = c = 118°$
(b) $d = 74°$, $e = 97°$, $f = 107°$
(c) $g = 245°$, $h = 50°$
(d) $k = 180°$, $t = 132°$, $l = 28°$
(e) $l = 90°$, $m = 68°$, $n = 225°$
(f) $p = 248°$, $q = 99°$, $r = 90°$, $s = 55°$
5. **(a)** 72° **(b)** 44° **(c)** 37°
(d) 26° **(e)** 83°
6. **(a)** 144° **(b)** 168° **(c)** 78°
(d) 9° **(e)** 92°
8. **(a)** 49° **(b)** 30°
(c) 90° **(d)** 60°
9. **(a)** 30 **(b)** 10 **(c)** 9
(d) 60 **(e)** $4\frac{2}{3}$
10. **(a)** $a + d = 90°$ **(b)** 90°
(c) 65°
11. 30°, 60°, 120°, 150°

12. **(a)** 50°
(b) 42°
(c) 106°
(d) 30°
(e) $p = 35°$, $q = 145°$
(f) 47°
(g) $x = 23$, $y = 69$, $z = 111$
(h) $x = 20$, $y = 120$
(i) $x = 18$
(j) $x = 15$

Exercise 13b *(Pg 301)*

1. **(a)** $b = y$, $h = r$, $e = p$
(b) $x = d$, $q = g$, $p = h$
2. $p = 106°$, $q = 74°$, $r = 106°$, $s = 74°$, $x = 73°$, $y = 107°$
3. **(a)** $a = 31°$, $b = 66°$
(b) $c = 45°$, $d = 60°$
(c) $e = 106°$
(d) $f = 104°$
(e) $x = 19$
(f) $x = 30$, $y = 65$
(g) $x = 145$, $y = 223$
(h) $x = 257$
(i) $x = 275$
(j) $x = 140$

Exercise 13c *(Pg 304)*

2. $a = 44°$, $b = 53°$; 5.6 cm, 7.0 cm, Yes
3. 12.3 cm, 11.3 cm, Yes
6. 6.6 cm

Review Questions 13 *(Pg 305)*

1. **(a)** $x = 36$
(b) $x = 22.5$
(c) $x = 29$, $y = 110$
(d) $x = 23$
(e) $x = 16$, $y = 48$
(f) $x = 120$
(g) $x = 80$
(h) $x = 24$
(i) $x = 30$
(j) $x = 56$
(k) $x = 44$, $y = 58$
(l) $x = 65$
(m) 150
(n) 124

Exercise 14a *(Pg 313)*

1. **(a)** 100°
(i) scalene △
(ii) obtuse-angled △

(b) 70°
(i) isosceles △
(ii) acute-angled △
(c) 60°
(i) equilateral △
(ii) acute-angled △
(d) 90°
(i) scalene △
(ii) right-angled △
(e) 65°
(i) isosceles △
(ii) acute-angled △
(f) 43°
(i) scalene △
(ii) obtused-angled △

2. **(a)** 96° **(b)** 16° **(c)** 144° **(d)** 52°
3. $A\hat{C}B = 48°$
5. 8.5 cm, 45°
6. BC = 8.0 cm, $\hat{B}$ = 58°; acute-angled triangle
7. PR = 6.8 cm, QR = 6.1 cm, $\hat{R}$ = 76°; acute-angled triangle
8. LN = 7.5 cm, $\hat{M}$ = 56°, $\hat{N}$ = 34°; right-angled triangle
9. **(a)** 35° **(b)** 25° **(c)** 22.5° **(d)** 60° **(e)** 102°
(f) $x = 40°$, $y = 55°$
(g) $a = 70°$, $b = 22°$
(h) $x = 127°$, $y = 58°$
(i) 20° **(j)** 69°
(k) $a = 35°$, $b = 108°$
(l) $x = 78°$, $y = 52°$
(m) $x = 35°$, $y = 125°$
(n) $x = 25°$, $y = 97°$
(o) $i = 101°$, $j = 42°$
(p) 138°
(q) $x = 24°$, $y = 96°$, $z = 48°$
(r) $p = 65°$, $q = 73°$
(s) $x = 47°$, $y = 65°$, $z = 112°$
(t) $x = 77°$, $y = 39°$
10. 100
11. 45°
12. $A\hat{B}C = 104°$, $C\hat{B}D = 76°$

Exercise 14b *(Pg 317)*

1. **(a)** $x = 36°$, $y = 36°$
(b) $x = 68°$, $y = 73°$
(c) $x = 54°$, $y = 36°$
(d) $x = 12°$, $y = 27°$
2. **(a)** $x = 20°$, $y = 80°$
(b) $x = 26°$, $y = 48°$
(c) $x = 60°$, $y = 65°$
(d) $x = 31°$, $y = 12°$
3. **(a)** $x = 40°$, $y = 58°$
(b) $x = 114°$, $y = 114°$
(c) $x = 65°$, $y = 46°$
(d) $x = 21°$, $y = 21°$
4. **(a)** $x = 33°$, $y = 114°$
(b) $x = 104°$, $y = 38°$
(c) $x = 42°$, $y = 48°$
(d) $x = 21°$, $y = 21°$, $z = 42°$
5. **(a)** $x = 37°$, $y = 127°$
(b) $x = 67.5°$, $y = 22.5°$
(c) $x = 22.5°$, $y = 108°$
(d) $x = 25°$, $y = 31°$
6. **(a)** 59° **(b)** 31°
7. **(a)** 110° **(b)** 28°
8. **(a)** 72° **(b)** 36° **(c)** 54°
9. **(a)** 58° **(b)** 96°
10. **(a)** 31° **(b)** 97°

Exercise 14c *(Pg 320)*

1. 16.9 cm **2.** 11.6 cm
3. 9 cm
4. 6.4 cm, 10.1 cm
5. 128 mm, 82°
6. 97 mm, 98 mm
7. 1.7 cm **8.** 7.0 cm
9. 65° **10.** 133°
11. 14.4 cm

Review Questions 14 *(Pg 322)*

1. **(a)** $x = 40°$, $y = 70°$
(b) $x = 39°$, $y = 63°$
(c) $x = 66°$, $y = 66°$
(d) 35°
(e) $x = 244°$, $y = 26°$
(f) $x = 39°$, $y = 63°$
(g) $x = 56$, $y = 31°$
(h) $x = 40°$, $y = 32°$
(i) $x = 122°$, $y = 58°$, $z = 64°$
(j) $x = 52°$, $y = 31°$, $z = 97°$
(k) 20°
(l) $x = 79°$

Exercise 15a *(Pg 330)*

1. **(a)** 2 : 1 **(b)** 66.7%
2. **(a)** 53 **(b)** 50%
3. **(a)** 1 000 **(b)** 900
(c) $\frac{4}{7}$
4. **(a)** Product 1 **(b)** April
(c) \$37 500 **(d)** \$45 000
(e) \$140 000 **(f)** \$125 000
(g) $\frac{28}{53}$
5. **(a)** 1999 – 260 000
(b) 1995 – 140 000
1996 – 160 000
1997 – 180 000
1998 – 220 000
(c) \$90 million
(d) 18.2%
8. **(a)** 4 : 5 **(b)** $33\frac{1}{3}\%$

Exercise 15b *(Pg 336)*

3. **(c)** A pie chart
(d) A bar chart
4. Hospital **(a)** \$324 m **(b)** 45%
Research **(a)** \$25 m **(b)** 3.5%
Public Health **(a)** \$25 m **(b)** 3.5%
Dental Clinics **(a)** \$36 m **(b)** 5%
Drugs **(a)** \$40 m **(b)** 5.6%
Doctors **(a)** \$130 m **(b)** 18.1%
Nursing Homes **(a)** \$80 m **(b)** 11.1%
Others **(a)** \$60 m **(b)** 8.3%
5. **(a)** 20
(b) **(i)** 540 **(ii)** 225
6. **(a)** Red
7. **(a)** 90° **(b)** 100° **(c)** 36 **(d)** 27.8%
8. 252°
9. 9
10. **(a)** **(i)** 50% **(ii)** 20%
(b) 63

Exercise 15c *(Pg 341)*

1. **(a)** 14 tonnes
(b) 4 tonnes
(c) 55 tonnes
2. **(b)** 39°, 38°
3. **(a)** 1999 and 2000
(b) $56\frac{1}{4}\%$

Exercise 15d *(Pg 344)*

1. **(c)** 43 minutes

(b) $26\frac{2}{3}\%$

2. **(c)** 90 kg **(d)** 92 kg

3. **(c)** 5.4 minutes

(d) $73\frac{1}{3}\%$

4. **(a)** XYZ school

(b) ABC school

(c) ABC school

Exercise 15e *(Pg 348)*

1. **(c)** 3 **(d)** 6

2. **(b)** 26 **(c)** 6 **(d)** 2

3. **(b)** Country A: 9; Country B: 8

(c) Country A: 349; Country B: 99

Review Questions 15 *(Pg 350)*

2. **(a)** \$55 million

(b) **(i)** 22% **(ii)** 64.8

(iii) $\$24\frac{3}{4}$ million

3. **(a)** 8

4. **(b)** 0

5. **(c)** 110 **(d)** 124

6. **(b)** 7.2 grams **(c)** 26%

Revision Exercise IV No 1 *(Pg 352)*

1. 50 **2.** 5.2 cm

3. 2.7 cm **4.** 1 h 32 min

5. $x = 137°$, $y = 81°$, $z = 99°$

6. **(a)** \$6 **(b)** 15%

7. **(a)** 3 years, 4 mths

(b) 96 km, 2 h

9. **(a)** 22.5 m **(b)** 6.9 m

10. **(a)** 48 **(b)** 64 **(c)** $\frac{2}{5}$

Revision Exercise IV No 2 *(Pg 353)*

1. 7.5 cm

2. 8.9 cm

3. **(a)** 52° **(b)** 116°

4. **(a)** 25° **(b)** 74°

5. 14 sec **6.** 8.2 cm

7. **(a)** 11

(b) **(i)** 84.375%

(ii) 84.4%

8. $p = 54°$, $q = 113°$, $r = 59°$, $s = 121°$

10. **(a)** 600 **(b)** 4 : 5

Revision Exercise IV No 3 *(Pg 353)*

1. 136 mm

2. 11.5 cm, 141°

3. **(a)** $p = 83°$ **(b)** $q = 104°$

(c) $r = 32°$

4. **(a)** 30 h **(b)** 120 days

5. 11.3 cm

6. 7.5%

7. **(a)** 500 **(b)** 24%

(c) \$27.20

8. $x = 55°$, $y = 82°$, $z = 38°$

9. 2 : 7

10. **(a)** 88 **(b)** 30%

(c) 2 024

Revision Exercise IV No 4 *(Pg 354)*

1. S\$218.40

2. $x = 70°$, $y = 60°$

3. 6 days **4.** 20%

5. 8.6 cm, 14.2 cm

7. 7.5 cm **8.** 52°

9. 108° **10.** 24%

Revision Exercise IV No 5 *(Pg 355)*

1. 78°

2. 4.6 cm, 5.7 cm

3. 21.5%

4. **(a)** 1 000 **(b)** \$15.50

(c) 8%

5. $x = 68°$, $y = 112°$

6. 25 **7.** 0.74%

8. $x = 24°$, $y = 96°$, $z = 48°$

9. **(a)** 15° **(b)** 8.7 cm, 76°

10. **(a)** 84 **(b)** 154

(c) 29.8%

End-of-Year Examination Specimen Paper 1 *(Pg 357)*

Part I

1. **(a)** $1\frac{3}{8}$ **(b)** $4\frac{13}{16}$

2. 31.744 kg **3.** 48 cm²

4. **(a)** 15.44 **(b)** 85.84

5. \$15, \$75, \$255

6. \$180 **7.** 12

8. **(a)** $11\frac{1}{2}$ **(b)** $4\frac{3}{7}$

9. **(a)** $14x - 30$ **(b)** $\frac{17 - 8x}{20}$

10. **(a)** 29° **(b)** 119°

11. **(a)** $1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25$

(b) 81 **(c)** 13

12. **(a)** 32 m **(b)** \$5 408

13. 6.8 cm

Part II

1. 42.24 m²

2. 18.48 l, 3 256 cm²

3. **(a)** 25 **(b)** 10

4. $x = 22°$, $y = 88°$, $z = 44°$

5. **(a)** 3 072 cm³ **(b)** 4.32 l

6. **(a)** 48.51 kg

(b) **(i)** 2.738 **(ii)** 0.978 5

7. **(a)** $x = 36°$, $y = 80°$

(b) 6 of each

8. **(a)** \$54 **(b)** M\$348

End-of-Year Examination Specimen Paper 2 *(Pg 358)*

Part I

1. **(a)** $-2\frac{1}{2}$ **(b)** $5\frac{7}{8}$

2. **(a)** $\frac{7}{200}$ **(b)** 1.225

(c) 2.00 **(d)** 0.025

3. **(a)** 1 200

(b) $18y - 6x - 21$

4. $\frac{3}{11}$

5. 60 cm

6. 144 000 l

7. 0.462 m³

8. \$4 760

9. $\$\left(\frac{1}{3}xz - xy\right)$

10. **(a)** 220 cm **(b)** 880 m

11. S\$3 158

12. 4.9 cm

13. **(a)** $11 \times 11 = 10 \times 12 + 1$

(b) 14 **(c)** 39 601

14. **(a)** 35 **(b)** $\frac{1}{5}$ **(c)** $\frac{2}{7}$

Part II

1. 145 g

2. 64 yrs, 16 yrs

3. **(a)** 308 cm² **(b)** 947

4. **(a)** **(i)** $x = 32°$, $y = 120°$

(ii) $x = 77°$, $y = 111°$

(b) 20

5. **(a)** \$24 000 **(b)** \$33 800

6. **(a)** 54 km/h **(b)** 3.75%

7. **(a)** **(i)** 156.4 **(ii)** 8.243

(b) \$8 400

8. 1.84 m³, 5 152 kg

End-of-Year Examination Specimen Paper 3 *(Pg 360)*

Part I

1. **(a)** $\frac{29}{36}$ **(b)** $5\frac{1}{6}$

2. **(a)** $23a$ **(b)** $3y$
(c) $-6a + 6b + 14c$

3. **(a)** $5\frac{1}{2}$ **(b)** 4 **(c)** 2

4. $x = 41°$, $y = 49°$, $z = 18°$

5. 126 cm^2

6. 112.5

7. \$1 175

8. 288 cm^2

9. S\$11.96

10. 20%

11. 2 yrs 3 mths

12. 6 km

13. 5.4 cm

Part II

1. 40

2. 1 220 l, 8

3. **(a)** 19, 21, 23
(b) 5 yrs, 35 yrs

4. **(a)** \$9.80 **(b)** \$31.80

5. **(a)** tv km, $\frac{s}{v}$ h
(b) \$17.10

6. **(a)** $x = 69°$, $y = 78°$, $z = 102°$
(b) 11.6 cm

7. 320 cm^3, 224 g

8. **(a)** **(i)** 20% **(ii)** \$30 200
(b) 9

End-of-Year Examination Specimen Paper 4 *(Pg 362)*

Part I

1. **(a)** 43 **(b)** 1.21

2. **(a)** **(i)** 0.09 **(ii)** 0.086
(b) $3^4 \times 5^2 \times 7^2$, 315

3. **(a)** $2\frac{12}{19}$ **(b)** 12

4. Q, 15 45

5. **(a)** 3.657 **(b)** 25 000
(c) 1 864 **(d)** 0.000 84

6. \$135

7. 20% gain

8. **(a)** 50
(b) There are school holidays in the months of March and June

9. **(a)** $3\frac{3}{5}$ **(b)** 43.75%

10. 21, 23

11. **(a)** $-7x + 6y$ **(b)** $\frac{130 - 7x}{60}$

12. **(a)** 68° **(b)** 24°

13. **(a)** $\frac{3x}{t}$ km **(b)** $\frac{xy}{t}$ km

Part II

1. $3\frac{3}{20}$, $\frac{22}{7}$, 3.142, $\frac{47}{15}$

2. \$1 638

3. 13.1 cm, 9.2 cm

4. **(a)** 22.5° **(b)** 20.5°

6. 8

7. 32 m, 512 m^2, 5.856 m^3

8. 81, 26 cm^3

9. **(a)** 6 252 cm^3 **(b)** 9.378 kg

End-of-Year Examination Specimen Paper 5 *(Pg 364)*

Part I

1. **(a)** $\frac{19}{60}$ **(b)** $4\frac{19}{24}$

2. **(a)** $-9x - 1$ **(b)** $20b - 5a$

3. **(a)** 63° **(b)** 15°

4. **(a)** 30 **(b)** 200
(c) 60 **(d)** 20

5. 26.5

6. **(a)** $\$\frac{c}{100}$ **(b)** $\frac{m}{1\,000}$
(c) 1 000k

7. £662

8. 220

9. **(a)** 81 cm^2 **(b)** 4 cm

10. 184 ha

11. 4 yrs, 32 yrs

12. 5 h

13. 140 mm, 97 mm

14. 81 cm^2

Part II

1. **(a)** 5 v km **(b)** $\frac{s}{v}$ h

2. $\frac{2}{9}$

3. **(a)** 52.5 **(b)** 34, 36, 38

5. **(a)** 2 **(b)** 4 860 l
(b) **(i)** 2 cm **(ii)** 16 m^2

6. **(a)** 336 **(b)** 1 yr 3 mths

7. **(a)** 3 696 cm^3 **(b)** 1 676 cm^2

8. **(a)** 298
(b) \$168.52, 45.8%

Notes