New Syllabus

Mathematics 1

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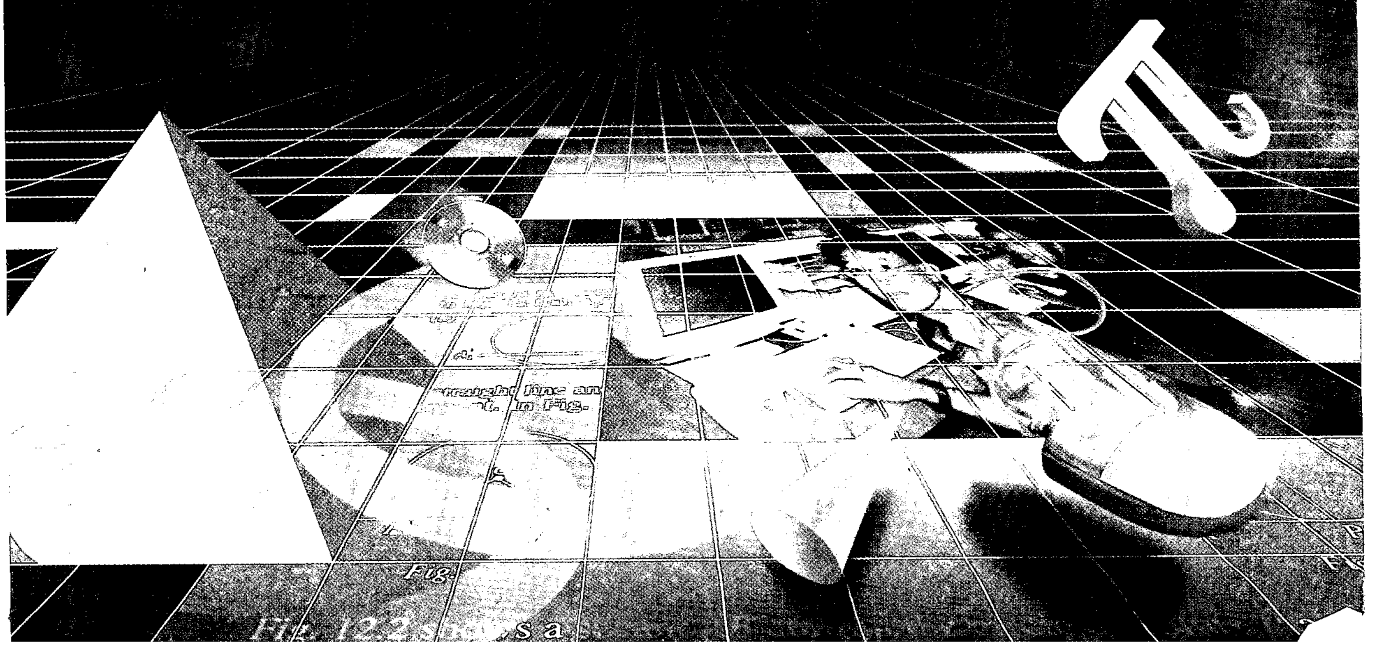

SHINGLEE PUBLISHERS PTE LTD

120 Hillview Avenue #05-06/07
Kewalram Hillview Singapore 669594
Tel: 7601388 Fax: 7625684
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First Published 1982
Reprinted 1983, 1984, 1985, 1986
Second Edition 1987
Reprinted 1987, 1988, 1989, 1990, 1991
Third Edition 1992
Reprinted 1992, 1993, 1994, 1995
Fourth Edition 1997
Reprinted 1997, 1999
Fifth Edition 2001
Reprinted 2002

ISBN 9971 61 971 7

Cover design by Dave Cheong Wan Kong

Acknowledgement

The authors and publishers are grateful to Suntec City Development Pte Ltd for the photograph on page 178.

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Printed in Singapore by KHL Printing Co Pte Ltd

PREFACE

New Syllabus Mathematics is a series of four books. These books follow the Mathematics Syllabus for Secondary Schools, implemented from 2001 by the Ministry of Education, Singapore. The whole series covers the complete syllabus for the Singapore-Cambridge GCE 'O' Level Mathematics.

The fifth edition of New Syllabus Mathematics 1 retains the goals and objectives of the previous edition, but has been revised to meet the requests of users of the fourth edition and to keep materials up-to-date as well as to give students a better understanding of the contents.

All topics are comprehensively dealt with to give students a firm grounding in the subject. Explanations of concepts and principles are concise and written in clear language with supportive illustrations and examples. Examples and exercises have been carefully graded to aid students in progressing within, as well as up, each level. Those exercises marked with a ✶ are either tricky or involve more calculations. "Problem Solving" and "Exploration", placed at the end of the chapter, contain more difficult and challenging questions requiring students to apply their knowledge and experience in solving them.

Numerous revision exercises are provided at appropriate intervals to enable students to recapitulate what they have learnt. In addition, there are mid-year and end-of-year examination specimen papers.

Important features which have been retained in this edition to facilitate learning are:

- an interesting introduction at the beginning of each chapter complete with photographs or graphics
- brief specific instructional objectives for each chapter
- in-class activities (investigation / discussion / problem solving)
- activities and interesting information in the marginal text (clip-notes, "Down Memory Lane", "Back In Time", "Investigate", "Check This Out!", "It's A Fact", "Just For Fun", "Are You Game Enough?", "For Your Information", "Library Corner", "Problems" and "IT")

Problem-solving heuristics are subsequently introduced at appropriate sections of the book to reinforce problem-solving skills. In addition, questions which call for problem-solving skills are also set in the margin for students to do at their own pace and time.

Ample opportunities are also provided for mathematical investigative and communicative activities.

It is hoped that these features will help students learn mathematics with more zest and excel in the subject.

CONTENTS

CHAPTER 1

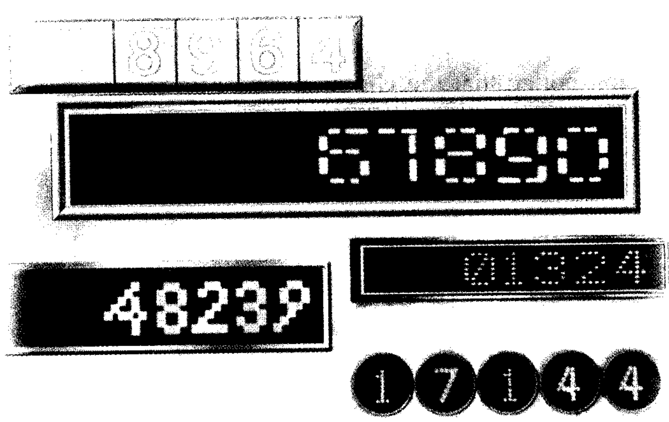
Whole Numbers

In this chapter, you will learn how to

- △ represent numbers on the number line and order them;
- △ use the symbols $=, \neq, >, <, \geqslant, \leqslant$;
- △ perform mental calculations with whole numbers;
- △ perform calculations with whole numbers using a calculator;
- △ check the accuracy of a calculation by estimation.

Preliminary Problem

When you log onto an internet website, have you ever noticed that the site will show that you are their number xxxxx visitor? The idea of counting is used to monitor the number of times a website has been visited.



The popularity of a website corresponds closely to the number of "hits" or the number of visitors it receives. Commercial websites boost their earnings through advertisements which depend on the number of visitors to the sites.

The Concept of Whole Numbers

Numeral System

We use numbers everyday. The number system that we use is called the **Hindu-Arabic** system. It is based on ten symbols called **digits**. They are

0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

They also form the ten smallest **whole numbers**. Any other whole number can be written using the ten digits and the idea of **place values**.

Using two or three digits, we can write the next nine hundred and ninety whole numbers

10, 11, … 20, 21, …, 98, 99, 100, 101, … 200, 201, …, 998, 999.

Very large whole numbers can be written with more digits. In July 1996, the estimated population of Singapore was 3 396 924 people or three million three hundred ninety-six thousand nine hundred and twenty-four people, in words.

Here are the place values for the seven digits of the number that represents the Singapore population.

millions (1 000 000)	hundred thousands (100 000)	ten thousands (10 000)	thousands (1 000)	hundreds (100)	tens (10)	ones (1)
3	3	9	6	9	2	4

$\therefore$ 3 396 924

$= (3 \times 1\ 000\ 000) + (3 \times 100\ 000) + (9 \times 10\ 000)$

$\times 10$ $\quad$ $\times 10$

$+ (6 \times 1\ 000) + (9 \times 100) + (2 \times 10) + (4 \times 1)$

$\times 10$ $\quad$ $\times 10$ $\quad$ $\times 10$

Did you notice that the Hindu-Arabic numeration system is built on groups of 1, $10 = 10 \times 1$, $100 = 10 \times 10$, $1\ 000 = 10 \times 100$ and so on?

Thus, the system is known as the **base ten** system or the **decimal** system.

Notes

In 3 396 924, from the right to the left
4 represents 4 ones or 4×1,
2 represents 2 tens or 2×10,
9 represents ____ or ____
6 represents 6 thousands or $6 \times 1\ 000$.
9 represents ____ or ____
3 represents ____ or ____
3 represents 3 millions or $3 \times 1\ 000\ 000$

Can you complete the above?

Use of Zero as a Place-holder
Can you imagine what will happen when zeros are omitted from the numerals 403 and 4 030?

Ordering of Whole Numbers

All the whole numbers can be arranged in the following **order**.

0, 1, 2, 3, 4, 5, …

They follow one another.

If we know a number, we know the next one that follows it. Can you think how we can obtain the next number?

The whole numbers get larger and larger in the above order. There is a first whole number. Is there a last whole number?

There is no largest whole number.

In Mathematics, we use the symbol '>' to denote 'is greater than' and the symbol '<' to denote 'is less than'. For example, we write '$7 > 4$' to express '7 is greater than 4' and we write '$3 < 6$' to express '3 is less than 6'.

We use the symbol '$\neq$' to denote 'is not equal to', for example, we write '$a \neq 8$' for 'a is not equal to 8'.

We use the symbol '$\geqslant$' to denote 'is greater than or equal to'. For example, we write '$a \geqslant 9$' for 'a is greater than or equal to 9'.

We use the symbol '$\leqslant$' to denote 'is less than or equal to'. For example, we write '$b \leqslant 10$' for 'b is less than or equal to 10'.

Notes

The Babylonians (around 2000 BC) used the symbols and to represent ten and one respectively.

represents 45.

The Romans (around 100 BC) used letters as symbols for numbers. The basic Roman numerals are I (one) V (five) X (ten) L (fifty) C (one hundred) D (five hundred) M (one thousand). The numerals are written in a definite order and the principles of subtraction and addition are employed. 1999 is represented by MCMXCIX. Since M represents 1 000, CM represents 1 000 – 100 or 900, XC represents 100 – 10 or 90, IX represents 10 – 1 or 9. Can you figure out how DCCCLXXXVIII represents 888? Now, can you see how efficient the Hindu-Arabic system is?

Natural, Even and Odd Numbers

The whole numbers 1, 2, 3, 4, 5, 6 … are **natural numbers**.
In other words all whole numbers except 0 are natural numbers.

The whole numbers 0, 2, 4, 6, 8, … are **even numbers**.
These numbers can be divided by 2 exactly.

The whole numbers 1, 3, 5, 7, 9 … are **odd numbers**.
These numbers cannot be divided by 2 exactly.

A whole number is either an even number or an odd number.
Do you know why this is so?

For your information

We use natural numbers in counting.

Example 1

List (a) all the natural numbers less than 5;
(b) all the even numbers between 20 and 43;
(c) all the odd numbers between 15 and 25.

Solution

(a) The natural numbers less than 5 are 1, 2, 3 and 4.
(b) The even numbers between 20 and 43 are 22, 24, 26, 28, 30, 32, 34, 36, 38, 40 and 42.
(c) The odd numbers between 15 and 25 are 17, 19, 21 and 23.

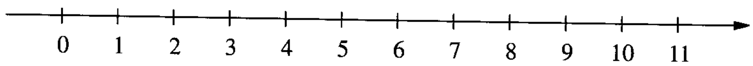
It's fact

Even numbers can be exactly divided by 2, eg

$$\begin{array}{r} 4 \\ 2\overline{)8} \\ \underline{8} \\ \underline{0} \end{array}$$

The Number Line

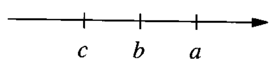
In Mathematics, it is often useful to represent whole numbers by points on a line called the **number line**.

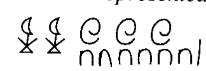


Draw a line. Choose any point on the line and label it 0. Starting with 0, mark off equal intervals of any suitable length. Label the points marked 1, 2, 3, 4, … as shown in the figure above. The arrow on the extreme right indicates that the list of numbers continues in the same way indefinitely.

A number on the number line is always greater than any number to its left and smaller than any number to its right, i.e. $4 > 3$ and $4 < 5$.

The number line below shows that $a > b > c$.



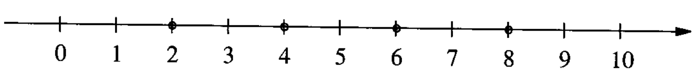
> **Back in Time**
>
> *Ancient civilizations used different systems of numerals.*
>
> *The early Egyptians (around 3000 BC) used the following symbols for different numerals.*
>
> *(one)*
>
> *(ten)*
>
> *(one hundred)*
>
> *(one thousand)*
>
> *(ten thousand)*
>
> *(one hundred thousand)*
>
> *(one million)*
>
> *2 362 is represented by*
>
> 

Example 2

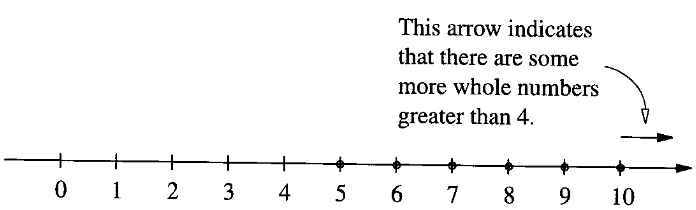
Draw a number line to represent the whole numbers
(a) 2, 4, 6 and 8; *(b) greater than 4.*

Solution

(a) We use dots to indicate the whole numbers, 2, 4, 6 and 8.



(b) We use dots to indicate the first few whole numbers that are greater than 4.

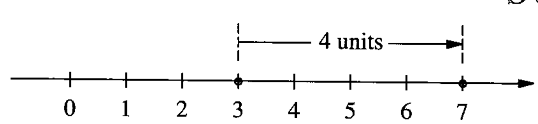


Example 3

Use a number line to find 3 + 4.

Solution

We start by drawing a number line.



Begin at 3 and then move 4 units to the right as shown in the above diagram.

We end at 7 and thus,

$$3 + 4 = 7.$$

Exercise 1a

1. In 1997, the Ministry of Education selected more than 80 000 students from schools and other educational institutions to obtain base-line indices on students' feelings and perceptions about the nation.

The survey findings indicated that more than 90% of students from Primary Six level onwards took pride in being Singaporeans and more than 80% of them accepted friends of different religious beliefs. 83% to 95% of Primary Three students answered three out of five general knowledge questions about the country correctly.

- **(a)** Given that exactly a students were selected for the survey, write an inequality statement for a.
- **(b)** If the actual percentages of students who take pride in being Singaporeans and accept friends of different religious beliefs were $b\%$ and $c\%$ respectively, write an inequality statement for each of b and c.
- **(c)** Given that $d\%$ of Primary Three students answered three out of five general knowledge questions about the country correctly, write an inequality statement for d.

2. List the following numbers:

- **(a)** Natural numbers less than 8.
- **(b)** Even numbers between 25 and 35.
- **(c)** Odd numbers between 40 and 53.
- **(d)** Whole numbers > 63 but < 70.
- **(e)** Whole numbers which are multiples of 3 and between 22 and 40.
- **(f)** Even numbers > 84 but < 99.
- **(g)** Odd numbers $\geqslant 55$ but < 65.
- **(h)** Even numbers > 72 but $\leqslant 90$.

3. Draw the number line to represent the following numbers:

- **(a)** 1, 3, 5 and 7.
- **(b)** 5, 7, 10 and 14.
- **(c)** Whole numbers < 6.
- **(d)** Natural numbers $\geqslant 7$.
- **(e)** Whole numbers $\geqslant 3$ but $\leqslant 9$.
- **(f)** Whole numbers < 19 but $\geqslant 8$.
- **(g)** Natural numbers > 2 but $\leqslant 7$.
- **(h)** Natural numbers > 3 but < 13.

4. Display each addition using the number line.

- **(a)** $2 + 6 = 8$
- **(b)** $7 + 4 = 11$
- **(c)** $3 + 5 + 8 = 16$

Addition and Subtraction of Whole Numbers

There are different ways to carry out addition and subtraction of whole numbers. Here is a general paper-and-pencil method. When adding and subtracting whole numbers, we write the numbers in vertical columns, aligning digits according to their place value. The following examples illustrate how to use the general method to do addition and subtraction of whole numbers.

Example 4

Evaluate (a) 68 + 27; (b) 523 + 268 + 95.

Solution

(a)

	tens	units	
	6	8	8 + 7 = 15
+	2	7	60 + 20 = 80
	1	5	(from the units column)
	8	0	(from the tens column)
	9	5	

or

	tens	units
	$^{1}6$	8
+	2	7
	9	5

$\therefore$ 68 + 27 = 95

(b)

	hundreds	tens	units
	$^{1}5$	$^{1}2$	3
	2	6	8
+		9	5
	8	8	6

$\therefore$ 523 + 268 + 95 = 886

3 + 8 + 5 = 10 + 6

Bring 10 to the tens column.

20 + 60 + 90 + 10 = 180 = 100 + 80

Bring 100 to the hundreds column.

500 + 200 + 100 = 800
800 + 80 + 6 = 886

Can you explain how the subtraction is carried out?

Example 5

Calculate (a) 658 – 436; (b) 835 – 479.

Solution

(a)

	hundreds	tens	units
	6	5	8
–	4	3	6
	2	2	2

Check:

$$\begin{array}{r} 436 \\ +\,222 \\ \hline 658 \end{array}$$

(b)

	hundreds	tens	units
	$^{7}8$	$^{12}3$	$^{15}5$
–	4	7	9
	3	5	6

Check:

$$\begin{array}{r} {}^{1}4\;{}^{1}7\;9 \\ +\;3\;\;5\;\;6 \\ \hline 8\;\;3\;\;5 \end{array}$$

Exercise 1b

1. Do the following additions:

(a)
$$\begin{array}{r} 934 \\ 715 \\ +\;\;86 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 4\,801 \\ 2\,191 \\ +\;8\,463 \\ \hline \end{array}$$

2. Do the following subtractions:

(a)
$$\begin{array}{r} 60\,152 \\ -\;\;1\,895 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 91\,346 \\ -\;88\,978 \\ \hline \end{array}$$

3. Copy and complete the following:

(a)
$$\begin{array}{r} 2\ 4\ 9 \\ +\ \square\ 8\ \square \\ \hline 8\ \square\ 5 \end{array}$$

(b)
$$\begin{array}{r} 2\ \square\ 4 \\ -\ \square\ 9\ \square \\ \hline 8\ 8 \end{array}$$

(c)
$$\begin{array}{r} \square\ 6\ 3 \\ 7\ \square\ 2 \\ +\ 5\ 8\ \square \\ \hline \square\ 0\ 4\ 2 \end{array}$$

(c)
$$\begin{array}{r} x\,5\,4\,8\,y\,z\,x \\ -\ \ y\,x\,4\,9\,8\,0 \\ \hline 8\,z\,z\,y\,5\,x \end{array}$$

(d)
$$\begin{array}{r} 2\,x\,y\,z \\ 2\,x\,y\,z \\ +\ \ 4\,x\,8 \\ \hline 4\,z\,y\,y \end{array}$$

4. In each of the following, find the digits represented by the letters x, y and z.

(a)
$$\begin{array}{r} 7\,x\,2 \\ -\ y\,3\,9 \\ \hline 4\,8\,z \end{array}$$

(b)
$$\begin{array}{r} 3\,x\,y\,z\,9 \\ +\ \ x\,y\,9\,z \\ \hline 4\,3\,z\,z\,x \end{array}$$

5. Using some of the single digit whole numbers, form:

(a) the smallest and the largest 5-digit numbers and find their difference;

(b) two 5-digit numbers such that their sum is

(i) the largest

(ii) the smallest;

(c) two 5-digit numbers such that their difference is the smallest.

Commutative and Associative Law of Addition

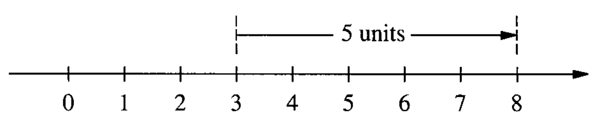
Commutative Law of Addition

We know that 3 + 5 = 8 and 5 + 3 = 8,

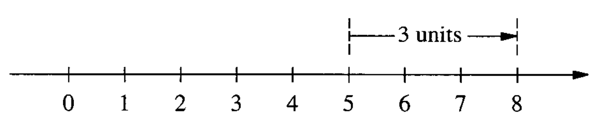
$\therefore\quad 5 + 3 = 3 + 5$

Using the number line, we obtain the same result for 3 + 5 and 5 + 3 as shown below:

Begin at 3 and then move 5 units to the right.



Begin at 5 and then move 3 units to the right.



We can see that the order of adding any two numbers does not affect the result. This is the **Commutative Law of Addition**.

If x and y represent two whole numbers, then

$$x + y = y + x$$

It is obvious that 8 – 5 and 5 – 8 will not give the same results.

Thus, subtraction is not commutative. In general, if x and y represent two whole numbers, then $x - y \neq y - x$, where $x \neq 0$, $y \neq 0$.

It's a Fact

If a cashier first scans in an item costing $3 and then an item costing $5, the total amount obtained must be the same as that which would be obtained if the item costing $5 were scanned in first.

In our everyday life, there are things that we must do in a certain order. For example, we have to put on our socks before we put on our shoes.

Can you give examples of things that we can do in any order and things that we must do in a definite order?

Associative Law of Addition

We know that

$$\begin{aligned} 2 + 3 + 5 &= (2 + 3) + 5 \\ &= 5 + 5 \\ &= 10 \end{aligned}$$

Also,

$$\begin{aligned} 2 + 3 + 5 &= 2 + (3 + 5) \\ &= 2 + 8 \\ &= 10 \end{aligned}$$

$$\therefore \quad (2 + 3) + 5 = 2 + (3 + 5)$$

This shows that the order of grouping numbers together in addition does not affect the answer. We say that addition is associative. This property is called the **Associative Law of Addition**.

If x, y and z represent three whole numbers, then

$$(x + y) + z = x + (y + z)$$

Do you think subtraction is associative? Why?

Applying the commutative law and associative law of addition can often help us add whole numbers more easily.

Example 6

Calculate (a) $9 + 17 + 3$; (b) $10 + 8 + 4$; (c) $18 + 5 + 2 + 6$.

Solution

(a) $9 + 17 + 3 = 9 + (17 + 3)$ (Associative law)
$= 9 + 20$
$= 29$

(b) $16 + 8 + 4 = 16 + 4 + 8$ (Commutative law, interchanging 8 and 4)
$= (16 + 4) + 8$ (Associative law)
$= 20 + 8$
$= 28$

(c) $18 + 5 + 2 + 6 = (18 + 2) + (5 + 6)$ (Commutative law, interchanging 5 and 2)
$= 20 + 11$
$= 31$

Exercise 1c

1. State each number represented by a.

(a) $15 + 39 = 39 + a$ **(b)** $a + 269 = 269 + 854$
(c) $32 + 75 + a = 18 + 32 + 75$ **(d)** $15 + a + 69 = 69 + 15 + 23$

2. Apply either the commutative law or associative law or both in the following mental calculations:

(a) $14 + 6 + 9$ **(b)** $14 + 21 + 9$ **(c)** $31 + 16 + 9$ **(d)** $25 + 28 + 15$
(e) $67 + 52 + 33$ **(f)** $123 + 66 + 77$ **(g)** $28 + 22 + 41 + 59$
(h) $49 + 51 + 101 + 99$ **(i)** $7 + 25 + 13 + 75$ **(j)** $11 + 26 + 4 + 89$

3. Calculate

(a) $25 + 43 + 75$ **(b)** $14 + 28 + 36 + 50$ **(c)** $145 + 80 + 55$ **(d)** $74 + 39 + 61 + 26$
(e) $9 + 25 + 41 + 125$ **(f)** $650 + 128 + 350 + 22$

Multiplication and Division of Whole Numbers

From the diagram given on the right we can see

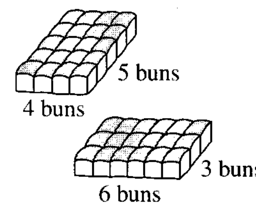
$$4 \times 5 = 5 + 5 + 5 + 5 = 20$$
$$6 \times 3 = 3 + 3 + 3 + 3 + 3 + 3 = 18$$

Multiplication is the same as **repeated addition** of the same number. From the above we can also get

$$20 \div 4 = 5 \quad \text{or} \quad 20 \div 5 = 4$$
$$18 \div 6 = 3 \quad \text{or} \quad 18 \div 3 = 6$$

As in the case of addition and subtraction of whole numbers, there is a general paper-and-pencil method to carry out multiplication and division of whole numbers, taking into consideration the place values of the digits. The following examples illustrate how to use this general method to carry out multiplication and division of whole numbers.

Notes



To find the total number of buns, all we need to know are the number of rows of buns and the number of buns in each row. Thus, in the diagram there are
$4 \times 5 = 20$ buns
and $6 \times 3 = 18$ buns
respectively.

Example 7

Calculate (a) 46×15; (b) 318×509.

Solution

(a)
$$\begin{array}{rl} 46 & \\ \times\ 15 & \leftarrow (15 = 10 + 5) \\ \hline 230 & \leftarrow (5 \times 46) \\ +\ 460 & \leftarrow (10 \times 46) \\ \hline 690 & \end{array}$$

(b)
$$\begin{array}{rl} 318 & \\ \times\ 509 & \leftarrow (509 = 500 + 0 + 9) \\ \hline 2862 & \leftarrow (9 \times 318) \\ 0000 & \leftarrow (0 \times 318) \\ +\ 159000 & \leftarrow (500 \times 318) \\ \hline 161862 & \end{array}$$

Example 8

Divide 64 by 4.

Solution

Using long division:

$$\begin{array}{r} 16 \\ 4\overline{)\ 64} \\ -4 \\ \hline 24 \\ -24 \\ \hline 0 \end{array}$$

Bring 4 down.

The remainder 2 tens is added to 4.

Using short division:

$$\begin{array}{r} {}^{2} \\ 4\overline{)\ 64} \\ 16 \end{array}$$

This 2 is the same as the circled 2 shown on the left.

Notes

In short division, working is done mentally.

Example 9

Divide 6 239 by 27.

Solution

Long division

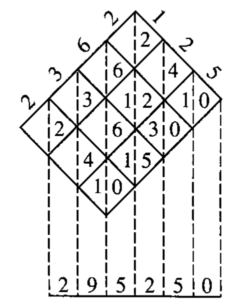
```
      231   quotient
27 )  6 239
    - 5 4
       83
     - 81
       29
     - 27
        2   remainder
```

Short division

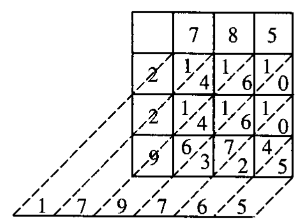
```
      8 2
27 ) 6 2 3 9
     2 3 1   remainder 2
```

Investigate

The following shows an increasing method of multiplication developed by the Arabs.



2 362 × 125 = 295 250



229 × 785 = 179 765

Can you figure out how the method works?

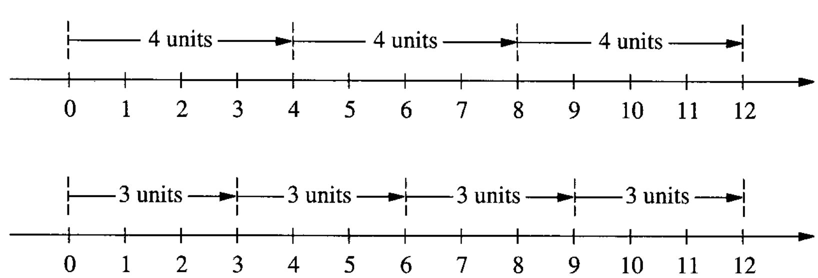
Exercise 1d

1. Do the following:
   (a) 326×19 (b) 537×160 (c) 671×407
2. Do the following using short division:
   (a) $992 \div 8$ (b) $6\,444 \div 9$ (c) $34\,566 \div 7$
3. Evaluate the following divisions:
   (a) $704 \div 22$ (b) $4\,446 \div 13$ (c) $6\,919 \div 11$

Laws of Multiplication of Whole Numbers

Commutative Law

Using the number line, the products of 3×4 and 4×3 are obtained as shown in the figure below.



The diagram above illustrates the **Commutative Law of Multiplication** by showing that

$$3 \times 4 = 4 \times 3$$

If x and y represent two whole numbers, then

$$x \times y = y \times x$$

Compute $27 \div 3$ and $3 \div 27$.

Is $27 \div 3$ equal to $3 \div 27$? Is the division commutative?

Just For Fun

999999999999999 = 1990
Insert +, –, × or ÷ in suitable places on the left-hand side of = so as to make the above equation true.

Associative Law

The product $2 \times 3 \times 4$ may be found in two different ways, i.e.,

$$(2 \times 3) \times 4 = 6 \times 4 = 24$$
$$2 \times (3 \times 4) = 2 \times 12 = 24$$
$$\therefore \quad (2 \times 3) \times 4 = 2 \times (3 \times 4)$$

In general, the property is called the **Associative Law of Multiplication**:

If x, y and z represent three whole numbers, then

$$(x \times y) \times z = x \times (y \times z)$$

Calculate $(9 \div 3) \div 3$ and $9 \div (3 \div 3)$.

Is $(9 \div 3) \div 3$ equal to $9 \div (3 \div 3)$? Is the division associative?

Distributive Laws

John did 5 hours and 9 hours of community work in the first and second half of the year respectively. He was awarded 4 points for each hour of community work. Let us find the total points John received for the whole year.

The points he received can be calculated in two ways as shown below:

(1) $4 \times (5 + 9) = 4 \times 14 = 56$

(2) $(4 \times 5) + (4 \times 9) = 20 + 36 = 56$

Clearly, $4 \times (5 + 9) = (4 \times 5) + (4 \times 9)$

In general, for any three whole numbers x, y and z we have $x \times (y + z) = x \times y + x \times z$. This is called the **Distributive Law of Multiplication over Addition**.

We also have the **Distributive Law of Multiplication over Subtraction**, that is for any three whole numbers, x, y and z, we have

$$x \times (y - z) = x \times y - x \times z.$$

Let us look at an example.

Notes

(1) John did 5 + 9 or 14 hours of community work. Hence, he received 4 × 14 = 56 points

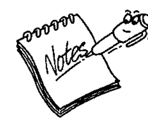
(2) For the first half of the year, he received 4 × 5, or 20, points. For the second half of the year, he received 4 × 9, or 36, points. Hence, for the whole year he received 20 + 36, or 56, points.

Ezar, John's classmate, performed 8 hours of community service for the first half of the year and a total of 15 hours for the whole year. The points Ezar was awarded for the second half of the year can be obtained as shown below:

(1) $4 \times (15 - 8) = 4 \times 7 = 28$

(2) $(4 \times 15) - (4 \times 8) = 60 - 32 = 28$

$\therefore \quad 4 \times (15 - 8) = (4 \times 15) - (4 \times 8)$



(I) Ezar did (15 – 8) or 7 hours of community work for the second half of the year. Hence, he received $4 \times 7 = 28$ points.

(II) For the whole year, he received 4×15 or 60 points. The first half of the year, he received 4×8 or 32 points. Hence, he received 60 – 32, or 28, points for the second half of the year.

In summary,

If x, y and z represent three whole numbers, then

$$x \times (y + z) = x \times y + x \times z$$
$$x \times (y - z) = x \times y - x \times z$$

Exercise 1e

1. Fill in the boxes with +, – or ×:

(a) $6 \square 4 = 4 \square 6 = 24$

(b) $23 \square 11 = 11 \square 23 = 34$

(c) $107 \square 33 = 33 \square 107 = 140$

(d) $6 \square 15 = 15 \square 6 = 90$

(e) $35 \square 7 = 7 \square 35 = 245$

(f) $263 \square 103 = 103 \square 263 = 366$

2. Put a numeral in each box to make the equation true:

(a) $6 \times (5 + \square) = 6 \times 5 + 6 \times 7$

(b) $10 \times (4 + 5) = 10 \times \square + 10 \times \square$

(c) $11 \times 12 + 6 \times 12 = (11 + 6) \times \square$

(d) $2 \times (\square - 5) = (2 \times 6) - (2 \times 5)$

(e) $(10 \times 13) - (9 \times 13) = (10 - 9) \times \square$

(f) $(6 - 5) \times 9 = (\square \times 9) - (5 \times \square)$

Applying the above laws of multiplication can often help us do the calculations more easily.

Example 10

Do the following sums mentally:

(a) $25 \times 29 \times 4$; *(b)* $45 \times 3 + 45 \times 7$;
(c) $33 \times 17 - 33 \times 7$; *(d)* 4×98;
(e) 25×16; *(f)* $5 \times 72 - 5 \times 12$.

$9 = 3 \times 3 = 3 + 3 + 3$
$= 2 + 3 + 4$

$\therefore$ 9 can be written as a sum of three consecutive numbers, 2, 3 and 4.

$40 = 5 \times 8 = 8 + 8 + 8 + 8 + 8$
$= 6 + 7 + 8 + 9 + 10$

$63 = 3 \times 21 = 21 + 21 + 21$
$= 20 + 21 + 22$

Solution

(a) $25 \times 29 \times 4 = 25 \times 4 \times 29$ (Commutative law)
$= 100 \times 29 = 2\,900$ (Associative law)

(b) $45 \times 3 + 45 \times 7 = 45 \times (3 + 7)$ (Distributive law)
$= 45 \times 10 = 450$

(c) $33 \times 17 - 33 \times 7 = 33 \times (17 - 7)$ (Distributive law)
$= 33 \times 10 = 330$

63 = 7 × 9 = 9 + 9 + 9 + 9 + 9 + 9 + 9 = 6 + 7 + 8 + 9 + 10 + 11 + 12

Can you write 30, 33, 42 and 22 each as a sum of consecutive numbers?

Can any one of them be written as a sum of consecutive numbers in more than one way?

(d) $4 \times 98 = 4 \times (100 - 2)$ (Write 98 as 100 – 2)
$= 4 \times 100 - 4 \times 2$ (Distributive law)
$= 400 - 8 = 392$

(e) $25 \times 16 = 25 \times 4 \times 4$ (Write 16 as 4 × 4)
$= 100 \times 4$ (Associative law)
$= 400$

(f) $5 \times 72 - 5 \times 12 = 5 \times 2 \times 36 - 5 \times 2 \times 6$ (Write 72 as 2 × 36 and 12 as 2 × 6)
$= 10 \times (36 - 6)$
$= 10 \times 30 = 300$ (Distributive law)

1, 10, 100 ... are friendly numbers.

Exercise 1f

1. **(a)** Find the product of 273 and 111 and then multiply the result by 8.
   (b) Find the product of 273 and 888 directly.
   (c) Will **(a)** or **(b)** give you the product 273 × 888 more easily?

2. Find the product of the three numbers 33, 444 and 99.

3. Calculate the following mentally.
   (a) $45 \times 7 + 45 \times 3$
   (b) $62 \times 4 + 62 \times 6$
   (c) $59 \times 19 - 59 \times 9$
   (d) $48 \times 77 - 38 \times 77$
   (e) $89 \times 15 + 11 \times 15$
   (f) $61 \times 123 - 23 \times 61$
   (g) $1\,291 \times 1\,291 - 1\,291 \times 1\,281$
   (h) $5 \times 816 \times 20$
   (i) $25 \times 1\,999 \times 4$
   (j) $2 \times 6\,505 \times 50$
   (k) 888×50
   (l) $8\,888 \times 25$
   (m) $8\,888 \times 125$
   (n) $4 \times 9 \times 9 \times 25$
   (o) $4 \times 8 \times 9 \times 5 \times 5$
   (p) $25 \times 7 \times 4 \times 11$
   (q) 8×999
   (r) $1\,999 \times 5$

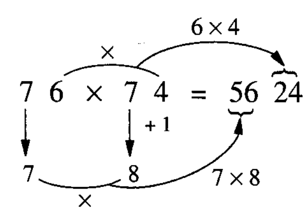
In-Class Activity

Mental calculations involving special 2-digit numbers

Work in pairs.

1. Calculate the following products:
   (i) 76×74 **(ii)** 32×38 **(iii)** 65×65
   (a) When you compare the tens digits and the units digits, what do you notice about each pair of numbers?
   (b) Did you obtain $76 \times 74 = 5\,624$?

2. Investigate how the digits of 76, 74 and 5 624 are related by studying the diagram below.



3. Investigate whether the same relationships exist for 1(ii) and 1(iii).

4. With a little practice, do you think you can use the relationships to calculate the product of such a pair of numbers mentally?

5. Find the product of each of the following mentally:
   (a) 67×63 **(b)** 96×94 **(c)** 58×52 **(d)** 85×85
   (e) 75×75 **(f)** 109×101 **(g)** 268×262

6. Practice with your partner by giving each other products of such pairs of numbers to work out mentally.

Order of Operations

Confusion arises when we try to evaluate $19 - 3 + 7$.

Which operation should we perform first?

If we perform the addition first, we obtain

$$\begin{aligned} 19 - (3 + 7) &= 19 - 10 \\ &= 9 \end{aligned}$$

If we perform the subtraction first, we obtain

$$\begin{aligned} (19 - 3) + 7 &= 16 + 7 \\ &= 23 \end{aligned}$$

Obviously, $9 \neq 23$ and hence $19 - (3 + 7) \neq (19 - 3) + 7$

Similarly, $(10 \times 6) + 4 \neq 10 \times (6 + 4)$ $\quad (10 \times 6) + 4 = 64$, $\; 10 \times (6 + 4) = 100$

and $(20 \div 5) - 3 \neq 20 \div (5 - 3)$ $\quad (20 \div 5) - 3 = 1$, $\; 20 \div (5 - 3) = 10$

From the discussion above, we know that some rules are needed for performing operations.

Some Simple Rules for Performing Arithmetical Operations

The following rules are applicable to arithmetical operations:

1. If an expression contains brackets, simplify the expression within the brackets first. For example:
$$3 + (5 - 3) = 3 + 2 = 5$$

2. If an expression contains more than one pair of brackets, that is, there are brackets within brackets, simplify the expression within the innermost pair of brackets first. For example:
$$\begin{aligned} 3 + [15 - (3 + 4)] &= 3 + [15 - 7] \\ &= 3 + 8 = 11 \end{aligned}$$

3. If an expression contains only additions and subtractions, work from left to right. For example:
$$28 + 12 - 9 = 40 - 9 = 31$$

4. If an expression contains only multiplications and divisions, work from left to right. For example:

$$125 \div 5 \times 15 = 25 \times 15 = 375$$

5. If an expression contains all the four operations (i.e.) addition, subtraction, multiplication and division), do multiplications or divisions before additions or subtractions. For example:

$$\begin{aligned} 12 + 3 \times 4 - 35 \div 7 &= 12 + 12 - 5 \\ &= 24 - 5 = 19 \end{aligned}$$

Exercise 1g

1. In each case, fill in the box with >, < or =:
   (a) $9 \times 3 - 11 \; \square \; 16$
   (b) $5 \times 11 + 6 \times 3 \; \square \; 60$
   (c) $56 \div 8 \times 4 - 15 + 9 \; \square \; 25$
   (d) $72 \div (18 - 60 \div 5) \; \square \; 10$
   (e) $2 \times [2 + 3 \times (2 + 5)] \; \square \; 45$
   (f) $24 \times 7 \times 8 \div 12 \; \square \; 112$
   (g) $2 \times 35 \div 5 + 96 \div 3 \; \square \; 50$
   (h) $2\,464 \div (1 + 3 + 3 \times 4) \; \square \; 54$
   (i) $105 + 27 \times 8 - 144 \div 9 \; \square \; 337$
   (j) $(8 \times 9 - 108 \div 12) \times 2 - 57 \; \square \; 70$

2. Insert parentheses in each of the following expressions to make the resulting statement true. For example, the statement $11 - 5 - 4 = 10$ will be true if brackets are inserted around $5 - 4$ because $11 - (5 - 4) = 11 - 1 = 10$.
   (a) $12 - 7 - 2 = 7$
   (b) $3 \times 5 + 7 = 36$
   (c) $3 \times 5 + 2 \times 4 = 39$
   (d) $3 \times 5 + 2 \times 4 = 84$
   (e) $3 \times 5 + 2 \times 4 = 68$
   (f) $4 \times 6 - 3 \times 5 = 60$

3. Calculate:
   (a) $98 - 32 - 15 + 21$
   (b) $24 \times 7 \times 8 \div 12$
   (c) $25 \div 5 \times 5 \div 25$
   (d) $(47 - 25) + (52 - 47) \times 8$
   (e) $(32 - 16) + (85 - 37) \div 2$
   (f) $[(12 + 18) \times 3 - 5] \div 17$
   (g) $(50 + 60) \times [40 \div (60 - 50)]$
   (h) $[50 \times 3 + (50 - 10) \times 3] \div 6$
   (i) $9\,000 \div [1\,500 \div 30 \div 10 \times (90 + 30)]$
   (j) $[5 \times 52 - (5 \times 52 + 5 \times 36) \div 2] \div 5$
   (k) $[567 - (175 - 132) \times 9] + (35 - 18) \times 6$
   (l) $[(325 + 45) \div 5 \times 7] - (78 - 65) \times 2$
   (m) $75 - 38 \div 2 + 75 \div 5 \times 7 + 81 \div 3 \div 9 \times 7 - 15 + 6 \times 7$

4. A shopkeeper buys 18 T-shirts and 12 skirts for \$144. If the T-shirts cost \$4 each, find the cost of each skirt by completing and calculating the following:

$$(144 - 18 \times \square) \div \square$$

5. A fruit seller buys 8 crates of oranges at \$18 per crate and 10 crates of apples at \$20 per crate. There are 72 oranges in each crate of oranges and 100 apples in each crate of apples. If he sells the oranges at 3 for a dollar and apples at 4 for a dollar, find his profit by completing and simplifying the following expression:

$$\square \times (\square \div 3 - 18) + 10 \times (100 \div \square - \square)$$

6. In a pet shop, two goldfish cost as much as five tropical fish. If Wei Meng pays \$20 for 10 tropical fish, how many goldfish can he buy with \$40? Obtain your answer by simplifying an expression like those in Question 3.

7. A shop charges the customers \$2 for binding a book. There will be no charge if customers are not satisfied with the service. In the last two days, the shop bound 38 and 45 books respectively and received \$156. How many books were not bound to the satisfaction of the customers? Obtain your answer by simplifying an expression like those in Question 3.

Rounding off Whole Numbers

Investigate

As of 1998, there are 702 100 foreigners in Singapore of which 5 000 are Australians.

Can you complete the following? 702 100 could have been obtained from a number which has been rounded off to the nearest 100. While 5 000 could also have been obtained when a number has been rounded off to the nearest 1 000.

(a) 702 100 could have been rounded off to the nearest ____.

(b) 5 000 could have been rounded off to the nearest ____.

Can you give other daily life examples in which you need to work with estimates rather than exact values?

It was mentioned at the beginning of this chapter that the estimated population of Singapore in July 1996 was 3 396 924 people.

It is usually not useful to give such an accurate estimate of the population of Singapore. We will get a better idea of how large or how small the population was if the estimate was to be given as 3.4 million or 3 400 000 people.

On 20th March 1999, *The Straits Times* published the population figure under the heading "Singapore with 4 million people" and gave our population as 3.87 million or 3 870 000.

3 400 000 (three million and four **hundred thousand**) has been obtained by **rounding 3 396 924** off to the nearest **hundred thousand** (100 000) and **3 870 000** (three million and eight hundred and **seventy thousand**) could have been rounded off to the nearest ten thousand (**10 000**).

By comparing the two rounded off figures, we grasp very quickly that the population has increased by about 470 000 people in less than three years.

There are other occasions where it is better to work with estimates. For example, we may budget $1 000 for a trip that would cost at least $850.

A certain type of printing paper is sold in reams of 500 sheets. If you need 2 345 sheets of printing paper, how many reams of paper must you buy?

In-Class Activity

Work with a partner.

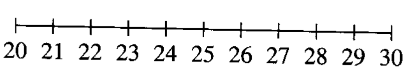
Notes

25 is exactly between 20 and 30. By convention, we round 25 to 30, correct to the nearest ten.

1. **(a)** Is 20 or 30 a better estimate of 24?

(b) Is 20 nearer to 24 than 30?

The diagram below shows the section of the number line from 20 to 30.



(i) Is 25 half way between 20 and 30?

(ii) Can you locate the point representing 24?

(iii) Can we say that 24 is nearer to 20 than 30 because the units digit of 24 is less than the units digit of 25, i.e. $4 < 5$?

(iv) 30 is nearer to 27 than 20. Why?

(v) Can we say that when 24 is rounded to 20 and 27 to 30, 24 and 27 have been rounded to the nearest 10?

2. Now, consider 824 and 827.

   (a) In each case, fill in the box with > or <:

      (i) 824 is nearer to 820 than 830 because 4 $\square$ 5.

      (ii) 827 is nearer to 830 than 820 because 7 $\square$ 5.

   (b) In each case, fill in the box with the correct number:

      (i) 824 = 8 $\square$ 0 (correct to the nearest 10)

      (ii) 827 = 8 $\square$ 0 (correct to the nearest 10)

3. Consider 845 and 855.

   (a) In each case, fill in the box with >, <, = or ≠:

      (i) 845 is nearer to 800 than 900 because the tens digit, 4 $\square$ 5.

      (ii) 855 is nearer to 900 than 800 because the tens digit, 5 $\square$ 5 and the units digit, 5 $\square$ 0.

   (b) In each case, find a digit for each ☆:

      (i) 845 = 8 ☆☆ (correct to the nearest 100)

      (ii) 855 = ☆☆☆ (correct to the nearest 100)

4. We say that 24 and 27 are approximately equal to 20 and 30 respectively and we write $24 \approx 20$ and $27 \approx 30$. In other words, 20 and 30 are estimates of 24 and 27 respectively.
   In each case, fill in the blank with the correct number:

   (a) $824 \approx$ ________ and $827 \approx$ ________.

   (b) $845 \approx$ ________ and $855 \approx$ ________.

From the above activity, we can conclude the following:

(i) To round off a whole number to the nearest ten, consider the units digit. If it is less than 5, simply replace it by 0. If it is 5 or more, add 1 to the tens digit and replace the units digit by 0.

(ii) To round off a whole number to the nearest hundred, consider the tens digit. If it is less than 5, simply replace the tens and the units digits by zeros. If it is 5 or more, add 1 to the hundreds digit and replace the tens and units digits by zeros.

(iii) The same procedure can be extended to round off a whole number to a specified place as shown below:

Step 1 Find the digit in the specified place.

Step 2 Consider the next digit to the right.

- **(a)** If the digit is less than 5, replace it and all the digits to its right by zeros.
- **(b)** If the digit is 5 or more, replace it and all digits to its right by zeros after adding 1 to the digit in the specified place.

Example 11

Write (a) 4 178 correct to the nearest 10; (b) 98 142 correct to the nearest 100;
(c) 48 653 correct to the nearest 1 000.

Solution

$$\text{(a)}\quad 4\ 1\ \overset{+1}{7\ 8} = 4\ 1\ 8\ 0 \quad \text{(correct to the nearest 10)}$$

This digit is more than 5.

(b) $9\,8\,\overset{+0}{1}\,4\,2 = 9\,8\,1\,0\,0$ (correct to the nearest 100)

This digit is less than 5.

(c) $4\,8\,\overset{+1}{6}\,5\,3 = 4\,9\,0\,0\,0$ (correct to the nearest 1 000)

This digit is more than 5.

Exercise 1h

1. With a population of 4 million and a land area of 585 sq km, Singapore has a population density of 6 838 people per sq km.
   (a) Round 585 off to the nearest
   (i) 10 (ii) 100
   (b) Round 6 838 off to the nearest
   (i) 10 (ii) 100 (iii) 1 000

2. It will take 10 076 Boeing 747s to ship out 4 million Singaporeans who would in turn fit into 2 222 MRT trains.
   (a) Round 2 222 off to the nearest
   (i) 100 (ii) 1 000
   (b) Round 10 076 off to the nearest
   (i) 10 (ii) 100 (iii) 1 000
   (iv) 10 000

3. A pop group sold 82 649 copies of their latest album. What is the number of copies sold to the nearest
   (a) ten (b) hundred
   (c) thousand (d) ten thousand?

4. Round the following figures correct to the
   (i) nearest ten
   (ii) nearest hundred
   (iii) nearest thousand
   (a) 55 726 (b) 380 014
   (c) 9 969 972 (d) 76 636 792

5. If a whole number n, rounded to the nearest 10, is 50, then the possible values of n are $45 \leqslant n \leqslant 54$ or $45 \leqslant n < 55$. Similarly if a whole number m, rounded off to the nearest 100, is 2 500, then the possible values of m are $2\,450 \leqslant m < 2\,550$.
   (a) In Singapore, about 15 000 National Service recruits are enlisted annually. If the actual number of recruits enlisted in a particular year is a and 15 000 is the value of a rounded off to the nearest 1 000, write down the possible values of a.
   (b) In Singapore, there are about 250 000 operationally ready servicemen in the SAF. If the actual number of operationally ready servicemen at a particular moment is b and 250 000 is the value of b rounded off to the nearest 10 000, write down the possible values of b.

Use of Calculators

There are different types of calculators we can use in Mathematics. Therefore, it is important that you follow the instructions given in the handbook that comes with your calculator.

Below are some important keys of the calculator.

$\boxed{0}$ to $\boxed{9}$ Numeral keys

$\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$ Operation keys

$\boxed{=}$ Equal key

$\boxed{\text{ON/C}}$ All clear key

$\boxed{(}$ $\boxed{)}$ Bracket keys

Before you use a calculator make sure that it is functioning properly. You can check this by performing some simple calculations to which you already know the answers.

For example,

3 $\boxed{\times}$ 2 $\boxed{\div}$ 3 must give 2 or 6 666 666 $\boxed{\div}$ 2 must give 3 333 333, etc.

"What is your favourite number?" John asked his little sister.

"Three", she said.

"I will use my calculator to work out a product to give you a string of your favourite number," John said.

John pressed 12345679 on his pocket calculator which has a 10-digit display. He multiplied this number by 27 and the product was 333333333. His sister was thrilled and told John, "I also like the number 9. Can you give me a string of 9's?"

"No problem," replied John. John pressed 12345679 again and this time he multiplied this number by 81 and the result was 999999999.

Can you multiply two numbers so as to give a string of 5's, 7's, 8's, and so on?

Example 12

Use the calculator to find the values of the following:

(a) 34 + 785; *(b) 357 × 174;* *(c) 966 ÷ 23.*

Solution

The steps	Final display
(a) 34 $\boxed{+}$ 785 $\boxed{=}$	819
(b) 357 $\boxed{\times}$ 174 $\boxed{=}$	62 118
(c) 966 $\boxed{\div}$ 23 $\boxed{=}$	42

NB: Many calculators available in the market observe rules for order of operations. Check your calculator for this by entering, say 7 $\boxed{+}$ 3 $\boxed{\times}$ 6. If you obtain 25, then your calculator does observe the rules for order of calculations. If you obtain 60, then it does not. It pays to obtain a calculator which observes such rules so that you can work from left to right without having to worry about the order of calculations.

Example 13

Evaluate using the calculator,

(a) 569 + 24 × 77; *(b) 3 255 ÷ 15 × 64;* *(c) (27 × 15 − 88) × 79.*

Solution

The steps	Final Display
(a) 569 $\boxed{+}$ 24 $\boxed{\times}$ 77 $\boxed{=}$ or $\boxed{\text{EXE}}$	2 417
(b) 3 255 $\boxed{\div}$ 15 $\boxed{\times}$ 64 $\boxed{=}$	13 888
(c) $\boxed{(}$ 27 $\boxed{\times}$ 15 $\boxed{-}$ 88 $\boxed{)}$ $\boxed{\times}$ 79 $\boxed{=}$	25 043

When Sumei opens a book, two pages face her. If the product of the two page numbers is 3 192, what are the two page numbers?

Many calculators provide a **memory storage space**. It is useful for computing complex expressions.

$\boxed{\text{STO}}$ or $\boxed{\text{Min}}$ Memory Entry Key

This key will transfer the number displayed to memory and it cancels the previous contents in the memory. To clear the memory, press zero $\boxed{\text{STO}}$.

$\boxed{\text{RCL}}$ or $\boxed{\text{MR}}$ Memory Recall Key

It will display the contents of the memory without clearing it.

For example, $\dfrac{323 + 7\,594}{285 - 198}$ *can be obtained by pressing*

$285 - 198$ $\boxed{=}$ $\boxed{\text{STO}}$ $323 + 7\,594$ $\boxed{=}$ $\boxed{\div}$ $\boxed{\text{RCL}}$ $\boxed{=}$ to get 91.

Can you work out the above with the use of bracket keys?

NB: Human and machine errors may affect the **accuracy** of a calculation. Hence, it is important to estimate mentally results of calculations for checking purposes. For example, $89 \times 68 \approx 90 \times 70 = 6\,300$. Since $90 > 89$ and $70 > 68$, the exact value of 89×68 should be around but less than 6 300. If you obtain an answer that is a lot less than 6 300 or one that is more than 6 300, then you know that you have obtained a wrong answer.

Example 14

Estimate mentally the results of the following calculations. State in each case whether the estimated answer is greater or less than the exact answer.

(a) $7\,800 \times 19$ *(b)* $425 \times 1\,015$ *(c)* $16\,800 \div 99$

Solution

(a) $7\,800 \times 19 \approx 7\,800 \times 20 = 156\,000$
$156\,000 > 7\,800 \times 19$

(b) $425 \times 1\,015 \approx 425 \times 1\,000 = 425\,000$
$425\,000 < 425 \times 1\,015$

(c) $16\,800 \div 99 \approx 16\,800 \div 100 = 168$
$168 < 16\,800 \div 99$

Exercise 1i

1. Make an estimate of each of the following and then use a calculator to get the exact answer.

	Estimate	Exact answer
(a) 84×103	______	______
(b) $2\,496 \div 48$	______	______
(c) 782×105	______	______
(d) $12\,883 \div 991$	______	______
(e) $3\,420 \times 998$	______	______

2. For each of the following, two answers are given. Only one answer is correct. Use estimation to identify the correct answer.

			Correct answer
(a) 298 × 11			
	(i) 4 268	**(ii)** 3 278	________
(b) 7 238 ÷ 77			
	(i) 94	**(ii)** 84	________
(c) 806 577 ÷ 87			
	(i) 8 421	**(ii)** 9 271	________
(d) 589 × 95			
	(i) 55 955	**(ii)** 62 945	________
(e) 673 × 108			
	(i) 72 684	**(ii)** 83 664	________

3. Use a calculator to compute the following:

(a) 43 749 888 + 9 046 302
(b) 26 070 000 – 58 999
(c) 87 415 × 738
(d) 745 153 ÷ 683
(e) 7 769 × 324 × 189
(f) 415 125 ÷ 45 ÷ 369
(g) 55 069 – 9 968 ÷ 178
(h) 49 138 – 89 × 397
(i) 3 007 × 518 + 475 × 70 562
(j) 8 318 × 978 – 1 547 × 739
(k) 5 097 × 1 574 – 47 827 ÷ 283
(l) 3 278 + 184 × 237 – 136 × 118
(m) 14 234 – 3 477 + 16 762 ÷ 29 – 5 436
(n) 7 532 × 7 156 – 31 188 ÷ 113 + 37 254
(o) 107 163 ÷ 189 × 6 051 – 31 779 ÷ 321
(p) (1 213 + 673) × (76 541 – 3 116)
(q) 131 × (3738 + 556 – 1 365) ÷ 29 × (543 + 6 351)

Summary

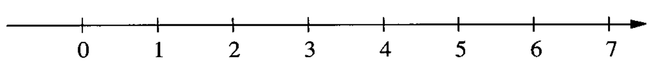
1. The numbers 1, 2, 3, 4, 5, … are **natural** numbers.

   The numbers 0, 1, 2, 3, 4, … are **whole** numbers.

   The numbers 0, 2, 4, 6, 8, … are **even** numbers.

   The numbers 1, 3, 5, 7, 9, … are **odd** numbers.

2. A **number line** is a straight line on which each point represents a number.
   A number on the number line is always greater than any number to its left and smaller than any number to its right.



3. **Commutative Laws**
   Addition and multiplication of whole numbers are commutative. For any two whole numbers x and y, we have

$$x + y = y + x \quad \text{and} \quad x \times y = y \times x.$$

   Subtraction and division are not commutative as $x - y \neq y - x$ (if $x \neq 0$; $y \neq 0$) and $x \div y \neq y \div x$ (if $x \neq 0$; $y \neq 0$) in general.

4. **Associative Laws**
   Addition and multiplication of whole numbers are associative. For any three whole numbers x, y and z, we have

$$(x + y) + z = x + (y + z) \quad \text{and} \quad (x \times y) \times z = x \times (y \times z).$$

5. Distributive Laws

Multiplication is distributive over addition and subtraction. For any three whole numbers x, y and z, we have:

$$x \times (y + z) = x \times y + x \times z; \; (x + y) \times z = x \times z + y \times z;$$
$$x \times (y - z) = x \times y - x \times z; \; (x - y) \times z = x \times z - y \times z.$$

6. Rule for Order of Operations

When an expression of arithmetic operations contains brackets, work with the expressions within the brackets first. (If there are brackets within brackets, work with the innermost pair of brackets first.)

When an expression contains addition, subtraction, multiplication and division, do multipliation and division before addition and subtraction.

When an expression contains only addition and subtraction or only multiplication and division, work from left to right.

Review Questions 1

1. Without using a calculator, find the value of each of the following. (a) and (b) have been done for you.

(a) $96 + 8 + 28 + 2 + 72 + 4 + 15$

$= (96 + 4) + (8 + 72) + (28 + 2) + 15$
$= 100 + 80 + 30 + 15$
$= 225$

(b) $3\,648 + 999$

$= 3\,648 + (1\,000 - 1)$
$= (3\,648 + 1\,000) - 1$
$= 4\,648 - 1$
$= 4\,647$

(c) $102 + 7 + 45 + 198 + 3$
(d) $1\,720 + 863 + 280 + 137$
(e) $135 + 798 + 465 + 202$
(f) $4\,685 + 3\,999$
(g) $999 + 99 + 9$
(h) $998 + 1\,246 + 9\,998$

2. Evaluate the following mentally:

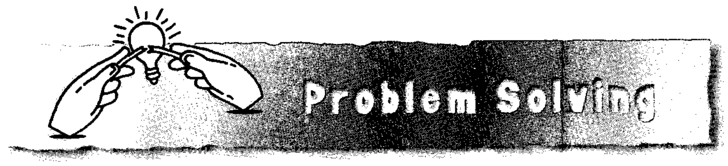
(a) 302×5 **(b)** $2 \times 135 \times 5$
(c) $2 \times 8 \times 9 \times 5$ ∗**(d)** $25 \times 7 \times 4 \times 11$
∗**(e)** $700 \div 25$ ∗**(f)** $1\,600 \times 25$
∗**(g)** $99\,800 \div 20 \div 5$
∗**(h)** 44×46 ∗**(i)** 197×193

3. Without using a calculator, evaluate the following:

(a) $40 \times 5 + 50 \times 6 - 7 \times 60$
(b) $(57 + 43 - 7 \times 5) \times 20 + 4 \times 90$
(c) $(325 - 127) \div 9 + 136 - 11 \times 11$
(d) $16 \div [18 - (32 - 100 \div 5) \div 6]$
(e) $1 \div 1 + 0 \div 26 + 26 \div 1$
(f) $7 \times 3 - 77 \div 11 + 16$
(g) $56 \div 8 + (47 - 17) \div 5 - 13$
(h) $(32 \times 5 - 60) \times 25 - 90 \times 15$
(i) $[4 \times 15 + 72 \div 8 - (47 - 23) \div 6] \times 2$
(j) $105 \div 5 + 15 \times 3 - (6 \times 12 - 54 \div 9)$
(k) $79 - 6 \times 81 \div 9 + 65 \div 13 - 11$
(l) $(640 \div 80 + 54 \div 6) \times 20 - 840 \div 7$

4. Estimate the results of the following calculations:

(a) $97 \times 1\,003$
(b) $3\,648 \times 999$
(c) $199 \times 21 \times 998$
(d) $19 \times 499 \div 51$
(e) $4\,201 \div 58$
(f) $160\,015 \div 801$
∗**(g)** $39 \div 5 + 51 \times 4 - 7 \times 19$
∗**(h)** $389 \div 13 + 2\,604 \div 13$
∗**(i)** $1\,999 \div 501 \times 49$
∗**(j)** $(599 + 402 - 298 \times 2) \times 49$



Problem Solving

1. Complete the following:

(a)
$$\begin{array}{r} \square\,2\,\square \\ \times \quad \square\,7 \\ \hline 2\,2\,\square\,8 \\ +\ \square\,6\,\square\,0 \\ \hline 1\,\square\,4\,6\,\square \end{array}$$

(b)
$$\begin{array}{r} \square\,6\,3 \\ 7\,\square\,2 \\ +\ 5\,8\,\square \\ \hline \square\,0\,4\,2 \end{array}$$

(c)
$$\begin{array}{r} \square\,0\,\square\,\square\,3 \\ -\ \ \square\,7\,2\,\square \\ \hline 7\,7\,7 \end{array}$$

(d)
$$\begin{array}{r} \square\,\square \\ 28\,\big)\,1\,\square\,\square\,4 \\ \square\,\square \\ \hline \square\,\square\,4 \\ \square\,\square\,4 \\ \hline 0 \end{array}$$

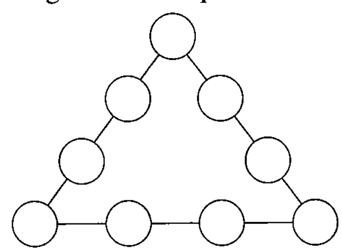
2. Fill each of the □ with a digit so as to make the long division correct.

(a)
$$\begin{array}{r} \square\,\square\,1 \\ \square\,\square\,\square\,\big)\,\square\,\square\,\square\,\square\,\square\,\square \\ 2\,3\,4 \\ \hline \square\,\square\,\square \\ 3\,5\,1 \\ \hline \square\,\square\,\square \\ \square\,\square\,\square \\ \hline 0 \end{array}$$

(b)
$$\begin{array}{r} \square\,\square\,\square\,8\,\square\,\square \\ \square\,\square\,\big)\,\square\,\square\,\square\,\square\,\square\,\square\,\square \\ \square\,\square\,\square \\ \hline \square\,\square \\ \square\,\square \\ \hline \square\,\square\,\square \\ \square\,\square\,\square \\ \hline 0 \end{array}$$

3. Given that a 6-digit number 199 6□□ is exactly divisible by 95, find the last two digits of the number.

4. Fill in the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 so that the sum of the numbers on each side of the triangle will be equal to 17.



5. Insert +, –, ×, ÷ and brackets to make the following sentences true. The first one has been done.

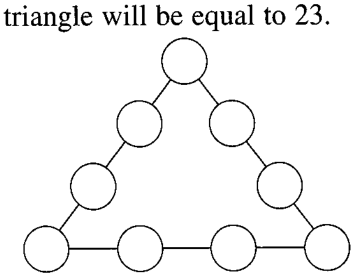
(a) (3 + 3) ÷ 3 – 3 ÷ 3 = 1
(b) 3 3 3 3 3 = 2 **(c)** 3 3 3 3 3 = 3
(d) 3 3 3 3 3 = 4 **(e)** 3 3 3 3 3 = 5
(f) 3 3 3 3 3 = 6 **(g)** 3 3 3 3 3 = 7
(h) 3 3 3 3 3 = 8 **(i)** 3 3 3 3 3 = 9
(j) 3 3 3 3 3 = 10

6. Insert +, –, ×, ÷ and brackets to make the following sentences true. The first one has been done.

(a) (5 + 5) ÷ 5 – 5 ÷ 5 = 1
(b) 5 5 5 5 5 = 2 **(c)** 5 5 5 5 5 = 3
(d) 5 5 5 5 5 = 4 **(e)** 5 5 5 5 5 = 5
(f) 5 5 5 5 5 = 6 **(g)** 5 5 5 5 5 = 7
(h) 5 5 5 5 5 = 8 **(i)** 5 5 5 5 5 = 10

7. Given that △ + □ + ◇ = 9 △ + □ + ○ = 8
△ + ◇ + ○ = 7 □ + ◇ + ○ = 6
Find the value of △, □, ◇ and ○.

8. Fill in the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 so that the sum of the numbers on each side of the triangle will be equal to 23.



9. Use eight 8's to make a total of 1 000.
(Use +, –, ×, ÷ or brackets as you see fit.)

CHAPTER 2

Factors and Multiples

In this chapter, you will learn

- about prime numbers;
- how to find the highest common factor (HCF) and the lowest common multiple (LCM) of two or more numbers;
- how to find squares, square roots, cubes and cube roots of numbers.

Preliminary Problem

In the picture, the alarm clock and the two watches display the same time. Suppose the alarm clock shows the correct time, while the two watches do not. If, for every minute, the watch on the left is slower by 5 seconds, while that on the right is faster by 5 seconds, how many minutes later do you think all three will display the same time again?

In-Class Activity

Work with a partner. You will need a box of toothpicks for the activity.

1. Start with 10 toothpicks. Can you arrange them into groups (more than 1) of equal numbers of toothpicks (more than 1) with no leftovers?
2. Repeat the above with 1, 2, 3, …, 9 toothpicks and also with 11, 12, 13, …, 20 toothpicks.
3. Classify the numbers 1, 2, 3, …, 20 according to how each number can be arranged by copying and completing the following table:

List I	List II	List III
The number 1: 1 group of 1 toothpick or $1 = 1 \times 1$	The number 2: 1 group of 2 toothpicks; 2 groups of 1 toothpick or $2 = 1 \times 2 = 2 \times 1$ The number 3: 1 group of 3 toothpicks; 3 groups of 1 toothpick or $3 = 1 \times 3 = 3 \times 1$	The number 4: 1 group of 4 toothpicks; 4 groups of 1 toothpick; 2 groups of 2 toothpicks or $4 = 1 \times 4 = 4 \times 1 = 2 \times 2$ The number 6: 1 group of 6 toothpicks; 6 groups of 1 toothpick; 2 groups of 3 toothpicks; 3 groups of 2 toothpicks or $6 = 1 \times 6 = 6 \times 1 = 2 \times 3 = 3 \times 2$

Factors and Multiples

From the above activity, we have

$$18 = \mathbf{1} \times 18 = \mathbf{2} \times 9 = \mathbf{3} \times 6 = \mathbf{6} \times 3 = \mathbf{9} \times 2 = \mathbf{18} \times 1.$$

We call each of the numbers 1, 2, 3, 6, 9 and 18 a **factor** of 18. Conversely, we call 18 a **multiple** of each of the numbers 1, 2, 3, 6, 9 and 18. Clearly, when 18 is divided by any one of its factors, the remainder is zero. We say that 18 is **divisible** by 1, 2, 3, 6, 9 and 18.

Example 1

(a) List the factors of 60. *(b) List the multiples of 5.*

Solution

(a) We have $60 = 1 \times 60 = 2 \times 30 = 3 \times 20 = 4 \times 15 = 5 \times 12 = 6 \times 10$.
Thus, the factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.

(b) By multiplying 5 with 1, 2, 3, 4 and so on, we obtain the multiples of 5, i.e. the multiples of 5 are 5, 10, 15, 20, ….

Prime Numbers and Composite Numbers

In Chapter 1, we classified natural numbers into even and odd numbers. In the activity above, we see another way of classifying natural numbers. We classify the numbers according to the number of factors they have.

Did you notice that each of your List II numbers on Page 25 has exactly **two** different factors, 1 and itself? Such natural numbers are called **prime numbers**.

A natural number which has only two different factors, 1 and the number itself, is a prime number.

For example: 2, 3, 5, 7, 11, ...

Each of the List III numbers has more than two different factors. Such natural numbers are known as **composite numbers**.

A natural number which has more than two different factors is a composite number.

For example: 4, 6, 8, 9, 10, ...

The number 1 is neither a prime number nor a composite number. Why?

In-Class Activity

You may work on this activity with a partner.

1. Copy on a piece of paper the numbers from 1 to 100 inclusive and arrange them in 10 rows as shown below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

   (a) Cross out the number 1.
   (b) Circle the number 2 and cross out all the other multiples of 2.
   (c) Circle the number 3 and cross out all the other multiples of 3.
   (d) Circle the number 5 and cross out all the other multiples of 5.
   (e) Circle the number 7 and cross out all the other multiples of 7.
   (f) Continue the process until all numbers are either circled or crossed out.

2. Answer the following questions:
   (a) What is the reason for crossing out the number 1?
   (b) What are the circled numbers?
   (c) What are the numbers crossed out?
   (d) List the number of prime numbers that are less than 100.

(e) What is the largest prime number less than 100?
(f) What are the first 20 composite numbers?
(g) Is every odd number a prime number?
(h) Is every even number a composite number?

The above process of finding all the prime numbers less than a given number is called the **Sieve of Eratosthenes** in honour of a Greek mathematician, Eratosthenes.

3. There is a statement which says that "every even number greater than 2 can be expressed as the sum of two prime numbers". For example, $4 = 2 + 2$, $8 = 3 + 5$ and $12 = 7 + 5$. This statement is called **Goldbach's Conjecture**. It is a conjecture because it has not yet been proven.

For each of the following numbers, verify Goldbach's Conjecture by expressing the number as a sum of two prime numbers:

(a) 16 **(b)** 36 **(c)** 64 **(d)** 98

4. Prime numbers such as 5 and 7 that differ by 2 are called **twin primes**. Mathematicians believe that there is an infinite number of pairs of twin primes. 1 000 000 061 and 1 000 000 063 are twin primes. List five other pairs of twin primes.

Exercise 2a

1. Write down all the factors of each of the following.

(a) 16 **(b)** 28 **(c)** 96
(d) 100 **(e)** 120 **(f)** 210

2. Write down the first six multiples of the following numbers.

(a) 4 **(b)** 7 **(c)** 9
(d) 12 **(e)** 17 **(f)** 21

3. Circle the numbers that have 18 as a factor.

54, 126, 198, 240, 320

4. Underline the numbers which are factors of 144.

1, 2, 3, 4, 8, 9, 12, 14, 16, 32, 48, 144

5. Identify the multiples of 8 from the following numbers.

14, 24, 32, 54, 56, 36, 72, 30, 64, 18, 40, 78, 96, 108, 120

6. State the numbers that have 224 as a multiple in the following.

3, 4, 12, 14, 24, 28, 32, 36, 56

7. Use a calculator to find all the factors of the following numbers.

(a) 480 **(b)** 600 **(c)** 960
(d) 936 **(e)** 1 080 **(f)** 1 200

8. John has 48 orange-flavoured sweets and Susan has 45 lime-flavoured sweets.

(a) John wishes to divide his sweets equally into bags. List all the possible ways he can do this. (For example, he can have 6 bags of 8 sweets.)
(b) Susan also wishes to divide her sweets equally into bags. List all the possible ways she can do this.
(c) Peter, their good friend, suggests that they combine the sweets and divide them equally into bags in such a way that each bag has equal number of orange-flavoured and lime-flavoured sweets. Explain how this can be done.

9. Determine whether each of the following is a prime number or a composite number:

(a) 2 **(b)** 15 **(c)** 17
(d) 21 **(e)** 27 **(f)** 29

10. Name the the next five prime numbers after 30.

11. To test whether a given number is a prime number.

(a) Is 221 a prime number? Use a calculator to divide 221 successively by the prime numbers 2, 3, 5, 7, ... You can stop the process when

(i) a prime number divides 221 exactly. If this happens, then 221 will not be a prime number (e.g. 25 is not a prime number because 5 divides 25 exactly).

(ii) a prime number does not divide 221 exactly and the number in the display is less than the prime number. If this occurs, then 221 will be a prime number (e.g. 13 is a prime number since $13 \div 2 = 6.5 > 2$; $13 \div 3 = 4.3 > 3$; $13 \div 5 = 2.6 < 5$).

(b) Repeat the above procedure to determine whether each of the following is a prime number.

(i) 101	**(ii)** 227	**(iii)** 323
(iv) 997	**(v)** 1 007	**(vi)** 2 761

12. Find two prime numbers whose sum is an odd number. Must one of the numbers be 2?

13. Can the product of two prime numbers be
(a) an odd number;
(b) an even number;
(c) a prime number?

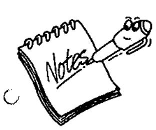
14. 37 and 73 are prime numbers with reversed digits. Name another pair of two-digit prime numbers with reversed digits?

Tests of Divisibility

Is 50 346 divisible by 2? Is 1 776 divisible by 3? Is 123 436 divisible by 4?
Is 17 325 divisible by 5? Is 738 divisible by 9? Are 6 721 and 8 162 divisible by 11?

We can find out whether a number is divisible by another by actually working out the division. However, this can be quite tedious. There are some short-cuts for determining whether one number is divisible by another number. These methods, called **tests of divisibility**, are shown below.

1. A number is divisible by 2 if it is even. 50 346 is even. Hence, it is divisible by 2.
2. A number is divisible by 3 if the sum of the digits is divisible by 3.
   1 776 is divisible by 3 as $1 + 7 + 7 + 6 = 21$ is divisible by 3.
3. A number is divisible by 4 if the number formed by the last two digits is divisible by 4.
   123 436 is divisible by 4 as 36 is divisible by 4.
4. A number is divisible by 5 if the last digit is 0 or 5.
   17 325 is divisible by 5.
5. A number is divisible by 9 if the sum of its digits is divisible by 9.
   738 is divisible by 9 as $7 + 3 + 8 = 18$ is divisible by 9.
6. A number is divisible by 10 if the last digit is 0.
   124 830 is divisible by 10.
7. A number is divisible by 11 if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is equal to 0 or is a multiple of 11.
   6 721 is divisible by 11 as $6 + 2 = 7 + 1$ or $6 - 7 + 2 - 1 = 0$.
   8 162 is divisible by 11 as $(8 + 6) - (1 + 2) = 11$
   or $8 - 1 + 6 - 2 = 11$.



111, 333, 555, 777, 999 are divisible by 3 while 222, 444, 666, 888 are divisible by 6. Why?

A palindromic number is one which reads the same forwards or backwards. Some examples are: 121, 2 332, 1 234 321, etc. Can you show that all palindromic numbers with an even number of digits are divisible by 11?

Example 2

Test each of the following numbers for divisibility by 2, 3, 4, 5, 10 and 11:

(a) 660 *(b) 510* *(c) 639* *(d) 49 610*

Solution

(a) 660 is divisible by 2 since it is even.
660 is divisible by 3 since $6 + 6 + 0 = 12$ is divisible by 3.
660 is divisible by 4 since 60 is divisible by 4.
660 is divisible by 5 and 10 as its last digit is 0.
660 is divisible by 11 as $(6 + 0) - 6 = 0$.
$\therefore$ 660 is divisible by 2, 3, 4, 5, 10 and 11.

(b) 510 is divisible by 2, 3, 5 and 10.

(c) 639 is divisible by 3 as $6 + 3 + 9 = 18$ is divisible by 3.

(d) Since $4 - 9 + 6 - 1 + 0 = 0$ or $(4 + 6 + 0) - (9 + 1) = 0$, 49 610 is divisible by 2, 5, 10 and 11.

Can you give the reasons for the answers to (b), (c) and (d)?

Han Sin, a Chinese general, devised a method to count the number of soldiers he had. First, he ordered his soldiers to form groups of 3 followed by groups of 5 and then groups of 7. In each case, he noted down the remainder. Using the three remainders, he was able to calculate the exact number of soldiers he had without doing the actual counting. Do you know how he did it?

Exercise 2b

1. Which of the following numbers are divisible by 2, 4 or 5?

(a) 10 **(b)** 24 **(c)** 60
(d) 108 **(e)** 135 **(f)** 189
(g) 240 **(h)** 315 **(i)** 648
(j) 756 **(k)** 1 024 **(l)** 2 410

2. Which of the following numbers are divisible by 3, 9 or 11?

(a) 18 **(b)** 72 **(c)** 126
(d) 441 **(e)** 649 **(f)** 825
(g) 1 419 **(h)** 9 372 **(i)** 666 633

3. Which of the following numbers are divisible by 6, 10, 12 or 15?

(a) 552 **(b)** 650 **(c)** 264
(d) 255 **(e)** 420 **(f)** 7 830

4. How would you test a number for divisibility by 30? Are 660, 540, 645 and 610 divisible by 30?

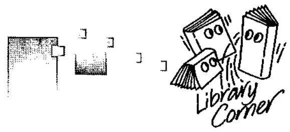
5. Test 4 237, 6 496, 7 770 and 8 514 for divisibility by 14.

6. If a number is divisible by 8, must it also be divisible by 2? By 4? Explain your answer.

7. If a number is divisible by 3, must it also be divisible by 9? Explain your answer.

Prime Factorisation

Every natural number (except 1) is either a prime number or a composite number. A composite number can be expressed as the product of two or more prime numbers, which are called **prime factors**. The process of **decomposition** of a composite number into prime factors is known as **prime factorisation**.



1. The early Greeks called some natural numbers "perfect". A perfect number is one that is equal to the sum of its factors excluding the number itself. For example, the factors of 6 are 1, 2, 3 and 6. We exclude 6 and we have 1 + 2 + 3 = 6. So 6 is a perfect number. Man has sought "perfect numbers" throughout the ages but found very few. Can you look up books in the library for more perfect numbers?

2. If you wish to convey a friendship day greeting to someone to express sincere friendship, you can send a card on which is written something like this:

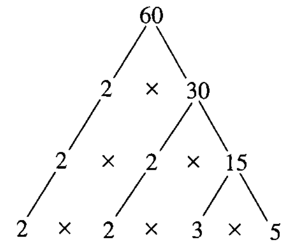
Fandi – 2620
Minghui – 2924

You can explain that 2620 and 2924 are amicable numbers. Find out more about amicable numbers from your library books and tell your friend what they are.

For example, $30 = 2 \times 3 \times 5$
and $60 = 2 \times 2 \times 3 \times 5$.

2, 3 and 5 are called the prime factors of 30. They are also the prime factors of 60. A **factor tree** can be used to express a composite number as a product of its prime factors.

The factor tree illustrates the prime factorisation of 60.



Index Notation

To be brief, we write 5×5 as 5^2 and read it as *5 squared* or *square of 5*.

$5 \times 5 \times 5$ is written as 5^3 and is read as *5 cubed* or *the cube of 5*.

$5 \times 5 \times 5 \times 5$ is written as 5^4 and is read as *5 to the power of 4*.

Similarly, $2 \times 2 \times 2 \times 2 \times 2 \times 2$ is written as 2^6 and is read as *2 is raised to the power of 6*.

In general, $\underbrace{a \times a \times \ldots \times a}_{n \text{ factors}}$ is written as a^n and is read as *a to the power of n*.

This index notation a^n gives us a more precise method of expressing the factors of a number.

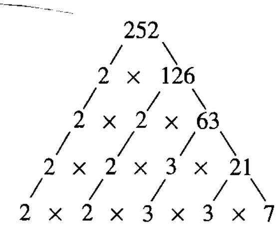
For example,
$12 = 2 \times 2 \times 3$ can be written as $12 = 2^2 \times 3$
$40 = 2 \times 2 \times 2 \times 5$ can be written as $40 = 2^3 \times 5$
$60 = 2 \times 2 \times 3 \times 5$ can be written as $60 = 2^2 \times 3 \times 5$
$72 = 2 \times 2 \times 2 \times 3 \times 3$ can be written as $72 = 2^3 \times 3^2$

Example 3

Express 252 in prime factors.

Solution

Method 1



From the above factor tree,
we have $252 = 2 \times 2 \times 3 \times 3 \times 7$
$= 2^2 \times 3^2 \times 7$ (using index notation)

Method 2

$$\begin{array}{r|r} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

(Start by dividing the number by the smallest prime factor and continue dividing until you get 1 as shown.)

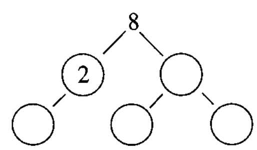
$\therefore \quad 252 = 2 \times 2 \times 3 \times 3 \times 7$
$= 2^2 \times 3^2 \times 7$

Exercise 2c

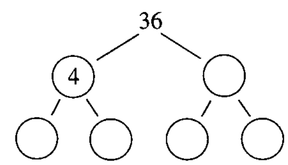
1. Express the following using index notation:
   (a) 7×7 (b) $2 \times 2 \times 5 \times 5$ (c) $3 \times 7 \times 7 \times 7$
   (d) $5 \times 5 \times 11 \times 11 \times 11$ (e) $2 \times 2 \times 13 \times 13 \times 31 \times 2$
   (f) $5 \times 5 \times 5 \times 19 \times 29 \times 19 \times 23 \times 29$

2. Express each of the following as a product of prime factors using index notation:
   (a) 28 (b) 48 (c) 54 (d) 88
   (e) 108 (f) 144 (g) 192 (h) 256

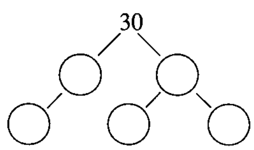
3. Complete the following factor trees:

(a)


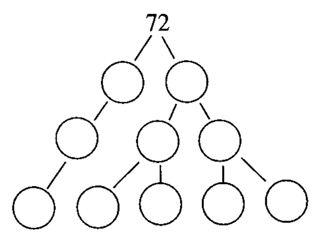
$8 = \_\_ \times \_\_ \times \_\_$

(b)


$36 = \_\_ \times \_\_ \times \_\_ \times \_\_$

(c)


$30 = \_\_ \times \_\_ \times \_\_$

(d)


$72 = \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_$

4. Factorise each of the following into prime factors using the 'factor tree' method:
   (a) 16 (b) 40 (c) 45 (d) 56
   (e) 60 (f) 84 (g) 114 (h) 120

5. Factorise each of the following numbers into prime factors:
   (a) 100 (b) 125 (c) 147 (d) 216
   (e) 225 (f) 360 (g) 567 (h) 648

Highest Common Factor (HCF)

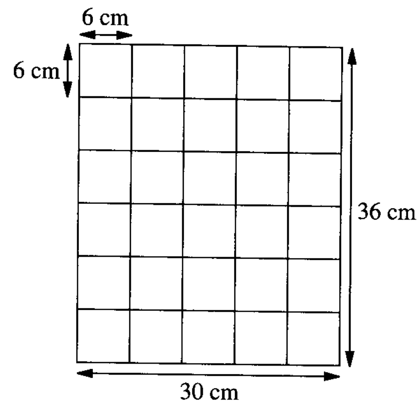
Suppose Mary, an art elective program student, is working on an assignment. She plans to cover a 30 cm by 36 cm sheet of paper completely with identical square patterns. Can you help her to find the side of the largest possible square?

First, consider dividing each side into groups of equal lengths. This is equivalent to finding the factors of 30 and 36. Making a complete list from the smallest to the largest, we have:

The factors of 30 are	1	2	3	5	6	10	15	30	
The factors of 36 are	1	2	3	4	6	9	12	18	36

The product of the ages of a group of teenagers is 705 600. Find the number of teenagers in the group and the sum of their ages.

1, 2, 3 and 6 are **common factors** to 30 and 36, the largest being 6. 6 is called the **Highest Common Factor (HCF)** of 30 and 36. Returning to the above problem, we now know that the side of the largest possible square is 6 cm.



The diagram illustrates that Mary's sheet of paper can be covered completely with 30 squares each of side 6 cm.

Listing all the possible factors of numbers to find the HCF of the numbers as shown above can be tedious. The following provides two alternative methods:

Method 1

The prime factorisation of 30 and 36 is shown below:

$$\begin{array}{rcccccl} 30 = & \boxed{2} & \times & \boxed{3} & \times & \boxed{5} & \\ 36 = & \boxed{2^2} & \times & \boxed{3^2} & \times & \boxed{1} & \\ & \downarrow & & \downarrow & & \downarrow & \\ & 2 & \times & 3 & \times & 1 & \end{array}\left.\vphantom{\begin{array}{c}1\\1\\1\end{array}}\right\} \begin{array}{l}\text{(Use index notation.)}\\ \text{Choose the smaller}\\ \text{number from each set.}\end{array}$$

$\therefore$ the HCF of 30 and 36 is $2 \times 3 \times 1 = 6$.

The HCF of a set of numbers will be less than the numbers or equal to one of the numbers.

Method 2

$$\begin{array}{lc|cc} \text{Step 1} & \boxed{2} & 30 & 36 \\ \hline \text{Step 2} & \boxed{3} & 15 & 18 \\ \hline \text{Step 3} & \downarrow & 5 & 6 \leftarrow \text{1 is their only common factor. Stop here.} \end{array}$$

$$\text{HCF} = \boxed{2 \times 3} = 6$$

In step 1, we know that 30 and 36 are divisible by 2 because they are both even numbers.

In step 2, we divide 30 and 36 by 2 and write the quotients (15 and 18). We know that 15 and 18 are divisible by 3.

In step 3, we divide 15 and 18 by 3 and write the quotients (5 and 6).

Since 5 and 6 have no common factors except 1, we stop our computation.

We multiply the numbers listed on the left side of the above problem: $2 \times 3 = 6$.

$\therefore$ the HCF of 30 and 36 is 6.

Example 4

Find the HCF of 60, 180 and 210.

Solution

$$\begin{array}{lcccccccc}
60 = 2 \times 2 \times 3 \times 5 & = & \boxed{2^2} & \times & \boxed{3} & \times & \boxed{5} & \times & \boxed{1} \\
180 = 2 \times 2 \times 3 \times 3 \times 5 & = & \boxed{2^2} & \times & \boxed{3^2} & \times & \boxed{5} & \times & \boxed{1} \\
210 = 2 \times 3 \times 5 \times 7 & = & \boxed{2} & \times & \boxed{3} & \times & \boxed{5} & \times & \boxed{7} \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 & & 2 & & 3 & & 5 & & 1
\end{array}$$

Choose the smallest number from each set.

$\therefore$ the HCF of 60, 180 and 210 is $2 \times 3 \times 5 \times 1 = 30$.

Alternatively,

$$\begin{array}{c|ccc}
2 & 60 & 180 & 210 \\ \hline
3 & 30 & 90 & 105 \\ \hline
5 & 10 & 30 & 35 \\ \hline
 & 2 & 6 & 7
\end{array}$$

2, 6 and 7 have no common factor except 1. Stop here.

HCF = $\boxed{2 \times 3 \times 5} = 30$

$\therefore$ the HCF of 60, 180 and 210 is 30.

Exercise 2d

1. Find all the common factors of:

(a) 6 and 9 **(b)** 12 and 16
(c) 15 and 18 **(d)** 21 and 28
(e) 27 and 36 **(f)** 30 and 45
(g) 36 and 60 **(h)** 45 and 75

2. Find the HCF of the following:

(a) 12 and 30 **(b)** 12 and 42
(c) 14 and 28 **(d)** 15 and 75
(e) 16 and 40 **(f)** 16 and 48
(g) 20 and 45 **(h)** 21 and 56
(i) 24 and 64 **(j)** 24 and 108
(k) 28 and 56 **(l)** 36 and 243
(m) 45 and 42 **(n)** 90 and 108
(o) 99 and 165 **(p)** 324 and 128

3. Find the HCF of:

(a) 27, 63 and 208 **(b)** 84, 63 and 126
(c) 192, 160 and 96 **(d)** 48, 72 and 132
(e) 112, 64 and 96
(f) 30, 75, 90 and 135
(g) 36, 168, 144 and 252
(h) $2^3 \times 3^2 \times 5$ and $2^2 \times 3^4 \times 5^3$
(i) $2 \times 5^2 \times 7$ and $2^3 \times 3^4 \times 5^3 \times 7^2$

4. James wants to cover a floor measuring 90 cm by 120 cm with square tiles of the same size. Given that he uses only whole tiles, find

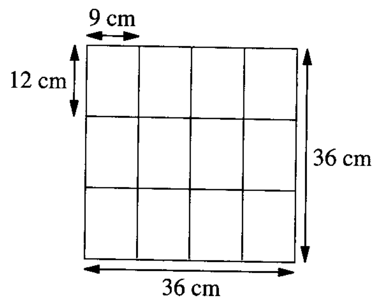
(a) the largest possible length of the side of each tile;
(b) the number of tiles that are needed to cover the floor.

5. Paul has three pieces of rope with lengths of 140 cm, 168 cm and 210 cm. He wishes to cut the three pieces of rope into smaller pieces of equal length with no remainders.

(a) What is the greatest possible length of each of the smaller pieces of rope?
(b) How many of the smaller pieces of rope of equal length can he get?

Least Common Multiple (LCM)

Mary, the art elective program student, is working on a second assignment. She first designs a rectangular pattern measuring 9 cm by 12 cm. She then makes copies of the rectangular pattern. Next she uses the rectangular patterns to form a square. How many rectangular patterns does she need to form the smallest square? What is the length of a side of this square?



Just For Fun

Each of the following represents a prime number. Find out what they are.

$$\begin{array}{r} aa \\ \times\ aaa \\ \hline aaa \\ aaa \\ aaa \\ \hline aaaaa \end{array}$$

The diagram shows that Mary needs 12 rectangular patterns to form the smallest square of side 36 cm.

The length, in cm, of the side of a square that can be formed is a multiple of both 9 and 12.

Can you think why?

Consider the possible multiples of 9 and 12.

The multiples of 9 are	9	18	27	36	45	54	63	72	81	90	99	108	..	..	..
The multiples of 12 are	12	24		36	48	60		72	84	96		108	..	..	..

The first three multiples common to 9 and 12 are 36, 72 and 108.

The smallest of all the common multiples of 9 and 12 is 36 and we call 36 the **Least Common Multiple (LCM)** of 9 and 12.

Returning to our problem, we now know that the side of the smallest square is 36 cm. The number of rectangular patterns needed to create such a square is $4 \times 3 = 12$.

Example 5

Find the LCM of 30 and 36.

Solution

Multiples of 30 = {30, 60, 90, 120, 150, 180, 210, ...}

Multiples of 36 = {36, 72, 108, 144, 180, 216, ...}

$\therefore$ the LCM of 30 and 36 is 180.

A group of people queued up for soccer tickets. If they formed in lines (L) of 2, there is a remainder (R) of 1.

If L is 3, R is 2;
If L is 4, R is 3;
If L is 5, R is 4;
If L is 6, R is 5;
If L is 7, R is 6;
If L is 8, R is 7;
If L is 9, R is 8;
If L is 10, R is 9.

What is the minimum number of people in the queue?

The above method of finding the LCM of two numbers is tedious. The following are two simpler methods:

Method 1

Using prime factorisation, we have

$$\begin{array}{rcccccl} 30 = & \boxed{2} & \times & \boxed{3} & \times & \boxed{5} & \\ 36 = & \boxed{2^2} & \times & \boxed{3^2} & \times & \boxed{1} & \\ & \downarrow & & \downarrow & & \downarrow & \\ & 2^2 & \times & 3^2 & \times & 5 & \end{array} \Bigg\} \text{ Choose the bigger number from each set.}$$

$\therefore$ the LCM of 30 and 36 is $2^2 \times 3^2 \times 5 = 180$.

Method 2

$$\begin{array}{r|rr} 2 & 30 & 36 \\ \hline 3 & 15 & 18 \\ \hline & 5 & 6 \end{array}$$

(Carry out the division as in the case of finding the HCF.)

→ Multiply the numbers on the left with the numbers at the bottom to obtain the LCM.

$\therefore$ the LCM of 30 and 36 is $2 \times 3 \times 5 \times 6 = 180$.

Example 6

Find the LCM of 18, 24 and 36.

Solution

Using prime factorisation, we have

$$\begin{array}{rcccc} 18 = 2 \times 3 \times 3 & = & \boxed{2} & \times & \boxed{3^2} \\ 24 = 2 \times 2 \times 2 \times 3 & = & \boxed{2^3} & \times & \boxed{3} \\ 36 = 2 \times 2 \times 3 \times 3 & = & \boxed{2^2} & \times & \boxed{3^2} \\ & & \downarrow & & \downarrow \\ & & 2^3 & & 3^2 \end{array} \Bigg\} \text{ Choose the largest number from each set.}$$

$\therefore$ the LCM of 18, 24 and 36 is $2^3 \times 3^2 = 72$.

Alternatively, we have

$$\begin{array}{r|rrr} 2 & 18 & 24 & 36 \\ \hline 3 & 9 & 12 & 18 \\ \hline 2 & 3 & \boxed{4} & \boxed{6} \\ \hline 3 & 3 & 2 & 3 \\ \hline & 1 & 2 & 1 \end{array}$$

(4 and 6 have a common factor 2. Divide 4 and 6 by 2.)
(Carry 3 to the next line.)
(Divide the two 3's by 3 and carry 2 to the next line.)
(Stop dividing when any two of the numbers have no common factors except 1.)

$\therefore$ the LCM of 18, 24 and 36 is $2 \times 3 \times 2 \times 3 \times 1 \times 2 \times 1 = 72$.

What do you notice about the LCM of a set of numbers? Can the LCM be smaller than one of the numbers?

Exercise 2e

1. Find the LCM of each of the following pairs of numbers:

(a) 3 and 7	**(b)** 5 and 13	**(c)** 6 and 9	**(d)** 6 and 15
(e) 7 and 21	**(f)** 8 and 12	**(g)** 12 and 9	**(h)** 15 and 25
(i) 24 and 18	**(j)** 30 and 25	**(k)** 65 and 135	**(l)** 81 and 54
(m) 100 and 75	**(n)** 120 and 135	**(o)** 144 and 36	**(p)** 225 and 105
(q) 250 and 125	**(r)** 400 and 160	**(s)** 63 and 490	

2. Find the LCM of:

(a) 6, 9 and 15	**(b)** 3, 12 and 16	**(c)** 8, 9 and 12	**(d)** 14, 18 and 21
(e) 28, 44 and 68	**(f)** 65, 175 and 135	**(g)** 18, 12, 6 and 8	**(h)** 6, 12, 9 and 24

3. Find the LCM of each of the following pairs of numbers:
   (a) $2^2 \times 3^3 \times 5^4$ and $2 \times 3^4 \times 5^3 \times 7$ **(b)** $2^3 \times 3^4 \times 5$ and $2^2 \times 3^3 \times 5^2$
   (c) $2^2 \times 5 \times 7$ and $2^3 \times 3^2 \times 5^2 \times 11$

4. Find the HCF and LCM of each of the following:

(a) 18 and 42	**(b)** 21 and 28	**(c)** 26 and 39
(d) 140 and 210	**(e)** 150 and 45	**(f)** 336 and 224

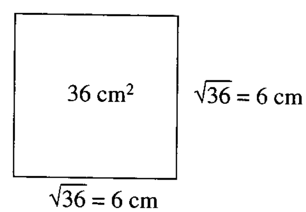
5. Two lighthouses flash their lights every 20 seconds and 30 seconds respectively. Given that they flash together at 8 p.m., when will they next flash together?

6. Three bells toll at intervals of 8 minutes, 15 minutes and 24 minutes respectively. If they toll together at 3 p.m., at what time will they next toll together again?

Squares and Square Roots

The area of a square of side 6 cm is given by

$6 \times 6 = 36$ cm^2.



Therefore 36 is said to be the **square** of 6. For short, we write $6^2 = 36$ and we read 'the square of 6 is 36' or simply '**6 squared is 36**'.

Clearly, to find the side of a square whose area is 36 cm^2 we find a positive number x such that $36 = x \times x$ or x^2.

Now, $x = 6$ and we say that 6 is the positive **square root** of 36 and we write $\sqrt{36} = 6$.

Similarly, $2 \times 2 = 2^2 = 4$ and $\sqrt{4} = 2$

$3 \times 3 = 3^2 = 9$ and $\sqrt{9} = 3$

$4 \times 4 = 4^2 = 16$ and $\sqrt{16} = 4$

$5 \times 5 = 5^2 = 25$ and $\sqrt{25} = 5$

$7 \times 7 = 7^2 = 49$ and $\sqrt{49} = \sqrt{7^2} = 7$

4, 9, 16 and 25 are perfect squares.

We use the symbol $\sqrt{}$ *to denote a square root.*

Notice that 4, 9, 16, 25 and 49 are squares of whole numbers. These numbers are called **perfect squares**.

Find the two-digit number which has the square of the sum of its digits equal to the number obtained by reversing its digits.

In general, if a number y can be expressed as $y = x^2$, we say that y is the square of x and x is a square root of y.

Example 7

Find the positive square root of (a) 784; (b) 2 025.

Solution

Working: Use prime factorisation

(a)
$$\begin{aligned} 784 &= (2 \times 2) \times (2 \times 2) \times (7 \times 7) \\ &= (2 \times 2 \times 7)^2 \\ \sqrt{784} &= \sqrt{(2 \times 2 \times 7)^2} \\ &= \sqrt{28^2} \\ &= 28 \end{aligned}$$

$$\begin{array}{r|r} 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

(b)
$$\begin{aligned} 2\,025 &= 5 \times 5 \times 3 \times 3 \times 3 \times 3 \\ &= (5 \times 3 \times 3)^2 \\ \sqrt{2\,025} &= \sqrt{(5 \times 3 \times 3)^2} \\ &= \sqrt{45^2} \\ &= 45 \end{aligned}$$

$$\begin{array}{r|r} 5 & 2025 \\ \hline 5 & 405 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Example 8

Find the square of (a) 15; (b) 35.

Solution

(a) $15^2 = 15 \times 15$
$= 225$

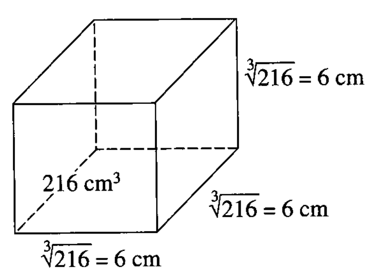
(b) $35^2 = 35 \times 35$
$= 1\,225$

Can you find 55^2 and 95^2 mentally?

Cubes and Cube Roots

The volume of a cube of side 6 cm is given by

$$6 \times 6 \times 6 = 216 \text{ cm}^3.$$



Therefore 216 is said to be the **cube** of 6. In short, we write $6^3 = 216$ and we read 'the cube of 6 is 216' or simply '**6 cubed is 216**'.

Clearly, to find the side of a cube whose volume is 216 cm^3, we find a number x such that $216 = x \times x \times x$ or x^3. Now, $x = 6$ and we say that 6 is the **cube root** of 216 and we write $\sqrt[3]{216} = 6$.

Similarly, $2 \times 2 \times 2 = 2^3 = 8$ and $\sqrt[3]{8} = 2$

$3 \times 3 \times 3 = 3^3 = 27$ and $\sqrt[3]{27} = 3$

$4 \times 4 \times 4 = 4^3 = 64$ and $\sqrt[3]{64} = 4$

$5 \times 5 \times 5 = 5^3 = 125$ and $\sqrt[3]{125} = 5$

We use the symbol $\sqrt[3]{}$ to denote a cube root.

Notice that 8, 27, 64 and 125 are cubes of whole numbers. These numbers are called **perfect cubes**.

In general, if a number y can be expressed as $y = x^3$, we say that y is the cube of x and x is the cube root of y.

Find the two-digit number which has the sum of the cubes of its digits equal to three times itself.

Example 9

Find the cube root of (a) 512; (b) 5 832.

Solution

(a) $512 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
$= 8 \times 8 \times 8$
$= 8^3$

$\therefore \sqrt[3]{512} = \sqrt[3]{8^3} = 8$

Working

$$\begin{array}{r|r} 2 & 512 \\ 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

(b) $5\,832 = (2 \times 3 \times 3) \times (2 \times 3 \times 3) \times (2 \times 3 \times 3)$
$= 18 \times 18 \times 18$
$= 18^3$

$\therefore \sqrt[3]{5\,832} = \sqrt[3]{18^3} = 18$

$$\begin{array}{r|r} 2 & 5\,832 \\ 2 & 2\,916 \\ 2 & 1\,458 \\ 3 & 729 \\ 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

Exercise 2f

1. Find all the perfect squares that are less than 150.
2. Copy the following numbers and circle those that are perfect squares:
   8, 16, 18, 25, 33, 49, 50, 72, 81, 100, 1, 125, 144, 200, 169, 111, 225, 400
3. Fill in the following table.

x	11	12	13	14	15	16	17	18	19	20
x^2										

4. Replace each box with the correct answer:

(a) Given that $8 \times 8 = 64$, then $\sqrt{64} = \square$.

(b) Given that $12 \times 12 = 144$, then $\sqrt{144} = \square$.

(c) Given that $17 \times 17 = 289$, then $\sqrt{289} = \square$.

(d) Given that $(3 \times 4 \times 5) \times (3 \times 4 \times 5) = 3\,600$, then $\sqrt{3\,600} = \square$.

5. Find the square root of each of the following numbers:

(a) 36 **(b)** 81 **(c)** 144 **(d)** 196
(e) 256 **(f)** 324 **(g)** 441 **(h)** 484

6. Use a calculator to factorise the following (into prime numbers) and hence find the square root of each of the following numbers:

(a) 1 156 **(b)** 1 296 **(c)** 1 764
(d) 9 801 **(e)** 11 025 **(f)** 34 596

7. Calculate the cubes of 3, 4, 7, 8, 9 and 10.

8. Replace each box with the correct answer:

(a) Given that $11 \times 11 \times 11 = 1\,331$, then $\sqrt[3]{1\,331} = \square$.

(b) Given that $19 \times 19 \times 19 = 6\,859$, then $\sqrt[3]{6\,859} = \square$.

(c) Given that $13^3 = 2\,197$, then $\sqrt[3]{2\,197} = \square$.

(d) Given that $(3 \times 4) \times (3 \times 4) \times (3 \times 4) = 1\,728$, then $\sqrt[3]{1\,728} = \square$.

(e) Given that $(4 \times 5 \times 9)^3 = 5\,832\,000$, then $\sqrt[3]{5\,832\,000} = \square$.

9. Using a calculator, completely factorise each of the following numbers and hence find its cube root:

(a) 3 375 **(b)** 4 096 **(c)** 13 824 **(d)** 21 952
(e) 46 656 **(f)** 91 125 **(g)** 262 144 **(h)** 373 248

10. Find the area of a square of side 56 cm.

11. What is the length of a side of a square whose area is 2 304 cm^2?

12. What is the volume of a cube of side 11 cm?

13. Given that the volume of a cube is 2 744 cm^3, find the length of its edge.

Mental Estimation

What is the square root of 48? What is the cube root of 65?

48 can be written as $48 = 4 \times 4 \times 3$.
65 can be written as $65 = 5 \times 13$.

Clearly, 48 and 65 cannot be expressed respectively as a^2 and b^3, where a and b are whole numbers. Thus, 48 is not a perfect square and 65 is not a perfect cube. $\sqrt{48}$ and $\sqrt[3]{65}$ are not whole numbers but decimals.

In this section, we are concerned with finding estimates of numbers such as $\sqrt{48}$ and $\sqrt[3]{65}$.

We observe that 48 is close to 49 which is a perfect square.

$\therefore \quad \sqrt{48} \approx \sqrt{49} = 7$ (Is the exact value of $\sqrt{48}$ greater or less than 7?)

The above process can also be done mentally.

Similarly, 65 is close to 64 which is a perfect cube.

$\therefore \quad \sqrt[3]{65} \approx \sqrt[3]{64} = 4$ (Is the exact value of $\sqrt[3]{65}$ greater or less than 4?)

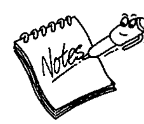
Example 10

Estimate each of the following:

(a) 29^2 *(b)* 19^3 *(c)* 401^2

(d) 104^3 *(e)* $\sqrt{99}$ *(f)* $\sqrt[3]{999}$

Solution



$30^2 = (3 \times 10) \times (3 \times 10)$
$= 3 \times 3 \times 10 \times 10$
$= 9 \times 100$
$= 900$

$20^3 = (2 \times 10) \times (2 \times 10) \times (2 \times 10)$
$= 2 \times 2 \times 2 \times 10 \times 10 \times 10$
$= 8 \times 1\,000$
$= 8\,000$

(a) $29^2 \approx 30^2 = 900$

Is $29^2 < 900$ or > 900?

(b) $19^3 \approx 20^3 = 8\,000$

Is $19^3 < 8\,000$ or $> 8\,000$?

Similarly, (c) and (d) can be worked out in the same way.

(c) $401^2 \approx 400^2 = 160\,000$

Is $401^2 < 160\,000$ or $> 160\,000$?

(d) $104^3 \approx 100^3 = 1\,000\,000$

Is $104^3 < 1\,000\,000$ or $> 1\,000\,000$?

(e) $\sqrt{99} \approx \sqrt{100} = \sqrt{10 \times 10} = 10$

Is $\sqrt{99} < 10$ or > 10?

(f) $\sqrt[3]{999} \approx \sqrt[3]{1\,000} = \sqrt[3]{10 \times 10 \times 10} = 10$

Is $\sqrt[3]{999} < 10$ or > 10?

The Use of Calculators

We can use scientific calculators to find the square, square root, cube and cube root of a number very easily. Below are some function keys for the purpose.

$\boxed{\sqrt{}}$	square root key
$\boxed{x^2}$	square key
$\boxed{y^x}$	power key
$\boxed{\sqrt[x]{}}$	x^{th} root key

To find $\sqrt{25}$, press 25 $\boxed{\sqrt{}}$ and the display gives 5, the square root of 25.

For calculators with Direct Algebraic Logic (DAL), the sequence of pressing the keys is $\boxed{\sqrt{}}$ 25 $\boxed{=}$.

We shall show the sequence following the DAL in all our examples.

Example 11

Use your calculator to evaluate the following:

(a) $14^2 + \sqrt[3]{2\,744} - \sqrt{529}$ *(b)* $\dfrac{\sqrt[3]{729} \times 39^2}{\sqrt{169}}$ *(c)* $\dfrac{\sqrt{65\,536} + 8^3}{11^2 - \sqrt[3]{15\,625}}$

Solution

Sequence of pressing keys:	Final display
(a) 14 $\boxed{x^2}$ $\boxed{+}$ 3 $\boxed{\sqrt[x]{}}$ 2 744 – $\boxed{\sqrt{}}$ 529 $\boxed{=}$	187
(b) 3 $\boxed{\sqrt[x]{}}$ 729 $\boxed{\times}$ 39 $\boxed{x^2}$ ÷ $\boxed{\sqrt{}}$ 169 $\boxed{=}$	1 053
(c) $\boxed{(}$ $\boxed{\sqrt{}}$ 65 536 $\boxed{+}$ 8 $\boxed{x^y}$ 3 $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ 11 $\boxed{x^2}$ $\boxed{-}$ 3 $\boxed{\sqrt[x]{}}$ 15 625 $\boxed{)}$ $\boxed{=}$	8
or	
11 $\boxed{x^2}$ – 3 $\boxed{\sqrt[x]{}}$ 15 625 $\boxed{=}$ $\boxed{\text{STO}}$ $\boxed{\sqrt{}}$ 65 536 $\boxed{+}$ 8 $\boxed{y^x}$ 3 $\boxed{=}$ $\boxed{\div}$ $\boxed{\text{RCL}}$ $\boxed{=}$	8

Note: Some calculators also have the $\boxed{\sqrt[3]{}}$ key for finding the cube root of a number. Try to use this key to work out the above calculations.

Exercise 2g

1. Estimate mentally the following:

   (a) 41^2 (b) 58^2 (c) 112^2 (d) 32^3
   (e) 39^3 (f) 98^3 (g) 198^2 (h) 301^3

2. Give an estimate of each of the following mentally:

   (a) $\sqrt{37}$ (b) $\sqrt[3]{26}$ (c) $\sqrt{63}$
   (d) $\sqrt[3]{124}$ (e) $\sqrt{84}$ (f) $\sqrt{142}$
   (g) $\sqrt[3]{1\,004}$ (h) $\sqrt{897}$

3. Evaluate the following using a calculator:

   (a) 26^2 (b) 37^2 (c) 78^2 (d) 99^2
   (e) 123^2 (f) 13^3 (g) 29^3 (h) 34^3
   (i) 67^3 (j) 109^3 (k) $\sqrt{961}$
   (l) $\sqrt{2\,209}$ (m) $\sqrt{3\,481}$
   (n) $\sqrt{11\,236}$ (o) $\sqrt{69\,169}$
   (p) $\sqrt[3]{4\,096}$ (q) $\sqrt[3]{68\,921}$
   (r) $\sqrt[3]{314\,432}$ (s) $\sqrt[3]{753\,571}$
   (t) $\sqrt[3]{1\,906\,624}$
   (u) $18^2 + 11^3 - \sqrt{484} + \sqrt[3]{4\,913}$
   (v) $\sqrt{676} \times 9^3 - 17^2 + \sqrt{2\,704}$
   (w) $24^3 \div \sqrt{4\,096} + \sqrt[3]{512} \times 44^2$
   (x) $\sqrt[3]{1\,331} \times \sqrt{2\,916} - 42^3 \div 21^2$
   (y) $\dfrac{7^3 \times \sqrt{576} + \sqrt[3]{512}}{\sqrt{7\,744} - 2^3}$

Summary

1. A **prime number** is a natural number which has only two different factors, 1 and the number itself. Prime numbers are 2, 3, 5, 7, 11, 13, 17, etc.
2. A **composite number** is a natural number which has more than two different factors. Composite numbers are 4, 6, 12, 15, 24, 32, etc.
3. A composite number can be expressed as the product of two or more prime numbers.
4. The process of expressing a composite number as the product of prime factors is called **prime factorisation.**
5. Divisibility:
   (a) A number is divisible by 2 if it is even.
   (b) A number is divisible by 3 if the sum of the digits is divisible by 3.
   (c) A number is divisible by 4 if the number formed by the last two digits is divisible by 4.
   (d) A number is divisible by 5 if the last digit is 0 or 5.
   (e) A number is divisible by 9 if the sum of its digits is divisible by 9.
   (f) A number is divisible by 11 if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is equal to 0 or is a multiple of 11.
6. Index notation:
   In general, $\underbrace{a \times a \times \ldots \times a}_{n \text{ factors}}$ is written as a^n and is read as a to power of n.
7. The largest of the factors common to two or more numbers is called the **Highest Common Factor** (HCF) of the numbers.
8. The smallest of the common multiples of two or more numbers is called the **Least Common Multiple** (LCM) of the numbers.

9. If a number y can be expressed as $y = x^2$, we say that y is the **square** of x and x is a **square root** of y.

   If x is a whole number, then y is a **perfect square**.

10. If a number y can be expressed as $y = x^3$, we say that y is the **cube** of x and x is the **cube root** of y.

Review Questions 2

1. State whether each of the following is true or false:
   (a) The prime numbers between 1 and 20 are 2, 3, 5, 7, 9, 11, 13, 17 and 19.
   (b) The HCF of 16, 20, 24 and 32 is 8.
   (c) The smallest number that is divisible by 10, 15 and 20 is 60.
   (d) 1 725 when expressed as a product of prime factors is $23 \times 15 \times 5$.
   (e) 9 996 is divisible by 11 because $9 + 9 + 9 + 6 = 33$ which is divisible by 11.
   (f) The HCF of 4×3^2, 3×4^2 and $2 \times 3 \times 5$ is $4^2 \times 3^2 \times 5$.
   (g) $\sqrt{802\ 500}$ lies between 800 and 900.
   (h) An approximation of 89^2 is 8 100 which is less than the exact value of 89^2.

2. Find the HCF of each of the following:
   (a) 144, 162 **(b)** 12, 18, 30 **(c)** 65, 78, 104 **(d)** 10, 15, 20, 30

3. Find the LCM of each of the following:
   (a) 12, 30 **(b)** 16, 18, 48 **(c)** 42, 63, 105 **(d)** 88, 220, 528

4. How many whole numbers are between each of the following pairs of numbers?
   (a) $\sqrt{7}$ and $\sqrt{80}$ **(b)** $\sqrt[3]{7}$ and $\sqrt[3]{215}$ **(c)** $\sqrt[3]{18}$ and $\sqrt{120}$

5. Evaluate each of the following using a calculator:
   (a) $\sqrt{27^2 + 36^2}$ **(b)** $\sqrt{136^2 - 64^2}$ **(c)** $29^2 + 19^3 + \sqrt{676} - \sqrt[3]{50\ 653}$
   (d) $58^2 \div \sqrt[3]{24\ 389} \times \sqrt{3\ 721} + 33^2$ **(e)** $\dfrac{26^3 \div \sqrt{2\ 704} \times \sqrt[3]{42\ 875}}{25 \times \sqrt{1\ 849} - 14^3 + 40^2 + 79}$.

6. Use your calculator to verify each of the following:
   (a) $718^2 + 1\ 199^2 = 145^2 + 1\ 390^2 = 625^2 + 1\ 250^2 = 1\ 953\ 125$
   (b) $3^2 + 16^2 + 24^2 = 29^2$
   (c) $3^3 + 4^3 + 5^3 = 6^3$
   (d) $55^2 + 56^2 + 57^2 + 58^2 + 59^2 + 60^2 = 61^2 + 62^2 + 63^2 + 64^2 + 65^2$

∗7. John, Peter and Paul were each given a piece of string of equal length. John cut his into equal lengths of 2 m; Peter cut his into equal lengths of 3 m; and Paul cut his into equal lengths of 5 m. If there was no remainder in each case, find the shortest length of string given to each of them.

∗8. What is the shortest length which can be divided into 4 cm, 8 cm or 2 cm portions without remainders?

∗9. Find two numbers if their LCM is 120 and their HCF is 4. (Give three possible answers.)

∗10. Find all the possible values of the digits X and Y if the six-digit number $123X4Y$ is divisible by 4 and 9.

☆**11.** A three-digit number is the product of four prime numbers. Given that the three digits of the number are all prime and different and that the sum of its prime factors is 30, find the number.

Exploration

1. Given the mathematical expression

$$\bigcirc\bigcirc \times \bigcirc\bigcirc = \bigcirc\bigcirc \times \bigcirc\bigcirc\bigcirc = 5\,568,$$

fill the nine circles with the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 to make it true.

2. Given a six-digit number $568XYZ$, find the values of X, Y and Z such that the number is divisible by 3, 4 and 5 and is also the smallest six-digit number starting with 568.

3. 1 and 49 are both perfect squares. Can you find a number

(a) n such that $13^2 + n$ and $13^2 - n$ are both perfect squares;

(b) m such that $41^2 + m$ and $41^2 - m$ are both perfect squares?

Note: $1 = 5^2 - 24$

$49 = 5^2 + 24$

4. There is a two-digit number x. 58 divided by x leaves a remainder of 2, 73 divided by x leaves a remainder of 3 and 85 divided by x leaves a remainder of 1. Find x.

5. Four wires with lengths of 126 cm, 140 cm, 154 cm and 238 cm are to be cut into pieces all of the same length. What is the greatest possible length for the pieces if there should be no wire left?

6. Four racing cars go round a track in 48 seconds, 1 minute, 1 minute 5 seconds and 1 minute 18 seconds respectively. If they start from the same point, how many minutes would have passed before they are side by side again?

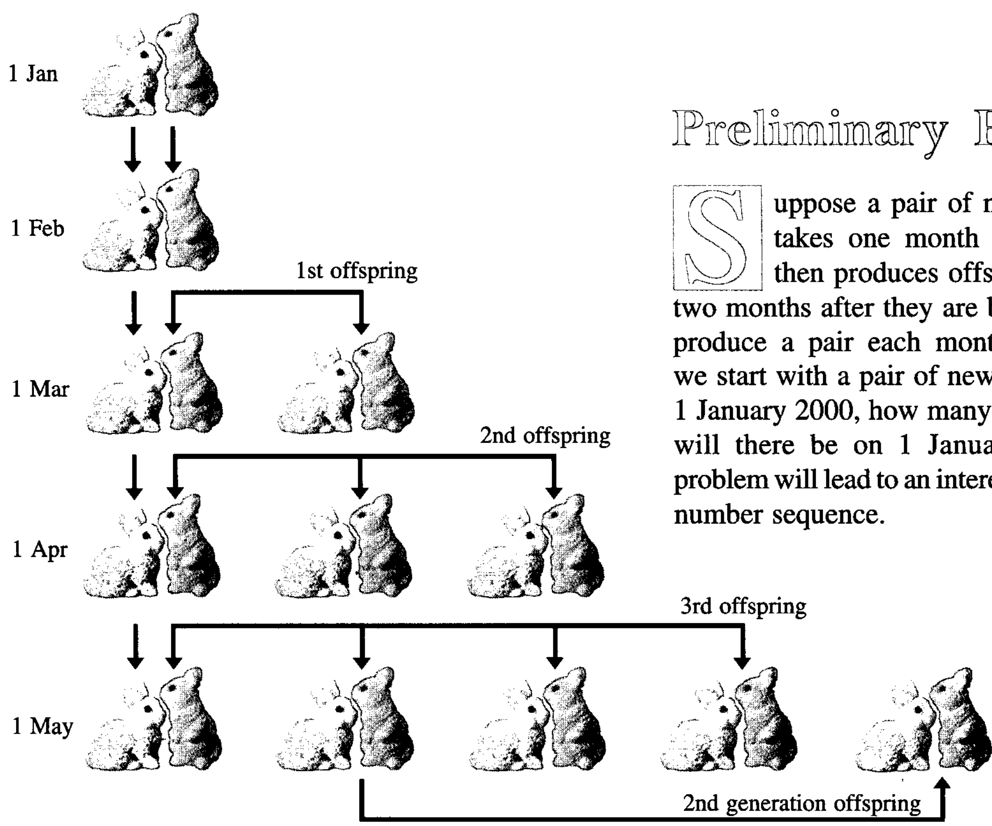
7. Can you explain why the sum of three consecutive whole numbers is always divisible by 3?

CHAPTER 3

Number Sequences and Problem Solving

In this chapter, you will learn

- how to recognise simple patterns from various number sequences;
- to continue a given number sequence using different strategies;
- about problem-solving heuristics.



Preliminary Problem

Suppose a pair of newborn rabbits takes one month to mature, and then produces offsprings in a pair two months after they are born. They then produce a pair each month after that. If we start with a pair of newborn rabbits on 1 January 2000, how many pairs of rabbits will there be on 1 January 2010? This problem will lead to an interesting and useful number sequence.

Number Sequences

Consider the following natural numbers:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...

We know that the numbers which come after 10 are 11, 12, 13, 14 and so on. We are able to continue writing down the numbers because each successive number follows the preceding one according to a specific rule. We say that the natural numbers form a **number sequence**. The numbers in a sequence are the **terms** of the sequence.

For the sequence of natural numbers, the rule is: start with 1, then add 1 to each term to get the next term.

Investigate

Can you continue this sequence?

$1 \times 8 + 1 = 9$
$12 \times 8 + 2 = 98$
$123 \times 8 + 3 = 987$
⋮

How about this one?

$0 \times 9 + 8 = 8$
$9 \times 9 + 7 = 88$
$98 \times 9 + 6 = 888$
⋮

Here are other examples of number sequences:

(a) Sequence of even numbers

2, 4, 6, 8, 10, 12, ...

Rule: Start with 2, then add 2 to each term to get the next term or multiply each term of the sequence 1, 2, 3, 4, 5, 6, ... by 2.

(b) Sequence of odd numbers

1, 3, 5, 7, 9, 11, ...

Rule: Start with 1, then add 2 to each term to get the next term or subtract 1 from each term of the sequence 2, 4, 6, 8, 10, 12, ...

(c) Sequence of powers of 2

1, 2, 4, 8, 16, 32

Rule: Start with 1, then multiply each term by 2 to get the next term.

(d) Sequence of squares

1, 4, 9, 16, 25, 36, ...

or $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, \ldots$

Rule: Square each term of the sequence 1, 2, 3, 4, 5, 6, ...

Try to find the pattern of the sequence below and state the rule for the pattern yourself.

1, 8, 27, 64, 125, 216, ...

Problems

Fill in the two missing numbers in the sequence below:

1, 4, 9, 61, 52, ____, 94, ____, 18, 1, 121, ...

State a rule for this sequence.

Example 1

For each sequence, state a rule and write the next three terms:

(a) 3, 8, 13, 18 ... *(b) 38, 32, 26, 20 ...*
(c) 2, 6, 18, 54 ... *(d) 128, 64, 32, 16 ...*

Solution

(a) **Rule:** Add 5 to each term to get the next term.
The next three terms are 23, 28 and 33.

(b) **Rule:** Subtract 6 from each term to get the next term.
The next three terms are 14, 8 and 2.

(c) **Rule:** Multiply each term by 3 to get the next term.
The next three terms are 162, 486 and 1 458.

(d) **Rule:** Divide each term by 2 to get the next term.
The next three terms are 8, 4 and 2.

Exercise 3a

1. Identify a rule and complete the following number sequences:
   - (a) 2, 5, 8, 11, ___, ___, ___
   - (b) 0, 10, 20, 30, ___, ___, ___
   - (c) 52, 59, 66, 73, ___, ___, ___
   - (d) 80, 72, 64, 56, ___, ___, ___
   - (e) 37, ___, 55, 64, ___, ___
   - (f) 59, ___, 51, 47, ___, ___

2. State a rule and write the next three terms of each sequence.
   - (a) 1, 3, 9, 27, …
   - (b) 6, 12, 24, 48, …
   - (c) 1 600, 800, 400, 200, …
   - (d) 4, 12, 36, 108, …

3. For each of the following sequences, state a rule and write down the next two terms:
   - (a) 14, 19, 24, 29, …
   - (b) 28, 39, 50, 61, …
   - (c) 73, 67, 61, 55, …
   - (d) 99, 90, 81, 72, …
   - (e) 15, 30, 60, 120, …
   - (f) 2 187, 729, 243, 81, …

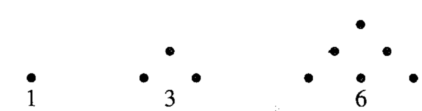
4. Identify a rule and complete the following:
   - (a) 1, 3, 6, 10, ___, ___, ___
   - (b) 17, 22, 27, 32, ___, ___, ___
   - (c) 50, 45, 44, 39, 38, ___, ___, ___
   - (d) 12, 10, 11, 9, ___, ___, ___
   - (e) 2, 5, 10, 13, 26, ___, ___, ___
   - (f) 1, 1, 2, 3, 5, 8, 13, ___, ___, ___

In-Class Activity

Carry out the activity with a partner.

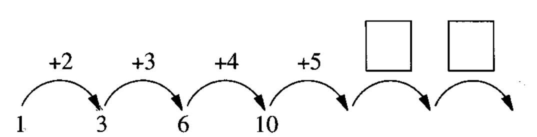
1. Triangular Numbers

(a) Draw a triangular array of dots on a piece of paper with 1 dot in the first row, 2 dots in the second row, 3 dots in the third row, and so on as shown on the right.



(b) The sequence of triangular array of dots corresponds to the number sequence 1, 3, 6, …. Add the next two triangles and then write down the two subsequent terms of the corresponding number sequence.

(c) Copy and complete the pattern below.



(d) These numbers are called **triangular numbers**. State the rule for writing down the numbers.

2. Fibonacci Sequence

(a) The number sequence resulting from the problem described at the beginning of the chapter: 1, 1, 2, 3, 5, … is known as a **Fibonacci sequence**. Write down the next two terms and figure out the rule for generating the next number in the sequence.

(b) Using the rule for generating a Fibonacci sequence, write down the next four terms of the Fibonacci sequence starting with
(i) 0, 1 **(ii)** 4, 2 **(iii)** 3, 3.

(c) The second and the third terms of a Fibonacci sequence are 3 and 10 respectively. Write down the first 8 terms of the sequence.

(d) Consider any three consecutive terms of the Fibonacci sequence 1, 1, 2, 3, 5, ... Multiply the end terms and square the middle term. Does the product differ from the square by 1? Check whether this is true for any other three consecutive terms.

(e) Consider the pattern:

$$1^2 + 1^2 = 1 \times 2$$
$$1^2 + 1^2 + 2^2 = 2 \times 3$$
$$1^2 + 1^2 + 2^2 + 3^2 = 3 \times 5$$
$$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 5 \times 8$$

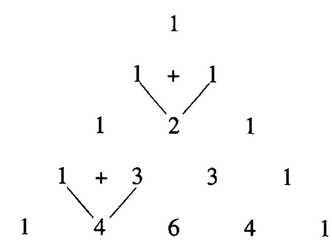
Write down the next three lines.

3. Pascal's Triangle

(a) The following triangle of numbers is called Pascal's triangle.

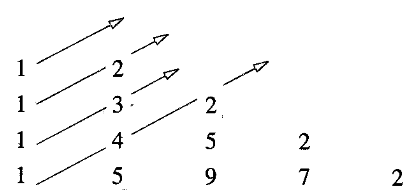
```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
            •
            •
            •
```

Each term is obtained by adding the two terms immediately above. For example, the number 2 in the third row is obtained by adding 1 and 1 which are above. The first 4 in the fifth row is obtained by adding 1 and 3 which are immediately above. (See diagram on the right)

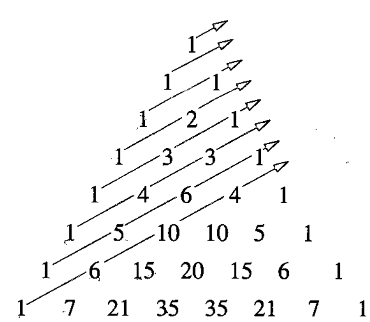


Write down the next two rows of the triangle.

(b) The table of numbers on the right is formed in the same way as the Pascal's triangle.
(i) Write down the next six rows.
(ii) Find the sum of the terms of the upward diagonals.
(iii) Name the sequence of numbers formed by these diagonal sums.
(iv) Write down the next five diagonal sums.



(c) In the diagram on the right, diagonals are drawn starting at each 1.
(i) Find the sum of the terms along the diagonals.
(ii) Do you find the Fibonacci sequence in the Pascal's triangle?



General Term in a Number Sequence

The sequence of even numbers 2, 4, 6, 8, 10, 12, … can be rewritten as

$$2 \times \mathbf{1},\ 2 \times \mathbf{2},\ 2 \times \mathbf{3},\ 2 \times \mathbf{4},\ 2 \times \mathbf{5},\ 2 \times \mathbf{6},\ \ldots,\ 2 \times \boldsymbol{n},\ \ldots.$$

The expression $2 \times n$ or $2n$ is the **formula** for the **nth term** or the **general term** of the number sequence. By varying the values of the letter n in the formula, we obtain corresponding values of the formula $2n$ and thus generating terms of the number sequence. The letter n is called a **variable**.

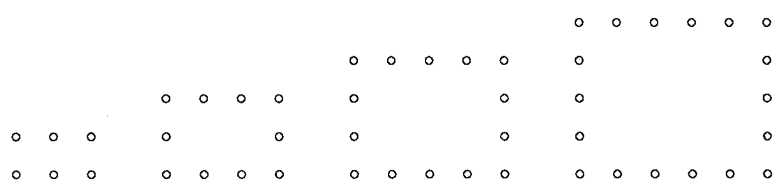
Similarly, the sequence of odd numbers 1, 3, 5, 7, 9, 11, … can be rewritten as

$$2 \times \mathbf{1} - 1,\ 2 \times \mathbf{2} - 1,\ 2 \times \mathbf{3} - 1,\ 2 \times \mathbf{4} - 1,\ 2 \times \mathbf{5} - 1,\ \ldots,\ 2\boldsymbol{n} - 1,\ \ldots.$$

The formula for the nth term of the sequence of odd numbers is $2n - 1$.

Clearly, the formula for the nth term of the number sequence, $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $25 = 5^2$, $36 = 6^2$, … is n^2.

Example 2



1 2 3 4

The diagram shows the first four of a sequence of figures. Figures 1 and 2 contain 6 dots and 10 dots respectively. The sequence continues as shown in Figures 3, 4 and so on. Let n denote the figure number and d the corresponding number of dots.

(a) Count the number of dots in each of the Figures 3 and 4 and write down the next 2 terms of the number sequence 6, 10, ….

(b) Find a formula that connects n and d, i.e., a formula for the nth term of the sequence in (a).

(c) Using the formula in (b), find

(i) the number of dots there will be in Figure 30;

(ii) the numbering of the figure that has 42 dots.

Solution

(a) The next two terms are 14 and 18 respectively.

(b) Notice that
$$\begin{aligned} 6 &= 4 \times \mathbf{1} + 2, && \text{(Figure } \mathbf{1}\text{)} \\ 10 &= 4 \times \mathbf{2} + 2, && \text{(Figure } \mathbf{2}\text{)} \\ 14 &= 4 \times \mathbf{3} + 2, && \text{(Figure } \mathbf{3}\text{)} \\ 18 &= 4 \times \mathbf{4} + 2, && \text{(Figure } \mathbf{4}\text{)} \end{aligned}$$

$\therefore$ for Figure n, the number of dots, $d = 4 \times \boldsymbol{n} + 2 = 4n + 2$.

(c) (i) To find the number of dots there will be in Figure 30, we find the value of d when n has a value 30, i.e. when $n = 30$.

Replacing n by 30 in the formula $d = 4n + 2$, we have

$$d = 4 \times 30 + 2 = 122.$$

$\therefore$ there are 122 dots in Figure 30.

(ii) By writing $42 = 4 \times 10 + 2$ and comparing with the formula $d = 4 \times n + 2$, we have $n = 10$ when $d = 42$.

$\therefore$ the numbering of the figure that has 42 dots is 10.

Example 3

Consider the pattern:

$$\begin{aligned} 2 &= 1 \times 2 \\ 6 &= 2 \times 3 \\ 12 &= 3 \times 4 \\ 20 &= 4 \times 5 \\ &\vdots \\ 110 &= k \times (k + 1) \\ &\vdots \end{aligned}$$

Write down (a) the 8th line in the pattern; (b) the value of k.

Solution

(a) From the pattern, the right-hand side of the 8th line is 8×9.

$\therefore$ the 8th line is $72 = 8 \times 9$.

(b) Since $110 = 10 \times 11 = 10 \times (10 + 1)$,

$\therefore$ $k = 10$.

Exercise 3b

1. Consider the pattern:

$$\begin{aligned} 1 + 3 &= 4 = 2^2 = (1 + 1)^2 \\ 1 + 3 + 5 &= 9 = 3^2 = (2 + 1)^2 \\ 1 + 3 + 5 + 7 &= 16 = 4^2 = (3 + 1)^2 \\ 1 + 3 + 5 + 7 + 9 &= 25 = 5^2 = (4 + 1)^2 \\ &\vdots \\ 1 + 3 + 5 + \dots + a &= b = c^2 = (d + 1)^2 \\ &\vdots \end{aligned}$$

(a) Write down the fifth and sixth lines in the pattern.
(b) Write down the 11th line in the pattern.
(c) Given that $b = 169$, find the values of a, c and d.

2. Consider the pattern:

$$\begin{aligned} 2 + 1^2 &= 3 \\ 2 + 2^2 &= 6 \\ 2 + 3^2 &= 11 \\ 2 + 4^2 &= 18 \\ &\vdots \\ 2 + x^2 &= 66 \\ &\vdots \end{aligned}$$

(a) Write down the 6th line in the pattern.
(b) Find the value of x.

3. Consider the pattern:
$$\begin{aligned}1^2 - 0^2 &= 1 = 1 + 0\\ 2^2 - 1^2 &= 3 = 2 + 1\\ 3^2 - 2^2 &= 5 = 3 + 2\\ 4^2 - 3^2 &= 7 = 4 + 3\\ &\vdots\\ m^2 - n^2 &= 81 = m + n\\ &\vdots\end{aligned}$$

(a) Write down the 10th line in the pattern. **(b)** Find the value of $598^2 - 597^2$.
(c) Find the values of m and n.

4. Consider the pattern:
$$\begin{aligned}\frac{1\times 2}{2} + (1-1)^2 &= 1\\ \frac{2\times 3}{2} + (2-1)^2 &= 4\\ \frac{3\times 4}{2} + (3-1)^2 &= 10\\ \frac{4\times 5}{2} + (4-1)^2 &= 19\\ &\vdots\\ \frac{110}{2} + (p-1)^2 &= q\\ &\vdots\end{aligned}$$

(a) Write down the 11th line in the pattern. **(b)** Find the values of p and q.

5.

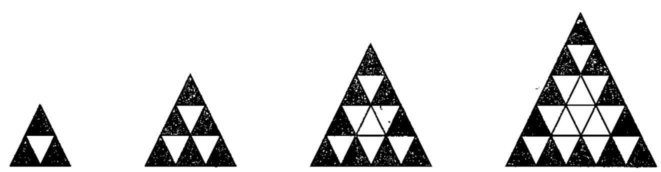


Diagram 1 Diagram 2 Diagram 3 Diagram 4

A sequence of diagrams consisting of shaded and unshaded small triangles is shown above. Diagrams 1 and 2 contain 3 and 6 shaded triangles respectively. The sequence continues as shown in Diagrams 3 and 4 and so on. Let n denote the diagram number and t the corresponding number of shaded triangles.

(a) By counting the number of shaded triangles in each of the Diagrams 3 and 4, write down the next 2 terms of the number sequence 3, 6, ….
(b) Find a formula that connects n and t.
(c) Using the formula in (b), find
  (i) the number of shaded triangles there will be in Diagram 50;
  (ii) the numbering of the diagram that has 87 shaded triangles.

6.

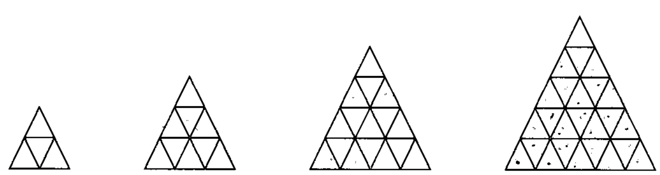


Figure 1 Figure 2 Figure 3 Figure 4

The above shows the first four of a sequence of figures. Figures 1 and 2 contain 4 and 9 small triangles respectively. The sequence continues as shown in Figures 3 and 4 and so on. Let N denote the figure number and T the corresponding number of small triangles.

(a) By counting the number of small triangles in Figures 3 and 4, write down the next 2 terms of the number sequence 4, 9, ….

(b) Find a formula that connects N and T.

(c) Using the formula in (b), find

  (i) the number of small triangles there will be in Figure 9;

  (ii) the numbering of the figure that has 121 small triangles.

Problem Solving

In-Class Activity

So far, in this chapter, you have learnt how to

(i) recognise simple patterns from various number sequences;
(ii) generalise the terms in the number sequences following the pattern;
(iii) associate patterns of figures with sequences of numbers.

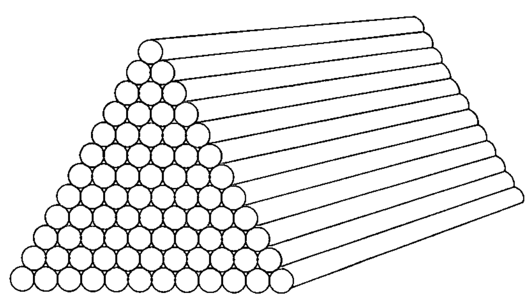
The above are useful for problem solving.

In the following activities, you shall explore various **heuristics** or strategies of solving problems.

You may do the first problem with a partner and the second problem with four others in a group.

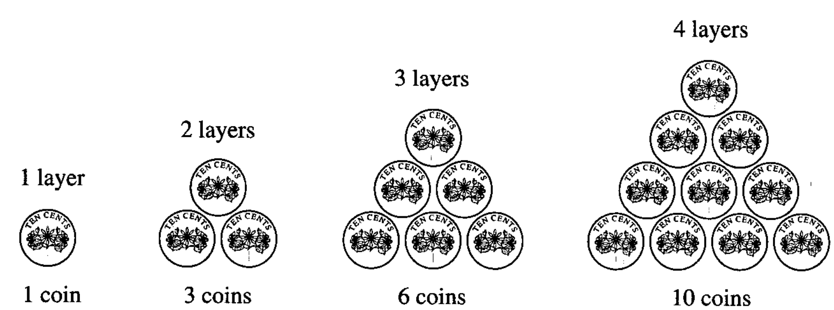
Problem 1

How many logs are there in the stack shown below?



Step 1: **Simplify the problem and use a model**

Use coins to model a different number of layers of logs from the top starting with 1, 2, 3 and 4 layers as shown below.



Step 2: **Look for a pattern**

(a) Use the piles of coins in the figure above to complete the following:

$$\begin{aligned} &1 \text{ layer} = 1 &&= 1 \\ &2 \text{ layers} = 1 + \square &&= 3 \\ &3 \text{ layers} = 1 + 2 + \square &&= 6 \\ &4 \text{ layers} = 1 + \square + 3 + \square &&= 10 \end{aligned}$$

(b) Does the following rule completely describe the sequence?

Rule: The rth term of the sequence is obtained by adding the first r natural numbers so that
the 5th term = $1 + 2 + 3 + 4 + 5 = 15$,
the 6th term = $1 + 2 + 3 + 4 + 5 + 6 = 21$,
the 7th term = $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$, and so on.

(c) Write down the 8th, 9th and 10th terms of the sequence.

(d) How many layers of logs are there in the stack?

(e) Which term of the sequence corresponds to the number of logs in the stack?

(f) Find the number of logs in the stack.

Alternatively:

The sequence of numbers 1, 3, 6, 10 can be written as follows:

$$1 \text{ layer} : \quad 1 = \frac{1 \times 2}{2} = \frac{1 \times (1 + 1)}{2}$$

$$2 \text{ layers} : \quad 3 = \frac{2 \times 3}{2} = \frac{2 \times (2 + 1)}{2}$$

$$3 \text{ layers} : \quad 6 = \frac{3 \times 4}{2} = \frac{3 \times (3 + 1)}{2}$$

$$4 \text{ layers} : \quad 10 = \frac{4 \times 5}{2} = \frac{4 \times (4 + 1)}{2}$$

(a) Do you notice an emerging pattern showing how the number of logs is connected to the number of layers in the stack?

(b) Can you state the rule for the pattern?

(c) Using the rule, find the number of logs when the number of layers in the stack is
(i) 8; **(ii)** 9; **(iii)** 10.

(d) Use the rule to find the number of logs in the stack shown. Does your answer agree with that obtained earlier?

(e) As a challenge, find the number of layers in such a stack of 820 logs.

Problem 2

John invites his Chinese, Malay and Indian friends for a Chinese New Year party. They shake hands to greet one another. Each person shakes hands with every other person exactly once. How many total handshakes are there if there are 20 people including John attending the party?

Step 1: **Simplify the problem and act it out**

Have two of you shake hands. Next, three of you shake hands exactly once with one another. Each time, count the number of handshakes. Repeat this with four of you and five of you respectively.

Step 2: Use tabulation and look for a pattern

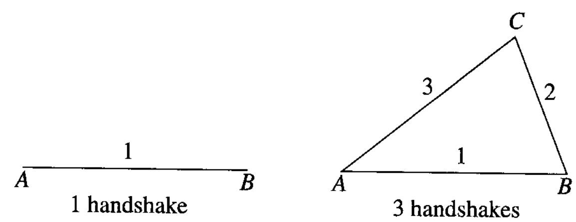
(a) Tabulate your results by copying and completing the table below.

Number of people (n)	**2**	**3**	**4**	**5**
Number of handshakes (N)	$1 = \frac{(2-1)\times 2}{2}$	$3 = \frac{(3-1)\times 3}{2}$		

(b) Write down the formula connecting n, the number of people and N, the number of handshakes.

(c) Use the formula to find how many handshakes there are when 20 people shake hands with one another exactly once.

Alternatively, in **Step 1** above, you can use **diagrams**.
The first two diagrams, involving two people, A and B, and three people, A, B and C, respectively, are shown below.



Add the next two diagrams, involving four people, A, B, C and D, and five people, A, B, C, D and E, respectively.

State a rule and write down the next twenty terms of the following sequence:

1, 3, 6, 10, 15, ...

(a) Multiply each number in the sequence by 8 and add 1. What do you notice?

(b) Choose any two consecutive numbers in the sequence and add them. What do you notice?

(c) Select any two consecutive numbers in the sequence. Square each of them and add. Do you get another number in the same sequence?

More Problem Solving

From the above activities, we see that a problem may be solved in more than one way and in solving the above problems, we have used different strategies or heuristics like:

- simplifying the problem
- using a model
- using tabulation
- acting it out
- looking for a pattern

Some other strategies you will find useful are:

- drawing a diagram
- making an organised list
- writing an equation
- using trial-and-error
- thinking of a related problem
- eliminating the unlikely possibilities
- solving a simpler problem
- changing your point of view
- working backwards
- making a supposition
- solving part of the problem

In the next few examples, we shall consider two of the above problem-solving strategies, i.e., drawing a diagram and changing your point of view. We shall introduce some of the other strategies in later chapters.

STRATEGY: Draw a diagram

Some problems are best approached by drawing diagrams. They help us to have a clearer picture of the problems.

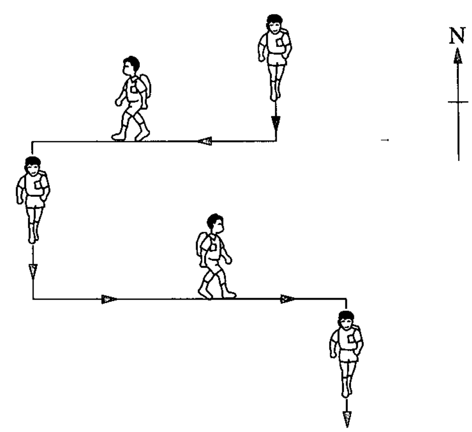
Example 4

A boy scout in a jungle is heading south. He takes a right turn and walks for 40 m. Then he takes a left turn and walks again for a further 50 m. He then takes a left turn and walks for another 45 m. Finally, he takes a right turn. In which direction is he heading now?

Solution

The following diagram illustrates the movement of the boy scout. Notice that the distance which the boy scout walks is not important in arriving at the answer.

The diagram tells us that the boy scout is heading south.

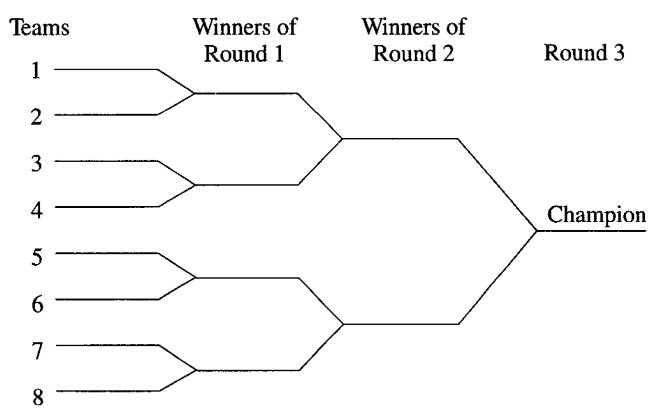


Example 5

Suppose in the Tiger Cup soccer tournament, 8 teams qualify for the final round. If the organisers adopt a single-elimination method for the final round, i.e., winners play against winners until only 1 team is left, what is the total number of games played? How many games would the eventual champion team have played?

Solution

Draw a diagram showing the progress of the tournament as shown below.



From the diagram, the number of games played = 4 + 2 + 1 = 7.

The champion is the only team that is left and therefore must survive each elimination round. From the diagram, there are 3 elimination rounds. Thus the champion team would have to play 3 games.

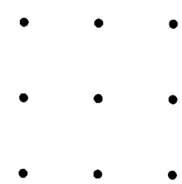
We can also use **logical reasoning** to find the total number of games played. Since 1 team is eliminated in each game played, we need 7 games to eliminate 7 teams to leave 1 champion team.

STRATEGY: **Change your point of view**

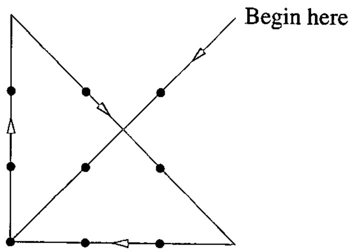
You probably have the experience of failing to solve a problem because of the way you think of the problem or because the method you use does not work. Therefore, if necessary, you must change your point of view so that you can think of more innovative ideas and suggestions.

Example 6

Use only 4 lines to join the 9 points without lifting your pencil.



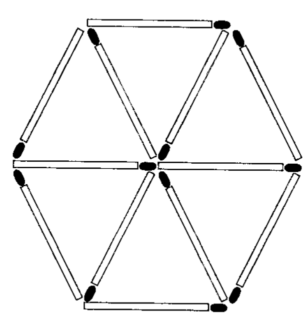
Solution



Many of us may fail when solving this problem because we restrict ourselves to drawing lines within the confines of the 9 points. If we extend our viewpoint, we would realise that we can draw lines beyond the confines of the 9 points as shown. We can then easily solve the problem.

Example 7

The diagram shows 12 matchsticks arranged to form 6 equilateral triangles. Can you rearrange these 12 matchsticks to form 8 equilateral triangles of the same size?



Solution

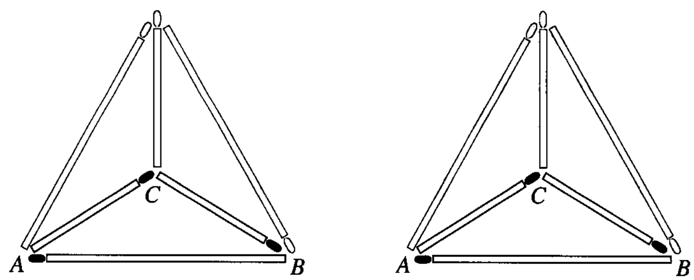
Most of us would probably first attempt to arrange these matchsticks on a flat surface such as the table-top or what mathematicians call a 2-dimensional plane. No matter how hard we try we will not succeed in the task.

There are seven colours in a rainbow: red, orange, yellow, green, blue, indigo and violet. These colours can be remembered as ROYGBIV.

A design consists of a circle divided in half. The top and bottom halves are to be painted with different colours from the seven colours of a rainbow such that the colours must be in the same order as those given above. For example, RO, RB, OI and so on are acceptable, while OR, GR, BY and so on are unacceptable. In how many ways can the circle be painted?

In another design there are three triangles in a row and each triangle is to be painted with a different colour in the same way. In how many ways can the triangles be coloured?

Have you ever thought of arranging these matchsticks in 3-dimension as shown in the diagram below (the matchsticks form a pyramid, with the triangle ABC as its base)? Each 3-dimensional figure consists of 6 matchsticks forming 4 equilateral triangles of the same size. Thus, we have succeeded in forming 8 equilateral triangles of the same size using 12 matchsticks.



From the above examples, we see that when solving problems, be it mathematical, scientific or social, or even everyday life problems, the solutions may not be too difficult to obtain if we can be open-minded and think of all possible options or strategies, such as drawing diagrams or looking at the problems from a totally different point of view.

Exercise 3c

1. How many segments will there be when 49 points divide a given line segment?

(a) Study the diagram below and complete the table that follows.

Figure 1 *Figure 2* *Figure 3* *Figure 4*

No. of points	1	2	3	4	5	6
No. of segments	1 + 1 = 2	2 + 1 = 3				

(b) Hence, write down the number of segments there will be when 49 points divide the given line segment.

(c) How many points are needed to divide a given line segment into 101 segments?

2. 20 people go to a restaurant for a buffet dinner. They request to be seated at the same table. The restaurant has only small square tables that can be joined end to end to form a large long table. If each small square table can seat only one person on each side, how many of such small tables are needed to seat this group of people?

(a) Study the diagram below and complete the tables (i) and (ii) that follow.

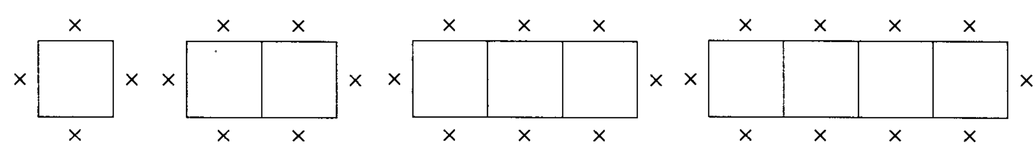


Figure 1 *Figure 2* *Figure 3* *Figure 4*

(i)

No. of people	4	6	8	10	12	14
No. of tables	$\frac{4-2}{2} = 1$	$\frac{6-2}{2} = 2$				

(ii)	No. of tables	1	2	3	4	5	6
	No. of people	2(1)+2=4	2(2)+2=6				

(b) How many tables will be needed to seat
(i) 20 people; **(ii)** 30 people?

(c) How many people can be seated if there are
(i) 22 tables; **(ii)** 36 tables?

3. The diagram shows a line segment, AB, on which 18 points ($P_1, P_2, \ldots, P_{18}$) are marked.

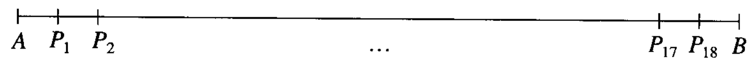


Fig. 3.1

(a) Study the diagram below and complete the table that follows.

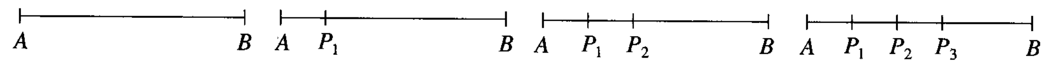


Figure 1 *Figure 2* *Figure 3* *Figure 4*

No. of points on the line segment AB (including the points A and B)	2	3	4	5	6	7
No. of possible line segments	$\frac{2 \times (2-1)}{2} = 1$	$\frac{3 \times (3-1)}{2} = 3$*				

* The three line segments are AP_1, P_1B and AB.

(b) What is the total number of possible line segments in Fig. 3.1?

4. Sixteen players participate in a table-tennis tournament. If the organisers adopt a round robin method, i.e., each player will meet each of the other players once, what will be the total possible number of tournament matches played?

[*Hint:* Model 16 players with 16 points on a line segment so that a line segment joining any two points represents a match played. How many possible triangles are there in Fig. 3.2? Since the triangles have a common vertex, V, the number of possible triangles is the same as the number of possible bases.]

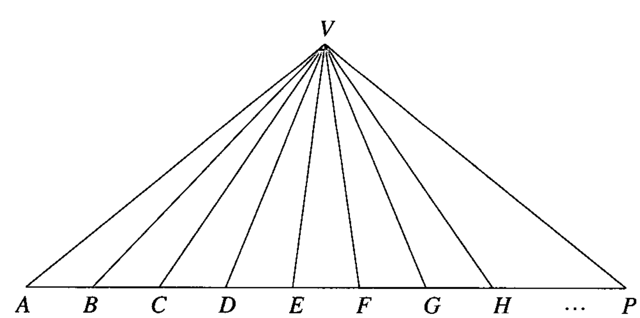


Fig. 3.2

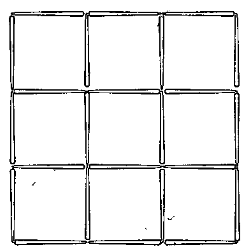
5. A man was trying to swim to a buoy placed at a distance of 200 m out in the sea. It took him 1 min to swim 20 m. Then a wave pushed him back 10 m and he rested for another 1 min before swimming again. He continued in this way for the rest of the journey. How long would it take the man to swim to the buoy?

6. Twenty-seven people entered a chess competition which runs in a single-elimination format. How many tournament games will the champion have to play? What is the total number of tournament games played?

7. Making cuts across the diameter, a saw-mill worker can cut a log into 3 pieces in 3 minutes. How long will it take the worker to cut a log of the same size into 11 pieces?

8. How can you place 15 cows into 4 pens so that there is an odd number of cows in each pen?

9. Remove 8 toothpicks from the following arrangement so that only 2 squares are left.



Summary

1. A **number sequence** is a set of numbers arranged in such a way that each successive number follows the preceding one according to a specific rule. The numbers in a sequence are known as the **terms** of the sequence.

2. Some heuristics for problem solving are:
   - change your point of view
   - make an organised list
   - work backwards
   - make a supposition
   - use trial-and-error
   - think of a related problem
   - simplify the problem
   - eliminate the unlikely possibilities
   - draw a diagram
   - look for a pattern
   - write an equation
   - solve a simpler problem
   - solve part of the problem
   - use tabulation
   - use a model
   - act it out

Review Questions 3

1. State the rule and write down the next three terms in the following sequences:

(a) 1, 7, 13, 19, … **(b)** 12, 15, 21, 30, … **(c)** 0, 2, 2, 4, 6, 10, …
(d) 101, 97, 93, 89, … **(e)** 5, 9, 13, 17, … **(f)** 3, 6, 11, 18, …

2. Fill in each box with an appropriate number. State the rule you used to obtain your answer.

(a) 1, 3, $\square$, 27, 81 **(b)** $\square$, 12, 24, 48, 96 **(c)** 2, 4, 7, 11, $\square$
(d) 60, 55, 54, 49, 48, 43, $\square$ **(e)** 1, 5, 6, 11, 17, $\square$ **(f)** 41, 40, 38, 35, 31, $\square$
(g) 2, 5, 11, 23, 47, $\square$

3. Consider the pattern:

$$\begin{aligned} 11 - 2 &= 3^2 \\ 1\,111 - 22 &= 33^2 \\ 111\,111 - 222 &= 333^2 \\ &\vdots \\ x - y &= 33\,333\,333^2 \end{aligned}$$

(a) Write down the 5th line in the pattern.
(b) Find the values of x and y.

4. **(a)** Consider the pattern:

$$\begin{aligned} 1 &= \frac{1 \times (1+1)}{2} \\ 1 + 2 = 3 &= \frac{2 \times (2+1)}{2} \\ 1 + 2 + 3 = 6 &= \frac{3 \times (3+1)}{2} \\ 1 + 2 + 3 + 4 = 10 &= \frac{4 \times (4+1)}{2} \\ &\vdots \\ 1 + 2 + 3 + 4 + \ldots + k = 45 &= \frac{k \times (k+1)}{2} \\ &\vdots \end{aligned}$$

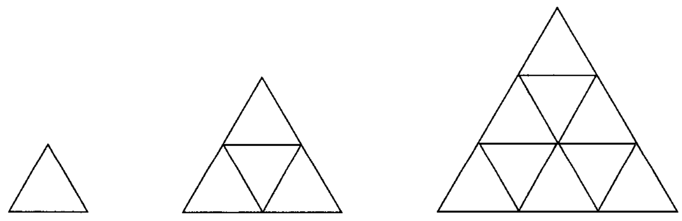
(i) Write down the 7th line in the pattern.
(ii) Find the value of k.

(b) Consider the pattern:

$$\begin{aligned} 1^3 = \quad 1 &= 1^2 \\ 1^3 + 2^3 = 1 + 8 = \quad 9 &= 3^2 \\ 1^3 + 2^3 + 3^3 = 1 + 8 + 27 = \quad 36 &= 6^2 \\ 1^3 + 2^3 + 3^3 + 4^3 = 1 + 8 + 27 + 64 = 100 &= 10^2 \\ &\vdots \\ 1^3 + 2^3 + 3^3 + \ldots + x^3 = 1 + 8 + 27 + \ldots + y = z &= 36^2 \\ &\vdots \end{aligned}$$

(i) Write down the 6th line in the pattern.
(ii) Making use of the pattern in (a), find the values of x, y and z.

★**5.** Using toothpicks, we can form patterns consisting of different layers of triangles as shown in the diagram below.



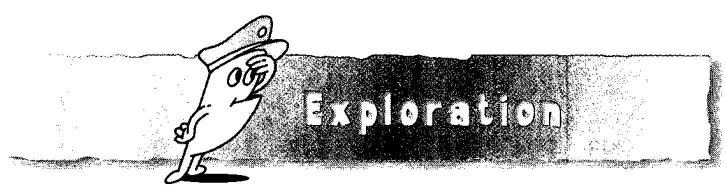
The number of toothpicks used form the following pattern:

Number of layers	Number of toothpicks
1	$1 \times 3 + 3 \times 0$
2	$2 \times 3 + 3 \times 1$
3	$3 \times 3 + 3 \times 3$
4	$4 \times 3 + 3 \times 6$
5	$5 \times 3 + 3 \times 10$
…	…
k	$k \times 3 + 3 \times \frac{k \times (k - 1)}{2}$
…	…

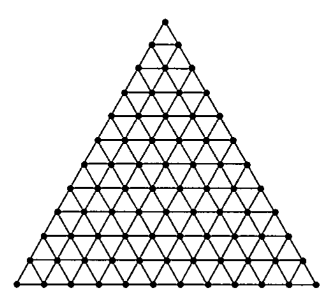
How many toothpicks are needed to form

(a) an 8-layer triangular pattern;

(b) a 20-layer triangular pattern and a 40-layer triangular pattern?



1. Each small triangle in the diagram on the right has three adjacent dots as vertices. Are there more dots than small triangles? Explain your reason.



2. In the diagram below, Figure 1 shows an equilateral triangle. Figure 2 is obtained from Figure 1 by adding new equilateral triangles all round the outside of the equilateral triangle. Figure 3 is obtained from Figure 2, and Figure 4 is obtained from Figure 3 in a similar manner.

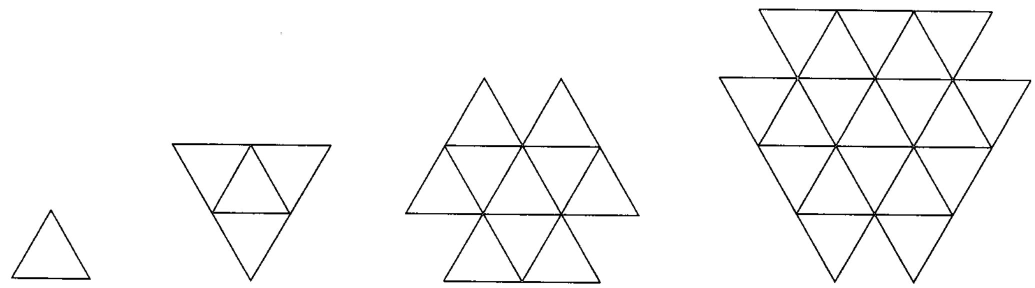


Figure 1 *Figure 2* *Figure 3* *Figure 4*

Show how many small triangles are in Figure 30. How many small triangles will there be in Figure 100?

3. Fill in the next line in the following sequence and explain your answer:

3
1 3
1 1 1 3
3 1 1 3
1 3 2 1 1 3
1 1 1 3 1 2 2 1 1 3
⋮

4. The following array of 800 natural numbers contain many squares of nine numbers. Three of them are shown below.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
793	794	795	796	797	798	799	800

(a) The sum of the numbers in one of these squares is 99. What are the sums of the numbers of the other two squares?
(b) Find the largest possible sum of a square of nine numbers in the above array.
(c) If a square is moved one place to the right, calculate the increase in value of the sum.
(d) If a square is moved one place upwards, calculate the decrease in value of the sum.
(e) Is it possible to find a square of nine numbers in the above array in which the sum is
(i) 1 060;
(ii) 2 871?

Explain your answer.

CHAPTER 4

Fractions and Decimals

In this chapter, you will learn how to

- △ interpret the meanings of fractions and decimals and use them;
- △ convert fractions to decimals and decimals to fractions;
- △ compare and arrange fractions and decimals;
- △ calculate with fractions and decimals, with or without the calculator;
- △ round off decimals to a specific degree of accuracy.

Preliminary Problem

A good picture requires sufficient light to enter the camera and fall on the film. The amount of light is controlled by the size of the aperture (an opening in front of the camera) and the duration it remains open. The aperture-size varies and is indicated by a sequence of numbers: 1.4, 2, 2.8, 4, 5.6, 8, 11, 16 and 22, while the duration follows a sequence of fractions of a second:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{15}, \frac{1}{30}, \frac{1}{60}, \frac{1}{125}, \frac{1}{250}, \frac{1}{500}, \frac{1}{1\,000}, \frac{1}{2\,000} \text{ and } \frac{1}{4\,000}.$$

The picture was taken with an aperture of size 5.6, left opened for $\frac{1}{250}$ of a second. The result is a sharp, frozen image of the moving basketballer. A longer duration of, say, $\frac{1}{60}$ may lead to a blurred picture which may be a deliberate attempt to show action.

When an object is divided into equal parts, each part is called a ***fraction*** *of the object.*

Fractions

If 5 boys were to share a cake equally, we need to cut the cake into 5 equal parts and each boy will receive one part (see Fig. 4.1).

$\frac{1}{5}$ of the cake

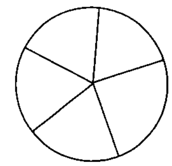


Fig. 4.1

What number expresses the part each boy receives? A whole number? Each part of the cake is called one-fifth of the cake, written as $\frac{1}{5}$. If 3 boys take $\frac{1}{5}$ of the cake each, then we have given away three-fifths of the cake, written as $\frac{3}{5}$. $\frac{1}{5}$ and $\frac{3}{5}$ are called fractions.

A **fraction** is a number written as a quotient, i.e., one number divided by another.

In the fraction $\frac{3}{5}$, **3** is the **numerator** and **5** is the **denominator**.

In the fraction $\frac{5}{8}$, which is the numerator and which is the denominator?

Exercise 4a

For all the exercises in this chapter, a calculator is not allowed unless otherwise stated.

1. Write the following fractions in numerals:

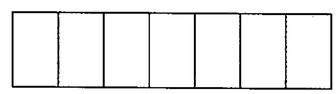
(a) one-sixth **(b)** two-ninths
(c) five-eighths **(d)** six-thirteenths
(e) five-twelfths **(f)** eleven-hundredths

2. Write the following fractions in words:

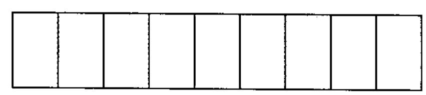
(a) $\frac{1}{9}$ **(b)** $\frac{2}{7}$ **(c)** $\frac{5}{20}$ **(d)** $\frac{35}{100}$

3. What fraction describes how much of each figure is shaded?

(a)

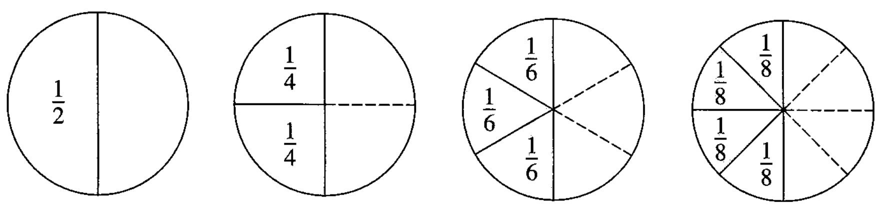


(b)



Equivalent Fractions

Equivalent fractions have the same value. For example, $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.



(a) (b) (c) (d)

Fig. 4.2

Fig. 4.2 shows that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$ and $\frac{4}{8}$ represent the same portion of a whole. $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$ and $\frac{4}{8}$ are called **equivalent fractions** and we have

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}.$$

Can you think of a few more fractions equivalent to $\frac{1}{2}$?

Notice that $\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ $\qquad \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ $\qquad \frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$

Hence, the value of a fraction remains unchanged if both the numerator and the denominator are multiplied or divided by the same number,

i.e. $$\frac{a}{b} = \frac{a \times c}{b \times c} \quad \textbf{and} \quad \frac{a}{b} = \frac{a \div c}{b \div c} \quad \textbf{where } c \neq 0$$

The above rules are useful for the conversion of equivalent fractions.

Simplifying Fractions

The ancient Egyptians were the first to use fractions. However, they only used fractions with a numerator of one.

Thus, they write $\frac{3}{8}$ as $\frac{1}{4} + \frac{1}{8}$, etc.

What do you think the Egyptians would write for the fractions $\frac{3}{5}$, $\frac{9}{20}$, $\frac{2}{3}$ and $\frac{7}{12}$?

We often simplify a fraction by **reducing** it to its **lowest terms**. A fraction in its **lowest terms** has a numerator and a denominator that have no common factor except 1. Thus, reducing a fraction to lowest terms is done by converting it to the simplest fraction equivalent to it.

For example,

$$\frac{120}{375} = \frac{120 \div 5}{375 \div 5} = \frac{24}{75}, \quad \frac{24}{75} = \frac{24 \div 3}{75 \div 3} = \frac{8}{25}.$$

In practice, this is done by **"cancelling"**. Thus,

$$\frac{\overset{\overset{8}{\cancel{24}}}{\cancel{120}}}{\underset{\underset{25}{\cancel{75}}}{\cancel{375}}} = \frac{8}{25}$$ (In "cancelling", we are doing the divisions in our minds.)

When the answer to a mathematical problem is in the form of a fraction, the result is usually expressed in its lowest terms.

Example 7

Reduce $\frac{70}{245}$ to its lowest terms.

Solution

$$\frac{70}{245} = \frac{70 \div 5}{245 \div 5} = \frac{14 \div 7}{49 \div 7} = \frac{2}{7}$$

Working:

$$\frac{\overset{\overset{2}{\cancel{14}}}{\cancel{70}}}{\underset{\underset{7}{\cancel{49}}}{\cancel{245}}} = \frac{2}{7}$$ (Divide first by 5, then by 7.)

Alternatively, using prime factorisation, $\frac{70}{245} = \frac{2 \times \cancel{5} \times \cancel{7}}{\cancel{5} \times \cancel{7} \times 7} = \frac{2}{7}$.

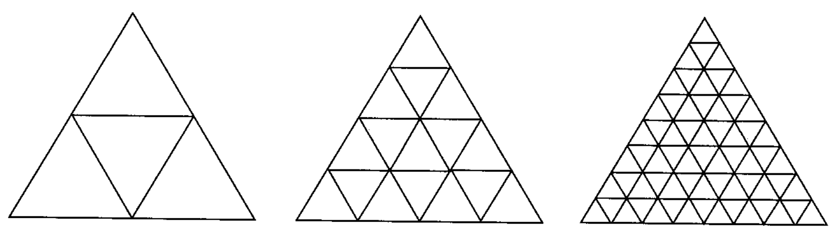
Exercise 4b

1. Copy and complete the following:

(a) $\frac{3}{5} = \frac{\square}{20}$ (b) $\frac{30}{100} = \frac{3}{\square}$ (c) $\frac{4}{13} = \frac{\square}{169}$

(d) $\frac{250}{750} = \frac{1}{\square}$ (e) $\frac{7}{9} = \frac{105}{\square}$ (f) $\frac{17}{9} = \frac{\square}{99}$

2. What set of equivalent fractions is shown by the coloured regions below?



3. Draw a diagram to show that **(a)** $\frac{1}{3} = \frac{3}{9}$ and **(b)** $\frac{2}{5} = \frac{6}{15}$.

4. Copy and complete the following:

(a) $\frac{2}{3} = \frac{8}{\square} = \frac{\square}{27} = \frac{20}{\square}$ (b) $\frac{3}{4} = \frac{\square}{8} = \frac{24}{\square} = \frac{21}{\square}$

5. Reduce the following fractions to their lowest terms:

(a) $\frac{29}{58}$ (b) $\frac{64}{88}$ (c) $\frac{66}{143}$ (d) $\frac{75}{90}$ (e) $\frac{625}{1\,000}$ (f) $\frac{3\,528}{6\,552}$

6. Indicate which of the following pairs of fractions are equivalent:

(a) $\frac{4}{6}$ and $\frac{10}{15}$ (b) $\frac{21}{28}$ and $\frac{12}{16}$ (c) $\frac{20}{48}$ and $\frac{35}{84}$ (d) $\frac{5}{16}$ and $\frac{34}{111}$

7. Reduce the following fractions to their lowest terms:

(a) $\frac{35}{50}$ **(b)** $\frac{26}{39}$ **(c)** $\frac{66}{143}$ **(d)** $\frac{29}{58}$

(e) $\frac{28}{42}$ **(f)** $\frac{84}{300}$ **(g)** $\frac{198}{462}$ **(h)** $\frac{525}{1\,155}$

Proper Fractions, Improper Fractions and Mixed Numbers

In each of the fractions $\frac{3}{4}$, $\frac{7}{9}$ and $\frac{123}{125}$, did you notice that the numerator is less than the denominator? They are examples of **proper fractions**. The fractions we have looked at so far are all proper fractions.

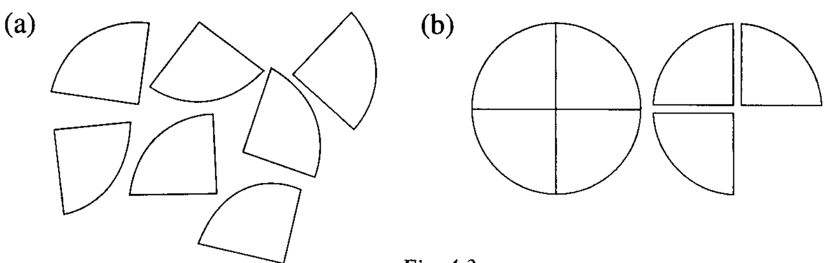


Fig. 4.3

For Your Information

*You know that $5\frac{1}{2}$ is read as 'five and a half': it means $5 + \frac{1}{2}$, but we leave out the addition sign and write what we call a **mixed number**. A mixed number is the sum of a whole number and a fraction.*

Fig. 4.3 illustrates that 7 quarters or $\frac{7}{4}$ can be written as

$$\frac{7}{4} = \left(1 \text{ and } \frac{3}{4}\right) = 1 + \frac{3}{4}.$$

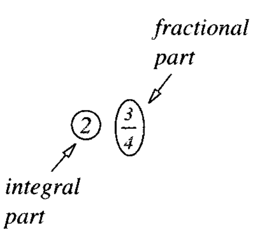
We write $\left(1 \text{ and } \frac{3}{4}\right)$ or $\left(1 + \frac{3}{4}\right)$ as $1\frac{3}{4}$.

$\frac{7}{4}$ is an example of an **improper fraction** which has the numerator *greater than* the denominator.

$\frac{7}{7}$ is also an improper fraction because its numerator is the *same as* the denominator.

$1\frac{3}{4}$ is an example of a **mixed number** which contains an **integral part** and a **fractional part**.

Notes



A mixed number can be expressed in fraction form as an improper fraction. The process of converting a mixed number into an improper fraction is illustrated by the following examples:

(a) $2\frac{3}{4} = \frac{2 \times 4}{4} + \frac{3}{4} = \frac{2 \times 4 + 3}{4} = \frac{11}{4}$

(b) $7\frac{5}{9} = \frac{7 \times 9}{9} + \frac{5}{9} = \frac{7 \times 9 + 5}{9} = \frac{68}{9}$

To express an improper fraction as a mixed number, we divide the numerator by the denominator. The quotient obtained is the integral part and the remainder is the numerator of the fractional part.

For example,

(a)

$$\frac{37}{14} = 2\frac{9}{14} \qquad \begin{array}{r} 2 \text{ (quotient)} \\ 14\overline{)\,37} \\ -28 \\ \hline 9 \text{ (remainder)} \end{array}$$

(b)

$$\frac{123}{7} = 17\frac{4}{7} \qquad \begin{array}{r} 17 \text{ (quotient)} \\ 7\overline{)\,123} \\ -7 \\ \hline 53 \\ -49 \\ \hline 4 \text{ (remainder)} \end{array}$$

Can you explain why the above procedure works?

Order of Fractions

Divide the distance between 0 and 1 into 4 equal parts.

In Chapter 1, we learnt how to represent a whole number on the number line. Can you also represent fractions on the number line?

Fig. 4.4 shows the fractions $\frac{1}{2}$ and $\frac{3}{4}$ on the number line.

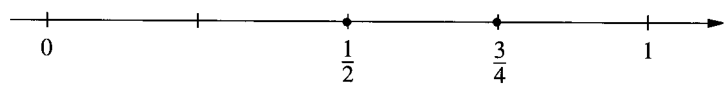


Fig. 4.4

From the number line, we know that $\frac{3}{4}$ is greater than $\frac{1}{2}$.

Where would you mark $\frac{1}{4}$, $\frac{7}{10}$ and $\frac{9}{10}$ on the above number line?

Comparing Fractions

The positions of $\frac{1}{4}$, $\frac{7}{10}$ and $\frac{9}{10}$ on the number line indicate that $\frac{9}{10} > \frac{7}{10} > \frac{1}{4}$. Clearly, $\frac{9}{10} > \frac{7}{10}$ because $9 > 7$. Comparing $\frac{1}{4}$ and $\frac{7}{10}$, two fractions with different denominators, can be done as follows:

$$\frac{7}{10} = \frac{7 \times 2}{10 \times 2} = \frac{14}{20}; \quad \frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20} \quad \text{(20 is the LCM of 4 and 10)}$$

$$\frac{7}{10} > \frac{1}{4} \text{ since } \frac{14}{20} > \frac{5}{20}.$$

In general, if two fractions have the same denominator, then the larger the numerator, the larger the fraction. In contrast, if two fractions have the same numerator, then the larger the denominator, the smaller the fraction.

For two fractions with different denominators or (numerators), we can convert the fraction to an equivalent fraction with the same denominator (or numerator) and then use the above rule to compare them.

Example 2

Arrange the fractions $\frac{5}{13}$, $\frac{3}{13}$, $\frac{7}{13}$ and $\frac{10}{13}$ in descending order.

Notes: *Descending order means from the largest to the smallest.*

Solution

$\frac{10}{13} > \frac{7}{13} > \frac{5}{13} > \frac{3}{13}$ because $10 > 7 > 5 > 3$.

Hence the arrangement of the fractions in descending order is $\frac{10}{13}, \frac{7}{13}, \frac{5}{13}, \frac{3}{13}$.

Example 3

Arrange the fractions $\frac{3}{4}$, $\frac{5}{9}$ and $\frac{7}{12}$ in ascending order.

Solution

The fractions $\frac{3}{4}$, $\frac{5}{9}$ and $\frac{7}{12}$ have different denominators. To compare the fractions, we convert them to equivalent fractions with the same denominator.

The LCM of 4, 9 and 12 is 36.

$$\frac{3}{4} = \frac{3 \times 9}{4 \times 9} = \frac{27}{36}; \quad \frac{5}{9} = \frac{5 \times 4}{9 \times 4} = \frac{20}{36}; \quad \frac{7}{12} = \frac{7 \times 3}{12 \times 3} = \frac{21}{36}.$$

$$\frac{20}{36} < \frac{21}{36} < \frac{27}{36} \quad \text{or} \quad \frac{5}{9} < \frac{7}{12} < \frac{3}{4}.$$

Hence, the arrangement of the fractions in ascending order is $\frac{5}{9}, \frac{7}{12}, \frac{3}{4}$.

Exercise 4c

1. Which of the following are
(i) proper fractions, **(ii)** improper fractions, **(iii)** mixed numbers?

(a) $\frac{4}{5}$ **(b)** $\frac{23}{7}$ **(c)** $\frac{11}{13}$ **(d)** $3\frac{1}{7}$
(e) $\frac{3}{14}$ **(f)** $2\frac{3}{4}$ **(g)** $\frac{9}{8}$ **(h)** $6\frac{4}{5}$

2. Express each of these mixed numbers as improper fractions.

(a) $2\frac{1}{3}$ **(b)** $1\frac{3}{11}$ **(c)** $7\frac{5}{9}$ **(d)** $4\frac{3}{5}$
(e) $5\frac{3}{4}$ **(f)** $3\frac{5}{6}$ **(g)** $2\frac{5}{13}$ **(h)** $14\frac{2}{11}$

3. Express each of these improper fractions as whole or mixed numbers.

(a) $\frac{22}{7}$ **(b)** $\frac{12}{4}$ **(c)** $\frac{35}{6}$ **(d)** $\frac{42}{9}$
(e) $\frac{84}{5}$ **(f)** $\frac{124}{8}$ **(g)** $\frac{111}{11}$ **(h)** $\frac{145}{13}$

4. Which of the two fractions is smaller?

(a) $\frac{4}{10}, \frac{9}{10}$ **(b)** $\frac{1}{16}, \frac{1}{4}$
(c) $\frac{5}{12}, \frac{1}{3}$ **(d)** $\frac{4}{7}, \frac{5}{9}$

5. Which of the three fractions is the largest?

(a) $\frac{1}{7}, \frac{6}{7}, \frac{4}{7}$ **(b)** $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$
(c) $\frac{3}{4}, \frac{7}{8}, \frac{3}{16}$ **(d)** $\frac{4}{15}, \frac{2}{3}, \frac{5}{9}$

6. Peter, the center on the basketball team, is $1\frac{7}{10}$ m tall. His rival for the position is $1\frac{5}{8}$ m tall. Who will more likely be selected for the position if they are equal in all aspects except in height?

7. Mary, Susan and Joan went fishing. Mary caught a sea bass that weighed $2\frac{2}{3}$ kg; Susan caught one that weighed $2\frac{7}{8}$ kg and Joan caught one that weighed $2\frac{3}{4}$ kg. Who caught the heaviest sea bass?

8. Arrange the following fractions in ascending order:
   (a) $\frac{11}{12}, \frac{5}{8}, \frac{3}{4}$ (b) $\frac{2}{3}, \frac{4}{9}, \frac{5}{6}$
   (c) $\frac{1}{3}, \frac{4}{7}, \frac{1}{2}$ (d) $\frac{7}{11}, \frac{5}{6}, \frac{2}{3}$

9. Arrange the following fractions in descending order:
   (a) $\frac{3}{4}, \frac{5}{6}, \frac{5}{9}, \frac{7}{12}$ (b) $\frac{3}{4}, \frac{4}{5}, \frac{7}{10}, \frac{11}{12}$
   (c) $\frac{2}{3}, \frac{5}{12}, \frac{1}{2}, \frac{5}{8}$ (d) $\frac{7}{9}, \frac{5}{6}, \frac{13}{18}, \frac{2}{3}$

Addition and Subtraction of Fractions with Same Denominators

We can only add the numerators together if they have the same denominator.

For example,

$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}.$

Fig. 4.5 shows that $\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$

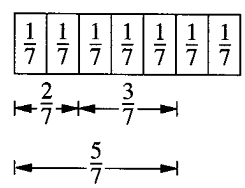


Fig. 4.5

Fig. 4.6 shows that $\frac{6}{7} - \frac{2}{7} = \frac{6-2}{7} = \frac{4}{7}$

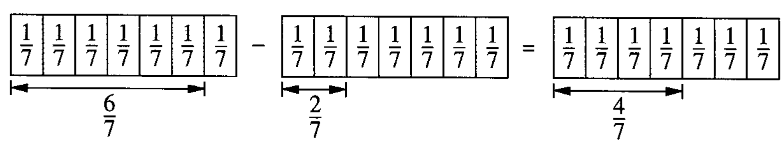


Fig. 4.6

The general rules for the addition and subtraction of fractions with the same denominator are

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad \textbf{and}$$

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}, \text{ where } a, b, c \text{ are whole numbers and } c \neq 0.$$

Addition and Subtraction of Fractions with Different Denominators

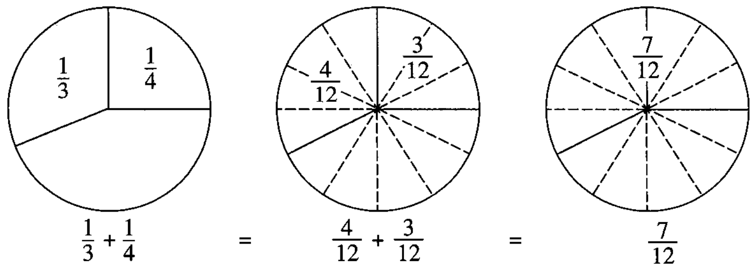


Fig. 4.7

For Your Information

If two fractions have different denominators, we must first change them to equivalent fractions which have a common denominator.

Fig. 4.7 shows $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$.

Notice that $\frac{1}{3}$ and $\frac{1}{4}$ are converted into $\frac{4}{12}$ and $\frac{3}{12}$, i.e., fractions with the same denominator.

Similarly,

$$\frac{4}{9} + \frac{5}{6} = \frac{4 \times 2}{9 \times 2} + \frac{5 \times 3}{6 \times 3}$$
$$= \frac{8}{18} + \frac{15}{18}$$
$$= \frac{8 + 15}{18}$$
$$= \frac{23}{18} = 1\frac{5}{18}$$

$$\frac{7}{8} - \frac{2}{5} = \frac{7 \times 5}{8 \times 5} - \frac{2 \times 8}{5 \times 8}$$
$$= \frac{35}{40} - \frac{16}{40}$$
$$= \frac{35 - 16}{40}$$
$$= \frac{19}{40}$$

Just For Fun

The sum of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of the enrolment of ABC school is exactly the enrolment of XYZ school. The sum of $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$ and $\frac{1}{8}$ of the enrolment of ABC school is exactly the enrolment of PQR school. What are the enrolments of these schools, assuming that no school has more than 1 000 pupils?

Note: The LCM of 9 and 6 is 18. The LCM of 8 and 5 is 40.

In adding or subtracting fractions with different denominators, we must first express the fractions in the same denominator. We always use the LCM of the denominators as the common denominator.

Example 4

Evaluate (a) $\frac{7}{10} + \frac{3}{8}$ *and (b)* $\frac{5}{8} + \frac{3}{4} + \frac{5}{6}$.

Solution

(a) $\frac{7}{10} + \frac{3}{8} = \frac{28}{40} + \frac{15}{40}$ (The LCM of 10 and 8 is 40.)

$= \frac{43}{40}$

$= 1\frac{3}{40}$

(b) $\frac{5}{8} + \frac{3}{4} + \frac{5}{6} = \frac{15}{24} + \frac{18}{24} + \frac{20}{24}$ (The LCM of 8, 4 and 6 is 24.)

$= \frac{15 + 18 + 20}{24}$

$= \frac{53}{24}$

$= 2\frac{5}{24}$

Addition and Subtraction of Mixed Numbers

Example 5

Evaluate (a) $3\frac{8}{15} + 1\frac{5}{8}$; *(b)* $6\frac{1}{6} - 1\frac{3}{4}$.

Solution

(a) $3\frac{8}{15} + 1\frac{5}{8} = (3 + 1) + \left(\frac{8}{15} + \frac{5}{8}\right)$ (Add the whole numbers and fractions separately.)

$= 4 + \frac{64 + 75}{120}$ (The LCM of 15 and 8 is 120.)

$= 4 + \frac{139}{120}$

$= 4 + 1\frac{19}{120}$

$= 5\frac{19}{120}$

(b) $6\frac{1}{6} - 1\frac{3}{4} = (5 - 1) + \left(1\frac{1}{6} - \frac{3}{4}\right)$

$= 4 + \left(\frac{7}{6} - \frac{3}{4}\right)$

$= 4 + \frac{14 - 9}{12}$

$= 4\frac{5}{12}$

Do you know why $6\frac{1}{6}$ is written as $5 + 1\frac{1}{6}$ instead of $6 + \frac{1}{6}$?

(c) $4\frac{7}{10} + \frac{7}{15} - 2\frac{5}{6} = (4 - 2) + \left(\frac{7}{10} + \frac{7}{15} - \frac{5}{6}\right)$

$= 2 + \frac{21 + 14 - 25}{30}$ (The LCM of 10, 15 and 6 is 30.)

$= 2 + \frac{10}{30}$

$= 2\frac{1}{3}$

Exercise 4d

1. Find the values of the following, giving your answers in the simplest form:

(a) $\frac{1}{4} + \frac{2}{4}$ (b) $\frac{14}{20} + \frac{7}{20}$

(c) $\frac{39}{50} + \frac{21}{50} + \frac{15}{50}$ (d) $\frac{3}{14} + \frac{9}{14} + \frac{11}{14}$

2. Evaluate the following, expressing your answers in the simplest form:

(a) $\frac{5}{6} - \frac{4}{6}$ (b) $\frac{5}{8} - \frac{3}{8}$

(c) $\frac{23}{30} - \frac{11}{30} - \frac{7}{30}$ (d) $\frac{37}{49} - \frac{17}{49} - \frac{6}{49}$

3. Calculate the following:

(a) $\frac{2}{7} + \frac{3}{6}$ (b) $\frac{7}{10} + \frac{12}{35}$

(c) $\frac{14}{35} - \frac{3}{10}$ (d) $\frac{19}{30} - \frac{9}{20}$

4. Evaluate the following:

(a) $3\frac{3}{5} + \frac{1}{5}$ (b) $3\frac{1}{6} + 2\frac{5}{9}$ (c) $2\frac{1}{7} + 3\frac{5}{8}$ (d) $6\frac{1}{100} + 3\frac{1}{8}$

(e) $2\frac{7}{100} + 1\frac{3}{40}$ (f) $6\frac{7}{8} - 3\frac{4}{8}$ (g) $7\frac{1}{5} - 3\frac{3}{10}$ (h) $6\frac{2}{3} - 4\frac{1}{4}$

(i) $9\frac{2}{5} - 5\frac{1}{3}$ (j) $6\frac{7}{12} - 3\frac{3}{8}$

5. Evaluate the following:

(a) $\frac{4}{7} + \frac{5}{21} + \frac{11}{42}$ (b) $\frac{5}{8} + \frac{21}{10} - \frac{17}{20}$ (c) $5\frac{27}{28} - 2\frac{3}{7} + \frac{13}{14}$

(d) $4\frac{2}{3} + 1\frac{3}{5} - 1\frac{1}{4}$ (e) $5\frac{3}{4} - 2\frac{5}{6} - 1\frac{8}{15} + 4\frac{7}{20}$ (f) $2\frac{1}{7} + 4\frac{5}{8} - 3\frac{9}{14} + 1\frac{1}{4}$

(g) $4\frac{3}{5} - 1\frac{2}{15} - 2\frac{7}{9} - \frac{2}{45}$ (h) $2\frac{19}{24} - \frac{1}{18} + \frac{61}{72} - \frac{1}{36}$

6. On Thursday, Jean ran $1\frac{2}{3}$ km. On Friday, she ran $2\frac{3}{5}$ km. Find the total distance she ran in two days.

7. John has $9\frac{1}{4}$ litres of paint. After using $4\frac{1}{2}$ litres for painting a room, how much paint has he left?

8. During the school's spring cleaning day, Robin and his classmates spent $1\frac{1}{4}$ hours and $1\frac{1}{12}$ hours cleaning two classrooms assigned to them respectively.
   (a) Find the total time they spent.
   (b) Which classroom took them longer to clean?
   (c) How much longer did they spend in cleaning one classroom than the other?

9. Mary has $6\frac{3}{4}$ cups of flour. She used $2\frac{1}{2}$ cups of flour in one recipe and $2\frac{1}{4}$ cups of flour in another.
   (a) How much flour did she use altogether?
   (b) How much flour has she left?

10. The police conduct regular anti-secret society operations among youths. On a Friday night, the officers spent $\frac{3}{4}$ hour, $1\frac{1}{2}$ hours and $2\frac{1}{4}$ hours checking on youths at three youth hangouts respectively. Find the total time the officers spent on their operation.

Multiplication of a Fraction by a Whole Number

We know that 3×4 can be written as $3 + 3 + 3 + 3 = 12$.

Similarly, $\frac{3}{5} \times 4$ can be written as $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{3+3+3+3}{5} = \frac{3 \times 4}{5}$

$= \frac{12}{5} = 2\frac{2}{5}$ (see Fig. 4.8)

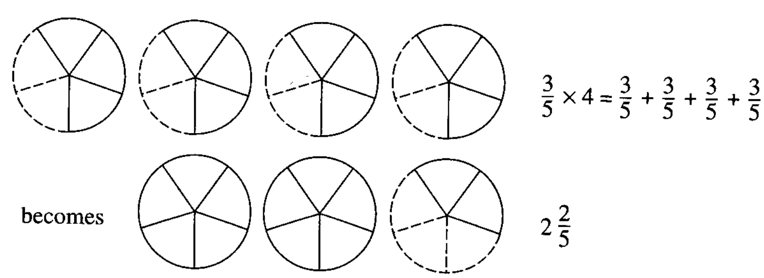


Fig. 4.8

This leads us to a rule for the multiplication of a fraction by a whole number.

$$\frac{a}{b} \times c = \frac{a \times c}{b}, \text{ where } a, b, c \text{ are whole numbers and } b \neq 0.$$

Example 6

John ordered 100 rewritable compact discs and 300 recordable compact discs for his computer store. His supplier delivered only $\frac{3}{4}$ of the rewritable compact discs and $\frac{3}{5}$ of the recordable compact discs he ordered. How many compact discs did he receive altogether?

Solution

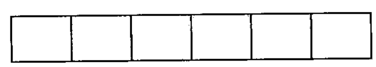
$\frac{3}{4}$ of $100 = \frac{3}{4} \times 100 = 75$; $\frac{3}{5}$ of $300 = \frac{3}{5} \times 300 = 180$.

Therefore, he received altogether $75 + 180 = 255$ compact discs.

Multiplication of Fractions

Mr. Lee has two bookstores. In October, his book distributor sent him only $\frac{5}{6}$ of an order for a school textbook, which he had expected to distribute equally between his two stores. What fraction of his original order will each store receive?

Each store will get $\frac{1}{2}$ of the books received, but the books received make up of only $\frac{5}{6}$ of the original order.



$\frac{5}{6}$ of the original order

Fig. 4.9 (i)

Each store will receive $\frac{1}{2}$ of $\frac{5}{6}$ of the original order. Fig. 4.9 illustrates that

$$\frac{1}{2} \text{ of } \frac{5}{6} = \frac{1}{2} \times \frac{5}{6} = \frac{1 \times 5}{2 \times 6} = \frac{5}{12}.$$

$\frac{1}{2}$ of $\frac{5}{6}$ of the original order $= \frac{5}{12}$ of the original order

(5 triangles out of a total of 12)

Fig. 4.9 (ii)

This leads us to a rule for the multiplication of fractions.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}, \text{ where } a, b, c, d \text{ are whole numbers and } b \neq 0, d \neq 0.$$

Where $d = 1$, we have $\frac{a}{b} \times c = \frac{a \times c}{b}$, which is the rule for the multiplication of a fraction and a whole number discussed earlier.

Example 7

Evaluate (a) $\frac{3}{8} \times \frac{5}{7}$ *and (b)* $2\frac{2}{3} \times \frac{5}{14}$.

Solution

(a) $\frac{3}{8} \times \frac{5}{7} = \frac{3 \times 5}{8 \times 7}$
$= \frac{15}{56}$

(b) $2\frac{2}{3} \times \frac{5}{14} = \frac{8}{3} \times \frac{5}{14}$
$= \frac{8 \times 5}{3 \times 14}$
$= \frac{40}{42} = \frac{20}{21}$

Note: Mixed numbers must be changed to improper fractions before multiplication. Sometimes, "cancellations" can be done before multiplication. This alternative method for (b) is shown below:

$$2\frac{2}{3} \times \frac{5}{14} = \frac{\cancel{8}^{4}}{3} \times \frac{5}{\cancel{14}_{7}}$$
$$= \frac{4 \times 5}{3 \times 7} = \frac{20}{21}$$

Example 8

Calculate (a) $\frac{5}{7} \times \frac{8}{15} \times \frac{14}{3}$ *and (b)* $2\frac{1}{3} \times 3\frac{3}{7} \times \frac{3}{4}$.

Solution

(a) $\frac{{}^{1}\cancel{5}}{{}_{1}\cancel{7}} \times \frac{8}{\cancel{15}_{3}} \times \frac{\cancel{14}^{2}}{3} = \frac{1 \times 8 \times 2}{1 \times 3 \times 3}$
$= \frac{16}{9} = 1\frac{7}{9}$

(b) $2\frac{1}{3} \times 3\frac{3}{7} \times \frac{3}{4} = \frac{{}^{1}\cancel{7}}{{}_{1}\cancel{3}} \times \frac{\cancel{24}^{6}}{\cancel{7}^{1}} \times \frac{\cancel{3}^{1}}{\cancel{4}^{1}}$
$= 6$

Division of a Fraction by a Whole Number

Let us consider, from another point of view, the fraction of Mr Lee's original order each of his two stores will receive.

Dividing $\frac{5}{6}$ of his original order by two, we have $\frac{5}{6} \div 2$.

From Page 75, we have found that each store will receive $\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$.

Thus, $\frac{5}{6} \div 2 = \frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$. $\left(\text{By commutative law, } \frac{1}{2} \times \frac{5}{6} = \frac{5}{6} \times \frac{1}{2}\right)$

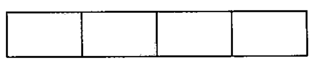
This leads us to a rule for the division of a fraction by a whole number.

$\frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c} = \frac{a}{b \times c}$, where a, b, c are whole numbers and $b \neq 0, c \neq 0$.

Division of a Fraction by Another Fraction

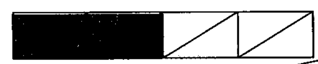
On another occasion, Mr. Lee's distributor delivered only $\frac{3}{4}$ of his original order. He satisfies one of his two stores first by giving the store the expected quantities, which make up of half of what he ordered. What fraction of the books delivered will this store receive?

(a) Method 1



$\frac{3}{4}$ of the original order

Fig. 4.10 (i)



4 triangles out of 6 triangles

$\frac{1}{2}$ of the original order $= \frac{4}{6}$ of the books delivered

$= \frac{2}{3}$ of the books delivered

Fig. 4.10 (ii)

Hence, the store will receive $\frac{2}{3}$ of the books delivered.

(b) Method 2

Now, consider

$$? \times \frac{3}{4} = \frac{1}{2}.$$

Thus, $? = \frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$.

However, we also have $\frac{1}{2} \times \frac{4}{3} = \frac{1 \times 4}{2 \times 3} = \frac{4}{6} = \frac{2}{3}$.

Therefore, $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3}$.

This leads to a rule for dividing a fraction by another fraction.

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$, where a, b, c, d are whole numbers and $b \neq 0, c \neq 0, d \neq 0$.

Example

Evaluate (a) $3\frac{1}{2} \div \frac{3}{4}$; (b) $\frac{24}{55} \div \frac{8}{11}$; (c) $1\frac{4}{13} \times 7\frac{4}{5} \div 11\frac{1}{3}$.

Solution

(a) $3\frac{1}{2} \div \frac{3}{4} = \frac{7}{2} \div \frac{3}{4}$

$= \frac{7}{2} \times \frac{4}{3} = \frac{14}{3} = 4\frac{2}{3}$

(b) $\frac{24}{55} \div \frac{8}{11} = \frac{24}{55} \times \frac{11}{8}$

$= \frac{3}{5}$

(c) $1\frac{4}{13} \times 7\frac{4}{5} \div 11\frac{1}{3} = \frac{17}{13} \times \frac{39}{5} \div \frac{34}{3}$

$= \frac{17}{13} \times \frac{39}{5} \times \frac{3}{34}$

$= \frac{9}{10}$

Note: In the above procedure to express each mixed number as an improper fraction, we changed '÷' to '×' and inverted the divisor.

$\frac{1}{3}$ of a number of animals are cows, $\frac{1}{4}$ are sheep, $\frac{1}{5}$ are horses, $\frac{1}{6}$ are deer and 4 are dogs. How many animals are there altogether?

Exercise 4e

1. Calculate the following:

(a) $20 \times \frac{4}{5}$ **(b)** $\frac{5}{6} \times 8$

(c) $\frac{2}{3} \times \frac{5}{7}$ **(d)** $\frac{4}{13} \times 3\frac{1}{4}$

(e) $10 \div \frac{2}{3}$ **(f)** $\frac{9}{28} \div \frac{6}{7}$

(g) $1\frac{5}{7} \div \frac{4}{21}$ **(h)** $8\frac{2}{7} \div 3\frac{9}{10}$

2. Evaluate the following, expressing your answers in the simplest form:

(a) $3\frac{1}{2} \times 4\frac{4}{5} \times \frac{5}{14}$

(b) $2\frac{1}{7} \times 1\frac{3}{46} \times 1\frac{5}{18} \times \frac{5}{7}$

(c) $5\frac{1}{4} \div 2\frac{4}{5} \div 1\frac{7}{9}$

(d) $3\frac{1}{9} \times 3\frac{3}{5} \div 2\frac{1}{10}$

(e) $\frac{8}{18} \times \frac{15}{20} \div \frac{24}{15} \times \frac{42}{35}$

3. Find the following:

(a) $\frac{1}{5}$ of 20 pupils

(b) $\frac{3}{5}$ of 15 oranges

(c) $\frac{4}{7}$ of 56 km

(d) $\frac{5}{8}$ of 24 hours

(e) $\frac{2}{9}$ of 36 kg

4. A school has $6\frac{3}{4}$ kg of detergent in stock. During the 'Use Your Hands' campaign, each class will be given $\frac{3}{8}$ kg of detergent. There are 28 classes in the school.

(a) What fraction of the school will be supplied with the detergent in stock?
(b) How much detergent will be required altogether for the whole school?
(c) How much more detergent does the school need to order?
(d) If the school gives out the detergent in stock to the 15 lower secondary classes first,
  (i) how much detergent will be given out;
  (ii) how much detergent in stock will be left?

5. (a) Last year Peter spent a total of $8\frac{1}{6}$ hours on community service. Visits to old folks' homes made up $\frac{4}{7}$ of the total time. How much time did he spend in visiting old folks' homes?

(b) This year Peter plans to spend $1\frac{1}{5}$ of his time spent last year on community service.
  (i) Find the time he will spend on community service.
  (ii) How much more time will he spend this year than last year?

Arithmetical Operations on Fractions

Keep in mind the following rules when doing arithmetic operations on fractions.

1. When an expression contains brackets, simplify the expression within the brackets first.
2. When an expression contains brackets within brackets, simplify the expression within the innermost pair of brackets first.
3. When an expression contains only additions and subtractions, work from left to right.
4. When an expression contains only multiplications and divisions, work from left to right.
5. When an expression contains addition, subtraction, multiplication and division, do multiplication and division before addition and subtraction.

Note: The above rules are the same as rules for whole numbers.

Can you spot the errors in each of the following?

(a) $2 = \frac{8}{4} = \frac{8}{2+2}$
$= \frac{8}{2} + \frac{8}{2} = 4 + 4 = 8$

(b) $\frac{3+4}{3} = \frac{3}{3} + 4$
$= 1 + 4 = 5$

(c) $\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15}$
$= \frac{10+12}{15+15} = \frac{22}{30} = \frac{11}{15}$

Example 10

Evaluate the following:

(a) $\left(\frac{1}{2} + \frac{1}{3}\right) \times \frac{1}{4}$

(b) $\frac{1}{3} \times \left(\frac{1}{2} + \frac{1}{4}\right) \div \frac{1}{6}$

(c) $\left(\frac{3}{4} \times \frac{1}{3} - \frac{1}{12}\right) \div \frac{1}{2}$

(d) $5\frac{1}{3} \times 4\frac{1}{2} - 3\frac{1}{4} \times \frac{8}{13}$

Solution

(a) $\left(\frac{1}{2} + \frac{1}{3}\right) \times \frac{1}{4}$ (Calculate the expression within the brackets first.)

$= \left(\frac{3+2}{6}\right) \times \frac{1}{4}$

$= \frac{5}{6} \times \frac{1}{4}$

$= \frac{5}{24}$

(b) $\frac{1}{3} \times \left(\frac{1}{2} + \frac{1}{4}\right) \div \frac{1}{6}$ (Calculate the expression within the brackets first. Work from left to right.)

$= \frac{1}{3} \times \left(\frac{2+1}{4}\right) \div \frac{1}{6}$

$= \frac{1}{3} \times \frac{3}{4} \times \frac{6}{1}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

(c) $\left(\frac{3}{4} \times \frac{1}{3} - \frac{1}{12}\right) \div \frac{1}{2}$ (Within brackets, do multiplication before subtraction.)

$= \left(\frac{1}{4} - \frac{1}{12}\right) \div \frac{1}{2}$

$= \left(\frac{3-1}{12}\right) \times \frac{2}{1}$

$= \frac{2}{12} \times 2$

$= \frac{1}{3}$

(d) $5\frac{1}{3} \times 4\frac{1}{2} - 3\frac{1}{4} \times \frac{8}{13}$ (Do multiplication before subtraction.)

$= \frac{16}{3} \times \frac{9}{2} - \frac{13}{4} \times \frac{8}{13}$

$= 24 - 2$

$= 22$

Exercise 4f

1. Evaluate the following:

(a) $\frac{1}{3} \times \left(\frac{1}{3} + \frac{1}{4}\right)$ **(b)** $\left(\frac{3}{4} - \frac{1}{2}\right) \times \frac{2}{3}$ **(c)** $\left(1\frac{3}{4} - \frac{1}{2}\right) \div \frac{1}{3}$

(d) $\left(\frac{4}{5} + \frac{1}{3}\right) \div \frac{2}{3}$ **(e)** $3\frac{3}{4} \times \left(4\frac{1}{5} - 2\frac{5}{9}\right)$ **(f)** $3\frac{3}{4} \div \left(2\frac{1}{3} - \frac{1}{4}\right)$

2. Find the values of the following, expressing your answers in the simplest form:

(a) $\frac{1}{3} \times \left(\frac{1}{3} + \frac{3}{4}\right) \div \frac{5}{2}$ **(b)** $\frac{1}{3} \times \left(\frac{1}{4} - \frac{1}{12} + \frac{1}{2}\right)$ **(c)** $\left(\frac{1}{2} + \frac{1}{3}\right) \div \left(\frac{2}{3} \times \frac{1}{8}\right)$

(d) $1\frac{3}{4} \times \left(\frac{4}{9} + \frac{2}{3}\right) \times \left(1\frac{1}{5} - \frac{1}{2}\right)$ **(e)** $\frac{2}{3} \times \frac{1}{4} - \frac{1}{12} \div \frac{1}{2}$ **(f)** $5\frac{1}{3} \times 4\frac{1}{2} - 3\frac{1}{4} \times 1\frac{5}{6}$

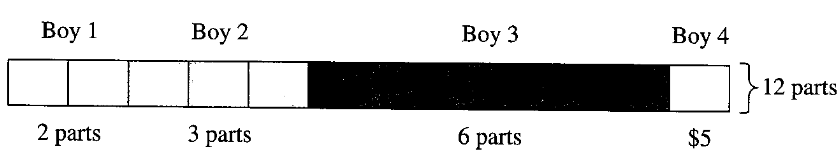
Problem Solving Involving Fractions

Example 11

Four boys shared a certain sum of money. The first received $\frac{1}{6}$ of it, the second $\frac{1}{4}$ and the third $\frac{1}{2}$. If the fourth boy received \$5, how much was the sum of money shared?

Solution

Use a model



From the model, the fourth boy shared 1 part of a total of 12 parts.

$\therefore$ 1 part = \$5 and thus, 12 parts = 12 × \$5 = \$60.

The sum of money shared is \$60.

Alternatively, the first three boys together received $\frac{1}{6} + \frac{1}{4} + \frac{1}{2} = \frac{2+3+6}{12} = \frac{11}{12}$ of the money. The fourth boy thus received $\left(1 - \frac{11}{12}\right) = \frac{1}{12}$ of the money.

$\frac{1}{12}$ of the money = \$5 or $\frac{1}{12}$ × the money = \$5.

$\therefore$ the money = \$5 $\div \frac{1}{12}$ = \$5 × 12 = \$60.

Example 12

Peter has 45 English and Chinese books. $\frac{4}{5}$ of the English books and $\frac{3}{4}$ of the Chinese books are fiction. The total number of fiction books he has is 35.

How many of his fiction books are in English?

Solution

Method 1: Make a list

Fraction of English fiction books $\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \mathbf{\frac{20}{25}}, \frac{24}{30}, \ldots$

Fraction of Chinese fiction books $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \mathbf{\frac{15}{20}}, \frac{16}{24}, \ldots$

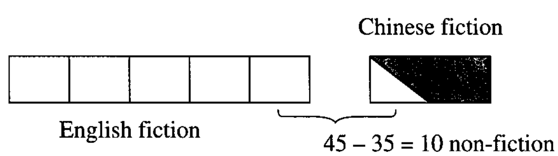
Study the fractions in bold from the lists.

The sum of the numerators = 20 (number of English fiction) + 15 (number of Chinese fiction)
= 35 (number of fiction Peter has)

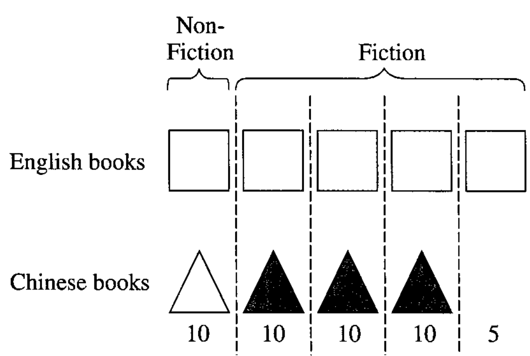
The sum of the denominators = 25 + 20 = 45 (Total number of books Peter has)

$\therefore$ 20 of his fiction books are in English.

Method 2: Use a model



Rearrange the squares and triangles as shown below.



Each pair of square and triangle represents 10 books.

A single square represents $45 - 4 \times 10 = 5$ books.

$\therefore$ The number of English fiction books $= 4 \times 5 = 20$.

Exercise 4g

1. James uses $\frac{1}{3}$ of his land for growing durians, $\frac{1}{4}$ for bananas, $\frac{3}{8}$ for guavas and the remaining 9 hectares for mangoes. What is the total area of his land?

2. A sum of money is shared among three brothers. The eldest receives $\frac{7}{13}$ of it and the next receives $\frac{2}{3}$ of the remainder. If the youngest brother receives \$6, find the sum of money shared.

3. There are 42 pupils in a class. $\frac{3}{4}$ of the boys and $\frac{2}{3}$ of the girls travel to school by bus. The total number of boys and girls who travel to school by bus is 30.
   (a) How many boys are there in the class?
   (b) How many girls travel to school by bus?

4. The Mathematics Club in a school has 53 members. $\frac{5}{7}$ of the girl members and $\frac{3}{5}$ of the boy members are from lower secondary. The total number of lower secondary members is 35. How many members of the club are boys?

5. Mary has two tanks of fish. If she transfers 15 fish from the larger tank to the smaller tank, then the number of fish in the smaller tank will be $\frac{5}{7}$ of the number of fish in the larger tank. Given that there are 35 fish in the smaller tank originally, find the number of fish in the larger tank before the transfer of fish.

6. As part of the 'Learning Journeys' program, 73 pupils travelled on two buses, one air-conditioned and one non-air-conditioned, to the Singapore Discovery Centre. Three-fifths of the pupils on the air-conditioned bus were girls. There were 17 boys in the non-air-conditioned bus. The number of girls on the two buses were equal. How many pupils were there on the air-conditioned bus?

Decimals

For Your Information

*A **decimal number** is a different way of writing fractions.*

In the decimal system, a number like 4 269 can be expressed as

$$4\,269 = 4 \times 1\,000 + 2 \times 100 + 6 \times 10 + 9 \times 1$$

÷ 10 ÷ 10 ÷ 10

Can we express a fraction like $4\,269\frac{39}{100}$ in the same manner?

We know that

$$\begin{aligned} 4\,269\frac{39}{100} &= 4\,269 + \frac{39}{100} \\ &= 4\,269 + \frac{30}{100} + \frac{9}{100} \\ &= 4 \times 1\,000 + 2 \times 100 + 6 \times 10 + 9 \times 1 + 3 \times \frac{1}{10} + 9 \times \frac{1}{100} \end{aligned}$$

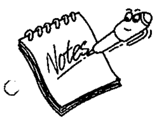
÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10

$4\,269\frac{39}{100}$ means 4 thousands, 2 hundreds, 6 tens, 9 ones, 3 tenths and 9 hundredths.

Similarly,

$$123\frac{456}{1\,000} = 1 \times 100 + 2 \times 10 + 3 \times 1 + 4 \times \frac{1}{10} + 5 \times \frac{1}{100} + 6 \times \frac{1}{1\,000}$$

and $$\frac{7\,891}{10\,000} = 7 \times \frac{1}{10} + 8 \times \frac{1}{100} + 9 \times \frac{1}{1\,000} + 1 \times \frac{1}{10\,000}$$



*A dot called the **decimal point** is placed after the units column to separate the whole number part from the fractional part.*

$4\,269\frac{39}{100}$ can be written as 4 269.39.

Insert a dot here to separate the fractional part which consists of the tenths, hundredths, etc. from the integral part which consists of the ones, tens, etc.

Similarly,

$123\frac{456}{1\,000} = 123.456$ and $\frac{7\,891}{10\,000} = 0.789\,1$

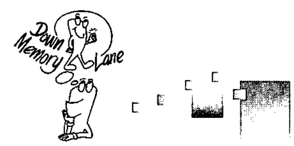
Insert a dot here.

Add a zero before the dot when the integral part is zero.

Also, $\frac{4}{100} = 0 \times \frac{1}{10} + 4 \times \frac{1}{100} = 0.04$

This zero indicates 0 tenths.

and $$3\frac{4}{1\,000} = 3 \times 1 + 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 4 \times \frac{1}{1\,000}$$
$$= 3.004$$



A decimal 123.456 would have been written as 123\456 by Francis Vieta in 1600, 123 (456 by Johannes Kepler in 1616, 123 : 456 by John Napier in 1617, 123^{456} by Henry Briggs in 1624 and 123|456 by William Aughtred in 1631.

The dot which we use to separate the fractional part from the integral part in a number is called the **decimal point**.

A number written with a decimal point is known as a **decimal**.

Examples of decimals are 4 269.39, 123.456, 0.789 1, 0.04 and 3.004.

In a decimal, the places occupied by the digits after the decimal point are called **decimal places**. 4 269.39, 123.456 and 0.789 1 have 2, 3 and 4 decimal places respectively.

Did you notice that the number of decimal places corresponds to the number of zeros in the denominators?

Note that

$$4\,269.39 = 4\,269\frac{39}{100} = \frac{426\,939}{100},\ 123.456 = 123\frac{456}{1\,000} = \frac{123\,456}{1\,000},$$

$$0.789\,1 = \frac{7\,891}{10\,000},\ 0.04 = \frac{4}{100} \text{ and } 3.004 = 3\frac{4}{1\,000} = \frac{3\,004}{1\,000}.$$

A decimal is a fraction whose denominator is 10 or a power of 10.

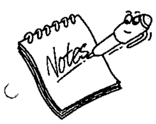
Conversion of Fractions into Decimals

Example 13

Express each of the following as a decimal:

(a) $\frac{87}{100}$ *(b)* $\frac{11}{10\,000}$ *(c)* $34\frac{97}{1\,000}$

Solution



To change a fraction into a decimal, divide the numerator by the denominator.

(a) $\frac{87}{\underbrace{100}_{\text{2 zeros}}} = 0.\underbrace{87}_{\text{2 decimal places}}$

(b) $\frac{11}{\underbrace{10\,000}_{\text{4 zeros}}} = 0.\underbrace{001\,1}_{\text{4 decimal places}}$

(c) $34\frac{97}{\underbrace{1\,000}_{\text{3 zeros}}} = 34.\underbrace{097}_{\text{3 decimal places}}$

Fractions whose denominators can be changed to 10 or powers of 10 can be converted to decimals mentally using the method shown in Example 13.

Example 14

Express each of the following as a decimal:

(a) $\frac{3}{5}$ *(b)* $\frac{3}{4}$ *(c)* $\frac{27}{25}$ *(d)* $\frac{21}{4}$

Solution

(a) $\frac{3}{5} = \frac{6}{10}$
$= 0.6$
(Multiply both the numerator and the denominator by 2 mentally. Write the decimal using the method in Example 13.)

(b) $\frac{3}{4} = \frac{75}{100}$
$= 0.75$
(Multiply both the numerator and the denominator by 25.)

(c) $\frac{27}{25} = \frac{108}{100}$
$= 1.08$

(d) $\frac{21}{4} = 5\frac{1}{4} = 5 + \frac{1}{4}$ $\left(\text{Note: } \frac{1}{4} = 0.25\right)$
$= 5 + 0.25$
$= 5.25$

For a fraction whose denominator cannot be changed to 10 or a power of 10 easily, the decimal form is obtained by dividing the numerator of the fraction by its denominator.

For example, $\frac{5}{8} = 5 \div 8$
$= 0.625$

$$\begin{array}{r} 0.625 \\ 8\,\overline{)\,5.000} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

Conversion of Decimals into Fractions

The Babylonians introduced the position system of numeral writing on which our decimal system is based. Although they introduced the decimal system into mathematics, it was not until the 9th century AD that this system was introduced into Europe by the Saracens.

Example 15

Express (a) 0.95 and (b) 0.015 as fractions in their lowest terms.

Solution

(a) $0.95 = \frac{95}{100} = \frac{19}{20}$

2 decimal places → 2 zeros in the denominator

(b) $0.015 = \frac{15}{1\,000} = \frac{3}{200}$

3 decimal places → 3 zeros in the denominator

Example 16

Express (a) 11.25 and (b) 31.75 as fractions in their lowest terms.

Solution

(a) $11.25 = 11 + 0.25$
$= 11 + \frac{25}{100}$
$= 11 + \frac{1}{4}$
$= 11\frac{1}{4} \quad \left(\text{Use } 0.25 = \frac{1}{4}\right)$

(b) $31.75 = 31 + 0.75$
$= 31 + \frac{75}{100}$
$= 31 + \frac{3}{4}$
$= 31\frac{3}{4} \quad \left(\text{Use } 0.75 = \frac{3}{4}\right)$

Exercise 4h

1. Express the following fractions as decimals:

(a) $\frac{43}{100}$ **(b)** $\frac{57}{1\,000}$ **(c)** $\frac{8}{10\,000}$ **(d)** $\frac{27}{20}$ **(e)** $\frac{33}{4}$

(f) $2\frac{3}{10}$ **(g)** $15\frac{96}{1\,000}$ **(h)** $7\frac{5}{10\,000}$ **(i)** $85\frac{3}{25}$ **(j)** $2\frac{31}{50}$

(k) $19\frac{3}{125}$ **(l)** $101\frac{11}{100\,000}$

2. Express the following as fractions in their lowest terms:

(a) 0.75 **(b)** 0.36 **(c)** 0.025 **(d)** 0.006 **(e)** 0.105

(f) 3.75 **(g)** 0.012 5 **(h)** 15.25 **(i)** 84.625

3. Change the following into decimals:

(a) $\frac{13}{8}$ **(b)** $\frac{67}{80}$ **(c)** $\frac{215}{80}$ **(d)** $\frac{245}{280}$ **(e)** $\frac{19}{32}$ **(f)** $\frac{94}{64}$

Recurring Decimals

In-Class Activity

1. Divide 1 by 8. Does the division process come to an end?
2. Divide 1 by 3. Does the division process come to an end? If it does not, do you observe anything interesting? Can you suggest a way of writing the answer?
3. Divide 7 by 12. Does any digit keep repeating itself? Suggest a way to write the answer.
4. Divide 13 by 99. How many digits keep repeating? Suggest a way to write the answer.
5. Divide 41 by 333. How many digits keep repeating? Suggest a way to write the answer.
6. Divide 1 by 7. Suggest a way to write the answer.

There is an end to the division process when we divide 1 by 8, i.e., $1 \div 8 = 0.125$. Therefore, 0.125 is called a **non-recurring** decimal.

When dividing 1 by 3, the digit 3 repeats itself and we get

$$1 \div 3 = \frac{1}{3} = 0.333\ldots = 0.\dot{3}.$$

$0.\dot{3}$ is called a **recurring decimal**. The dot above '3' indicates that '3' is the repeating digit. Similarly, we write

$$\frac{7}{12} = 0.583\,333\ldots = 0.58\dot{3} \qquad \frac{13}{99} = 0.131\,313\ldots = 0.\dot{1}\dot{3}$$

$$\frac{41}{333} = 0.123\,123\,123\ldots = 0.\dot{1}2\dot{3} \qquad \frac{1}{7} = 0.142\,857\,142\,857\ldots = 0.\dot{1}42\,85\dot{7}$$

It is more convenient to write $0.\dot{1}\dot{2}\dot{3}$ as $0.\dot{1}2\dot{3}$ and $0.\dot{1}\dot{4}\dot{2}\,\dot{8}\dot{5}\dot{7}$ as $0.\dot{1}42\,85\dot{7}$. Hence, the first dot and the last dot mark the beginning and end of a repeating block of digits.

Some numbers such as $\frac{5}{27}$ (= 0.185 185 185 ...) cannot be written as decimal with a finite number of decimal places, but result in a number of digits that repeat infinitely. These are called **recurring decimals**.

Exercise 4i

1. Express the following fractions as recurring decimals:

   (a) $\frac{2}{3}$ (b) $\frac{5}{11}$ (c) $\frac{29}{33}$ (d) $\frac{5}{22}$ (e) $\frac{4}{7}$ (f) $\frac{35}{72}$

2. Express each of the following as a decimal and indicate whether it is recurring or non-recurring:

   (a) $\frac{4}{9}$ (b) $\frac{7}{3}$ (c) $\frac{61}{90}$ (d) $\frac{59}{99}$ (e) $\frac{97}{125}$ (f) $\frac{13}{44}$

Order of Decimals

We can represent decimals on the number line. Fig. 4.13 shows the number line with an interval of 1 unit divided into 10 equal parts. The values 0.2, 0.4 and 0.8 are marked as shown below:

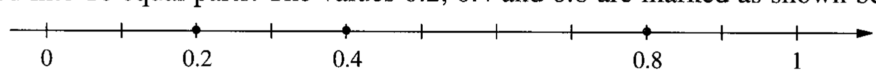


Fig. 4.13

Since 0.2 is to the left of 0.4 and 0.4 is to the left of 0.8, we have $0.2 < 0.4 < 0.8$.

In-Class Activity

1. **(a)** Without using the number line, how can you tell that $0.2 < 0.4 < 0.8$?
   (b) Do you agree that $0.2 < 0.4 < 0.8$ because $2 < 4 < 8$?
2. Is it reasonable to say that $0.7 < 1.4$ because $0 < 1$ and $1.4 < 1.8$ because $4 < 8$?
3. Do you agree that $2.02 < 2.04 < 2.08$ because $2 < 4 < 8$? What about $2.08 < 2.12$? Do you agree that $2.08 < 2.12$ because $0 < 1$? Similarly, can you see that $2.16 > 2.12$ because $6 > 2$?

The rule for comparing decimals is as follows:

> **Compare the digits which have the same place value from left to right, skipping the equal digits. Examine the first pair of unequal digits, the greater decimal being the decimal with the greater digit.**

Consider the following recurring decimals:

$\frac{1}{7} = 0.\dot{1}42\ 85\dot{7}$

$\frac{1}{11} = 0.\dot{0}\dot{9}$

$\frac{1}{13} = 0.\dot{0}76\ 92\dot{3}$

$\frac{1}{17} = 0.\dot{0}58\ 823\ 529\ 411\ 764\ \dot{1}$

(a) Are the denominators of the fractions prime numbers?
(b) Are there even numbers of digits in the repeating blocks?
(c) Divide each repeating block in half and add the two parts. For example, 142 857 gives

$$\begin{array}{r} 142 \\ +\ 857 \\ \hline 999 \end{array}$$

(d) Do you notice an interesting general pattern?

Example 17

Arrange the following sets of decimals in descending order:
(a) 1.209, 1.234 *(b) 7.3, 6.5, 6.9*

Solution

(a) $1\,.\,2\,\boxed{3}\,4 > 1\,.\,2\,\boxed{0}\,9$ since $3 > 0$

$\therefore$ the correct descending order is 1.234, 1.209.

(b) $\boxed{7}\,.\,3 > \boxed{6}\,.\,5$ and $\boxed{6}\,.\,9$ since $7 > 6$

$6\,.\,\boxed{9} > 6\,.\,\boxed{5}$ since $9 > 5$

$\therefore$ the correct descending order is 7.3, 6.9, 6.5.

Example 18

Arrange $\frac{9}{4}$, 2.23 and 2.232 in ascending order.

Solution

$\frac{9}{4} = 2\frac{1}{4} = 2.25$ $\left(\text{Express } \frac{9}{4} \text{ as a decimal using } \frac{1}{4} = 0.25.\right)$

$2\,.\,2\,\boxed{5}\,0 > 2\,.\,2\,\boxed{3}\,2$ and $2\,.\,2\,\boxed{3}\,0$ since $5 > 3$

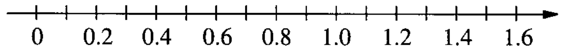
Add 0 to get the same number of decimal places.

$2\,.\,2\,3\,\boxed{2} > 2\,.\,2\,3\,\boxed{0}$ since $2 > 0$

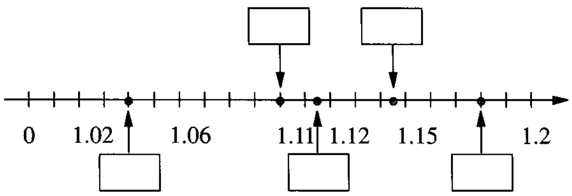
$\therefore$ the correct ascending order is 2.23, 2.232, $\frac{9}{4}$.

Exercise 4j

1. Copy the number line given below and indicate 0.3, 0.6, 1.1, 0.15 and 1.45 on it.



2. Fill in the boxes with the correct decimals.



3. Represent the following decimals on the number line and arrange them in ascending order:

(a) 2.7, 2.4, 2.1
(b) 3.03, 3.16, 3.12
(c) 0.02, 0.08, 0.035, 0.065
(d) 1.22, 1.28, 1.31, 1.25, 1.38, 1.345

4. Fill in each box with '<' or '>':

(a) 4.13 ☐ 4.17 **(b)** $\frac{16}{5}$ ☐ 3.19

(c) $5\frac{1}{4}$ ☐ 6.04

(d) 0.005 4 ☐ 0.005 19

(e) $\frac{9}{100}$ ☐ 0.075 ☐ 0.044

(f) 27.11 ☐ 28.01 ☐ 29.001

5. Arrange the following in descending order:

(a) 0.3, 0.8, 0.4 **(b)** $\frac{5}{4}$, 1.54, $1\frac{3}{4}$

(c) 1.88, 1.13, 1.9

(d) $\frac{13}{20}$, $0.\dot{6}\dot{5}$, $0.\dot{6}0\dot{5}$, $0.6\dot{5}$

(e) 3.14, $\frac{22}{7}$, $3.\dot{1}\dot{4}$, $3.1\dot{4}$

(f) 2.102, 2.012, 2.201, $2.0\dot{2}$

Addition and Subtraction of Decimals

When two decimals are added together or subtracted from each other, the decimal points must be placed directly one below the other.

Example 19

Evaluate (a) 137.45 + 145.25 + 12.106; (b) 733.75 – 123.98;
(c) 123.14 + 52.76 – 152.75.

Solution

Decimal points aligned.

(a)
```
  137.450 } Fill in empty
  145.250 } spaces with zeros.
+  12.106
  -------
  294.806
```

(b)
```
  733.75
- 123.98
  ------
  609.77
```

(c)
```
  123.14
+  52.76
  ------
  175.90
- 152.75
  ------
   23.15
```

Decimal point in the answer.

Decimals are added and subtracted in the same way as whole numbers. It is simplest to work in columns when adding decimals. Keep the decimal points under each other and write the digits in the correct place value columns.

Exercise 4k

1. Do the following additions:

(a)
$$\begin{array}{r} 0.825 \\ 0.073 \\ +\ 0.46 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 63.008 \\ 18.7 \\ 9.82 \\ +\ 245.2 \\ \hline \end{array}$$

(c)
$$\begin{array}{r} 828 \\ 17.9 \\ 61.38 \\ +\ 9.182 \\ \hline \end{array}$$

(d)
$$\begin{array}{r} 83.4 \\ 6.78 \\ 135.009 \\ +\ 42.014 \\ \hline \end{array}$$

2. Do the following subtractions:

(a)
$$\begin{array}{r} 649.08 \\ -\ 51.63 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 300 \\ -\ 28.09 \\ \hline \end{array}$$

(c)
$$\begin{array}{r} 9.06 \\ -\ 8.999 \\ \hline \end{array}$$

3. Do the following additions:
   - **(a)** 3.45 + 15.52
   - **(b)** 0.872 + 56.43 + 239.8
   - **(c)** 83.72 + 16.43 + 1.4 + 25.63
   - **(d)** 11.42 + 9.865 + 3.1 + 7.98

4. Do the following subtractions:
   - **(a)** 7.02 – 4.55 **(b)** 20 – 6.72
   - **(c)** 9.6 – 4.751 **(d)** 10 – 0.366
   - **(e)** 610.57 – 602.57 **(f)** 325.5 – 18.674

5. **(a)** Find the sum of 79.8, 7.98 and 0.798.
   - **(b)** Subtract 29.7 from 244.93.
   - **(c)** What is the difference between the sum of 93.71 and 8.51 and the sum of 79.93 and 33.509?

General Multiplication of Decimals

Notes

To find the product of decimals, multiply the numbers in the same way as for whole numbers first. Then put in the decimal point. The number of decimal places in the answer must correspond to the total number of decimal places in the decimals being multiplied.

Consider the product of 23.45 and 2.3.

$$23.45 \times 2.3 = \frac{2\,345}{100} \times \frac{23}{10} = \frac{2\,345 \times 23}{1\,000}$$

The product of 2 345 and 23 is obtained as shown below:

$$\begin{array}{r} 2\,345 \\ \times\quad 23 \\ \hline 7\,035 \\ 46\,90 \\ \hline 53\,935 \end{array}$$

$$\therefore\quad 23.\underbrace{45}_{\text{2 decimal places}} \times 2.\underbrace{3}_{\text{1 decimal place}} = \frac{53\,935}{1\,000} = 53.\underbrace{935}_{\text{3 decimal places}}$$

Example 20

Evaluate (a) 46.75 × 2.12, (b) 256.7 × 0.005 6 and (c) 7.06 × 72.675.

Solution

(a)
$$\begin{array}{r} 4\,675 \\ \times\quad 212 \\ \hline 9\,350 \\ 46\,75 \\ 935\,0 \\ \hline 991\,100 \end{array}$$

$$46.\underbrace{75}_{\text{2 decimal places}} \times 2.\underbrace{12}_{\text{2 decimal places}} = 99.\underbrace{110\,0}_{\text{4 decimal places}}.$$

We may omit the two zeros at the end as they have no value. Hence, the answer is 99.11.

(b)

$$\begin{array}{r} 2\,567 \\ \times \quad 56 \\ \hline 15\,402 \\ 128\,35 \\ \hline 143\,752 \end{array}$$

$$\underbrace{256.7}_{\text{1 decimal place}} \times \underbrace{0.005\,6}_{\text{4 decimal places}} = \underbrace{1.437\,52}_{\text{5 decimal places}}$$

The answer in this case is 1.437 52.

(c)

$$\begin{array}{r} 706 \\ \times \quad 72\,675 \\ \hline 3\,530 \\ 49\,42 \\ 423\,6 \\ 1\,412 \\ 49\,42 \\ \hline 51\,308\,550 \end{array} \quad \text{or} \quad \begin{array}{r} 72\,675 \\ \times \quad 706 \\ \hline 436\,050 \\ 000\,00 \\ 50\,872\,5 \\ \hline 51\,308\,550 \end{array} \quad \text{or simply as} \quad \begin{array}{r} 72\,675 \\ \times \quad 706 \\ \hline 436\,050 \\ 50\,872\,5 \\ \hline 51\,308\,550 \end{array}$$

$\therefore \quad 7.06 \times 72.675 = 513.085\ 5$

It's Fact

The Dewey decimal system of classifying library books was invented by the American librarian Melvil Dewey. In this system, books are divided into 10 main categories. Each category is then further divided into 10 smaller categories and so on. Thus, a mathematics book may be numbered 510.7.

Multiplication of a Decimal by Powers of 10

Example 21

Evaluate (a) 2.75 × 10, (b) 2.75 × 100 and (c) 2.75 × 1 000.

Solution

(a) $275 \times 10 = 2\ 750$

$\therefore \quad 2.75 \times 10 = 27.50 = 27.5$

(b) $275 \times 100 = 27\ 500$

$\therefore \quad 2.75 \times 100 = 275.00 = 275$

(c) $275 \times 1\ 000 = 275\ 000$

$\therefore \quad 2.75 \times 1\ 000 = 2\ 750.00 = 2\ 750$

As a useful rule,

When we multiply a decimal by 10, 100, 1 000, etc., we move the decimal point 1, 2, 3, etc. places respectively to the right.

Investigate

Compare 2.75 and 27.5. Notice that if we move the decimal point in 2.75 one place to the right, we will get 27.5.

If we move the decimal point in 2.75 two places to the right, we will get 275.

What will we get if we move the decimal point in 2.75 three places to the right?

Division of a Decimal by Powers of 10

How will the position of the decimal point in a decimal change when the decimal is divided by 10, 100, 1 000, etc.?

The rule for division of a decimal by powers of 10 is as follows:

When we divide a decimal by 10, 100, 1 000 etc., we move the decimal point 1, 2, 3, etc. places respectively to the left.

Example 22

Evaluate (a) 48.6 ÷ 10, (b) 45.62 ÷ 100 and (c) 34.81 ÷ 1 000.

Solution

(a) $48.6 \div 10 = 4.86$ (Move the decimal point 1 place to the left.)

(b) $45.62 \div 100 = 045.62 \div 100$ (Move the decimal point 2 places to the left.)

$= 0.456\ 2$

(c) $34.81 \div 1\ 000 = 0\ 034.81 \div 1\ 000$ (Move the decimal point 3 places to the left.)

$= 0.034\ 81$

Division of a Decimal by a Decimal

Just For Fun

Can you form 100 by writing an equation using four 8's?

Dividing a decimal by a whole number is relatively easy.

For example, consider 24.5 ÷ 5.

Remember, 24.5 is the **dividend** and 5 is the **divisor**.

Line up the decimal points.

$$\begin{array}{r|r} & 4.9 \\ \hline 5 & 24.5 \\ & -20 \\ \hline & 4\,5 \\ & 4\,5 \\ \hline \end{array}$$

$\therefore\ 24.5 \div 5 = 4.9$

When dividing a decimal by a decimal, it is easier to use the idea of equivalent fractions to convert the divisor to a whole number.

For example, $\frac{2.345}{0.05} = \frac{2.345}{0.05} \times \frac{100}{100}$

$= \frac{234.5}{5}$

$= 46.9$ (see working)

Line up the decimal points.

$$\begin{array}{r} 46.9 \\ 5\overline{)234.5} \\ \underline{20} \\ 34 \\ \underline{30} \\ 4\,5 \\ \underline{4\,5} \end{array}$$

The rule for dividing one decimal by another decimal is as follows:

Multiply the divisor and the dividend by the same power of 10 so that the divisor becomes a whole number, then perform a long division, remembering to line up the decimal points.

Note: It is easier to shift the decimal points the same number of places in the dividend and divisor to make the divisor a whole number.

For example, consider 3.439 8 ÷ 0.49.

3.439 8 ÷ 0.49 = 343.98 ÷ 49 = 7.02 (see working)

(Move the decimal points in the dividend and divisor two places to the right.)

$$\begin{array}{r} 7.02 \\ 49\overline{)343.98} \\ \underline{343} \\ 98 \\ \underline{98} \end{array}$$

Exercise 4I

1. Evaluate the following:

(a) 0.5×0.6 **(b)** 8.41×0.3 **(c)** 0.82×0.03
(d) 7.3×0.9 **(e)** 0.08×0.09 **(f)** 2.33×0.32

2. Find the exact value of the following:

(a) $6 \times 0.004\,75$ **(b)** 13.75×43 **(c)** 7.89×3.2
(d) 15.68×102 **(e)** $120 \times 0.2 \times 3.2$ **(f)** 3.418×0.45
(g) 3.94×0.023 **(h)** $0.5 \times 0.4 \times 0.07$

3. Evaluate the following:

(a) 0.736×10 **(b)** 18.517×100 **(c)** 15.029×100
(d) $17.9 \times 1\,000$ **(e)** $0.006\,6 \times 1\,000$ **(f)** $10\,000 \times 0.012\,4$

4. Evaluate the following:

(a) $753.8 \div 10$ **(b)** $0.029 \div 10$ **(c)** $624 \div 100$
(d) $0.006\,6 \div 100$ **(e)** $4\,000 \div 1\,000$ **(f)** $86.5 \div 1\,000$

5. Evaluate the following:

(a) $63.6 \div 6$ **(b)** $1.71 \div 0.3$ **(c)** $0.165 \div 1.5$
(d) $720 \div 0.09$ **(e)** $0.444 \div 0.04$ **(f)** $7.647 \div 0.25$

6. Evaluate:

(a) $(0.01 \div 0.005)^2$ **(b)** $(0.3)^2 \div (0.1)^2$ **(c)** $0.8^3 \div 0.2^2$
(d) $\frac{0.055 \times 8.1}{4.5}$ **(e)** $\frac{0.44 \times 12^2}{0.055}$ **(f)** $\frac{0.04 \times 0.25^3}{0.008}$

Money

We know that,

100 cents = \$1
75¢ can be written as \$0.75.
\$3 and 75¢ can be written as \$3.75.
\$3 and 5¢ can be written as \$3.05.
\$3 and 50¢ can be written as \$3.50.

Common denominations of coins and notes used in Singapore are:

Coins: 1¢, 5¢, 10¢, 20¢, 50¢, \$1
Notes: \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500

Example 23

I spent \$19.90, \$26.95 and \$46.50 on three different sports items. How much change would I get back if two \$50 notes were used to pay for the goods?

Solution

Amount of money spent = \$19.90 + \$26.95 + \$46.50
= \$93.35

$$\begin{array}{r} \$19.90 \\ \$26.95 \\ +\ \$46.50 \\ \hline \$93.35 \end{array}$$

Amount of change = (2 × \$50) − \$93.35
= \$100 − \$93.35 = \$6.65

Example 24

Bars of soap of a particular brand are sold at \$5.40 for 8 in store A and \$3.60 for 5 in store B. Which store offers a cheaper price?

Solution

Store *A*: 8 for \$5.40 or 1 for \$5.40 ÷ 8 = 67.5 cents
Store *B*: 5 for \$3.60 or 1 for \$3.60 ÷ 5 = 72 cents

∴ store *A* offers a cheaper price.

Alternatively,
Store *A*: 8 for \$5.40 or 8 × 5 = 40 for \$5.40 × 5 = \$27
Store *B*: 5 for \$3.60 or 5 × 8 = 40 for \$3.60 × 8 = \$28.80

∴ store *A* offers a cheaper price.

Exercise 4m

1. How many 5-cent coins will give \$3?
2. If 4 pears cost as much as 5 oranges and an orange costs 32¢, how much does a pear cost?
3. Find the total cost of 3 mangoes at \$1.40 each, 2 nectarines at \$1.95 each and 15 apples at \$0.79 for 3.

4. Find the total bill for the following:

$1\frac{1}{2}$ kg of T-bone steak at $9.98 per kg,

$\frac{3}{4}$ kg of beef cubes at $6.96 per kg,

$\frac{1}{2}$ kg of tiger prawns at $14.14 per kg.

$\frac{1}{4}$ kg of boneless chicken breast at $8.28 per kg.

What will be the change if a $50 note is used to pay the bill?

5. Facial tissues are sold in packs of 3 for $1.95 or packs of 5 for $2.80. Which is the better buy?

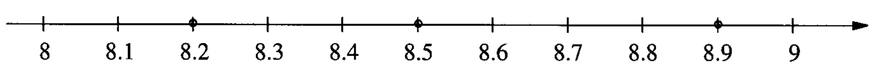
Rounding Off Decimals

Sometimes it is impractical or meaningless to have too many decimal places in a decimal. For example, it is absurd to give the speed of a car as 48.234 56 km/h. Here, we will learn how to round off a decimal to a desired number of decimal places.

Example 25

Round off (a) 8.2, (b) 8.9 and (c) 8.5 to the nearest whole number.

Solution



(a) The number line above shows that 8.2 lies between 8 and 9 but is closer to 8.

$\therefore$ $8.2 \approx 8$ (correct to the nearest whole number)

(b) The number line shows that 8.9 lies closer to 9 than to 8.

$\therefore$ $8.9 \approx 9$ (correct to the nearest whole number)

(c) Similarly, it shows that 8.5 is exactly halfway between 8 and 9. For such cases, it is common to round up.

$\therefore$ $8.5 \approx 9$ (correct to the nearest whole number)

Notes

The symbol $\approx$ represents approximation. For example, when we say that 8.1 is approximately equal to 8, we can write $8.1 \approx 8$.

The following are steps to follow when rounding off a decimal:

1. Include one extra digit for consideration.
2. Simply drop the extra digit if it is less than 5. If it is 5 or more, add 1 to the previous digit before dropping the extra digit.

Example 26

Write the following numbers correct to (i) the nearest whole number, (ii) 2 decimal places and (iii) 3 decimal places:

(a) 9.716 8 *(b) 19.214 7* *(c) 0.825 14*

Solution

(a) (i) $9.716\,8 \approx 10$ — This digit is more than 5.

(ii) $9.716\,8 \approx 9.72$ — This digit is more than 5.

(iii) $9.716\,8 \approx 9.717$ — This digit is more than 5.

(b) (i) $19.214\,7 \approx 19$ — This digit is less than 5.

(ii) $19.214\,7 \approx 19.21$ — This digit is less than 5.

(iii) $19.214\,7 \approx 19.215$ — This digit is more than 5.

(c) (i) $0.825\,14 \approx 1$ — This digit is more than 5.

(ii) $0.825\,14 \approx 0.83$ — This digit is 5.

(iii) $0.825\,14 \approx 0.825$ — This digit is less than 5.

Example 27

Express (a) $\frac{5}{14}$ as a decimal correct to 3 decimal places, and

(b) $\frac{14}{37}$ as a decimal correct to 4 decimal places.

Solution

(a)

$$\begin{array}{r} 0.357\,1 \\ 14\,\overline{)\,5.000\,0} \\ \underline{4\,2} \\ 80 \\ \underline{70} \\ 100 \\ \underline{98} \\ 2\,0 \\ \underline{1\,4} \\ 6 \end{array}$$

This division is carried out up to 4 decimal places, 1 decimal place more than required.

$\therefore \quad \frac{5}{14} \approx 0.357$

(correct to 3 decimal places)

(b)

$$\begin{array}{r} 0.378\,37 \\ 37\,\overline{)\,14.000\,00} \\ \underline{11\,1} \\ 2\,90 \\ \underline{2\,59} \\ 310 \\ \underline{296} \\ 14\,0 \\ \underline{11\,1} \\ 2\,90 \\ \underline{2\,59} \\ 31 \end{array}$$

This extra digit is greater than 5.

$\therefore \quad \frac{14}{37} \approx 0.378\,4$

(correct to 4 decimal places)

Exercise 4n

1. Write the following correct to **(i)** the nearest whole number and **(ii)** 2 decimal places:

(a) 5.424 67 **(b)** 15.824 **(c)** 7.862 **(d)** 130.829

2. Write the following correct to 3 decimal places:

(a) 712.892 6 **(b)** 0.002 72 **(c)** 0.827 4 **(d)** 7.024 489

3. Express $\frac{2}{7}$ as a decimal and give your answer correct to 3 decimal places.

4. Express the following fractions as decimals correct to 2 decimal places:

(a) $\frac{4}{9}$ **(b)** $\frac{7}{11}$ **(c)** $\frac{9}{14}$ **(d)** $\frac{11}{15}$

Use of Calculator

To do operations involving fractions and decimals, we need the Decimal Point Key $\boxed{.}$ and the Fraction Key $\boxed{a^b/c}$.

For example,

(a) To find 14.7×8.74, press 14 $\boxed{.}$ 7 $\boxed{\times}$ 8 $\boxed{.}$ 74 $\boxed{=}$ to get 128.478.

(b) To find $3\frac{2}{5} \div \frac{3}{4}$, press 3 $\boxed{a^b/c}$ 2 $\boxed{a^b/c}$ 5 $\boxed{\div}$ 3 $\boxed{a^b/c}$ 4 $\boxed{=}$ and the display screen shows 4 ⌋ 8 ⌋ 15, i.e., $3\frac{2}{5} \div \frac{3}{4} = 4\frac{8}{15}$.

(c) To find $1\frac{1}{5} + 3\frac{2}{3} \times 1\frac{3}{4}$, press 1 $\boxed{a^b/c}$ 1 $\boxed{a^b/c}$ 5 $\boxed{+}$ 3 $\boxed{a^b/c}$ 2 $\boxed{a^b/c}$ 3 $\boxed{\times}$ 1 $\boxed{a^b/c}$ 3 $\boxed{a^b/c}$ 4 $\boxed{=}$ to get 7 ⌋ 37 ⌋ 60, i.e. $7\frac{37}{60}$.

Some calculators might have different keys for displaying decimals and fractions. Check the manual of your calculator before using it.

Example 28

Evaluate each of the following using a calculator:

(a) $2.6 \times 2.7 \times (3.5 + 6.1)$ *(b)* $\dfrac{3.2^3 + 4.3^2}{\sqrt{47.5} - 2.74}$ *(c)* $\dfrac{\frac{2}{5} + \frac{3}{4}}{1\frac{2}{3} + \frac{3}{4} - \frac{5}{6}}$

Solution

The steps:	Final display:
(a) 2 $\boxed{.}$ 6 $\boxed{\times}$ 2 $\boxed{.}$ 7 $\boxed{\times}$ $\boxed{(}$ 3 $\boxed{.}$ 5 $\boxed{+}$ 6 $\boxed{.}$ 1 $\boxed{)}$ $\boxed{=}$	67.392
(b) 3.2 $\boxed{y^x}$ 3 $\boxed{+}$ 4.3 $\boxed{x^2}$ $\boxed{=}$ $\boxed{\div}$ $\boxed{(}$ $\boxed{\sqrt{}}$ 47.5 – 2.74 $\boxed{)}$ $\boxed{=}$	12.34530324

(c) 1 $\boxed{a^b/c}$ 2 $\boxed{a^b/c}$ 3 $\boxed{+}$ 3 $\boxed{a^b/c}$ 4 $\boxed{-}$ 5 $\boxed{a^b/c}$ 6 $\boxed{=}$ $\boxed{\text{STO}}$
2 $\boxed{a^b/c}$ 5 $\boxed{+}$ 3 $\boxed{a^b/c}$ 4 $\boxed{=}$ $\boxed{\div}$ $\boxed{\text{RCL}}$ $\boxed{=}$ $\qquad \dfrac{69}{95}$

Making words with your calculator.

Most calculators have a liquid crystal display panel. Some of the numbers displayed look like letters when viewed upside down. The letters available are shown below.

0 = O	*5 = S*
1 = l	*6 = g*
3 = E	*7 = L*
4 = h	*8 = B*

Enter 0.773 4 and turn your calculator upside down. What do you see?

Can you try to form some words with your calculator? Take note that you have to reverse the order of the digits to be entered.

Example 29

John uses his calculator to compute (a) 43.958 12 − 28.340 75 + 41.823 58 and (b) $\frac{502 \times \sqrt{24.98}}{9.96}$. He obtains the following answers: (a) 57 and (b) 346, both correct to the nearest whole number. Estimate the results of John's calculations and state whether John has obtained the correct answer.

Solution

(a) $43.958\,12 - 28.340\,75 + 41.823\,58 \approx 44 - 28 + 42$
$= 58$

Since John's answer is close to the estimated value, it is likely that he has obtained the correct answer.

(b) $\dfrac{502 \times \sqrt{24.98}}{9.96} \approx \dfrac{500 \times \sqrt{25}}{10}$
$= 250$

Since John's answer is not very close to the estimated value, it is likely that he has not obtained the correct answer.

Note: The correct answer to $\frac{502 \times \sqrt{24.98}}{9.96}$ is 252 (correct to the nearest whole number).

Exercise 4o

1. Use your calculator to evaluate each of the following, giving your answer correct to 3 decimal places if it is not exact:

(a) 37.56×12.45 **(b)** $44.75 \div 1.25$ **(c)** $121.35 - 12.75 + 46.32$
(d) $52.35 \times 6.78 \div 13.57$ **(e)** $5.69 + 3.64 \times 2.79$ **(f)** $55.69 - 6.94 \div 1.78$

2. Use your calculator to evaluate each of the following, giving your answer as **(i)** a fraction and **(ii)** a decimal correct to 2 decimal places:

(a) $\frac{1}{5} + \frac{3}{4}$ **(b)** $3\frac{2}{3} - 1\frac{5}{6}$ **(c)** $3\frac{1}{5} \times 1\frac{2}{3} - 1\frac{1}{6}$
(d) $\left(3\frac{1}{3}\right)^2 \times 4\frac{5}{6}$ **(e)** $7\frac{1}{8} \div 4\frac{3}{4} + 1\frac{5}{6}$ **(f)** $\left(5\frac{3}{4}\right)^3 \div \left(4\frac{3}{5}\right)^2$

3. Make an estimate of the following and then use your calculator to work out the answers correct to 2 decimal places.

	Estimate	Answer (correct to 2 decimal places)
(a) $97.45 + 20.15 - 49.89$	______	______
(b) $80.17 \div 7.91 \times 99.93$	______	______
(c) $0.996\,3 + 101.11 \times 30.96$	______	______
(d) $300.972 - 99.983\,2 \div 10.106\,7$	______	______
(e) $\dfrac{9.879 \times 46.071}{22.998\,2}$	______	______

Summary

1. A fraction is a number of the form $\frac{a}{b}$ where $b \neq 0$ and a, b are whole numbers. a is called the **numerator** and b is called the **denominator**.

2. $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$ and $\frac{5}{10}$ have the same value as $\frac{1}{2}$. They are equivalent fractions of $\frac{1}{2}$.

3. A **proper fraction** is one whose numerator is less than the denominator, e.g. $\frac{2}{5}$, $\frac{3}{7}$.

4. An **improper fraction** is one whose numerator is the same as or is greater than the denominator. e.g. $\frac{8}{8}$, $\frac{7}{4}$.

5. A **mixed number** is one that contains an integral part and a fractional part, e.g. $1\frac{3}{4}$, $2\frac{1}{2}$.

6. In general, if two fractions have the same denominator, then the larger the numerator, the larger the fraction. In contrast, if two fractions have the same numerator, the larger the denominator, the smaller the fraction.

7. The value of a fraction remains unchanged if both the numerator and denominator are multiplied or divided by the same number, e.g. $\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$ and $\frac{16}{40} = \frac{16 \div 4}{40 \div 4} = \frac{4}{10}$.

8. To **simplify** a fraction is to reduce it to its **lowest terms** or simplest form, e.g. $\frac{16}{40} = \frac{4}{10} = \frac{2}{5}$.

9. A decimal is a fraction whose denominator is 10 or a power of 10.

10. When we **multiply** a decimal by 10, 100, 1 000, etc., we move the decimal point 1, 2, 3, etc. places respectively to the **right**.

11. When we **divide** a decimal by 10, 100, 1 000, etc., we move the decimal point 1, 2, 3, etc. places respectively to the **left**.

12. Steps for rounding off a decimal:
    (a) Include one extra digit for consideration.
    (b) Drop the extra digit if it is less than **5**. Otherwise, add 1 to the previous digit before dropping the extra digit.

9 12 8 24

Review Questions 4

1. Arrange the following fractions in ascending order:

(a) $\frac{3}{4}, \frac{7}{10}, \frac{4}{5}$ **(b)** $\frac{2}{3}, \frac{7}{12}, \frac{5}{8}$ **(c)** $\frac{7}{12}, \frac{4}{7}, \frac{9}{14}, \frac{25}{42}$

2. Arrange the following fractions in descending order:

(a) $\frac{5}{8}, \frac{7}{12}, \frac{5}{9}, \frac{13}{24}$ **(b)** $\frac{6}{35}, \frac{5}{21}, \frac{2}{7}, \frac{4}{15}$ **(c)** $\frac{2}{5}, \frac{6}{11}, \frac{7}{15}, \frac{9}{20}, \frac{13}{25}$

3. Yuanwei weighs $45\frac{13}{24}$ kg. Xinyu weighs $45\frac{5}{9}$ kg. Who is heavier?

4. Evaluate the following, expressing your answers in the simplest form:

(a) $2\frac{1}{3} + 1\frac{5}{12}$ **(b)** $4\frac{5}{7} - 2\frac{5}{21}$ **(c)** $\frac{41}{12} - \frac{23}{8}$

(d) $\frac{2}{3} - \frac{1}{6} + \frac{4}{5}$ **(e)** $8\frac{1}{3} - 1\frac{1}{9} - \frac{5}{18} - 2\frac{5}{6}$ **(f)** $2\frac{17}{27} - \frac{1}{18} + \frac{61}{72} - \frac{1}{54}$

5. Calculate the following:

(a) $2\frac{3}{11} \times 1\frac{8}{25}$ **(b)** $3\frac{3}{4} \div 2\frac{1}{2}$ **(c)** $1\frac{6}{15} \times 3\frac{8}{9} \div \frac{7}{8}$

(d) $\dfrac{2\frac{3}{5} - 1\frac{1}{2}}{3\frac{2}{3}}$ **(e)** $1\frac{1}{4} - \left(\frac{2}{3} \times \frac{1}{3} + \frac{1}{9}\right) \div \frac{4}{9}$ **(f)** $\frac{3}{49} \div \frac{1}{7} + \left(\frac{1}{5} + \frac{7}{10}\right) \times \frac{4}{21}$

(g) $8\frac{1}{2} - 2\frac{1}{3} - 1\frac{6}{7} \times \frac{3}{13}$ **(h)** $2 \times 2\frac{2}{5} \times \left(3\frac{1}{4} + 1\frac{7}{16}\right)$

6. Evaluate the following without using a calculator:

(a) $17.2 + 13.8$ **(b)** $16.83 - 7.57$ **(c)** 1.72×0.091
(d) $0.042 \div 0.35$ **(e)** $80 \times 0.6 \times 2.5$ **(f)** $16.52 \div 0.04$

7. Use a calculator to evaluate each of the following, giving your answer correct to 2 decimal places if it is not exact:

(a) $\left(1\frac{1}{4}\right)^3 \times \frac{6}{25} \div \frac{5}{6}$ **(b)** $\dfrac{9.35^2 + 96.1^3}{65.9^3 - 38.4^2}$ ✡**(c)** $\dfrac{\frac{4}{7} + \frac{2}{9}}{2\frac{3}{5} - 1\frac{5}{8} + 5\frac{3}{4}}$

(d) $522.76 \times 647.15 \div 322.11 + 631.7 \div 524.38$

(e) $\frac{4}{9} \times \left(\frac{5}{8} + \frac{5}{6} - \frac{2}{3}\right) \div \frac{2}{5} \times \left(\frac{3}{4} + \frac{1}{5}\right)$

8. John paid \$9.25 for a number of packets of instant noodles costing \$1.85 for 5. How many packets of noodles did John buy? If he were to buy 40 packets of the same noodles, would \$15 be enough to pay for them?

Problem Solving

1. Which of the two fractions $\frac{1993}{1994}$ and $\frac{1995}{1996}$ is larger? Explain the reason without making the denominators of the two fractions the same or converting the fractions into decimals.

2. We can write $\frac{1}{6}$ as the sum of two fractions with numerators equal to 1. For example, $\frac{1}{6} = \frac{1}{12} + \frac{1}{12}$ and $\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$.
There are three other ways of writing $\frac{1}{6}$ in the same form. Can you find them?

3. Calculate $\left(5\frac{1}{3} + \frac{2\frac{1}{4} - 1\frac{1}{5}}{1\frac{1}{3} + 1\frac{1}{2}}\right) \times \frac{1\frac{1}{3} + 1\frac{1}{2}}{5\frac{1}{3} \times \left(1\frac{1}{3} + 1\frac{1}{2}\right) + 2\frac{1}{4} - 1\frac{1}{5}}$.

4. Evaluate $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{1995 \times 1996}$.

5. Evaluate $0.0\dot{1} + 0.1\dot{2} + 0.2\dot{3} + 0.3\dot{4} + 0.4\dot{5} + 0.5\dot{6}$.

6. $\frac{4}{7}$, $0.\dot{5}\dot{1}$, $\frac{23}{47}$, $0.5\dot{1}$, $\frac{13}{25}$ and $\frac{5}{9}$ are 6 of 8 numbers. When these numbers are arranged in ascending order the fourth number is $0.5\dot{1}$. If these numbers are arranged in descending order, which is the fourth number?

7. Identify a rule and then write the next three terms for the sequence $1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \frac{63}{32}, \dots$. What can you say about the value of the term as it continues?

8. In a boys' school, $\frac{5}{8}$ of the boys play football and $\frac{4}{7}$ play rugby. If every boy plays at least one of the two games, find the fraction of the boys who play both.

Revision Exercise I No. 1

1. Evaluate the following mentally:
   - (a) $12 + 4 + 18$ (b) $2 \times 16 \times 5$
   - (c) 32×25 (d) 66×64
2. Estimate the following mentally, giving your answer correct to 1 significant figure:
   - (a) $101 \times \sqrt{81}$
   - (b) $\sqrt[3]{26} \times 502 \div 49$
   - (c) $\sqrt{65} \times \sqrt[3]{63} \div 17$
3. Find the HCF and LCM of 12, 15 and 18.
4. Evaluate the following:
   - (a) $18 - 18 \times \frac{5}{6}$
   - (b) $\left(7\frac{1}{6} - 2\frac{7}{15}\right) \div 9\frac{2}{5}$
   - (c) $\dfrac{2}{2 + \dfrac{1}{1 + \frac{1}{2}}}$
5. Write down the next two terms in the following number sequences:
   - (a) 5, 9, 13, 17, … (b) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$
   - (c) $1 \times 3, 2 \times 4, 3 \times 5, 4 \times 6, \ldots$
6. Use a calculator to evaluate each of the following, giving your answer correct to 2 decimal places:
   - (a) $\sqrt{89} + \sqrt[3]{116} - \sqrt{41}$
   - (b) $12.79^3 \div (5.83)^2$
   - (c) $\sqrt{9.69^3 - 5.67^2}$ (d) $\dfrac{172.68 \times (12.93)^2}{13.84^3}$
7. Find the value of the following:
   - (a) $\dfrac{\sqrt{9} + \sqrt{144}}{\sqrt{25}}$
   - (b) $\dfrac{2 \times \sqrt{64} + 3 \times \sqrt[3]{8}}{\sqrt{121}}$
   - (c) $\dfrac{1}{8} + \dfrac{1}{\sqrt[3]{125}} - \dfrac{1}{4}$
   - (d) $\sqrt{\dfrac{7}{8} + \dfrac{4}{5} + \dfrac{9}{8} + \dfrac{6}{5}}$
8. (a) Round off 2 876 to the nearest hundred.
   - (b) Express $\frac{2}{3}$ as a decimal correct to 2 decimal places.
   - (c) Change 0.425 into a fraction in its simplest form.
   - (d) Which of the decimals, 2.9, 2.84 and 2.847, is the smallest?
9. A family of 2 adults and 4 children visited Sentosa during the June school holidays. An adult ticket costs \$18.90 and a child-ticket cost \$12.80 during the promotional period. If the family enjoyed the promotion price,
   - (a) what is the total cost of the tickets for the family?
   - (b) what would be the change if a \$100-note was used to purchase the tickets?
10. A teacher distributes 255 pencils, 425 sheets of graph paper and 595 sheets of writing paper equally to a group of students.
    - (a) Calculate the smallest and largest possible number of students in the group.
    - (b) Calculate the smallest and largest number of pencils, graph paper and writing paper each student can receive.

Revision Exercise I No. 2

1. Write down the next three terms in the following number sequences:
   - (a) 91, 87, 82, 76, …
   - (b) 1, 4, 3, 6, 5, …
   - (c) −48, −36, −24, −12, …
2. Evaluate each of the following:
   - (a) $3 \times 0.74 - \frac{3}{4} + 4.006$
   - (b) $\frac{2}{5} \times 2.5 + 4.5 \div 1.5$
3. $\frac{4}{9}$ of the passengers in an MRT train are men, $\frac{2}{5}$ of them are women and the rest are children. If there are 72 women, find
   - (a) the total number of passengers in the train;
   - (b) how many more men than children there are in the train.
4. (a) Find the LCM of the following:
   - (i) 56, 63 (ii) 8, 56, 140
   - (iii) 35, 45, 55

(b) Find the HCF of the following:
  (i) 32, 48, 72 **(ii)** 54, 90, 240

5. Evaluate

(a) $45 \times (6 + 2) \div 18$;
(b) $[66 - 6 \times 6 \div (6 + 6)] - 6$;
(c) $180 \div \{23 - [30 \div (3 \times 7 - 15)]\}$.

6. **(a)** Express $\frac{5}{16}$ as a decimal.
(b) Change 0.225 into a fraction in its lowest form.
(c) Express 9.906 correct to
  (i) 2 decimal places;
  (ii) 2 significant figures.

7. Cane sugar is sold at $2.88 for a 900 g pack or at $1.65 for a 500 g pack. Which is more expensive and by how much per kilogram?

8. Find the exact value of

(a) 0.28×3.5 **(b)** $17.9 - 5.67$
(c) $0.816 \div 0.4$ **(d)** $\sqrt{25} - \sqrt[3]{27}$

9. Consider the number pattern:

$$\begin{aligned} 1^2 - 2 \times 1 &= -1, \\ 2^2 - 2 \times 2 &= 0, \\ 3^2 - 2 \times 3 &= 3, \\ 4^2 - 2 \times 4 &= 8, \\ 5^2 - 2 \times 5 &= 15, \\ &\vdots \\ x^2 - 2x &= 63, \\ &\vdots \end{aligned}$$

(a) Write down the 6th line in the number pattern.
(b) Find the value of x.

10. Evaluate each of the following, giving your answer as a fraction in its lowest term:

(a) $2\frac{1}{5} + 3\frac{4}{5} \times \frac{1}{2}$ **(b)** $4\frac{1}{2} \times \frac{3}{4} + \frac{1}{4}$
(c) $5\frac{1}{3} - 2\frac{1}{2} \div \frac{1}{2}$

Revision Exercise I No. 3

1. Find the difference between $5^2 - \left(3\frac{1}{2}\right)^2$ and $\left(5 - 3\frac{1}{2}\right)^2$.

2. Find the exact value of

(a) $33.559 \div 0.037$
(b) $2 \times 2.41 \times (3.27 + 1.44)$
(c) $0.2 \times 0.3 \div 0.001\,2$

3. **(a)** Express 0.56 as a fraction in its lowest terms.
(b) Express $\frac{2}{7}$ as a decimal, giving your answer correct to 2 decimal places.

4. Evaluate the following giving each answer as a fraction in its lowest terms:

(a) $3\frac{1}{4} - 2\frac{3}{5}$ **(b)** $\frac{1}{4} \div \left(\frac{1}{3} + \frac{3}{4}\right)$
(c) $\sqrt{\frac{7}{8} \times \frac{3}{4} \div 1\frac{1}{6}}$ **(d)** $\dfrac{2\frac{3}{4} - \frac{3}{8}}{\frac{8}{9} \times \frac{3}{4} \div \frac{4}{17}}$

5. If $3\,920 = 2^x \times 5^y \times 7^z$, find the possible values of x, y and z.

6. **(a)** Express **(i)** 48 and **(ii)** 324 as a product of prime factors.
(b) Find the HCF of
  (i) 49, 63; **(ii)** 36, 54, 75.
(c) Find the LCM of
  (i) 56, 72; **(ii)** 21, 35, 42.

7. Write down the next two terms in each of the following number sequences:

(a) –9, –6, –3, 0, …
(b) 33, 44, 55, 66, …
(c) 13, 26, 39, 52, …

8. List all the prime numbers between 15 and 45.

9. Mrs Lee was given $1 200 every month for the household expenses. She used $\frac{17}{24}$ of it to buy food for the family, $\frac{5}{7}$ of the remainder for clothes and other expenses. She puts the remaining money into the bank as savings for the family. Find the amount she put into the bank.

10. **(a)** Arrange the fractions $\frac{3}{4}, \frac{5}{6}, \frac{11}{12}, \frac{8}{9}$ in ascending order.
(b) Arrange the fractions $\frac{2}{5}, \frac{1}{4}, \frac{3}{8}, \frac{1}{3}$ in descending order.

Revision Exercise I No. 4

1. Evaluate

   (a) $3.09 \div 1.03$ (b) $52.6 - 3.5 \times 1.4$

   (c) $12.3 - 2 \times (3 - 5.4)$

   (d) $3.5 \div 4 - 2.6$

2. Simplify

   (a) $\frac{11}{32} + \frac{9}{16} - \frac{5}{48}$

   (b) $\frac{7}{16} + \frac{1}{4} \times \frac{2}{3} - \frac{3}{8}$

3. Is 212 345 divisible by

   (a) 2; (b) 3; (c) 4;

   (d) 5; (e) 9; (f) 11?

4. (a) Express the following fractions as decimals:

   (i) $\frac{7}{40}$ (ii) $\frac{33}{80}$ (iii) $\frac{9}{25}$ (iv) $\frac{21}{16}$

   (b) Express each of the following decimals as a fraction in its lowest terms:

   (i) 0.066 (ii) 0.575

   (iii) 0.875 (iv) 0.437 5

5. A water pump can fill a tank at the rate of 750 litres per hour. A tank has a capacity of 9 000 litres. Find the time needed to fill $\frac{7}{15}$ of the tank.

6. The entrance fee to an amusement park during a promotional period was \$4.75 for an adult and \$2.85 per child. Mr Ong took all his little nephews to the park and paid a total of \$30.40 as entrance fee. How many children did Mr Ong bring?

7. Mrs Goh bought 4 bags of rice at \$5.28 per bag, 5 packets of biscuits at \$1.25 per packet and 2.8 kg of meat at \$7.40 per kg. Assuming that GST was absorbed by the retailer, calculate the total amount she spent. What will be the change if she paid for all these with a \$50-note?

8. Find the LCM and HCF of

   (a) 12, 16, 32 (b) 20, 24, 140.

9. Write down the next two terms in the following number sequences:

   (a) 1, 8, 27, 64, …

   (b) 4, 16, 64, 256, …

   (c) 1, 2, 4, 7, 11, …

10. (a) Which of the fractions, $\frac{1}{2}$, $\frac{3}{7}$, $\frac{2}{5}$ and $\frac{4}{9}$, is the largest?

    (b) Which of the fractions, $\frac{5}{6}$, $\frac{3}{5}$, $\frac{2}{9}$ and $\frac{3}{10}$, is the smallest?

Revision Exercise I No. 5

1. Simplify the following:

   (a) $\left(3\frac{1}{4} + 2\frac{1}{6} - 4\frac{3}{8}\right) \div 1\frac{2}{3}$

   (b) $\left(2\frac{1}{2} + \frac{1}{7}\right) \div \left(3\frac{1}{2} - 2\frac{1}{13}\right)$

   (c) $2\frac{2}{3} \times 1\frac{3}{4} + 1\frac{7}{8} \times \frac{1}{2}$

   (d) $1\frac{3}{4} + 1\frac{5}{16} \times \frac{4}{9} - 1\frac{5}{8}$

2. 3 cars leave Town A for Town B on a straight road. Car A stops every 60 m, Car B stops every 80 m and Car C stops every 180 m. After how many metres will the 3 cars stop at the same place?

3. Evaluate each of the following showing each step clearly:

   (a) $(15 - 6)^2 + (18 - 15)^3 - \sqrt[3]{64}$

   (b) $26 - 46 \times 0.5 + 2^3$

4. Evaluate

   (a) $\sqrt{6\,400}$ (b) $\sqrt{1\frac{9}{16}}$

   (c) $\sqrt[3]{2\frac{10}{27}}$ (d) $\sqrt[3]{2\frac{93}{125}}$

5. Use a calculator to evaluate

   (a) $\sqrt{88} + \sqrt[3]{117} - \sqrt{39}$;

   (b) $\frac{6\sqrt[3]{10} + 5\sqrt{17}}{9\sqrt{5}}$;

   (c) $\sqrt[3]{36.05^3 - (4.85 + 0.28)^2}$;

   giving each answer correct to 2 decimal places.

6. Write down
   (a) all the positive even integers less than 15;
   (b) all the positive odd integers less than or equal to 11;
   (c) all the prime numbers less than 25;
   (d) all the factors of 32.

7. Find the HCF and LCM of the following:
   (a) 48, 64
   (b) 54, 63, 84
   (c) 60, 72, 120
   (d) 64, 84, 112

8. Tickets to an Arts festival concert are priced at $35 each for seats at the front rows and at $15 each for seats at the back rows. There are 12 front rows with 26 seats in each row and 18 back rows with 28 seats in each row. If $\frac{5}{6}$ of all the front row seats and $\frac{6}{7}$ of the back row seats are sold, how much is the total in ticket sales?

9. (a) Express, as a fraction, the difference between the largest and the smallest fraction of the following:
$$\frac{4}{7}, \frac{5}{8} \text{ and } \frac{7}{12}.$$
   (b) Which of the fractions, $\frac{2}{5}, \frac{6}{11}, \frac{7}{15}, \frac{9}{20}$ and $\frac{13}{25}$, is the smallest?
   (c) Arrange the following numbers in ascending order:
$$0.57\dot{1}\dot{4},\ 0.571\dot{4},\ \frac{4}{7},\ 0.5\dot{7}1\dot{4}$$

10. Write down the next three terms in the following number sequences:
   (a) 2, 5, 10, 17, ...
   (b) 10, 2, 0.3, 0.04, ...
   (c) 10, 8, 6, 4, ...
   (d) 1, 4, 5, 9, 14, ...
   (e) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

CHAPTER 5

Real Numbers

In this chapter, you will learn how to

- △ use negative numbers in practical situations;
- △ perform calculations with integers;
- △ recognise rational and irrational numbers;
- △ perform calculations with rational numbers;
- △ use a calculator to find an approximate value of an irrational number.

Preliminary Problem

The main peak of the Jade Dragon Snow Mountain in China is at an altitude of 5 596 metres above sea level. Riding in a cable car, the two young ladies reached the altitude of 4 500 metres, from which they climbed another 150 metres to arrive at the Dragon Spruce Meadow. At a temperature of 10°C below freezing point, the ladies enjoyed having a photo taken with their fruit of labour.

In this chapter you will learn about negative numbers, which were first used by the Indians for accounting purposes in the 6th and 7th centuries. If we take the altitude of the Dragon Spruce Meadow to be 0 metre altitude, then we use +946 m to indicate that the peak is 946 m above the Dragon Spruce Meadow and –150 m to indicate that the cable car terminal is 150 m below the Dragon Spruce Meadow. Similarly, we use –10°C to represent a temperature that is 10°C below 0°C.

Negative Numbers

The concept of negative numbers is derived from many of our real life situations. Here are some examples.

We normally record the temperature 10°C above zero as 10°C, and correspondingly, we record the temperature 10°C below zero as –10°C.

Sea level is usually taken as zero altitude. If a bird flies 50 m above zero altitude, we use 50 m to represent how high the bird is flying above sea level.

Correspondingly, if a submarine dives in 50 m, we will use –50 m to represent its depth below sea level.

In business, if a store sells an item $3 above the cost price and another item $3 below the cost price, then we can use +$3 and –$3 to record how much profit and loss the store makes on the two transactions respectively.

Tom and Johnson walk away from the same place but in opposite directions. If Tom walked 15 m heading towards the East, and Johnson walked 12 m heading towards the West, then we can use 15 m and –12 m to record how far and in which directions they walked respectively.

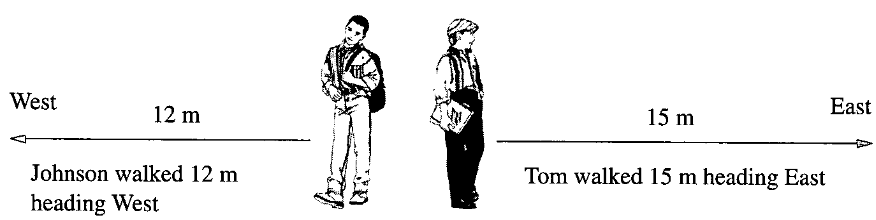


Fig. 5.1

In the above examples, 10, 50, 3 and 15 are called positive numbers, and –10, –50, –3 and –12 are called negative numbers. Can you give some other examples where both positive and negative numbers are used?

In Mathematics, numbers with the sign "–" are called negative numbers. "–" is read as "negative".

Integers

Each positive number (natural numbers) corresponds to a negative number. For example, +1 (for emphasis, we attach '+' sign to a positive number) corresponds to –1, +2 to –2, +3 to –3, and so on.

The whole numbers (zero and the natural numbers) together with the negative numbers, are called **integers**.

The integers can be displayed on the number line as shown in Fig. 5.2.

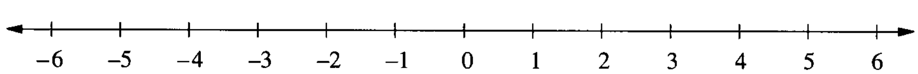


Fig. 5.2

The natural numbers 1, 2, 3, 4, ... are also called **positive integers**. The corresponding negative numbers $-1, -2, -3, -4, \ldots$ are called **negative integers**. Zero is an integer but is neither positive nor negative.

From the number line, $3 < 4$ but $-3 > -4$. Why?

Numerical Value or Absolute Value of an Integer

Positions of -2 and 2 on the number line indicate that $-2 < 2$ but they are of the same **distance** from zero. -2 and 2 are said to have the same **numerical value**, or **absolute value**.

The numerical or absolute value of a number is its distance from zero on the number line.

The numerical or absolute value of a number is **always positive**.

We write the numerical or absolute value of a number x as $|x|$. Thus, $|-2| = |2| = 2$.

Example 7

Given the numbers −8, 9 and −20,

(a) arrange the numbers in ascending order; *(b) find the numerical values of the numbers;*
(c) arrange the numerical values of the numbers in descending order.

Solution

(a) The required arrangement of the numbers is $-20, -8, 9$ since $-20 < -8 < 9$.

(b) Distances between -8, 9, -20 and zero on the number line are 8 units, 9 units, 20 units respectively.

$\therefore \quad |-8| = 8, |9| = 9, |-20| = 20.$

(c) The required arrangement of the numerical values is 20, 9, 8.

Exercise 5a

1. (a) Dennis withdrew $50 from his savings account. Is this transaction considered positive or negative?

(b) Copy and complete the following:
- **(i)** An ascent of −20 m means a descent of ________ m.
- **(ii)** A clockwise rotation of −90° means an anticlockwise rotation of ________°.
- **(iii)** Walking 9 km from East to West means walking ________ km from West to East.

2. Copy and complete the following:
- **(a)** If −6 represents 6 m below sea-level, then +30 represents ________.
- **(b)** If +40 represents depositing $40 in the bank, then a withdrawal of $35 is ________.
- **(c)** If +60° represents rotating 60° clockwise, then −30° represents ________.
- **(d)** If +45 represents a speed of 45 km/h of a car moving to the East, then −45 represents ________.

3. Suppose time is measured in years and zero stands for the year 2000.
   - **(a)** What number stands for the year 1999? **(b)** What number stands for the year 2002?
   - **(c)** What number stands for the year 1996?

4. Put the correct sign, $<$ or $>$, between each pair of temperatures below.
   (a) $-4°, 4°$ **(b)** $8°, -2°$ **(c)** $-13°, -5°$ **(d)** $-1°, 5°$ **(e)** $-3°, -4°$ **(f)** $-20°, -19°$

5. Translate each of the following into a mathematical statement using either a $<$ or a $>$ sign:
   - **(a)** A temperature of $-5°$ is colder than a temperature of $12°$.
   - **(b)** A gain of \$200 is better than a loss of \$120.
   - **(c)** A depth of 40 m below sea level is lower than a depth of 25 m below sea level.

6. Arrange the numbers in each group in ascending order.
   (a) $-2, -6, 0, -60, -90$ **(b)** $1, -1\,500, 2, -1$

7. Use a number line to illustrate each of the following:
   - **(a)** $-5, -2, 0, 5, -3$ **(b)** $\ldots, -10, -8, -6, -4, \ldots$
   - **(c)** The set of integers greater than -4 and less than 2.
   - **(d)** The set of integers between -2 and $+6$.

8. Fill in the boxes with $>$ or $<$:
   (a) $8 \square -8$ **(b)** $-11 \square -6$ **(c)** $0 \square -2$ **(d)** $(-12)^2 \square -200$
   (e) $-\sqrt{64} \square \sqrt[3]{125}$ **(f)** $\sqrt[3]{-27} \square -\sqrt{16}$

Addition of Integers

You have learnt how to add positive integers. You have also learnt how to add them using the number line. Below is an example.

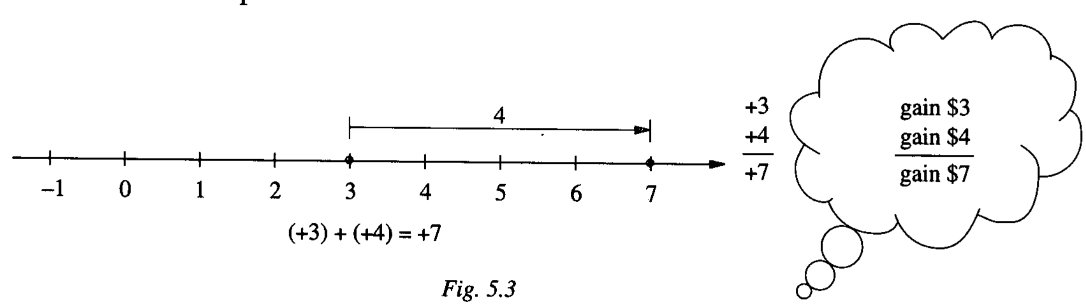


Fig. 5.3

Now, let us use the number line to add two negative integers e.g. -2 and -7. (See Fig. 5.4)

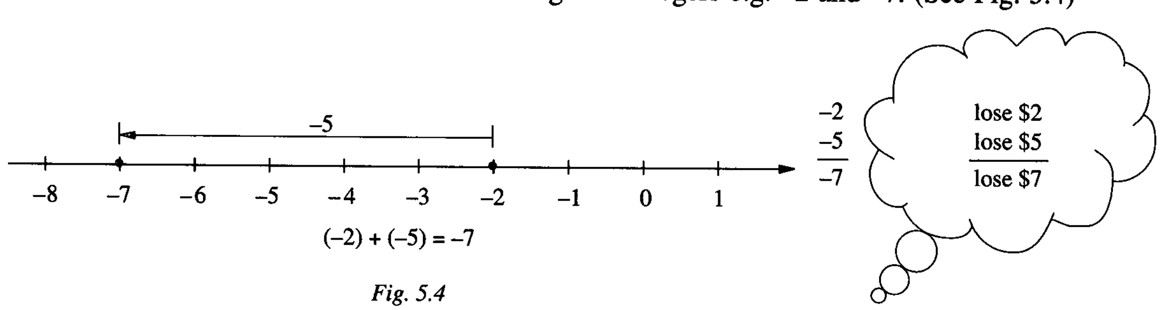


Fig. 5.4

The above discussion suggests the following rules for adding two numbers with the **same signs**.

1. To add two positive numbers, add their numerical values. The result is positive.

$$(+x) + (+y) = +(x + y)$$

To make the rule easy for remembering, you may simply remember

$$\begin{array}{ccccc} (+) & + & (+) & = & (+) \\ \text{(gain)} & + & \text{(gain)} & = & \text{(gain)} \end{array}$$

2. To add two negative numbers, add their numerical values and place a negative sign before the result.

$$(-x) + (-y) = -(x + y)$$

To make the rule easy for remembering, you may simply remember

$$\begin{array}{ccccc} (-) & + & (-) & = & (-) \\ \text{(loss)} & + & \text{(loss)} & = & \text{(loss)} \end{array}$$

Example 2

Find the sum of the following:

(a) $(-3) + (-8)$ *(b)* $(-71) + (-43)$

Solution

(a) $(-3) + (-8) = -(3 + 8) = -11$ $(|-3| = 3 \text{ and } |-8| = 8)$

(b) $(-71) + (-43) = -(71 + 43) = -114$ $(|-71| = 71 \text{ and } |-43| = 43)$

Exercise 5b

1. Use the number line to find the values of the following:

(a) $(-1) + (-4)$ **(b)** $(-2) + (-3)$ **(c)** $(-6) + (-1)$
(d) $(-4) + (-2)$ **(e)** $(+3) + (+5)$ **(f)** $(-2) + (-7)$

2. Evaluate the following:

(a) $(-5) + (-6)$ **(b)** $(-9) + (-7)$ **(c)** $(-3) + (-11)$
(d) $(+12) + (+8)$ **(e)** $(-10) + (-11)$ **(f)** $(-4) + (-7) + (-9)$
(g) $(-8) + (-17) + (-5)$

Next, let us add a positive integer and a negative integer.

Fig. 5.5 shows the addition of +6 and –2.

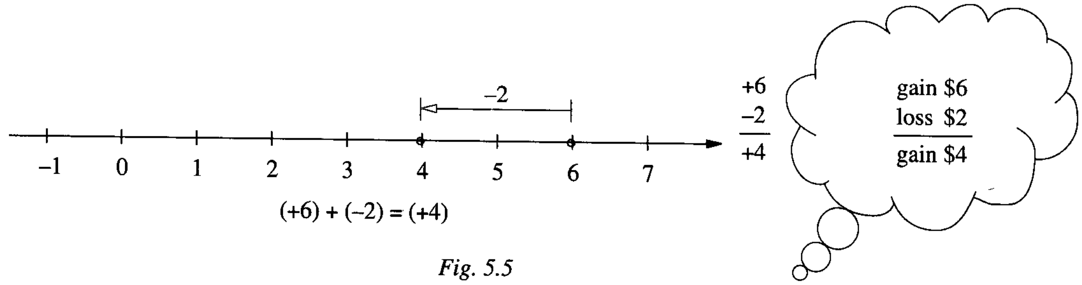


Fig. 5.5

Fig. 5.6 shows the addition of –7 and +2.

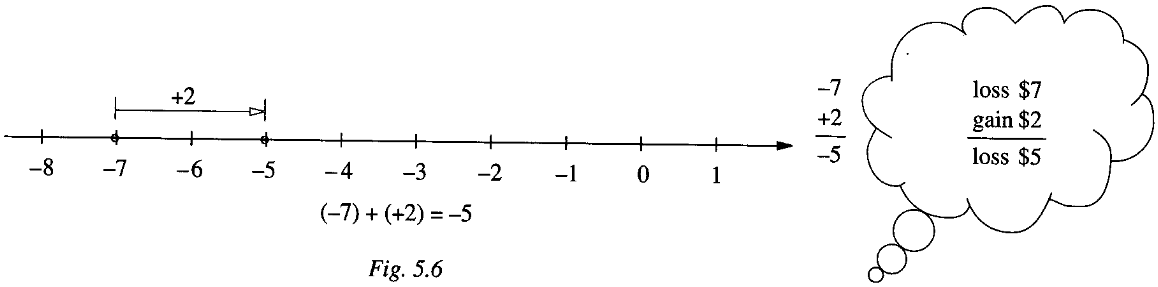


Fig. 5.6

The above discussion suggests the following rules for adding two numbers with the **different signs.**

To add a positive and a negative number, find the difference of their numerical values by subtracting the smaller numerical value from the larger numerical value. Place the sign of the number having the larger numerical value before the result.

For any $x > 0$, $y > 0$,

$$\begin{aligned} (+x) + (-y) &= +(x - y) \quad &\text{if } x > y \\ (+x) + (-y) &= -(y - x) \quad &\text{if } y > x \\ (-x) + (+y) &= -(x - y) \quad &\text{if } x > y \\ (-x) + (+y) &= +(y - x) \quad &\text{if } y > x \end{aligned}$$

You may simply remember

$$\begin{pmatrix} + \\ \text{gain} \end{pmatrix} + \begin{pmatrix} - \\ \text{loss} \end{pmatrix} = \begin{pmatrix} + \\ \text{gain} \end{pmatrix} \quad \text{if} \begin{pmatrix} + \\ \text{gain} \end{pmatrix} > \begin{pmatrix} - \\ \text{loss} \end{pmatrix}$$

$$\begin{pmatrix} + \\ \text{gain} \end{pmatrix} + \begin{pmatrix} - \\ \text{loss} \end{pmatrix} = \begin{pmatrix} - \\ \text{loss} \end{pmatrix} \quad \text{if} \begin{pmatrix} - \\ \text{loss} \end{pmatrix} > \begin{pmatrix} + \\ \text{gain} \end{pmatrix}$$

Example 3

Find the following sums: (a) $-6 + 18$ (b) $23 + (-68)$

Solution

(a) $-6 + 18 = (-6) + (+18) = +(18 - 6) = +12 = 12.$ (+18 has a larger absolute value of 18)

(b) $23 + (-68) = (+23) + (-68) = -(68 - 23) = -45.$ (–68 has a larger numerical value of 68)

Example 4

Find the following sums:

(a) [–5 + (–9)] + [16 + (–21)] *(b) –13 + 14 + (–7)*

Solution

(a) $[-5 + (-9)] + [16 + (-21)]$
$= [(-5) + (-9)] + [(+16) + (-21)]$
$= [-(5 + 9)] + [-(21 - 16)]$
$= (-14) + (-5) = -(14 + 5) = -19$

(b) $-13 + 14 + (-7)$
$= [(-13) + (+14)] + (-7)$
$= [+(14 - 13)] + (-7)$
$= (+1) + (-7) = -(7 - 1) = -6.$

Exercise 5c

1. Use the number line to evaluate the following:
(a) $-1 + 4$ **(b)** $-3 + 2$
(c) $3 + (-4)$ **(d)** $2 + (-6)$
(e) $5 + (-1)$ **(f)** $-5 + 4$

2. Calculate the following.
(a) $-5 + 13$ **(b)** $-11 + 19$
(c) $14 + (-7)$ **(d)** $23 + (-12)$
(e) $-37 + 22$ **(f)** $-45 + 19$
(g) $25 + (-66)$ **(h)** $74 + (-89)$
(i) $101 + (-200)$

3. Evaluate the following.
(a) $-7 + (-11) + 9$
(b) $-10 + 17 + (-21)$
(c) $34 + (-18) + 9$
(d) $81 + (-6) + (-62)$
(e) $51 + 14 + (-100)$
(f) $-27 + 71 + 12$
(g) $[-7 + (-14)] + [-45 + 92]$
(h) $[31 + (-48)] + [-16 + (-120)]$

4. The temperature of a piece of meat was –8°C when it was taken out of a freezer. After several minutes of warming, its temperature rose by 17°C. What is the new temperature of the piece of meat?

5. A company's profits and losses for the first quarter of a certain year are as shown below.

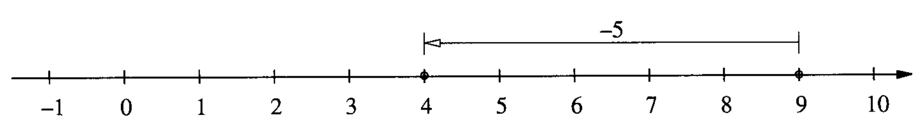
January: $6 000 (profit)
February: $2 000 (loss)
March: $5 500 (loss)

(a) Translate the company's performance for the first quarter into a mathematical statement.
(b) How much profit or loss did the company make in the first quarter?

Subtraction of Integers

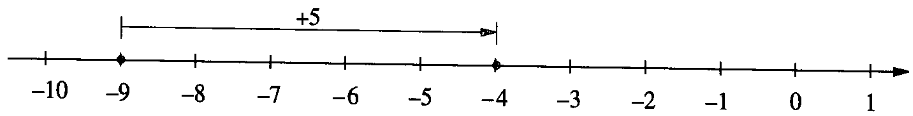
We think of subtraction (–) as the **reverse** or **opposite** of addition (+).

On the number line, to **add** 5, we move 5 units to the **right**. Thus, to **subtract** 5, we move 5 units to the **left**. Recall also that to add –5, we move 5 units to the left. Thus, to **add** –5, we move 5 units to the **left** and to **subtract** –5, we move 5 units to the **right**.



$9 - 5 = 9 + (-5) = 4$

Fig. 5.7



$(-9) - (-5) = (-9) + (+5) = -4$

Fig. 5.8

Fig. 5.7 represents both $9 - 5$ and $9 + (-5)$.

Fig. 5.8 represents both $(-9) - (-5)$ and $(-9) + (+5)$.

Hence, $9 - 5 = 9 + (-5) = 4$

$(-9) - (-5) = (-9) + (+5) = -4$

Similarly, $(-9) - 5 = (-9) + (-5) = -14$

$9 - (-5) = 9 + 5 = 14.$

The above discussion suggests the following rule for subtracting integers.

To subtract integers, change the sign of the integer being subtracted and add according to the rules for addition of integers.

$$x - y = x + (-y)$$

Example 5

Evaluate the following:

(a) $9 - 14$ *(b)* $-12 - 6$ *(c)* $-10 - (-7)$ *(d)* $4 - (-11)$

Solution

(a) $9 - 14$ is the same as $9 - (+14)$. Change $+14$ to -14. Add 9 and -14.

$\therefore\ 9 - 14 = 9 + (-14) = -(14 - 9) = -5.$

(b) $-12 - 6$ is the same as $-12 - (+6)$. Change $+6$ to -6. Add -12 and -6.

$\therefore\ -12 - 6 = -12 + (-6) = -(12 + 6) = -18.$

(c) Change -7 in $-10 - (-7)$ to $+7$. Add -10 and $+7$.

$\therefore\ -10 - (-7) = -10 + (+7) = -(10 - 7) = -3.$

(d) Change -11 in $4 - (-11)$ to $+11$ or simply 11. Add 4 and 11.

$\therefore\ 4 - (-11) = 4 + 11 = 15.$

Example 6

Do the following:

(a) $280 + (-120) - (-320) + 50$ *(b)* $-(-700) + (-500) - 130 + 70$

Solution

(a) $280 + (-120) - (-320) + 50$
$= (280 - 120) + 320 + 50$
$= 160 + 320 + 50 = 530$

(b) $-(-700) + (-500) - 130 + 70$
$= 700 - 500 - 130 + 70$
$= 200 - 130 + 70$
$= 70 + 70 = 140$

Exercise 5d

1. Use the number line to do the following.
   (a) $-2 - 3$ (b) $1 - 5$
   (c) $0 - 4$ (d) $-3 - 3$
   (e) $1 - (-3)$ (f) $-2 - (-2)$
   (g) $5 - (-1)$ (h) $-7 - (-3)$

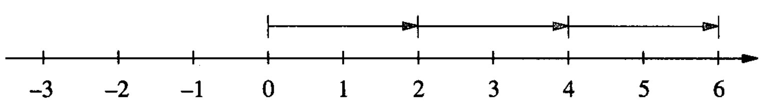
2. Evaluate the following.
   (a) $8 - 3$ (b) $2 - 9$
   (c) $-4 - 7$ (d) $-6 - 11$
   (e) $10 - (-5)$ (f) $7 - (-7)$
   (g) $-6 - (-7)$ (h) $-23 - (-34)$
   (i) $-42 - 23$ (j) $16 - (-54)$
   (k) $-127 - 143$ (l) $106 - 144$

3. Evaluate the following.
   (a) $-3 - (6 - 9)$ (b) $-4 - (2 - 5)$
   (c) $-6 - (-3) - 1$ (d) $4 - (-9) - 5$
   (e) $-2 - (-6) - 3$ (f) $5 - 7 - (-10)$
   (g) $4 - (-8) - 6 - (-11)$
   (h) $-12 - 7 - (-9) - 3$
   (i) $15 - 20 - 13 - 32$

4. Find the values of the following expressions.
   (a) $-12 - (-24) + (-36)$
   (b) $-40 + 15 - 27 + 11$
   (c) $(-5 - 6) - (15 - 5)$
   (d) $-324 + 12 - 56$
   (e) $[-2 + (-10)] - [15 + (-20)]$
   (f) $146 - (-200) + (-100) - 150$
   (g) $-176 + (-123) - (-167) + 103$
   (h) $-14 + 26 - 37 - 45 + 56 - 67 - 71$
   (i) $19 - 27 + 34 + 43 - 58 - 66 - 76 + 81$

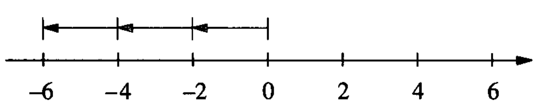
Multiplication of Integers

We already know the rule for multiplying positive integers using a number line. For example, $3 \times 2 = 6$.



$3 \times 2 = 2 + 2 + 2 = 6$.

Similarly, for multiplying a positive integer by a negative number, for example $3 \times (-2)$, we have



$3 \times (-2) = (-2) + (-2) + (-2) = -6 = -(3 \times 2)$.

We know $2 \times 3 = 3 \times 2 = 6$. Similarly we have $(-2) \times 3 = 3 \times (-2) = -(3 \times 2) = -(2 \times 3)$.

In general, if both x and y represent positive integers, then

$$x \times (-y) = -(x \times y) \text{ or } (-x) \times y = -(x \times y).$$

In other words, the product of a positive integer and a negative integer is a negative integer.

It's a Fact

Scientists have found that the lowest possible temperature is $-273°C$. The temperature is referred to as the absolute zero. The boiling points of some gases are given below:

Hydrogen	*$-253°C$*
Nitrogen	*$-196°C$*
Oxygen	*$-183°C$*

If mixtures of the above three gases are cooled to $-260°C$ and then allowed to warm, which of the gases will escape first?

Notes

Simply remember the rule for signs as:

$(+)(-) = (-)$
$(-)(+) = (-)$

Example 7

Evaluate the following:

(a) $9 \times (-4)$ (b) $(-8) \times 6$

Solution

(a) $9 \times (-4) = -(9 \times 4) = -36$ (b) $(-8) \times 6 = -(8 \times 6) = -48$

We know that the product of two positive integers is a positive integer. Will the product of two negative integers be a negative integer or a positive integer?

Let us consider $(-1) \times (-1)$ first, we know $1 \times (-1) = -1$. It is reasonable to have $(-1) \times (-1) = -(1 \times (-1)) = +1$. Similarly for $(-2) \times (-3)$, we have $(-2) \times (-3) = (-1) \times 2 \times (-1) \times 3 = -(-1) \times 2 \times 3 = +2 \times 3$.

In general, if x and y are any two positive integers, then

$$x \times y = +(x \times y) \text{ or } (-x) \times (-y) = +(x \times y).$$

In other words, the product of two negative integers is a positive integer.

Notes

Simply remember the rule for signs as:

$(+)(+) = (+)$
$(-)(-) = (+)$
$(+)(-) = (-)$
$(-)(+) = (-)$

Example 8

Evaluate the following:

(a) $(-8) \times (-3)$ (b) $(-7) \times (-13)$

Solution

(a) $(-8) \times (-3) = +(8 \times 3) = 24$

(b) $(-7) \times (-13) = +(7 \times 13) = 91$

Example 9

Evaluate the following:

(a) $(-8) \times (4) - (-5) \times 3$ (b) $(-7 + 6) \times (-11)$
(c) $9 \times (-2) \times (-2) \times 10$

Solution

Notes

$-(-15)$ means $(-1)(-15)$

(a) $(-8) \times 4 - (-5) \times (3) = -32 - (-15)$
$= -32 + 15$
$= -17$

(b) $(-7 + 6) \times (-11) = (-1) \times (-11)$
$= 11$

(c) $9 \times (-2) \times (-2) \times 10 = (-18) \times (-2) \times 10 = 36 \times 10 = 360$

Division of Integers

We know division is the inverse operation of multiplication. For example, $3 \times 2 = 6$, $6 \div 3 = 2$ and $6 \div 2 = 3$ and similarly as $3 \times (-2) = -6$ we naturally have $(-6) \div 3 = -2 = -(6 \div 3)$ and $-(6) \div (-2) = 3$.

In general, if x and y are any two positive integers, then

(1) $(-x) \div y = -(x \div y) = x \div (-y)$.
(2) $(-x) \div (-y) = +(x \div y)$.

Notes

Simply remember the rule for signs as:

$\frac{(-)}{(+)} = (-)$

$\frac{(+)}{(+)} = (+)$

Simply remember the rule for signs as:

$\frac{(+)}{(-)} = (-)$

$\frac{(-)}{(-)} = (+)$

We know that if x is a positive integer $0 \times x = 0$.

Therefore, $0 \div x = 0$. Similarly, as $0 \times (-x) = 0$, we say $0 \div (-x) = 0$.

Think what should be $-(x) \div 0$?

Can we divide an integer by 0? Consider $5 \div 0$ and assume that $5 \div 0 = x$. If it is true, by the relationship of division and multiplication then $5 = 0 \times x = 0$ but this is impossible. Therefore we say an integer divided by 0 is undefined.

It's Fact

"≠" denotes "is not equal to"

There is no answer for the division of any number by 0.

Example 10

Evaluate the following:

(a) $-100 \div (-20)$ *(b) $625 \div (-25)$*
(c) $0 \div (-14)$ *(d) $16 \div 0$*

Solution

(a) $-100 \div (-20) = 100 \div 20 = 5$

(b) $625 \div (-25) = -(625 \div 25) = -25$

(c) $0 \div (-14) = 0$

(d) $16 \div 0$ is undefined.

Exercise 5e

Evaluate the following:

(a) $3 \times (-7)$ **(b)** $(-3) \times (-2)$ **(c)** $(-4) \times 8$ **(d)** $0 \times (-5)$ **(e)** $(-20) \times (-14)$
(f) $-4 \times (0)$ **(g)** $(-4) \div 2$ **(h)** $(-122) \div (-2)$ **(i)** $(-144) \div 24$ **(j)** $275 \div (-5)$
(k) $0 \div 25$ 12 −16 **(l)** $(-16) \div (-2)$ **(m)** $0 \div (-13)$ **(n)** $480 \div (-30)$
(o) $(-3) \times (-4) \times (-8) \times (-2)$ **(p)** $(-2) \times 0 \times (-7) \times (-4) \times 5$
(q) $(-3) \times 0 \times (-8) \times (-5) \times 4$ **(r)** $(-2) \times (-7) \times (-2) \times (-1)$

Rules for Operating on Integers

1. Are subtraction and division of integers commutative?

2. Are subtraction and division of integers associative?

Addition and multiplication of integers obey the commutative law.

e.g. $2 + (-5) = -3 = (-5) + 2$ $\quad 2 \times (-5) = -10 = (-5) \times 2$

$(-3) + (-4) = -7 = (-4) + (-3)$ $\quad (-3) \times (-4) = 12 = (-4) \times (-3)$

Addition and multiplication of integers obey the associative law.

e.g. $[2 + (-5)] + 7 = -3 + 7 = 4$ $\quad [2 \times (-5)] \times 7 = -10 \times 7 = -70$

$2 + (-5 + 7) = 2 + 2 = 4$ $\quad 2 \times (-5 \times 7) = 2 \times (-35) = -70$

For integers, multiplication is distributive over addition and subtraction.

e.g.
$$-2 \times (-3 + 4) = -2$$
$$-2 \times (-3) + (-2) \times 4 = -2$$
$$\therefore \quad -2 \times (-3 + 4) = -2 \times (-3) + (-2) \times 4$$
$$(-3 - 4) \times (-2) = 14$$
$$-3 \times (-2) - 4 \times (-2) = 14$$
$$\therefore \quad (-3 - 4) \times (-2) = -3 \times (-2) - 4 \times (-2)$$

The rules for order of operations on integers are the same as those for whole numbers.

For example,

(a) $-3 \times (5 - 3) = -3 \times 2$
$= -6$ (Simplify the expression within the brackets first.)

(b) $-3 \times [-15 + (7 - 2)] = -3 \times (-15 + 5)$
$= -3 \times (-10)$
$= 30$ (Simplify the expression within the innermost pair of brackets first.)

(c) $-28 + 12 - 9 = (-28 + 12) - 9$
$= -16 - 9 = -25$ (Work from left to right.)

(d) $-125 \div 5 \times (-10) = (-125 \div 5) \times (-10)$
$= (-25) \times (-10) = 250$ (Work from left to right.)

(e) $-12 + (-3) \times 4 - 35 \div (-7) = -12 + [(-3) \times 4] - [35 \div (-7)]$
$= -12 + (-12) - (-5)$ (Do multiplication and division first.)
$= -24 + 5 = -19$

(f) $2 - (-3)^2 = 2 - (9) = -(9 - 2)$ $\quad [(-3)^2 = (-3) \times (-3) = 9]$
$= -7$

Exercise 5f

1. Copy and complete the following by filling in an appropriate operation symbol in each box:

(a) $(-5) \square 3 = 3 \square (-5) = -15$ **(b)** $23 \square (-11) = -11 \square 23 = 12$

(c) $-64 \square (-36) = -36 \square (-64) = -100$ **(d)** $-6 \square (-15) = -15 \square (-6) = 90$

2. Replace each $\square$ with an integer.

(a) $(-3) \times (\square + 8) = (-3) \times (-28) + (-3) \times 8 = \square$

(b) $(-16) \times 12 + 6 \times 12 = (-16 + 6) \times \square = \square$

(c) $(-5) \times (-4 - \square) = [(-5) \times (-4)] - [(-5) \times (-14)] = \square$

(d) $(-11) \times (8 - 9) = [(-11) \times \square] - [(-11) \times \square] = \square$

(e) $(-11 \times 10) - (-9 \times 10) = [-11 - (-9)] \times \square = \square$

(f) $[-15 - (-5)] \times (-9) = [\square \times (-9)] - [(-5) \times \square] = \square$

***3.** Evaluate the following:

(a) $5 \times 2 - (-3)$ **(b)** $5 \times [3 \times (-2) - 10]$

(c) $\sqrt{10 - 3 \times (-2)}$ **(d)** $[3 - (-2)]^3$

(e) $24 \times (-2) \times 5 \div (-6)$ **(f)** $(16 - 24) - (57 - 77) \div (-2)$

(g) $[(12 - 18) \div 3 - 5] \times (-4)$ **(h)** $160 \div (-40) - 20 \div (-5)$

(i) $2 \times (-2) + (-2) \times (-3) + 2 \times (-3)$ **(j)** $\{[(-15 + 5) \times 2 + 8] - 32 \div 8\} - (-7)$

(k) $[3 \times (-3) - 4 \times (-2) + 5 \times 2] + [-2 \times (-3) + 8 \times (-2) - 8 \times 2]$

Rational Numbers

When we add two whole numbers, the result is always a whole number. However, if we subtract one whole number from another, do we always obtain a whole number? We know that $5 - 3 = 2$ and $3 - 5 = \underbrace{-2}_{\text{not a whole number}}$.

Thus subtraction of whole numbers does not always produce a whole number. This explains the need to include negative numbers to form the set of integers.

When we multiply two integers, we always obtain an integer. However, if we divide one integer by another, do we always obtain an integer?

We know that $6 \div 2 = 3$ and $2 \div 6 = \underbrace{\frac{1}{3}}_{\text{not an integer}}$.

Thus, the division of two integers does not necessarily result in an integer. Hence, there is a need to include fractions to form a bigger set of numbers known as the set of **rational numbers**.

For Your Information

Rational Numbers *are the set of numbers that includes integers and fractions.*

The word 'rational' comes from the word 'ratio'. Thus 'irrational' means 'not ratio'.

It's a Fact

Adding or subtracting integers will give integers.

A rational number is a number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

From the definition of a rational number, we see that fractions such as $-\frac{3}{4}$, $\frac{1}{2}$ and $\frac{5}{3}$ are rational numbers. If we replace b in $\frac{a}{b}$ by the integer 1, we have $\frac{a}{b} = \frac{a}{1} = a$. Hence, the integers $6 = \frac{6}{1}$, $-5 = \frac{-5}{1}$, $-10 = \frac{-10}{1}$, $0 = \frac{0}{1}$ and so on are rational numbers.

Notes

$\frac{-3}{4}$ and $\frac{3}{-4}$ have the same value and are commonly written as $-\frac{3}{4}$.

Example 11

Evaluate the following:

(a) $2 - \frac{10}{3}$ *(b)* $3 - 5\frac{3}{4}$ *(c)* $-\frac{4}{5} \times \left(-\frac{15}{28}\right)$ *(d)* $\left(-2\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^3$

Solution

(a) $2 - \frac{10}{3} = \frac{6-10}{3} = -\frac{4}{3} = -1\frac{1}{3}$

(b) $3 - 5\frac{3}{4} = 3 - \frac{23}{4}$ $\left(\text{Write } 5\frac{3}{4} \text{ as } \frac{23}{4}.\right)$

$= \frac{12-23}{4} = -\frac{11}{4} = -2\frac{3}{4}$

Alternatively, $3 - 5\frac{3}{4} = (3-5) - \frac{3}{4}$ $\left(-5\frac{3}{4} = -5 - \frac{3}{4}\right)$

$= -2 - \frac{3}{4} = -2\frac{3}{4}$

(c) $-\frac{4}{5} \times \left(-\frac{15}{28}\right) = \frac{4}{5} \times \frac{15}{28} = \frac{3}{7}$

(d) $\left(-2\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^3 = \left(-\frac{5}{2}\right)^2 - \left(-\frac{1}{8}\right)$ $\left[\left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = -\frac{1}{8}\right]$

$= \frac{25}{4} + \frac{1}{8}$ $\left[\left(-\frac{5}{2}\right)^2 = \left(-\frac{5}{2}\right) \times \left(-\frac{5}{2}\right) = \frac{25}{4}\right]$

$= \frac{50+1}{8}$

$= \frac{51}{8} = 6\frac{3}{8}$

Example 12

Evaluate

(a) $\left(\frac{1}{5} - \frac{1}{2}\right) \div \left(-\frac{2}{3} \times \frac{1}{8}\right)$ *and (b)* $-\frac{1}{2} \div \frac{3}{18} - \left(\frac{1}{5} - \frac{7}{10}\right) \times \left(-\frac{4}{21}\right)$.

Solution

(a) $\left(\frac{1}{5} - \frac{1}{2}\right) \div \left(-\frac{2}{3} \times \frac{1}{8}\right) = \left(\frac{2-5}{10}\right) \div \left(-\frac{1}{12}\right)$

$= \frac{-3}{10} \div \left(-\frac{1}{12}\right) = \frac{3}{10} \times \frac{12}{1} = \frac{18}{5} = 3\frac{3}{5}$

(b) $-\frac{1}{2} \div \frac{3}{18} - \left(\frac{1}{5} - \frac{7}{10}\right) \times \left(-\frac{4}{21}\right) = -\frac{1}{2} \div \frac{3}{18} - \left(\frac{2-7}{10}\right) \times \left(-\frac{4}{21}\right)$

$= -\frac{1}{2} \times \frac{18}{3} - \left(-\frac{5}{10}\right) \times \left(-\frac{4}{21}\right)$

$= -3 - \frac{2}{21} = -\left(3 + \frac{2}{21}\right) = -3\frac{2}{21}$

Exercise 5g

1. Evaluate the following. Express each answer in its lowest terms.

(a) $\frac{5}{8} - \left(-\frac{1}{8}\right)$ (b) $-\frac{13}{15} + \left(-\frac{2}{15}\right)$ (c) $2\frac{1}{5} - 3\frac{3}{10}$ (d) $-7\frac{1}{5} - \left(-3\frac{3}{10}\right)$

(e) $-\frac{1}{2} + \frac{1}{4} - \frac{2}{3}$ (f) $-\frac{1}{10} + \left(-\frac{4}{5}\right) - \left(-\frac{1}{4}\right)$ (g) $-\frac{1}{2} - \frac{1}{4} - \left(-\frac{9}{10}\right)$

(h) $-2\frac{3}{7} + \frac{13}{14} - 5\frac{27}{28}$ (i) $\frac{1}{3} + \left(\frac{-1}{2}\right)^2$ (j) $2\frac{1}{2} + \left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^4$

2. Evaluate the following. Express each answer in its lowest terms.

(a) $-5 \times \left(-\frac{9}{10}\right)$ (b) $\frac{4}{9} \times \left(-\frac{3}{10}\right)$ (c) $\frac{3}{4} \div \left(\frac{-2}{3}\right)^2$ (d) $\frac{11}{12} \times \left(-\frac{23}{33} + \frac{7}{11}\right)$

(e) $-3\frac{1}{4} \times 1\frac{3}{5} \times \left(-1\frac{2}{13}\right)$ ∗(f) $\frac{3}{5} \times \left(-\frac{1}{4} - \frac{1}{6}\right) \div \left(-2\frac{1}{3} + 1\frac{1}{4}\right)$

∗(g) $\left(-1\frac{1}{6} \times 2\frac{1}{2} \times 1\frac{3}{5}\right) \div \left[1\frac{1}{4}\left(-2\frac{3}{10}\right) \times 1\frac{2}{3}\right]$

Irrational Numbers

It's Fact

π may be regarded as the most famous irrational number. We normally take π to be $\frac{22}{7}$ or 3.14.

Fig. 5.9 shows some rational numbers represented on the number line.

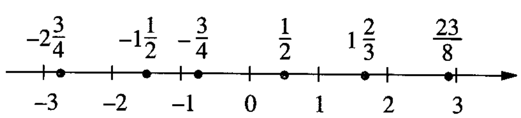


Fig. 5.9

Are there any points on the number line which do not represent rational numbers? The answer is "yes". There are points on the number line which represent numbers like $\sqrt{2}$, $\sqrt{7}$, π, $\sqrt{11}$ and so on. These numbers cannot be expressed as ratios of two integers, so they are not rational. Such numbers are called **irrational numbers**.

An irrational number cannot be expressed as a ratio of two integers.

Real Numbers

The rational numbers and the irrational numbers **completely** fill the number line and form the set of **real numbers**.

The relationship between the various sets of numbers we have discussed is shown in Fig. 5.10.

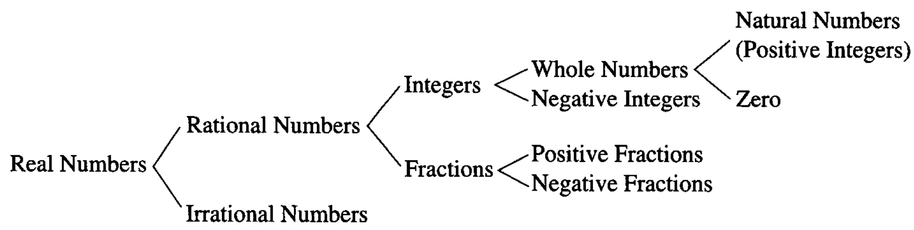
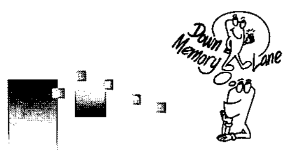


Fig. 5.10



The following are approximations of π:

Archimedas (287–212 BC): $3\frac{10}{71} < \pi < 3\frac{1}{7}$

Zu Chongzhi (5th century): $3.1415926 < \pi < 3.1415927$

Vieta (1593): 3.14159265 358979323

Ludoff von Ceulen (1615): 35 decimal places

Abraham Sharp (1717): 72 places

W. Shanks (1873): 707 places

Computer (1958): 10 000 places

Ludoff and Shanks both devoted their entire life trying to calculate π more accurately. In contrast, a modern computer (1958) took only 1 hour and 40 minutes to find the value of π correct to 10 000 decimal places. The Chuduovky brothers set the one billion mark in 1989 by computing 1 011 196 691 digits of π.

How many digits of π has the latest computer calculated?

Use of Calculators

The following keys are available on a calculator for calculations involving negative numbers and irrational numbers:

$\boxed{+/-}$ Sign Change Key

$\boxed{\pi}$ Pi Key, which is 3.141 592 654 correct to 9 decimal places.

$\boxed{\sqrt{\ }}$ Square Root Key

$\boxed{\sqrt[x]{\ }}$ Root Key

For example,

(a) to find $-65 + 47 \times (-79)$, press $\boxed{+/-}$ 65 $\boxed{+}$ 47 $\boxed{\times}$ $\boxed{+/-}$ 79 $\boxed{=}$ to get $-3\ 778$,

(b) to find $-3\frac{2}{3} \div \left(-1\frac{5}{6}\right)$, press $\boxed{+/-}$ 3 $\boxed{a^b/_c}$ 2 $\boxed{a^b/_c}$ 3 $\boxed{\div}$ $\boxed{+/-}$ 1 $\boxed{a^b/_c}$ 5 $\boxed{a^b/_c}$ 6 $\boxed{=}$ to get 2,

(c) to find $-23.4 \div \pi$, press $\boxed{+/-}$ 23.4 $\boxed{\div}$ $\boxed{\pi}$ $\boxed{=}$ to get $-7.448\ 451\ 33$.

Like π, irrational numbers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{4}$ and so on cannot be determined exactly. We can only find approximate values of these irrational numbers using a calculator.

For example, to find $\sqrt{2}$, press $\boxed{\sqrt{\ }}$ 2 $\boxed{=}$ to get 1.414 213 562,

to find $\sqrt{3}$, press $\boxed{\sqrt{\ }}$ 3 $\boxed{=}$ to get 1.732 050 808,

to find $\sqrt[3]{4}$, press 3 $\boxed{\sqrt[x]{\ }}$ 4 $\boxed{=}$ to get 1.587 401 052,

to find $\sqrt[3]{5}$, press 3 $\boxed{\sqrt[x]{\ }}$ 5 $\boxed{=}$ to get 1.709 975 947.

As your calculator might be different, please check your calculator manual before using it.

Example 13

Use a calculator to evaluate (a) $\frac{\sqrt{3}}{2}$, *(b)* $\frac{1}{\sqrt{7}-1}$ *and (c)* $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{2}-\sqrt{5}}$ *correct to 2 decimal places.*

Solution

Sequence of pressing keys:	Final display
(a) $\boxed{\sqrt{\ }}$ 3 $\boxed{\div}$ 2 $\boxed{=}$	0.866 025 403
(b) 1 $\boxed{\div}$ $\boxed{(}$ $\boxed{\sqrt{\ }}$ 7 $\boxed{-}$ 1 $\boxed{)}$ $\boxed{=}$	0.607 625 218
(c) $\boxed{\sqrt{\ }}$ 5 + $\boxed{\sqrt{\ }}$ 2 $\boxed{=}$ $\boxed{\div}$ $\boxed{(}$ $\boxed{\sqrt{\ }}$ 2 − $\boxed{\sqrt{\ }}$ 5 $\boxed{)}$ $\boxed{=}$	−4.44 151 844

The answers are thus (a) 0.87, (b) 0.61 and (c) −4.44.

Exercise 5h

1. State whether each of the following is a rational or irrational number:

(a) $\frac{1}{5}$ **(b)** -4 **(c)** 0.6
(d) $\sqrt{5}$ **(e)** $\sqrt{6}$ **(f)** $\sqrt{9}$
(g) 0 **(h)** 2π **(i)** $\sqrt[3]{8}$
(j) 3.142 **(k)** $\frac{22}{7}$ **(l)** $\sqrt{100}$
(m) $\sqrt{1\,000}$ **(n)** $\sqrt[3]{1\,000}$ **(o)** $\frac{\pi}{2}$

2. State whether each of the following statements is true or false:

(a) An integer is a rational number.
(b) A rational number is an integer.
(c) A real number is either a rational or an irrational number.
(d) The set of real numbers consists of positive numbers, negative numbers and zero.
(e) Every irrational number is a real number.
(f) A fraction is an irrational number.

3. Use a calculator to compute each of the following:

(a) $-5\,165 + 2\,844 + 8\,416$
(b) $-4\,715 \times (-78)$ **(c)** $-29\,187 \div 69$
(d) $597 \times (-57) - 4\,648 \div (-83)$
(e) $-\frac{2}{21} + \frac{1}{43}$ **(f)** $-\frac{47}{98} \times \frac{518}{77}$
(g) $-\frac{51}{23} \div \left(-\frac{48}{62}\right)$
(h) $\dfrac{-\frac{7}{19} + \frac{5}{18}}{-\frac{6}{13} - \left(-\frac{11}{7}\right)}$ **★(i)** $\dfrac{\left(-\frac{4}{7}\right)^2 - \left(-\frac{2}{5}\right)^3}{-\sqrt{\frac{64}{625}} \div \sqrt[3]{-\frac{8}{125}}}$

4. Use a calculator to evaluate the following. Give your answer correct to 2 decimal places.

(a) $\sqrt{129}$ **(b)** $\sqrt[3]{81}$
(c) $-\frac{\pi^2}{4}$ **(d)** $\sqrt{\frac{45}{8}}$
(e) $\sqrt{5} - \sqrt{3}$ **(f)** $\sqrt{14^2 + 19^2}$
(g) $\pi \times (79.67)^2$
(h) $\frac{1}{3} \times \pi \times (43.6)^2 \times 56.9$
★(i) $\dfrac{\sqrt[3]{12} - \sqrt{6}}{\sqrt{50} - \sqrt[3]{111}}$ **★(j)** $\sqrt{\dfrac{46^2 + 83^2 - 65^2}{2 \times 46 \times 83}}$

Summary

1. The set of **integers** is Z = {…, −4, −3, −2, −1, 0, 1, 2, 3, 4, …}.

2. Addition of Integers

(a) For any two negative integers $-x$ and $-y$,

$$-x + (-y) = -(x + y).$$

(b) For a positive integer x and a negative integer $-y$,

$$x + (-y) = x - y \text{ if } x > y \text{ and } x + (-y) = -(y - x) \text{ if } y > x.$$

3. **Subtraction of Integers**
For any two integers a and b, $a - b = a + (-b)$.

4. **Multiplication of Integers**
For any two positive integers x and y,
(a) $x \times (-y) = -(x \times y)$ and $(-x) \times y = -(x \times y)$,
(b) $x \times y = +(x \times y)$ and $(-x) \times (-y) = +(x \times y)$.

5. **Division of Integers**
For any two positive integers x and y,
(a) $0 \div x = 0$ and $0 \div (-x) = 0$,
(b) $(-x) \div y = -(x \div y)$ and $x \div (-y) = -(x \div y)$,
(c) $x \div y = +(x \div y)$ and $-x \div (-y) = +(x \div y)$.

6. **Rational Numbers**
A rational number is a number which can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

7. **Irrational Numbers**
An irrational number cannot be expressed as a ratio of two integers.

8. **Real Numbers**
The rational numbers and the irrational numbers together form the set of real numbers. The diagram below shows the relationship between the various sets of numbers we have discussed.

- Real Numbers
  - Rational Numbers
    - Integers
      - Whole Numbers
        - Natural Numbers (Positive Integers)
        - Zero
      - Negative Integers
    - Fractions
      - Positive Fractions
      - Negative Fractions
  - Irrational Numbers

Review Questions 5

1. Evaluate each of the following:
(a) $13 - (-54)$ (b) $(-74) - (-46)$ (c) $11 + (-33)$ (d) $-12 - 88$
(e) $[-13 + (-15)] + (-8)$ (f) $500 - (-200) - 210 - 100$
(g) $777 - (-111) - (-222) + 20$ (h) $-(-129) + (-119) - 301 + 12$

2. Evaluate each of the following:
(a) $(-4) \times (-5) \times (-6)$ (b) $(-3 + 6) \times (-4)$ (c) $(-3 - 5) \times (-3 - 4)$
(d) $(-3 - 15) \div (-6)$ (e) $4 \times (-5) \div (-2)$ (f) $-5 \times 6 - 18 \div (-3)$
(g) $2 \times (-3)^2 - 3 \times 4$ (h) $[-3 \times (-2)] \times (2 - 5)^2$ (i) $3 \times 5^2 - 2 \times (-3) \times 2$

3. Evaluate the following:
(a) $(-2)^2 - (-2 \times 3) + (2 \times 3^2)$ (b) $5 \times (-2)^3 + (-4)^2 \times (-3)$

(c) $(-4)^2 \div (-8) + 3 \times (-2)^3$

(d) $(-2)^3 - \left(-2 \times \frac{1}{2}\right) + 3(-1)^2$

(e) $4 \times 3^2 \div (-6) - (-1)^3 \times (-3)^2$

(f) $-2 \times (-2)^3 \times (-2) \times 3 + (-2) \times 3 \times (-1)^2$

★4. Evaluate each of the following and express the result in its lowest terms:

(a) $\frac{1}{5} - \left(\frac{1}{3} + \frac{1}{2}\right)$

(b) $-2\frac{3}{4} + \left(-\frac{1}{2} \times 1\frac{1}{3}\right)$

(c) $\left(-2\frac{1}{2} \div 2\frac{1}{4}\right) - \left(-\frac{2}{3}\right)$

(d) $\frac{1}{\sqrt{64}} - \sqrt{16} + \sqrt{\frac{4}{9}}$

(e) $\dfrac{\frac{2}{3} - 4\frac{1}{2}}{4\frac{1}{2} \times \left(-\frac{2}{3}\right)}$

(f) $(-5)^2 + \left(3\frac{1}{2}\right)^2 - \left(-5 + 3\frac{1}{2}\right)^2$

5. Which of the fractions $-\frac{5}{6}, -\frac{9}{11}, -\frac{11}{13}$ is the smallest?

★6. Arrange the following numbers in ascending order:

(a) $1.428, 1.42\dot{8}, 1.\dot{4}2\dot{8}, 1\frac{3}{7}$

(b) $-3\frac{7}{11}, -3.6\dot{3}, -3.63, -3.\dot{6}$

(c) $-1.43\dot{5}, -1.4\dot{3}\dot{5}, -1\frac{4}{9}, -1.\dot{4}3\dot{5}$

1. Write down any three real numbers and the integer –1. Find the sum s_1 of these four numbers. Multiply any two of the four numbers at a time. Add the six possible products to obtain the sum s_2. Now, multiply any three of the four numbers at a time. Add the four possible products to obtain the sum s_3. Multiply all the four numbers to obtain the product s_4. Calculate the sum $s_1 + s_2 + s_3 + s_4$. What do you obtain?

Repeat the above process with another three real numbers and the integer –1. What do you notice? Have a few of your friends work at the same time with other real numbers and the integer –1. Compare the results you and your friends obtain. Are you all surprised with the outcome? Discuss with your friends to obtain an explanation for the outcome.

2. If n is an integer and the numbers $n + 1$, $2n + 1$ and $8n + 1$ are divisible by 3, 5 and 7 respectively, what is the largest negative value of n?

3. Identify a rule and then write the next three terms of each of the following sequence:

(a) $-\frac{1}{1}, \frac{2}{1}, -\frac{3}{2}, -\frac{5}{3}, \frac{8}{5}, -\frac{13}{8}, \ldots$

(b) $\frac{9}{2}, \frac{11}{9}, -\frac{20}{11}, \frac{31}{20}, \frac{51}{31}, -\frac{82}{51}, \ldots$

4. The boiling point of alcohol is 82°C and the boiling point of water is 100°C. A mixture of alcohol and water is heated to a temperature of 95°C and by then there is only water left. The boiling point of liquid nitrogen is –196°C, that of xenon is –108°C and of oxygen –183°C. A mixture of liquid nitrogen, xenon and oxygen is at a temperature of –215°C. The mixture is then warmed to a temperature of –185°C. Which of the liquidified gas has evaporated?

CHAPTER 6

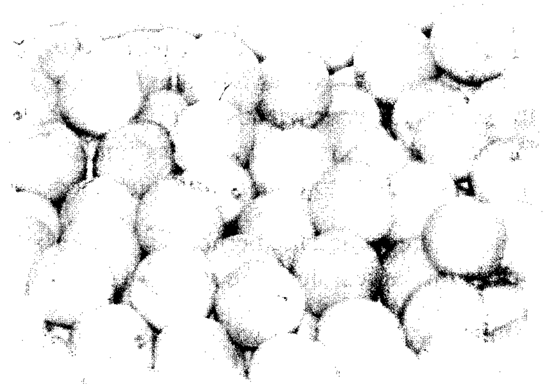
Estimation and Approximation

In this chapter, you will learn how to

- △ round off numbers and measures to a specified degree of accuracy;
- △ make estimates of numbers and measures.

Preliminary Problem

Can you estimate the number of oranges in the picture shown? Many a time we have to make estimates and approximation. A methodical way of making estimates is a far better method than making wild guesses.



Estimation and Rounding

In our daily life, we often need to use estimation when getting a precise answer is impossible, unnecessary, or inconvenient.

Estimation often involves rounding. In rounding, we may **round up**, **round down** or round off to the nearest. For example, we round up 850 to 1 000 when we budget for a trip that will cost at least \$850. At the car park, the car park charges are often rounded up. For example, you pay \$1.80 for 1 hour and 50 minutes of parking at 45 cents every half hour. At NTUC supermarkets, the bill is round down to the nearest 5 cents. For example, if your bill is \$12.03, you pay \$12.00 and if your bill is \$12.09, you pay \$12.05. In Mathematics we round off a number to the nearest. For example, to round 3.824 and 2.815 each to 2 decimal places, we round down 3.824 to 3.82 and round up 2.815 to 2.82.

Example

Estimate the cost of 8 copies of a textbook at \$10.49 each.

Solution

To estimate the cost of 8 copies of the textbook, we may choose to round \$10.49 to the nearest 10 cents and obtain the result as shown below. The actual cost is also provided for comparison.

Estimate cost:
$$\begin{array}{r} \$10.50 \\ \times \quad 8 \\ \hline \$84.00 \end{array}$$

Actual cost:
$$\begin{array}{r} \$10.49 \\ \times \quad 8 \\ \hline \$83.92 \end{array}$$

For a quick estimate, which can be done mentally, round \$10.49 to the nearest dollar and the estimate is $\$10 \times 8 = \$80$.

In-Class Activity

You may carry out this activity individually.

It is useful to estimate the total sum of a bill to avoid overpayment.

For example you can estimate the total amount of a supermarket receipt. This is done by rounding the cost of each item to the nearest 50 cents and keeping a running total mentally from the first item to the last item.

The table below is drawn up from a supermarket receipt. Observe how the estimation is done.

SALE	Actual Cost ($)	Estimated Cost ($)	Running total ($)
P BUTTER	4.50	4.50	4.50
B/SARDINE	3.50	3.50	8.00
COD FISH	1.30	1.50	9.50
M+M PLAIN	0.60	0.50	10.00
ALMOND CHO	2.85	3.00	13.00
HI LO MILK	2.85	3.00	16.00
F/P KAYA	1.60	1.50	17.50
F/SPREAD	2.85	3.00	20.50
TAPIOCA CRISP	2.40	2.50	23.00
SUBTOTAL	22.45		
TOTAL	22.45		

$\therefore$ The estimated total amount = $23.00

You notice that the estimated total is very close to the actual total.

1. Collect supermarket receipts with at least 5 items. Without seeing the actual total, estimate the total amount by rounding the cost of each item and adding them mentally. Compare your estimate with the actual total.

2. Exchange the receipts with other students and repeat the process.

3. Choose a receipt with more than 10 items. Use it to start a friendly competition among your classmates. Give a round of applause to the winner, the student who takes the shortest time to estimate the total mentally.

Example 2

Make an estimate and pick the nearest answer in each of the following cases:

		(i)	(ii)	(iii)	(iv)	(v)
(a)	$\dfrac{3\,902}{23\,839}$	0.02	0.2	2	20	200
(b)	$\dfrac{4.19 \times 0.030\,9}{0.022\,2}$	0.006	0.06	0.6	6	60
(c)	$\dfrac{52.41 \times 0.044}{0.001\,18}$	20	200	2 000	20 000	200 000
(d)	$\sqrt{990}$	10	30	100	300	1 000
(e)	$\sqrt{\dfrac{8.05 \times 24.78}{1.984}}$	0.1	1	10	100	1 000

Solution

(a) $\dfrac{3\,902}{23\,839}$ is roughly $\dfrac{4\,000}{20\,000}$, i.e., 0.2. Therefore the answer is (ii).

(b) $\dfrac{4.19 \times 0.030\,9}{0.022\,2}$ is roughly $\dfrac{4 \times 0.03}{0.02}$, i.e., 6. Therefore the answer is (iv).

(c) $\dfrac{52.41 \times 0.044}{0.001\,18}$ is roughly $\dfrac{50 \times 0.04}{0.001}$, i.e., 2 000. Therefore the answer is (iii).

(d) $\sqrt{990}$ is roughly $\sqrt{900}$, i.e., 30. Therefore, the answer is (ii).

(e) $\sqrt{\dfrac{8.05 \times 24.78}{1.984}}$ is roughly $\sqrt{\dfrac{8 \times 25}{2}}$, i.e., $\sqrt{100}$ which is equal to 10. Therefore the answer is (iii).

Exercise 6a

1. During a sale, one kilogram of fish was sold for \$4.95. Estimate how many kilograms of fish you could buy with \$20.

2. Without doing an exact calculation, determine whether you can afford all the items below if you have only \$30.

- 1 two-kilogram bottle of corn oil for \$6.95.
- 5 cans of peach at \$1.95 per can.
- 300 g of beef at \$1.02 per 100 g.
- 24 packets of recombined milk at \$2.85 for 6.

3. Given below are the prices of three brands of soap. Determine which brand is the cheapest.

Brand	No. of bars	Net weight of each bar	Price
A	3	100 g	\$1.30
B	6	100 g	\$2.35
C	4	125 g	\$2.85

4. Estimate each of the following mentally and choose the correct answer in each case:

(a) $3.14 \times 80.5 =$
(i) 2.527 7 **(ii)** 25.277
(iii) 252.77 **(iv)** 2 527.7

(b) $91.44 \div 0.36 =$
(i) 2.54 **(ii)** 25.4
(iii) 254 **(iv)** 2 540

(c) $\sqrt{917} =$
(i) 3.028 2 **(ii)** 30.282
(iii) 302.82 **(iv)** 3 028.2

(d) $\sqrt[3]{998} =$
(i) 9.993 **(ii)** 31.591
(iii) 3.159 **(iv)** 99.93

(e) $\dfrac{4\,927}{9\,835} =$
(i) 0.5 **(ii)** 5
(iii) 50 **(iv)** 500

5. Estimate each of the following and pick the closest answer in each case:

(a) $4\,831.9 \times 229.78 =$
(i) 10 000 **(ii)** 100 000
(iii) 1 000 000 **(iv)** 10 000 000

(b) $17\,913 \times 0.963 =$
(i) 180 **(ii)** 1 800
(iii) 18 000 **(iv)** 180 000

(c) $\dfrac{52.47 \times 0.083}{0.001\,98} =$
(i) 200 **(ii)** 2 000
(iii) 20 000 **(iv)** 200 000

(d) $\dfrac{2\,857 \times (0.5)^2}{0.004\,9} =$
(i) 1 500 **(ii)** 15 000
(iii) 150 000 **(iv)** 1 500 000

(e) $\sqrt{0.081\,5} =$
(i) 0.09 **(ii)** 0.03
(iii) 0.9 **(iv)** 0.3

Approximations in Measurements and Accuracy

In most of our daily situations we do not need to use highly sensitive measuring devices. How accurate our measurements are depends on what we need the information for. For example, if we use a compass to guide us from one end of the school to the other, it would not be a serious error if we are 1° off course. However, 1° off course on a journey from the earth to the moon will mean an error of 644 000 km!

Besides the errors arising from the use of different instruments, human error is another source of error. In an athletics meet, the time for the first placing of a 100-m race given by two timekeepers may be slightly different. That is why in a school athletics meet there are usually two or more timekeepers for the first few placings.

In fact, all physical measurements such as mass, length, time, area and volume can never be absolutely accurate. They are only approximations. The accuracy of a measurement depends on the measuring instruments and the person taking the measurement. Both of them can never be absolutely accurate.

> *It's a Fact*
>
> *Light travels approximately 870 000 times faster than sound and sound travels approximately 33 times faster than the best runner in the world.*

In-Class Activity

You can do this activity with a partner.

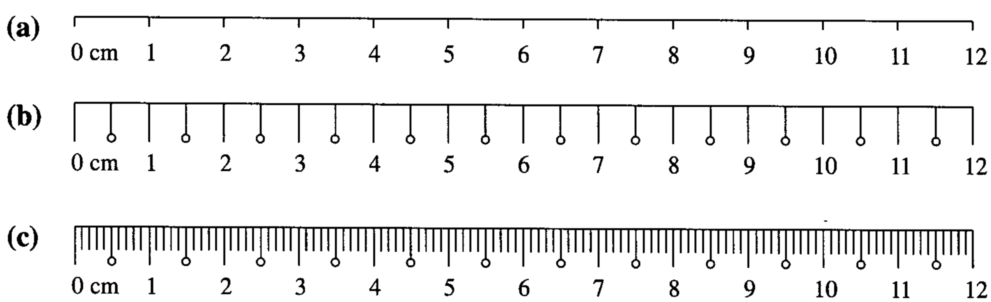


Fig. 6.1

1. Make a photocopy of Fig. 6.1 and paste it on a piece of vanguard sheet. Cut out the three strips and use them as rulers for this activity.

2.

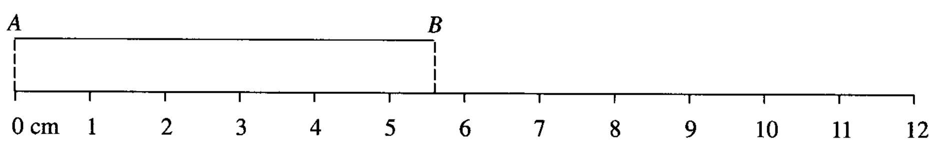


Fig. 6.2

In Fig. 6.2, we say that the length of *AB* is **6 cm to the nearest cm** as the end point is nearer to 6 cm.

(a) Draw three lines of lengths between 5 cm and 10 cm each.
(b) Use the first ruler (Fig. 6.1(a)) to measure the lengths of the line drawn by the other.
(c) What are the lengths of the lines to the nearest cm?

3. Fig. 6.3 shows the same line *AB* measured using the second ruler (Fig. 6.1(b)).

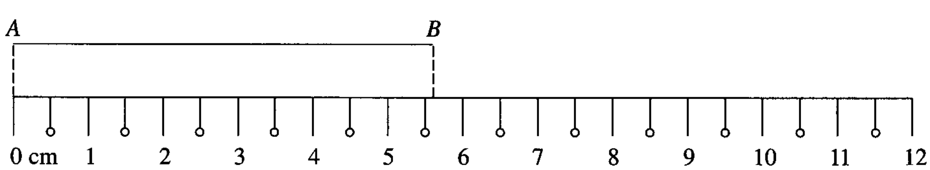


Fig. 6.3

Notice that the end point B is nearer to 5.5 cm. We say that the length of AB is **5.5 cm to the nearest 0.5 cm**. Do you agree that we achieve a greater accuracy by using the second ruler to measure the length of AB.

(a) Use your second ruler to measure the lengths of your lines.
(b) What are their lengths to the nearest 0.5 cm?

4. If we divide each centimetre (cm) into 10 equal parts like the third ruler (Fig. 6.1(c)), we can measure AB even more accurately.

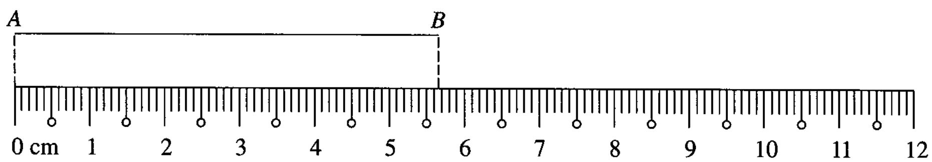


Fig. 6.4

Fig. 6.4 shows that the end point B lies between the sixth-tenth and seventh-tenth of a centimetre mark. However, it is nearer to 5.6 cm. We say that the length of AB is **5.6 cm to the nearest $\frac{1}{10}$ or 0.1 cm**.

(a) Use your third ruler to measure the lengths of the lines you have drawn.
(b) What are their lengths to the nearest 0.1 cm?

Rounding of Decimals (Revision)

The rules for rounding a decimal to a required number of decimal places have been dealt with in Chapter 4. The following examples serve as a revision.

For example,
(a) 3.128 cm $\approx$ 3.1 cm, rounded off to the nearest 0.1 cm.
(b) 2.765 cm $\approx$ 2.8 cm, rounded off to the nearest 0.1 cm.
(c) 45.7 kg $\approx$ 46 kg, rounded off to the nearest kg.
(d) 12.45 kg $\approx$ 12 kg, rounded off to the nearest kg.
(e) 42.449 kg $\approx$ 42.4 kg, rounded off to the nearest 0.1 kg.
(f) 528 g $\approx$ 530 g, rounded off to the nearest 10 g.
(g) 21.85 cm $\approx$ 22 cm, rounded off to the nearest cm.
(h) 15.22 s $\approx$ 15.2 s, rounded off to one decimal place.

Note: If we round 5.01 to the nearest tenth, the answer should be 5.0 and not 5, because 5 can be interpreted as an approximation of 5.01 to the nearest whole number but not to the nearest 0.1. Although 5.0 and 5 are equal as numbers they indicate different degrees of accuracy.

Accuracy and Significant Figures

The accuracy of a measurement is indicated by the number of figures or digits, called **significant figures or digits**, that it contains. Suppose that a line PQ, with an actual length of 5.01 cm, is measured using ruler 1 (Fig. 6.1(a)) and ruler 3 (Fig. 6.1(c)) respectively. The measurements of PQ would be 5 cm (to the nearest cm) and 5.0 (to the nearest 0.1 cm) respectively.

We say that

5 cm is the length of PQ (measured) correct to **one** significant figure

and 5.0 cm is the length of PQ correct to **two** significant figures.

Significant Figures and Estimation

Estimation can be done by rounding a number to a specified decimal place. It can also be done by rounding to a specified number of significant figures. Consider a HDB flat priced at $485 500. To get a quick idea of the affordability of the flat, we may estimate the price by rounding it to $500 000, an approximation of $485 500 to one significant figure.

Rounding a Number to a Given Number of Significant Figures

We have the following **rules** for rounding a number to a given number of significant figures:

1. Consider the place values of the number from left to right, starting with the first non-zero figure. Include one extra figure for consideration.
2. If the extra figure is less than 5, drop the extra figure and all other following figures to the right. Use zeros to keep the place values if necessary. (e.g. correct to 4 significant figures, 2.040 45 = 2.040 and not 2.04; correct to 3 significant figures, 0.400 127 = 0.400 and not 0.4.)
3. If the extra figure is 5 or more, add 1 to the previous figure before dropping the extra figure and all other following figures. Use zeros to keep the place values if necessary.

To find how many number of significant figures there are in a number we have the following rules:

1. The following figures in a number are significant:
   - **(a)** All non-zero figures (e.g. 7.12 has three significant figures).
   - **(b)** All zeros between significant figures (e.g. 2003 has four significant figures).
   - **(c)** All zeros at the end of a decimal (e.g. 22.300 has five significant figures).
2. The following figures in a number are not significant:
   - **(a)** All zeros at the beginning of a decimal less than 1 (e.g. 0.000 325 has three significant figures).
   - **(b)** All zeros at the end of a number may or may not be significant. It depends on how the estimation is made (e.g. in 0.020 25 correct to 1 significant number, 0.02, the zeros are not significant).

For example,

1. 0.060 52 = 0.06 correct to 1 significant figure.
2. 0.065 2 = 0.065 correct to 2 significant figures.
3. 0.003 824 = 0.004 correct to 1 significant figure.
4. 0.003 824 = 0.003 8 correct to 2 significant figures.
5. 0.003 824 = 0.003 82 correct to 3 significant figures.

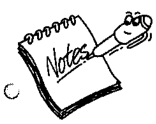
The zeros in 1–5 are not significant.

6. 2.005 7 = 2.0 correct to 2 significant figures.
7. 2.005 7 = 2.01 correct to 3 significant figures.
8. 2.005 7 = 2.006 correct to 4 significant figures.

The zeros in 6–8 are significant.

9. 0.834 = 0.83 correct to 2 significant figures.
10. 0.600 27 = 0.600 correct to 3 significant figures.

11. 0.059 002 = 0.059 00 correct to 4 significant figures.
12. 4 276 = 4 000 correct to 1 significant figure.
13. 4 276 = 4 300 correct to 2 significant figures.
14. 4 276 = 4 280 correct to 3 significant figures.
15. 40 004 = 40 000 correct to 1 significant figure.
(if the estimation is made, correct to the nearest 10 000)
16. 40 004 = 40 000 correct to 2 significant figures.
(if the estimation is made, correct to the nearest 1 000)
17. 40 004 = 40 000 correct to 3 significant figures.
(if the estimation is made, correct to the nearest 100)
18. 40 004 = 40 000 correct to 4 significant figures.
(if the estimation is made, correct to the nearest 10)



When we are asked to estimate to 1 significant figure, we normally estimate to 2 significant figures in the working and then round off to 1 significant figure for the final answer. Similarly, for 2 significant figures, work with 3 significant figures before rounding off and so on.

Example 3

Estimate the following, giving your answers correct to 1 significant figure:

(a) $\dfrac{74.97}{2.52}$ *(b)* $\sqrt{\dfrac{12.02 \times 24.99}{3.001}}$

Solution

(a) $\dfrac{74.97}{2.52} \approx \dfrac{75}{2.5} = \dfrac{750}{25} = 30$

$\therefore \dfrac{74.97}{2.52} \approx 30$ (correct to 1 significant figure)

(b) $\sqrt{\dfrac{12.02 \times 24.99}{3.001}} \approx \sqrt{\dfrac{12 \times 25}{3}} = \sqrt{100} = 10$

$\therefore \sqrt{\dfrac{12.02 \times 24.99}{3.001}} \approx 10$ (correct to 1 significant figure)

Example 4

(a) Express 0.061 54 correct to
(i) three decimal places and state the number of significant figures in the result;
(ii) three significant figures and state the number of decimal places in the result.

(b) Express 12.205 7 correct to
(i) two decimal places and state the number of significant figures in the result;
(ii) five significant figures and state the number of decimal places in the result.

Solution

(a) (i) 0.061 54 = 0.062 (correct to three decimal places)
0.062 has two significant figures.
(ii) 0.061 54 = 0.061 5 (correct to three significant figures)
0.061 5 has four decimal places.

(b) (i) 12.205 7 = 12.21 (correct to two decimal places)
12.21 has four significant figures.

(ii) 12.205 7 = 12.206 (correct to five significant figures)
12.206 has three decimal places.

Exercise 6b

1. Round off the following:

(a) 456 g to the nearest 10 g
(b) 722 g to the nearest 100 g
(c) 3.27 cm to the nearest cm
(d) 123.452 cm to 1 decimal place
(e) 18.2 to the nearest whole number
(f) 31.256 m to the nearest 10 m
(g) 12.35 cm to the nearest 0.1 cm
(h) 4 325 pupils to the nearest 100 pupils
(i) 845 km to the nearest 10 km
(j) 22.58 mm to the nearest $\frac{1}{10}$ mm

2. State the number of significant figures in each of the following:

(a) 15.0	**(b)** 27.3
(c) 30 756	**(d)** 4.02
(e) 63	**(f)** 9.5
(g) 30.08	**(h)** 48.20
(i) 6 000 000	**(j)** 0.028 3
(k) 0.74	**(l)** 0.000 65

3. Express the following numbers correct to the number of significant figures indicated within the brackets:

(a)	3.084	(2)	**(b)**	1.483 56	(4)
(c)	0.003 46	(1)	**(d)**	16.047	(1)
(e)	3.141 59	(2)	**(f)**	0.574 38	(2)
(g)	0.056 78	(3)	**(h)**	217.006	(5)
(i)	3.598	(3)	**(j)**	15.703 7	(4)
(k)	0.003 56	(2)	**(l)**	5.98	(2)
(m)	0.035 1	(2)	**(n)**	17.97	(3)
(o)	8.353	(2)	**(p)**	120.408	(5)
(q)	4.826	(3)	**(r)**	12.096	(2)
(s)	0.049 72	(2)	**(t)**	0.080 46	(2)
(u)	0.010 10	(3)	**(v)**	0.103 49	(3)

4. (a) Estimate the value of $\frac{7.94}{2.01}$ correct to 1 significant figure.
(b) Use your result to estimate the value of $\frac{79\,400}{0.000\,201}$.

5. Estimate the value of $\frac{21.83 \times 0.498}{220.1}$, giving your answer correct to 1 significant figure.

6. Use a calculator to evaluate the following, giving your answer correct to 3 significant figures.

(a) $(0.218\,7)^3$ **(b)** $\sqrt[3]{0.086\,42}$
(c) $\sqrt{25.6^2 + 17.89^2}$
(d) $\sqrt{\frac{1\,976 \times (14.98)^2}{(59.87)^2}}$

7. Express 28.136 275 correct to two significant figures. How many decimal places are there in the result?

8. Express 0.211 087 94 correct to three significant figures. Write down the number of decimal places in the result.

9. Express 0.008 345 7 correct to four decimal places. State the number of significant figures in the answer.

10. Express 117.964 8 correct to two decimal places. How many significant figures are there in the answer?

Summary

Rules for rounding a number to a given number of significant figures:

(a) Count the given number of significant figures from left to right, starting with the first non-zero figure. Include one extra figure for consideration.

(b) If the extra figure is less than 5, drop the extra figure and all other following figures. Use zeros to keep the place value if necessary.

(c) If the extra figure is 5 or more, add 1 to the previous figure before dropping the extra figure and all other following figures. Use zeros to keep the place value if necessary.

Rules for determining the number of significant figures:

(a) The following figures in a number are significant:
- **(i)** All non-zero figures.
- **(ii)** All zeros between significant figures.
- **(iii)** All zeros at the end of a decimal.

(b) The following figures in a number are not significant:
- **(i)** All zeros at the beginning of a decimal less than 1.
- **(ii)** All zeros at the end of a whole number may or may not be significant. It depends on how the estimation is made.

Review Questions 6

1. Estimate each of the following mentally and pick the correct answer in each case:

(a) $4.07 \times 6.998 =$

(i) 0.284 8	**(ii)** 2.848
(iii) 28.48	**(iv)** 284.8

(b) $29.7 \div 5.03 =$

(i) 0.590 5	**(ii)** 5.905
(iii) 59.05	**(iv)** 590.5

(c) $\sqrt{899} + \sqrt[3]{1\,029} =$

(i) 4.008	**(ii)** 40.08
(iii) 400.8	**(iv)** 4 008

(d) $\dfrac{6.003 - 5.12 \times 0.992}{5.97 \div 3.103} =$

(i) 0.048	**(ii)** 0.48
(iii) 4.8	**(iv)** 48

(c) $\dfrac{45.117 \div 8.97}{4.983 \times 0.002\,104} =$

(i) 50	**(ii)** 500
(iii) 5 000	**(iv)** 50 000

(d) $\dfrac{3.989\,7 \times 2.100\,5}{(0.019\,9)^3} =$

(i) 1 000 000	**(ii)** 100 000
(iii) 10 000	**(iv)** 1 000

2. Estimate each of the following and pick the closest answer in each case:

(a) $4\,962.8 \times 312.93 =$

(i) 1 500 000	**(ii)** 150 000
(iii) 15 000	**(iv)** 1 500

(b) $\dfrac{9.103\,4 - 7.990\,2}{(10.012\,3)^2} =$

(i) 0.001	**(ii)** 0.01
(iii) 0.1	**(iv)** 1.0

3. Express the following correct to the number of decimal places or significant figures indicated within the brackets:

(a)	0.085 67	(3 decimal places)
(b)	0.085 67	(3 significant figures)
(c)	5.096	(3 significant figures)
(d)	726 990	(2 significant figures)
(e)	0.058 76	(2 decimal places)
(f)	0.058 76	(1 significant figure)
(g)	0.006 138	(3 significant figures)
(h)	0.003 549	(3 decimal places)

4. Estimate, correct to 1 significant figure, the value of 52.976 03 − 31.321 86.

5. Estimate, correct to 1 significant figure, the value of

(a) $\dfrac{79.81}{1.62}$; **(b)** $\dfrac{66.4}{0.031\,9}$.

Problem Solving

1. Find the approximate value of $\frac{31.98 \div 8.03}{48.109 - 29.989 \times 0.995}$, giving your answer correct to 1 significant figure.

2. Estimate the value of $20.02 \times 9.99 - 6.112 \times \frac{16.027}{(1.977)^3}$ correct to 2 significant figures.

3. Estimate, correct to 1 significant figure, the value of $\sqrt{136.05 - (2.985 + 7.001)^2}$.

4. A Singaporean has assets of \$5 billion (\$5 000 000 000). If he spends \$10 every second, how long will it take for him to spend all his money?

CHAPTER 7

Basic Algebra

In this chapter, you will learn how to

- △ use letters to represent numbers;
- △ express basic arithmetical processes algebraically;
- △ substitute numbers for letters in formulae and expressions;
- △ manipulate simple algebraic expressions.

Preliminary Problem

Petrol costs \$1.21 per litre.

How much will it cost to fill up a full tank of petrol if the tank can hold up to 45 litres of petrol?

How much will it cost to fill up 12 full tanks if each tank can hold up to 50 litres of petrol?

How much will it cost to fill up x full tanks if each tank can hold up to y litres of petrol?

Algebraic Expressions

Consider the following statements:

(a) I think of a number and when I add 5 to it the result is 9.

(b) I would like to buy 5 mangoes and 4 durians. If the price of a durian is twice that of a mango, I would have to pay $18.

(c) Twice my age plus three times the age of my mother will add to 120 years.

Translate the above statements using mathematical statements:

(a) We shall use $\square$ to represent the number I think of. The statement can be written simply as $\square + 5 = 9$.

We can make the above mathematical statement correct by filling in the correct number in $\square$.

(b) We can use $\triangle$ to represent the price of a mango, then the price of a durian will be $(2\ \triangle)$. We write $5 \times \triangle + 4 \times (2\ \triangle) = \$18$.

(c) We let $\square$ to represent my age and $\triangle$ to represent the age of my mother. We write $2 \times \square + 3 \times \triangle = 120$.

Example 1

Use mathematical symbols, $\square$ and/or $\triangle$, to rewrite the following statements:

(a) I think of a number, and if I subtract 3 from it the result is 15.
(b) I think of two numbers; twice the first number when added to the second number is less than 50.
(c) I think of two numbers. Adding 5 to the first number and dividing the result by the second number gives 5.

Solution

(a) $\square - 3 = 15$ (b) $2 \times \square + \triangle < 50$ (c) $(\square + 5) \div \triangle = 5$

Example 2

Use mathematical symbols $\square$ and/or $\triangle$ to rewrite the following statements:

(a) I bought 5 exercise books and the total cost is $2.75.
(b) Aunty Tan gave each of the 5 boys some sweets and each of the 6 girls 2 more sweets than the boys. Altogether she gave away 100 sweets.
(c) Mrs Kunna bought 3 blouses and 4 skirts for her daughter. She paid a total of $245.

Solution

(a) $5 \times \square = \$2.75$
(b) $5 \times \square + 6 \times (\square + 2) = 100$
(c) $3 \times \square + 4 \times \triangle = \$245$

Exercise 7a

1. Use □ and/or △ and mathematical symbols to rewrite the following statements:

(a) I think of a number, multiply it by 7 and the result is 91.
(b) I think of a number, subtract 5 from it and multiply the result by 4. The final answer is 28.
(c) I think of a number, multiply it by 5 and add 4 to the result. The final answer is 19.
(d) I think of a number, subtract 2 from it and multiply the result by 3 to give a final result of 12.
(e) I think of a number and add 15 to it. The result multiplied by another number gives a final answer of 84.
(f) I buy 2 toy cars and the total cost is $32.
(g) Amy bought a meal and two soft toys from a fast-food outlet for a total cost of $8.40.
(h) Twice Jeffrey's age and five times Anita's age will add up to 47 years.
(i) The Tan family bought 5 music CDs and 3 VCDs for a total cost of $98.
(j) The total labour cost to transport a total of 320 chairs and 450 tables is $128.
(k) An apprentice painter can work at only $\frac{3}{4}$ the rate of a master painter. Two apprentice painters and 4 master painters together can paint a block of flats in 6 days.
(l) A company allocates a budget of not more than $3 000 per month to maintain a car and two lorries.
(m) It takes 2 hours and 40 minutes for Jason to complete 3 exercises for Maths and 2 exercises for Geography.

Fundamental Algebra

We used different shapes to represent numbers in our earlier discussion. As the number of unknowns increases we may find it inconvenient to use many different shapes. It will be easier if we use letters to represent these unknowns.

In algebra, we use numbers as well as letters such as A, B, C, a, b and c to stand for any numerical values we choose. Algebra is an extension of arithmetic.

Notations in Algebra

The signs +, −, ×, ÷, =, etc, are used in algebra as in arithmetic.

1. In arithmetic, $5 + 4 = 9$ means that the sum of 5 and 4 is equal to 9. In algebra, $x + y = z$ means that the sum of two numbers represented by x and y is equal to the number represented by z.

   If $x = 4$, $y = 3$, then z stands for 7.
   If $x = 4$, $z = 8$, then y stands for 4.

   Similarly, $x - y = z$ means that the difference between two numbers represented by the letters x and y is equal to the number represented by the letter z.

   Note: if $x + y = z$ stands for 9, then x and y may stand for any pair of numbers whose sum is 9, for example, 4 and 5, 7 and 2, and 1.7 and 7.3.

2. In arithmetic, $5 \times 4 = 20$ means that the product of 5 and 4 is 20. In algebra $a \times b = c$ means that the product of two numbers a and b is equal to the number represented by the letter c. Normally we write $a \times b = c$ simply as $ab = c$, omitting the multiplication sign. Similarly $x = a \times b \times c$ is simply written as $x = abc$.

   Note: (1) When we multiply y by 3, we normally write it as $3y$ and **not** $y3$.
   (2) When we multiply x by 1, we write it simply as x and **not** $1x$ or $x1$.

   Can we write 5×4 simply as 54?

3. In arithmetic, $36 \div 4 = 9$ means that 36 divided by 4 gives 9. In algebra $a \div b = c$ means that the number represented by a is divided by the number represented by the letter b to give the result represented by the letter c. Also $a \div b$ is normally written as $\frac{a}{b}$.

4. In arithmetic, $3 \times 3 \times 3 \times 3$ may be written as 3^4, and 7^5 means $7 \times 7 \times 7 \times 7 \times 7$. In algebra $a \times a \times a \times a$ is written as a^4 and y^5 means $y \times y \times y \times y \times y$.

Example 3

Write an algebraic expression for each of the following:

(a) Subtract $2x$ from y and multiply the difference by z.
(b) Divide the sum of x and y by the difference when x is subtracted from y.

Solution

(a) When $2x$ is subtracted from y, we have $y - 2x$. Multiplying the difference by z gives the result $(y - 2x)z$.

(b) The sum of x and y is $(x + y)$. When x is subtracted from y, the difference is $(y - x)$. The final result is $\frac{x+y}{y-x}$.

Example 4

Is $3a^2 = (3a)^2$? Explain your answer.

Solution

$3a^2$ means the product of 3 and a^2 i.e., $3 \times a \times a$.
$(3a)^2$ means the product of $3a$ and $3a$ i.e., $3a \times 3a$.

$$3a \times 3a = 3 \times a \times 3 \times a$$
$$= 9a^2$$

Hence,
$$3a^2 \neq (3a)^2$$

Polynomials, Variables, Coefficients and Constant Terms

An algebraic expression involves numbers, and operational signs such as +, –, × and ÷. The + and – signs in an algebraic expression separate it into terms.

For example,	$8y + 7z$	consists of 2 terms,
	$7x^2 - 2xy + 7y^2$	consists of 3 terms,
while	ab	consists of only 1 term.

A **polynomial** is an algebraic expression consisting of one or more terms. For example, the expressions $x + 5$, $x + 3y$, $5x + 7y$ and $x^2 + 4x - 3$ are polynomials.

Consider the algebraic expressions: (a) $4x$ (b) $5x + 7$

The polynomial in (a) will take on different values for different values of x given.

For example, when $x = 2$, $4x = 4 \times 2 = 8$
$x = 3$, $4x = 4 \times 3 = 12$
$x = 5$, $4x = 4 \times 5 = 20$ and so on.

Since the value of $4x$ varies according to the value given to x, x is called a **variable**.

In the term $4x$, the constant factor 4 is called the **coefficient** of the term. Thus, the coefficient of x in $5x$ is 5, the coefficient of xy in $7xy$ is 7, the coefficient of abc in $23abc$ is 23, and so on.

What is the coefficient of x^2 in $25x^2$?

The polynomial in (b) will take on different values for different values of x given.

For example, when $x = 2$, $5x + 7 = 5 \times 2 + 7 = 17$
$x = 3$, $5x + 7 = 5 \times 3 + 7 = 22$
$x = 5$, $5x + 7 = 5 \times 5 + 7 = 32$ and so on.

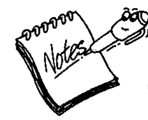
Notice that the value of the polynomial depends on x; the numeral 7 always remains unchanged. We call this numeral a **constant term** or simply a **constant**.

Each polynomial has a **degree** which is given by the **highest power** of the variable.

For example, in $8x^3 - 7x^2 + 5x + 3$, the highest power of the variable x is 3. Therefore, the degree of the polynomial $8x^3 - 7x^2 + 5x + 3$ is 3.

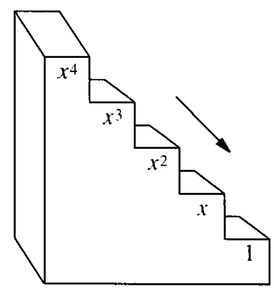
What is the degree of the polynomial of $5x^2 - 3x^3 + 5x^4 - 7x + 2$?

Usually, a polynomial is expressed such that the degrees of the terms appear in **descending order**, for example, $3x^3 + 2x^2 - 4x + 7$. Sometimes it is expressed with the degrees of the terms appearing in **ascending order**, for example, $3 + 5x - 7x^2 + 8x^3$.

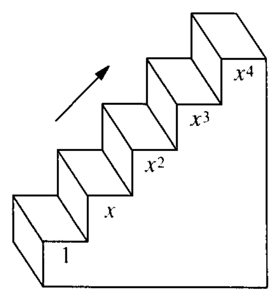


1. A ***variable*** *is a changing quantity, usually denoted by a letter in algebraic equations and expressions, that might have any one of a range of possible values.*

2. A ***coefficient*** *is a multiplying factor. In $2x^2 + 3x = 0$, x is a variable, and the coefficient of x^2 is 2 and that of x is 3.*



Descending



Ascending

Example 5

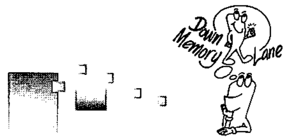
Write an algebraic expression for each of the following:

(a) Add 2x to 3y. *(b) Subtract 5x from 12.* *(c) Multiply u by 3v.*
(d) Divide 3k by 7x. *(e) Subtract the product of x and 3y from the sum of p and q.*

Solution

(a) $2x + 3y$ (b) $12 - 5x$ (c) $u \times 3v = 3uv$

(d) $3k \div 7x = \dfrac{3k}{7x}$ (e) $p + q - x \times 3y = p + q - 3xy$



In 820 A.D., the Muslim mathematician, Al-Khwarizmi, wrote the algebraic text entitled Al-Jabr Wa-Al-Muqabala (the science of cancellation and reduction). A Latin translation of this text became known in Europe under the title Al-Jabr. Thus the Arabic word for reduction, al-Jabr, became the word algebra. Al-Khwarizmi was known as the "Father of Algebra".

Example 6

Ali is three times as old as Beng and Beng is five years older than Chandra. If Beng is x years old, write algebraic expressions for each of the following:

(a) Ali's age today. *(b) Chandra's age today.*
(c) Ali's age in 5 years' time.
(d) The sum of Ali's and Beng's ages in 2 years' time.
(e) The sum of Beng's and Chandra's ages 4 years ago.

Solution

(a) Ali's age is $3x$ years old.
(b) Chandra's age is $(x - 5)$ years old.
(c) Ali will be $(3x + 5)$ years old.
(d) In 2 years' time, Ali will be $(3x + 2)$ years old and Beng will be $(x + 2)$ years old. The sum of their ages is $(3x + 2) + (x + 2) = (4x + 4)$.
(e) Four years ago, Beng was $(x - 4)$ years old and Chandra was $(x - 5 - 4)$, i.e., $(x - 9)$ years old. The sum of their ages then was $(x - 4) + (x - 9) = (2x - 13)$ years old.

Example 7

Write an algebraic expression for each of the following:

(a) The sum of three consecutive integers, of which x is the middle integer.
(b) The product of two consecutive odd integers, of which x is the smaller integer.
(c) The total cost of x 10¢ stamps and 25 y¢ stamps.

Solution

(a) The integer after x is $(x + 1)$ and the one before x is $(x - 1)$. The sum of the three consecutive integers is $(x - 1) + x + (x + 1) = 3x$.
(b) The greater of the two consecutive odd integers is $(x + 2)$. The product of the two integers is $x \times (x + 2)$ or $x(x + 2)$.
(c) The cost of x 10¢ stamps = $(x \times 10)$¢ = $10x$¢.
The cost of 25 y¢ stamps = $(25 \times y)$¢ = $25y$¢.
The total cost = $(10x + 25y)$¢.

Example 8

Evaluate $2x + 5y$ when

(a) $x = 3$ and $y = 5$; *(b) $x = 2$ and $y = -1$;* *(c) $x = -2$ and $y = 3$.*

Solution

(a) When $x = 3$ and $y = 5$, $2x + 5y = 2(3) + 5(5) = 6 + 25 = 31$.
(b) When $x = 2$ and $y = -1$, $2x + 5y = 2(2) + 5(-1) = 4 - 5 = -1$.
(c) When $x = -2$ and $y = 3$, $2x + 5y = 2(-2) + 5(3) = -4 + 15 = 11$.

For your information

It has been said that the language of science is mathematics and the grammar of mathematics is algebra.

Example 9

Evaluate the following when $a = 3$, $b = 4$ and $c = 6$:

(a) $a(2c - b)$ *(b) $ab[(3b - c) - a]$*
(c) $(a + b)(c - b) + ab$ *(d) $\frac{a+b}{c} + \frac{a+c}{c-b}$*

Solution

(a) $a(2c - b) = 3(2 \times 6 - 4) = 3(12 - 4) = 3(8) = 24$

(b) $ab[(3b - c) - a] = 3 \times 4[(3 \times 4 - 6) - 3]$
$= 12[(12 - 6) - 3]$
$= 12[6 - 3] = 12(3) = 36$

(c) $(a + b)(c - b) + ab = (3 + 4)(6 - 4) + 3 \times 4$
$= 7 \times 2 + 12 = 14 + 12 = 26$

(d) $\frac{a+b}{c} + \frac{a+c}{c-b} = \frac{3+4}{6} + \frac{3+6}{6-4} = \frac{7}{6} + \frac{9}{2}$
$= \frac{7 + 9 \times 3}{6} = \frac{7 + 27}{6} = \frac{34}{6} = 5\frac{2}{3}$

Exercise 7b

1. Express the following polynomials so that the degrees of the terms are in descending order:
(a) $3x + 7x^2 + 4 - 5x^3$
(b) $7x^4 - 4x + 7x^3 - 5x^2$
(c) $4x^2 + 5x^3 - 7x + 4$
(d) $7x^2 + 5x^5 - 6x^3 + 7$

2. Express the following polynomials so that the degrees of the terms are in ascending order:
(a) $7x + 4x^3 + 4 - 3x^2$
(b) $8x^3 - 9x^2 + 4x^5 - 4x$
(c) $2a^2 - 4a + 3a^5 - 4a^6$
(d) $4b^3 - 3b + 7b^5 - 4b^2$

3. Write an algebraic expression for each of the following:
(a) Add $2x$ to 14.
(b) Subtract 14 from $5a$.
(c) Multiply 4 by $2k$.
(d) Divide $8x$ by $24y$.
(e) Add $2x$ to twice $3y$.
(f) Subtract $5x$ from half of y.

4. Translate each of the following word expressions into algebraic expressions:
(a) The sum of a number $2x$ and a number y.
(b) The product of 7 and a number k.
(c) Fifteen subtracted from twice the number t.

(d) Three times the number u decreased by four.
(e) Eight more than half of a number v.
(f) The total value of h 50¢ coins and k \$2 notes in dollars.
(g) The total cost of buying x sets of stamps to commemorate the 50th Anniversary of the Inter-Religious Organisation (IRO) at \$1.82 per set, and y sets of the Rabbit Zodiac series at \$2.22 per set.

5. If $a = 2$, $b = -3$, $c = 4$, $d = 5$ and $e = -6$, find the value of each of the following:

(a) $3a - 3(2c - e)$ **(b)** $4(a - 3b) - 5c$
(c) $4c - (a - 2b - e)$ **(d)** $9c - 3(2d + c)$
(e) $7e - 5b^2 + 4ac$ **(f)** $5abe - 4(e + c)^2$

6. Write an algebraic expression for each of the following:

(a) The cost of x litres of petrol at \$1.10 per litre.
(b) Three times the variable x divided by the sum of 3 and k.
(c) Five times the number which is 3 more than h.
(d) One quarter of the number which is 4 less than m.
(e) The total number of eggs in k cartons where each carton contains n eggs.
(f) The total distance travelled by a car for x hours at a constant speed of z km/h.

7. Mary is x years old. Write an algebraic expression for each of the following:

(a) Three times Mary's age next year.
(b) Five times Mary's age six years ago.
☆**(c)** The present age of Mary's aunt if her aunt is four times as old as Mary will be 2 years from now.
☆**(d)** The present age of Mary's niece if her niece is 3 years less than one-third Mary's age 5 years ago.

8. If $a = 2$, $b = -3$ and $c = 4$, evaluate each of the following:

(a) $\dfrac{5ac - 2b^2}{2ab}$ **(b)** $\dfrac{3a^3 + 2b^2 - 4c}{2a + 4b}$

(c) $\dfrac{5a + 3bc - c^2}{2ac - 4b}$ **(d)** $\dfrac{7a + 8b + 2c}{3c - 2b - 4a}$

(e) $\dfrac{2c - 4b + 5ab}{(c + a)(c - a)}$ **(f)** $\dfrac{\left(\frac{a}{b}\right)\left(\frac{c}{b}\right)}{\frac{a}{b} - \frac{a}{c}}$

(g) $\dfrac{\frac{2}{b} + \frac{c}{a}}{\frac{c}{b} - \frac{a}{c}}$ **(h)** $\dfrac{\frac{1}{a} - \frac{2}{b}}{\frac{b}{a} \div \frac{1}{c}}$

Three boxes of fruit, the first containing oranges, the second containing apples and the third containing half of oranges and half of apples are labelled O, A and OA to indicate their contents. However, you are told that the labels have been switched so that every box is now incorrectly labelled.

You are to re-label the boxes so that the labels are correct. You are allowed to draw only one fruit from any of the 3 boxes.

Some Rules in Algebra

1. In algebra, terms of the same kind, called **like terms**, can be combined into a single term; added to or subtracted from one another. For example,

(a) $3a + 5a = 8a$ (b) $7b - 3b = 4b$
(c) $2a + 5b + 4a + 8b = (2a + 4a) + (5b + 8b) = 6a + 13b$
(d) $9c + 7d - 4c - 5d = (9c - 4c) + (7d - 5d) = 5c + 2d$
(e) $7a + 9b - 5a - 4b + 2a - b = (7a - 5a + 2a) + (9b - 4b - b)$
$= 4a + 4b$

Can you simplify $5x^2 + 3x$ or $2x^3 - 3x^2$?

2. In multiplication and division, the coefficients and the variables are multiplied or divided. For example,

(a) $3 \times 6a = 3 \times 6 \times a = 18a$
(b) $2a \times 5a = 2 \times a \times 5 \times a = 10a^2$
(c) $12m \times 3n = 12 \times m \times 3 \times n = 36mn$
(d) $3(a - b) = 3 \times a - 3 \times b = 3a - 3b$

(e) $-a \times (-2ab) = (-1) \times a \times (-2) \times a \times b$
$= (-1) \times (-2) \times a \times a \times b = 2a^2b$

(f) $14m \div 7 = \frac{14m}{7} = 2m$

(g) $18a \div 10b = \frac{18a}{10b} = \frac{9a}{5b}, b \neq 0$

(h) $\sqrt{9a^4} = \sqrt{3 \times 3 \times a \times a \times a \times a} = 3a^2$

3. The terms $3a^2$ and $2a$ are unlike terms. Therefore they cannot be combined into a single term by adding or subtracting.

Exercise 7c

1. Simplify the following expressions:

(a) $4x + 7x + 3x$ **(b)** $5c - 7c + 2c$
(c) $4a - 6b - 3a$ **(d)** $2c + 4d - 5d$
(e) $13x + 6y - 6x$ **(f)** $11pq - 7pq$
(g) $ef - 3ef + 4ef$ **(h)** $5de + 8de - bc$
(i) $p^2 + 4p^2 - 3p^2$ **(j)** $9a^2 - 7a^2 + 2a^2$
(k) $q^3 - q^3 - q^3$ **(l)** $3t^3 + t - 2t^3$

2. Simplify the following expressions:

(a) $5n \times 12$ **(b)** $-2 \times 6a$
(c) $-4 \times (-2k)$ **(d)** $\frac{1}{3} \times 15a$
(e) $2 \times \frac{3m}{4}$ **(f)** $-16b \times \frac{1}{4}$
(g) $-\frac{2}{5} \times 20n$ **(h)** $-\frac{3}{7}a \times \left(-4\frac{2}{3}\right)$
(i) $51u \div 17$ **(j)** $(-11c) \div 121$
(k) $(-27v) \div (-3)$
(l) $18 \div (6a) \quad (a \neq 0)$
(m) $45 \div (-15d) \quad (d \neq 0)$
(n) $(-32) \div 8 \times (-5m)$

3. Simplify the following expressions:

(a) $2k \times (-7k)$ **(b)** $-4b \times (-8b)$
(c) $-1\frac{1}{2}x \times 6x$ **(d)** $-\frac{3}{4}y \times \left(-\frac{8}{9}y\right)$
(e) $3a \times 5b$ **(f)** $2m \times (-7n)$
(g) $-\frac{3}{5}u \times (-20v)$
(h) $54b \div 9a \quad (a \neq 0)$
(i) $(-24m) \div (-18n) \quad (n \neq 0)$
(j) $2a \times 7a \times (-5b)$
(k) $\sqrt{d^4e^2}$
(l) $2xy \div 3y^2 \times 5x^2 \quad (y \neq 0)$
(m) $\frac{c^2}{5} \div \frac{cd}{25} \quad (c, d \neq 0)$
(n) $3d \times 2de \times def$ **(o)** $\sqrt{25c^6d^4}$

Use of Brackets in Simplification

For Your Information

1) $a(b + c) = ab + ac$
2) $a(b - c) = ab - ac$
3) $-a(b + c) = -ab - ac$
4) $-a(b - c) = -ab + ac$
5) $a(-b + c) = -ab + ac$
6) $a(-b - c) = -ab - ac$
7) $-a(-b + c) = ab - ac$
8) $-a(-b - c) = ab + ac$

When brackets occur in an algebraic expression, the rules by which operations are performed apply exactly as in arithmetic:

(a) Simplify the expression within the brackets first.

(b) When an expression contains more than one pair of brackets, simplify the expression within the innermost pair of brackets first,
for example, $[2c - 4(c - 1)] = [2c - 4c + 4] = 4 - 2c$

(c) If an expression in brackets is multiplied by a number, **each term** within the brackets must be multiplied by that number when the brackets are removed,
for example, $4(a - 2b + 3c) = 4a - 8b + 12c$

For Your Information

$4p - 3(m + p) \neq 4p - 3m + 3p$ or $4p - 3m - p$

Also,

$(+) \times (-) = (-)$
$(-) \times (+) = (-)$
$(-) \times (-) = (+)$

(d) The expression $\frac{x}{y}$ $(y \neq 0)$ is called an algebraic fraction.

In the fractions $\frac{2x-3}{4}$ and $\frac{5x-7}{6}$, the numerators can be placed within brackets.

Thus, $\frac{2x-3}{4}$ can be written as $\frac{(2x-3)}{4}$ or $\frac{1}{4}(2x-3)$ and $\frac{5x-7}{6}$ can be written as $\frac{(5x-7)}{6}$ or $\frac{1}{6}(5x-7)$.

The procedure for simplifying algebraic fractions is similar to that of simplifying ordinary fractions.

Example 10

Simplify (a) $3a + 5b - 3c - 2b + 7c - a + 4c - 6b + 8a$;
(b) $3x + 2(x + 4) - (2x - 3) + 5x - 7$;
(c) $2[4p - 3(m + p)]$.

Solution

(a) $3a + 5b - 3c - 2b + 7c - a + 4c - 6b + 8a$
$= (3 - 1 + 8)a + (5 - 2 - 6)b + (-3 + 7 + 4)c = 10a - 3b + 8c$

(b) $3x + 2(x + 4) - (2x - 3) + 5x - 7 = 3x + 2x + 8 - 2x + 3 + 5x - 7$
$= (3 + 2 - 2 + 5)x + (8 + 3 - 7)$
$= 8x + 4$

(c) $2[4p - 3(m + p)] = 2(4p - 3m - 3p)$
$= 2(p - 3m)$
$= 2p - 6m$

Notes

$-\frac{2x-6}{9} = -\frac{(2x-6)}{9}$

The numerator is placed within brackets.

Example 11

Simplify the following expressions:

(a) $\frac{2x-3}{5} + \frac{x-3}{3}$ (b) $\frac{2x-7}{3} - \frac{2x-6}{9}$

(c) $\frac{x+y}{2} + \frac{3x-y}{5} - \frac{7(2x-4)}{6}$

Solution

(a) The LCM of 5 and 3 is 15.

$$\frac{2x-3}{5} + \frac{x-3}{3}$$
$$= \frac{3(2x-3)}{15} + \frac{5(x-3)}{15}$$
$$= \frac{3(2x-3) + 5(x-3)}{15}$$
$$= \frac{6x - 9 + 5x - 15}{15}$$
$$= \frac{11x - 24}{15}$$

(b) The LCM of 3 and 9 is 9.

$$\frac{2x-7}{3} - \frac{2x-6}{9}$$
$$= \frac{3(2x-7)}{9} - \frac{(2x-6)}{9}$$
$$= \frac{3(2x-7) - (2x-6)}{9}$$
$$= \frac{6x - 21 - 2x + 6}{9}$$
$$= \frac{4x - 15}{9}$$

(c) $\frac{x+y}{2} + \frac{3x-y}{5} - \frac{7(2x-4)}{6} = \frac{15(x+y) + 6(3x-y) - 35(2x-4)}{30}$

$= \frac{15x + 15y + 18x - 6y - 70x + 140}{30}$

$= \frac{-37x + 9y + 140}{30}$

Exercise 7d

1. Simplify the following expressions:
   - (a) $5(a + 2b) - 3b$
   - (b) $4u - 3(2u - 5v)$
   - (c) $-2a - 3(a - b)$
   - (d) $6x - 2(4y + x)$
   - (e) $7m - 2n - 2(3n - 2m)$
   - (f) $-3(2h - k) + 4(k - 3h)$
   - (g) $5x(a - 6b + 5c) - 2x(b - c)$
   - (h) $3(5x - 4y) - 2(x - 4y)$
   - (i) $-4(a - 3b) - 5(a - 3b)$
   - (j) $5(3p - 2q) - 2(3p + 2q)$
   - (k) $a + 3(2a - 3b + c) + 7c$
   - (l) $5k - 3(b + 3k) - 3b$
   - (m) $(x + y) - 2(3x - 4y + 3)$
   - (n) $3(p - 2q) - 4(2p - 3q - 5)$
   - (h) $5[3c - \{d - 2(c + d)\}]$
   - (i) $11x - \{5y - 3(2x + y)\}$
   - (j) $-a - [b - \{a - (b - a)\}]$
   - (k) $(w + r) - 4(3t - 2w) + 6(t - 2r)$
   - (l) $\frac{1}{2}(4x - 2y) - \frac{2}{3}(9x - 3y) - 2(x - y)$
   - (m) $-4[5(2x + 3y) - 4(x + 2y)]$
   - (n) $8[3(4x + 5y) - 2(6x - 5y)]$
   - (o) $3a - 4[3(1 - 2a) - 2(3 - 5a)]$
   - (p) $7a - 2[3(5a - b) - 2(4a + b)]$
   - (q) $5a - [3a - 2(a - 2b)]$
   - (r) $2[y - 3(2y + x)] - 3x$

2. Write down the simplest forms for the following:
   - (a) $\frac{1}{2}\left[2x + \frac{1}{2}(4x - 12)\right]$
   - (b) $\frac{2}{5}[12p - (5 + 2p)]$
   - (c) $a - \{b - (c + d)\}$
   - (d) $4\{2x + 5 - (3x - 2)\}$
   - (e) $a - \{6a + 2(1 - 3a)\}$
   - (f) $2\{(3p - 2q) - (p - q)\}$
   - (g) $-2[3a - 4\{a - (2 + a)\}]$

3. Simplify the following algebraic fractions:
   - (a) $\frac{x}{5} + \frac{2x-4}{7}$
   - (b) $\frac{2x+7}{3} + \frac{6x-3}{5}$
   - (c) $\frac{4x+1}{5} + \frac{3x-1}{2}$
   - (d) $\frac{2x-7}{4} - \frac{x-6}{7}$
   - (e) $\frac{3(x-2)}{4} - \frac{4(2x-3)}{5}$
   - (f) $\frac{2(x+3)}{5} - \frac{1}{2} + \frac{3x-4}{4}$

Addition and Subtraction of Polynomials

Example 12

Find the sum of $4x^2 - 9x + 3$ and $x^2 - 2x - 8$.

Solution

The expressions are arranged so that the like terms are grouped in the same columns. Then, each column is added.

$$
\begin{array}{r}
4x^2 - 9x + 3 \\
+ \quad x^2 - 2x - 8 \\
\hline
5x^2 - 11x - 5
\end{array}
$$

$\therefore$ the sum is $5x^2 - 11x - 5$.

Alternatively, if we use brackets, we write

$$
\begin{aligned}
(4x^2 - 9x + 3) + (x^2 - 2x - 8) &= 4x^2 - 9x + 3 + x^2 - 2x - 8 \\
&= 5x^2 - 11x - 5
\end{aligned}
$$

Example 13

Find the sum of $2a + 3b - 4c$, $3a - 2b$, $-4a + 5c$ and $a - 2b + c$.

Solution

The expressions are arranged so that the like terms are grouped in the same columns. Then, each column is added.

$$
\begin{array}{rrrr}
 & 2a & +3b & -4c \\
 & 3a & -2b & \\
 & -4a & & +5c \\
+ & a & -2b & +c \\
\hline
 & 2a & -b & +2c
\end{array}
$$

$\therefore$ the sum is $2a - b + 2c$.

Alternatively, $(2a + 3b - 4c) + (3a - 2b) + (-4a + 5c) + (a - 2b + c)$

$$
\begin{aligned}
&= 2a + 3b - 4c + 3a - 2b - 4a + 5c + a - 2b + c \\
&= 2a - b + 2c
\end{aligned}
$$

Example 14

Subtract $x^3 - 5x^2 + 6x - 7$ from $2x^3 - 7x^2 + 11x + 6$.

Solution

Again, the expressions are arranged in order. The expression to be subtracted is placed below the other expression and like terms are grouped in the same columns.

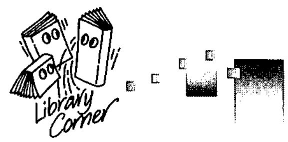
$$
\begin{array}{r}
2x^3 - 7x^2 + 11x + 6 \\
-\ (x^3 - 5x^2 + \ 6x - 7) \\
\hline
x^3 - 2x^2 + \ 5x + 13
\end{array}
$$

$\therefore$ the result is $x^3 - 2x^2 + 5x + 13$.

This shows that if the expressions are written down as in arithmetic, the result is obtained by changing the sign of each term in the lower line and then adding. Alternatively, if we use brackets, we write

$$
\begin{aligned}
&(2x^3 - 7x^2 + 11x + 6) - (x^3 - 5x^2 + 6x - 7) \\
&= 2x^3 - 7x^2 + 11x + 6 - x^3 + 5x^2 - 6x + 7 \\
&= x^3 - 2x^2 + 5x + 13
\end{aligned}
$$

$$
\begin{array}{r}
2x^3 - 7x^2 + 11x + 6 \\
+\ -x^3 + 5x^2 - \ 6x + 7 \\
\hline
x^3 - 2x^2 + \ 5x + 13
\end{array}
$$



When a ship is in distress, it sends out an S.O.S. signal for help. This signal was first invented by Samuel Morse in 1837 for use with his telegraph. It was adopted in 1912 for international telegraphic use and is widely known as the "Morse code". The Morse code is a system of dots, dashes and spaces. A combination of dots ∘ (one-unit length) and dashes — (three-unit length) make up the system of codes. The distance between two single characters is a space of one-unit length and the distance between two alphabets is a space of three-unit length. The space between two words is given a distance of seven-unit length.

The following are some of the words used in the Morse code:

—	....	.—	—.	—.—
T	*H*	*A*	*N*	*K*

—.——	———	..—
Y	*O*	*U*

Can you find out the symbols for all the alphabets?

Example 15

Subtract $3a + 4b - 2c - 5d$ from $2a + 5b - d$.

Solution

$$\begin{array}{r} 2a + 5b \qquad - d \\ -\ (3a + 4b - 2c - 5d) \\ \hline -a + b + 2c + 4d \end{array}$$

$\therefore$ the result is $-a + b + 2c + 4d$.

Note: Since there is no term involving c in the first line, there is a gap.

Exercise 7e

1. Find the sum of the following expressions:

(a) $x^2 - 3x - 1,\ 3x^2 + 2x + 9$
(b) $x^3 + 5x^2 + 2,\ 4x^2 - 3x - 10$
(c) $-2a^3 - 3a^2 + 4a + 6,\ 2a^3 + 5a^2 + 7$
(d) $4a + 6b + 5c,\ -3a - 9b,\ a + 3b - 4c$
(e) $p + 2q - 4r,\ 2p - 3q + 5r,\ 3p - q + 2r$
(f) $5x - 4y,\ 6y - 7z,\ 3z - 4x$
(g) $2x^3 + 3x^2 + 1,\ 2x^3 - 2x^2 + 6x,\ 4x^2 - 2x + 9,\ -3x^3 + 5$
(h) $9p + 12q - 3r - 4s,\ -8q + 4s,\ -7p + q + 2r,\ p + 4r - 5s$
(i) $5xy - 6yz + 7zx,\ xy + 5yz - 6zx,\ -6xy + yz + zx$
(j) $x^3 - 5x^2 + 4x - 7,\ x^4 + 2x^3 + x^2 - 7x + 4,\ x^4 - 7x + 8$
(k) $x^5 - 3x^4 + 5x^2 - 2x + 3,\ 2x^5 + 7x^2 - 8,\ -3x^5 + 7x^4 - 4x^2 + 5x - 9$
(l) $3x^2y + xy - xy^2,\ 5x^2y + 2xy - 7xy^2,\ 2x^2y + 7xy + 9xy^2$

2. Subtract

(a) $3x^2 - x - 1$ from $4x^2 + 3x - 3$;
(b) $3x^2 - 5x$ from $2x^2 - 4x - 5$;
(c) $a - 2b + 6c$ from $3a + 3b - 4c$;
(d) $2q - 3r - s$ from $p - 4q - 6r$;
(e) $3x^2 + 2x - 4$ from $x^3 - 3x^2 - 5x + 6$;
(f) $2a^3 + 3a^2 - 6a + 7$ from $a^3 - 4a + 5$;
(g) $8a - 3b + 5c - 2d$ from $10a - b - 4c - 6d$;
(h) $2a^5 - 3a^4 + 7a^3 - 6$ from $7a^5 + 4a^4 - 2a^3 + 3a + 2$;
(i) $a^4 + 4a^2 + 7$ from $5a^5 + 2a^4 - 3a^3 + 2a^2 - 9$;
(j) $2a^5 + 3a^4 - 7a^3 + 4a^2 + 8$ from $2a^6 - 3a^5 - 7a^4 + 4a^3 - 8a^2 + 3a$.

3. Simplify the following expressions:

(a) $(3a^2 + 7a) + (2a^2 - 9a)$
(b) $(-2a + 7b) - (a + 4b)$
(c) $6(2a + 3b - 7ab) - 4(5a - 2b + 5ab)$
(d) $(3a + 4b - 5c) + (2a - 7b - 6c) + (8a - 5b + 9c)$
(e) $2(a + b - 3c) - 4(a - b + c) + 5a$
(f) $5(b + a - 6c) - 7(c - b + 6a)$
(g) $3(a - 5c) - 4(b - a) + 3(c - b)$
(h) $6(a - 3b + 5c) - 4(5b - c + 2a) - 5(2a - 4c + 3b)$
(i) $8(3a - 4b + c) + 5(2a - 3b + c) - 3(2c - 9a + 7b)$
(j) $9(2a - 7c + 4b) - 4(b - c) - 7(-c - 4b)$

Summary

1. In algebra, we use symbols, e.g. a, x^2 and xy, to represent numbers and variables. We add or subtract the like terms by adding or subtracting the coefficients, e.g. $2a + 5a = 7a$ and $7b - 3b = 4b$. We do not add the coefficients of unlike terms, so adding $3x$ and $4y$ gives $3x + 4y$.

2. When an expression of arithmetic operations contains brackets, work with the expressions within the brackets first. (If there are brackets within brackets, work with the innermost pair of brackets first.)

3. If an expression in brackets is multiplied by a number, each term within the brackets must be multiplied by that number when the brackets are removed.

Review Questions 7

1. Given that $a = -2$ and $b = 7$, evaluate the following expressions:
   (a) $4a + 5b$ **(b)** $2a^2$ **(c)** $3a - 4b$
   (d) $a(b - a)$ **(e)** $b - a^2$ **(f)** $(b - a)^2$

2. Given that $a = \frac{1 + b}{1 - b}$, calculate the value of a when $b = -3$, giving your answer as a fraction in its lowest terms.

3. Given that $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, find the value of f when $v = 10$ and $u = 15$.

4. Simplify each of the following expressions:
   (a) $4[e - 3\{f - 6(f - e)\}]$
   (b) $-3 + m - \{2 - (m - 4)\}$
   (c) $10 + \frac{1}{3}[4k - (18 + 7k)]$
   (d) $1 - 3(1 + x) + \{2 - (4x - 7)\}$
   (e) $18 - [10 - x - (9 - x)]$
   (f) $-2[3x - (4 - 5) - (6 - 8)x]$
   (g) $\frac{1}{2}\left[16x - \frac{2}{3}(6x - 12)\right]$
   (h) $a - [b - \{c - (d + e)\}]$
   (i) $9[5a - 2\{3a - (7 - 2a)\}]$
   (j) $5x - [2x - \{3x - 3(x - 2y) + y\}]$
   (k) $5a - 2[3a - 7(a - 2) - 5]$
   (l) $3[4x - \{2x + 5(x - 2y) + 3x\}]$
   (m) $\frac{a + b}{3} - \frac{b + c}{2} + \frac{4a - c}{5}$
   (n) $\frac{2(3a + b)}{a} + \frac{4(2a - b)}{3a}$
   (o) $\frac{6x - y}{5} + \frac{3x - 4}{10} - \frac{5(x - 2)}{6}$
   (p) $\frac{4(x - 5)}{7} - \frac{5(x - y)}{6} + \frac{7x - z}{21}$

5. Work out the polynomial we must use to subtract $(3p^2 + 2pq + 7q^2)$ from, to get $(7p^2 + 5pq - 3q^2)$.

6. Subtract the sum of $(3x^2 - 4x + 3)$ and $(2x^2 + 7x - 5)$ from $(4x^2 + 2x - 17)$.

7. Subtract the sum of $(a^2 + 5ab + b^2)$ and $(2a^2 - 4ab + 5b^2)$ from the sum of $(5a^2 - 7ab + 4b^2)$ and $(7a^2 - 2ab + 3b^2)$.

Problem Solving

1. If $a = 3$, $b = -4$ and $c = -2$, evaluate each of the following:

(a) $\dfrac{3a - b}{2c} + \dfrac{3a - c}{c - b}$

(b) $\dfrac{2c - a}{3c + b} - \dfrac{5a + 4c}{c - a}$

(c) $\dfrac{a + b + 2c}{3c - a - b} - \dfrac{5c}{4b}$

(d) $\dfrac{b - c}{3c + 4b} \div \left(\dfrac{bc}{a} + \dfrac{ac}{b}\right)$

2. The average salary of m male employees and f female employees of a company is \$$A$. If the average salary of the male employees is \$$B$, find an expression for the average salary of the female employees.

3. At a famous "roti prata" shop, for every two people who order egg prata, there are five people who order plain prata.

(a) If a people ordered egg prata, how many people ordered plain prata?
(b) If b people ordered plain prata, how many people ordered egg prata?
(c) If there are a total of c people in the shop, how many of them ordered egg prata?

4. A collection of coins contain only 10-cent and 5-cent coins. There are x 5-cent coins in the collection. Write an algebraic expression for each of the following:

(a) The total value of the 5-cent coins.
(b) The total value of the 10-cent coins if there are three times as many 10-cent as 5-cent coins.
(c) The total value of the coins if for every three 10-cent coins there are five 5-cent coins.

CHAPTER 8

Algebraic Equations

In this chapter, you will learn how to

- △ solve simple algebraic equations;
- △ construct simple linear equations from given situations and solve these equations.

Preliminary Problem

Albert Einstein derived the simple yet elegant formula, $E = mc^2$, to measure the amount of energy released when a quantity of matter is destroyed. This idea has been used by other scientists to develop the atomic bomb. An atomic explosion produces a 'mushroom-shaped' cloud which may look majestic but is actually very destructive and harmful.

Open Sentences

Consider the following sentences and state whether each of them is true or false:

(a) 5 is greater than 4.
(b) 4 is a factor of 32.
(c) $4 + 5 = 7$
(d) London is an island.

Clearly, we can conclude that sentences (a) and (b) are true while sentences (c) and (d) are false.

Now, consider the following sentences:

(e) $7 + \square = 13$.
(f) 3 is a factor of $\triangle$.
(g) $\bigcirc$ is the capital of Malaysia.

We cannot say whether sentences (e), (f) and (g) are true or false because it is not given in the sentence what the symbols $\square$, $\triangle$ and $\bigcirc$ stand for. We call such sentences **open sentences** and the symbols $\square$, $\triangle$ and $\bigcirc$ **unknowns** or **variables**. An open sentence is a sentence which contains one or more unknowns. An open sentence can be true or false depending on what we replace the unknown(s) in the sentence with.

Normally, we use letters such as a, b, c, x, y and z to represent unknowns. Open sentences that include numbers, variables and operation symbols in mathematics are called **mathematical sentences**. In particular, (e) and (f) are called open mathematical sentences.

Simple Equations

An open mathematical sentence which contains an equal sign "=" is called an equation. The following are some simple equations:

(a) $x - 5 = 7$ (b) $2x + 7 = 26$ (c) $\frac{x+5}{2} = 3x - 2$ (d) $x^2 + x = 6$

Equations like $3x - 5 = x + 7$ which contain only one unknown or variable are called equations in one unknown.

To solve an equation means to find the value of the unknown so that the equation becomes a true or correct sentence. The value found is called the **solution** of the equation.

Example

What does x stand for if (a) $x + 5 = 11$; (b) $x - 6 = 14$?

Solution

By observation, we deduce that

(a) x stands for 6 because $6 + 5 = 11$. (b) x stands for 20 because $20 - 6 = 14$.

Problems

Find two different integers x and y such that $x^y = y^x$.

Example 2

What does x stand for if (a) $3x = 18$; (b) $\frac{x}{5} = 7$?

Solution

By observation, we deduce that

(a) x stands for 6 because $3 \times 6 = 18$.

(b) x stands for 35 because $\frac{35}{5} = 7$.

Exercise 8a

1. Find the solution of each of the following equations by observation:

(a) $3a - 4 = a$ **(b)** $3 \times 47 = 3a$ **(c)** $2a + 8 = 16$ **(d)** $50 - 5a = 10$

(e) $\frac{24}{a} = 3$ **(f)** $\frac{a}{3} = \frac{25}{5}$ **(g)** $\frac{1}{2}a - \frac{1}{3}a = 2$ **(h)** $\frac{3}{4}a - 3 = \frac{1}{2}a$

(i) $\frac{1}{3}a - 3 = 0$ **(j)** $1.5a + 2 = 5$ **(k)** $0.5a - 1 = 4$ **(l)** $0.1a + 1.5 = 2$

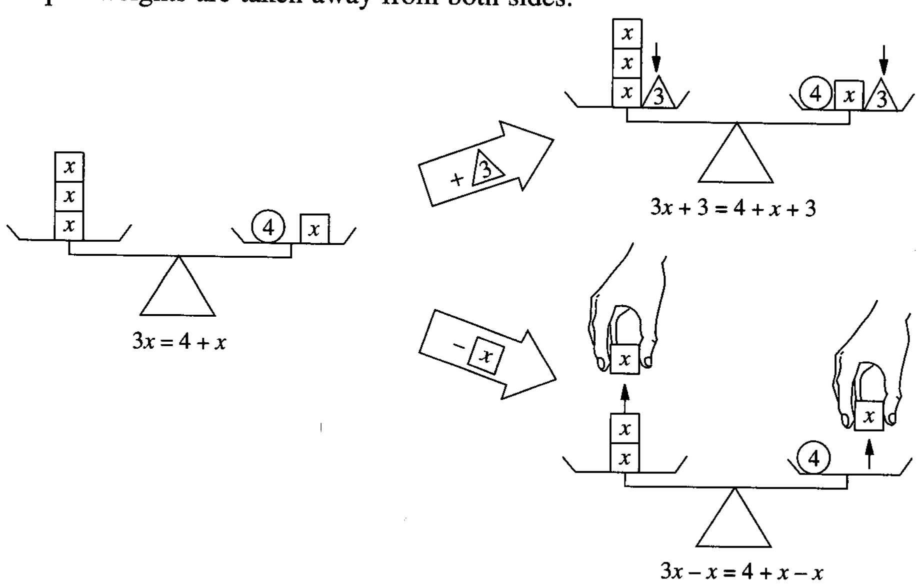
2. If a is an integer, find the possible solutions, if any, for each of the following by observation:

(a) $a^2 = 4$ **(b)** $9 - a^2 = 0$ **(c)** $a^2 + 16 = 0$ **(d)** $\sqrt{a} = 5$

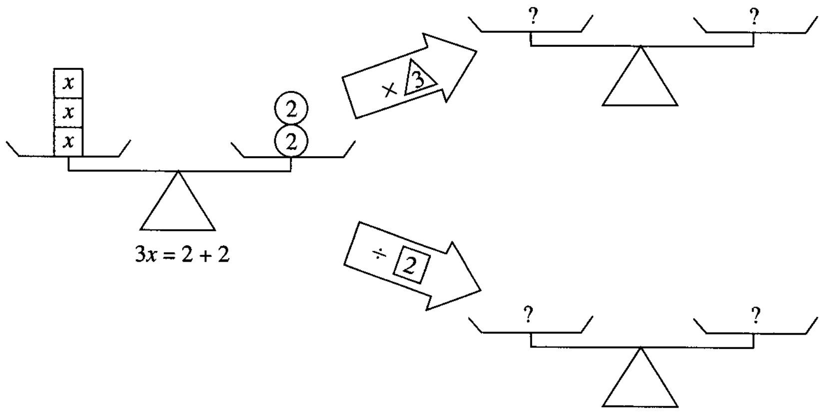
(e) $\sqrt[3]{a} = 3$ **(f)** $\sqrt[3]{a} = -2$ **(g)** $\sqrt[3]{a + 4} = 4$ **(h)** $\sqrt{2a + 1} = 7$

Solving Simple Equations

We can use the idea of a balance to help us solve equations. Consider the case where the contents of the two scale pans balance each other. They will remain balanced if equal weights are added to both sides or if equal weights are taken away from both sides.



What will happen to the scales below if the weights of the contents of both sides are trebled or halved?

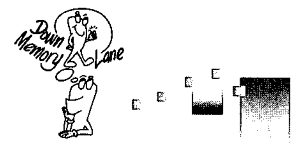


The above discussion leads to the following rules:

To balance an equation,

1. equal numbers may be added to each side;
2. equal numbers may be subtracted from each side;
3. each side may be multiplied by equal numbers;
4. each side may be divided by equal numbers except zero.

We shall use the above rules to solve simple equations.



Albert Einstein (1879–1955) published three famous papers in 1905. The first suggested the Quantum Theory for light, the second suggested the special Theory of Relativity and the third related the conversion of mass and energy in the famous equation $E = mc^2$ where m is the mass in kg, E is the energy in Joules and c is the speed of light in metres per second. This equation eventually led to man's use of atomic energy and the creation of atomic bombs.

Example 3

Solve (a) $n - 4 = 6$, (b) $3x + 2 = 8$.

Solution

(a) $n - 4 = 6$

$n - 4 + 4 = 6 + 4$ (Add 4 to both sides.)

$\therefore \quad n = 10$

(b) $3x + 2 = 8$

$3x + 2 - 2 = 8 - 2$ (Subtract 2 from both sides.)

$3x = 6$

$\frac{3x}{3} = \frac{6}{3}$ (Divide both sides by 3.)

$\therefore \quad x = 2$

Notes

Check: $10 - 4 = 6$

Check:
$3(2) + 2 = 6 + 2 = 8$

Example 4

Solve (a) $3a - 1 = 5$; (b) $5(x + 3) + 2(x + 1) = 5 - 4x$.

Solution

(a) $3a - 1 + 1 = 5 + 1$ (Add 1 to both sides.)

$3a = 6$

$\therefore \quad a = 2$

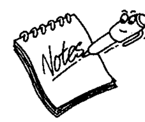
(b) $5(x + 3) + 2(x + 1) = 5 - 4x$

$$
\begin{aligned}
5x + 15 + 2x + 2 &= 5 - 4x \\
7x + 17 &= 5 - 4x \\
7x + 17 + 4x &= 5 - 4x + 4x && \text{(Add } 4x \text{ to both sides.)} \\
11x + 17 - 17 &= 5 - 17 && \text{(Subtract 17 from both sides.)} \\
11x &= -12 \\
\frac{11x}{11} &= -\frac{12}{11} && \text{(Divide both sides by 11.)} \\
\therefore \quad x &= -1\frac{1}{11}
\end{aligned}
$$

Can you check to see if the solution obtained is correct?

Example 5

Solve $2x - (x + 5) = 2 - (7 - x)$.



$2 - (7 - x) \neq 2 - 7 - x$

Solution

$$
\begin{aligned}
2x - (x + 5) &= 2 - (7 - x) \\
2x - x - 5 &= 2 - 7 + x \\
x - 5 &= x - 5 \\
x - 5 + 5 &= x - 5 + 5 && \text{(Add 5 to both sides.)} \\
x &= x
\end{aligned}
$$

Notice that the above equation is true for all values of x. An equation such as this is called an **identity**.

Exercise 8b

1. Find the value of the unknown in each equation:

(a) $2a = 10$ **(b)** $11x = 66$ **(c)** $2y = 0$
(d) $4a = -20$ **(e)** $-12e = 36$ **(f)** $-9p = 0$
(g) $x - 7 = 0$ **(h)** $a + 11 = 0$ **(i)** $3d - 1 = 0$

2. Solve the following equations:

(a) $2x + 15 = 27 - 4x$ **(b)** $15 - 5x = 24 - 8x$ **(c)** $2(c - 4) = 3(c - 2)$
(d) $3(2a + 3) = 4a + 3$ **(e)** $5x = x + 4$ **(f)** $-d + 3d = 14$
(g) $a + 4 = 7 - a$ **(h)** $3d - 12 = d + 2$

3. Solve the following equations and indicate whether they are identities.

(a) $5x - 7 = 2x + 3x - 7$ **(b)** $6(a - 1) - 2(a + 3) = 4(a - 3)$
(c) $5x + 7 = 4(x + 3) + 2x - 4$ **(d)** $5m - 52 = 7(m + 2) + 2m$
(e) $3(4x + 13) - 5(2x + 3) = 2x + 24$ **(f)** $2(5x - 7) - 4(x + 2) = 18(x - 4)$
(g) $(5m - 2) - 2(m + 1) = (3m - 4)$

Equations Involving Fractional and Decimal Coefficients

Example 6 (optional)

Solve the equation $\frac{x-1}{3} = \frac{2x+5}{7}$.

Solution

$$\frac{x-1}{3} = \frac{2x+5}{7}$$
$$7(x-1) = 3(2x+5) \quad \text{(Multiply both sides by the LCM of 3 and 7 ie. 21)}$$
$$7x - 7 = 6x + 15$$
$$7x - 7 + 7 = 6x + 15 + 7 \quad \text{(Add 7 to both sides)}$$
$$7x - 6x = 6x + 22 - 6x \quad \text{(Subtract } 6x \text{ from both sides)}$$
$$\therefore \quad x = 22$$

Notes

It is useful to check your answer by substituting the value obtained for the unknown into the equation.

LHS (Left Hand Side)
$= \frac{x-1}{3} = \frac{22-1}{3} = \frac{21}{3} = 7$

RHS (Right Hand Side)
$= \frac{2x+5}{7} = \frac{2(22)+5}{7} = \frac{49}{7}$
$= 7$

Example 7

Solve the equation $1\frac{4}{5}x - \frac{3}{5} = 1\frac{1}{5}x + 10\frac{1}{5}$.

Solution

$$1\frac{4}{5}x - \frac{3}{5} = 1\frac{1}{5}x + 10\frac{1}{5}$$
$$1\frac{4}{5}x - \frac{3}{5} + \frac{3}{5} = 1\frac{1}{5}x + 10\frac{1}{5} + \frac{3}{5} \quad \left(\text{Add } \frac{3}{5} \text{ to both sides}\right)$$
$$1\frac{4}{5}x - 1\frac{1}{5}x = 1\frac{1}{5}x + 10\frac{4}{5} - 1\frac{1}{5}x \quad \left(\text{Subtract } 1\frac{1}{5}x \text{ from both sides}\right)$$
$$\frac{3}{5}x = 10\frac{4}{5}$$
$$x = 10\frac{4}{5} \times \frac{5}{3} \quad \left(\text{Multiply both sides by } \frac{5}{3}\right)$$
$$\therefore \quad x = 18$$

Notes

Check:
LHS $= 1\frac{4}{5}x - \frac{3}{5}$
$= 1\frac{4}{5}(18) - \frac{3}{5}$
$= 31\frac{4}{5}$

RHS $= 1\frac{1}{5}x + 10\frac{1}{5}$
$= 1\frac{1}{5}(18) + 10\frac{1}{5}$
$= 31\frac{4}{5}$

Example 8

Solve the equation $2.4x - 3.2 = 1.6x + 1.12$

Solution

$$2.4x - 3.2 = 1.6x + 1.12$$
$$2.4x - 3.2 + 3.2 = 1.6x + 1.12 + 3.2 \quad \text{(Add 3.2 to both sides)}$$
$$2.4x - 1.6x = 1.6x + 4.32 - 1.6x \quad \text{(Subtract } 1.6x \text{ from both sides)}$$
$$0.8x = 4.32$$
$$x = \frac{4.32}{0.8} \quad \text{(Divide both sides by 0.8)}$$
$$x = 5.4$$

Notes

Check:
LHS $= 2.4(5.4) - 3.2$
$= 9.76$
RHS $= 1.6(5.4) + 1.12$
$= 9.76$

Note: It is not necessary to write down the statements in the brackets when you are solving the equations.

Example 9

Solve the equation $2.3(2x - 7) = 3.3x - 4.6$ giving your answer correct to 3 significant figures.

Solution

$$
\begin{aligned}
2.3(2x - 7) &= 3.3x - 4.6 \\
4.6x - 16.1 &= 3.3x - 4.6 \\
4.6x - 16.1 + 16.1 &= 3.3x - 4.6 + 16.1 && \text{(Add 16.1 to both sides)} \\
4.6x &= 3.3x + 11.5 \\
4.6x - 3.3x &= 3.3x + 11.5 - 3.3x && \text{(Subtract } 3.3x \text{ from both sides)} \\
1.3x &= 11.5 \\
x &= \frac{11.5}{1.3} && \text{(Divide both sides by 1.3)} \\
x &= 8.85 && \text{(Correct to 3 significant figures)}
\end{aligned}
$$

Can you check to see if the solution obtained is correct?

Exercise 8c

1. Solve the following equations: (* indicate that these are optional questions.)

(a) $x + \frac{x}{5} = 12$ **(b)** $\frac{2}{5} = \frac{3}{x}$ **(c)** $\frac{2}{3}x + 14 = 0$

(d) $\frac{3}{5}x - \frac{3}{7} = \frac{5}{7}$ **(e)** $2\frac{1}{2}y = 10 - 1\frac{2}{3}y$ **(f)** $\frac{d}{3} - \frac{d}{4} = 1$

(g) $\frac{n}{2} + \frac{n}{3} = 30$ **(h)** $\frac{1}{2}a - 2 = \frac{3}{5}a + 4$ ***(i)** $m + 2 = \frac{2 - m}{3} - 2$

(j) $\frac{k}{2} - \frac{k}{5} - \frac{k}{6} = 2$ **(k)** $\frac{x}{5} - \frac{x}{4} + \frac{x}{6} = 3$ **(l)** $\frac{2}{3}x + 4 = x - \frac{1}{3}$

(m) $\frac{5x}{2} = \frac{4}{3} + 2x$ ***(n)** $\frac{2x - 1}{5} + \frac{x + 3}{8} = 0$ **(o)** $\frac{2x + 3}{4} - \frac{x - 5}{6} = 0$

***(p)** $\frac{1}{2} = \frac{1}{y + 2} - 1$ ***(q)** $\frac{2}{x} + 1 = \frac{5}{x} + 2\frac{1}{2}$ ***(r)** $\frac{5}{a - 2} = \frac{7}{a + 6}$

2. Solve the following equations, giving your answers correct to 3 significant figures where necessary:

(a) $0.2x = 0.5x + 3$ **(b)** $3 + 0.4x = 1.9x$ **(c)** $2.3 - 0.3x = 1.7x + 2$

(d) $3.5x - 7.8 = 1.6x - 0.2$ **(e)** $5x + 1.6 = 7.5x + 3.2$ **(f)** $4(0.7x + 1.3) = 8.6$

(g) $5(0.6x + 3.4) = 3.5x$ **(h)** $1.2(2x - 3) = 1.45$ **(i)** $2.4(x + 1) = 3.7x - 1.4$

(j) $3.4(3x - 2) = 4.8x - 1.9$

Formulae

A **formula** expresses a rule in algebraic terms. It uses variables to write instructions in short form for performing a calculation. For example, to find the area of a rectangle, we use the formula: $A = l \times b$ or lb. We have used the letter A to stand for the area, l for the length and b for the breadth.

This is a short form of a rule. In words, this rule can be written as: "The area of a rectangle is equal to the product of its length and breadth".

If P represents the perimeter of the rectangle, then we have $P = 2(l + b)$. If the values of l and b are known, we can find the values of A and P.

For example, if $l = 4$ and $b = 3$, then

$$A = 4 \times 3 = 12$$
$$P = 2(4 + 3) = 14$$

The above example shows that if we know the values of l and b, then we can find the corresponding values of A and P. We can also find the value of l if A and b are given. This is an example of the process of replacing letters by numbers and this process is called **substitution**.

Example 10

The formula for the volume V of a cuboid is $V = lbh$ where l is the length, b is the breadth and h is the height. Find the volume of the cuboid where

(a) $l = 5$ cm, $b = 4$ cm and $h = 3$ cm; *(b) $l = 8$ cm, $b = 6$ cm and $h = 5$ cm.*

Solution

(a) $V = lbh$
$= 5 \times 4 \times 3 = 60$
$\therefore$ volume of the cuboid = 60 cm^3

(b) $V = lbh$
$= 8 \times 6 \times 5 = 240$
$\therefore$ volume of the cuboid = 240 cm^3

Example 11

If $a = \dfrac{b}{c - b}$, find (a) a when $b = 5$ and $c = 7$; (b) b when $a = 3$ and $c = 10$.

Solution

(a)
$$a = \frac{5}{7 - 5} = \frac{5}{2} = 2\frac{1}{2}$$

(b)
$$3 = \frac{b}{10 - b}$$
$$3(10 - b) = b$$
$$30 - 3b = b$$
$$30 = 4b$$
$$\therefore \quad b = 7\frac{1}{2}$$

Exercise 8d

Take the value of π as $3\frac{1}{7}$, where necessary.

1. If $V = \frac{1}{3}Ah$, find V when $A = 43$ and $h = 6$.
2. If $F = \frac{9c}{5} + 32$, find F when $c = 30$.
3. If $S = 4\pi r^2$, find S when $r = 10\frac{1}{2}$.
4. If $T = \pi(R^2 - r^2)$, find T when $R = 4$ and $r = 3$.
5. If $a = \frac{y^2 - xz}{5}$, find a when $x = 4$, $y = 7$ and $z = 6$.
6. If $k = \frac{x + y}{3}$, find x when $k = 12$ and $y = 4$.
7. If $t = \frac{v - u}{a}$, find a when $t = 1$, $u = 1\frac{2}{3}$ and $v = 3\frac{1}{2}$.

8. If $U = \pi(r + h)$, find r when $U = 16\frac{1}{2}$ and $h = 2\frac{3}{4}$.

9. If $v^2 = u^2 + 2gs$, find s when $v = 20$, $u = 10$ and $g = 10$.

10. If $\frac{m}{x + q} = N$, find q when $m = 9$, $x = 2$ and $N = 1\frac{4}{5}$.

11. If $n - y = \frac{4y - n}{m}$, find n when $y = 3$ and $m = 7$.

12. If $\frac{a}{b} + e = \frac{c}{b}$, find c when $a = 4$, $b = 12$ and $e = -\frac{1}{6}$.

***13.** If $y + b = \frac{ay + c}{b}$, find c when $y = 12$, $a = 14$ and $b = 3$.

***14.** If $\frac{1}{a} = \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$, find c when $a = 2$, $b = 3$ and $d = 5$.

***15.** If $\frac{m(ny - x^2)}{z} + n = 5n$, find y when $n = 4$, $x = 2$, $m = 6$ and $z = 8$.

***16.** If $c = \frac{a}{b} - \frac{d - e}{f - d}$, find f when $a = 2$, $b = 3$, $c = 4$, $d = 5$ and $e = 6$.

Construction of Formulae

To construct a formula, choose letters to represent the quantities. Usually, the first letter of the word is used. Then express the rule in algebraic terms. For example, the sum, S kg, of the weights of two boys, one weighing m kg and the other n kg, is expressed as $S = m + n$.

Example 12

Find a formula for the sum (S) of any three consecutive even numbers.

Solution

Let the smallest number be n.

The next number is $(n + 2)$ and the biggest number is $(n + 4)$.

$$S = n + (n + 2) + (n + 4)$$
$$S = n + n + 2 + n + 4$$
$$\therefore \quad S = 3n + 6 \text{ or } 3(n + 2)$$

Exercise 8e

1. Using the letters suggested, construct a simple formula in each case:

(a) The sum (S) of three numbers a, b and c.
(b) The product (P) of two numbers x and y.
(c) The difference (D) between the ages of two boys; one being a years old and the other e years old.
(d) The area (A) of a semicircle whose radius is r.
(e) The cost ($\$C$) of m eggs at 12 cents each.
(f) The average age (A) of 4 boys whose ages are m, n, p and q years.
(g) The vertical angle ($x°$) of an isosceles triangle whose base angle is $y°$.

(h) The time (T) in minutes for a train journey of a hours b minutes.

(i) The total cost ($$T$) of d chairs at $$p$ each and c tables at $$q$ each.

$\ast$**(j)** A car travels x km/h for p km and makes another journey at y km/h for q km.

Find

(i) the total distance (r km) the car travels;

(ii) the formula for the average speed.

Writing Algebraic Expressions

The sum of the ages of two brothers is 21. One is 20 years older than the other. How old are the brothers?

Example 13

How many days are there in (a) 5 weeks; (b) n weeks?

Solution

There are 7 days in a week.

(a) There are $(7 \times 5) = 35$ days in 5 weeks.
(b) There are $(7 \times n) = 7n$ days in n weeks.

Example 14

A girl is now 10 years old. How old will she be (a) 5 years later; (b) t years later?

How old was she (c) 3 years ago; (d) x years ago?

What are the possible values of x? Can x be 15?

Solution

The girl becomes 1 year older every year.

One year later, she will be $(10 + 1) = 11$ years old.

(a) 5 years later, she will be $(10 + 5) = 15$ years old.
(b) t years later, she will be $(10 + t)$ years old.
(c) 3 years ago, she was $(10 - 3) = 7$ years old.
(d) x years ago, she was $(10 - x)$ years old.

The possible values of x are 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 and 0.
x cannot be 15 since the girl is now only 10 years old.

Example 15

A car travels at a speed of 55 km/h. How far will it travel in (a) 2 hours; (b) t hours?

Solution

In 1 hour, the car travels 55 km.

(a) In 2 hours, the car will travel (55×2) km, i.e., 110 km.
(b) In t hours, the car will travel $(55 \times t)$ km, i.e., $55t$ km.

Exercise 8f

1. How many grams are there in 5 kg? How many grams are there in x kg?

2. What is the cost of 6 magazines at $4 each? What is the cost of p magazines at $4 each? Find also the cost of p magazines at $$q$ each.

3. Four tennis balls have a total mass of m kg. Find the mass of each ball.

4. If a horse runs at b km/h, how far can it go in 2 hours if it keeps the same speed?

5. How many minutes are there in m hours?

6. How many weeks are there in y days?

7. If Dan has x dollars, how many marbles can he buy if each marble costs five cents?

8. Write the number which is half as big as b.

9. Find the time taken by a cyclist to travel 21 km if he is travelling at v km/h.

10. A shopkeeper buys an armchair for $$a$ and then sells it at a profit of $$b$. What is the selling price of the armchair?

11. A boy is b years old and his father is 6 times as old as him. Find the father's age. Find also the sum of their ages in y years' time.

12. Mrs Jones's age is equal to the sum of the ages of her two daughters. If the younger daughter is x years old and the elder is 4 years older, how old is Mrs Jones?

13. Find the three consecutive numbers in which n is the middle number.

14. A motorist drives for 5 hours at u km/h and for 3 hours at v km/h. Find the total distance travelled.

Problem Solving with Algebra

One of the most powerful mathematical tools to solve problems is by **using an equation and solving it**. We shall look at this problem-solving method more closely. (Alternative methods will be shown too if they are shorter and easier than the method of using an equation).

Setting Up Equations

In order to solve word problems in mathematics we often need to translate them into mathematical equations first. First consider these problems:

1. "When I subtract 3 from a number, the result is the same as if I had halved the number. Find the number."
2. The sum of two consecutive odd numbers is 64. Find the two numbers.

For the first question if we let x represent the number, we can write the given question mathematically as:

$$x - 3 = \frac{1}{2}x$$

For the second question, let x be the smaller of the two consecutive odd numbers, then the larger one will be $x + 2$. We can then write the mathematical equation as:

$$x + (x + 2) = 64$$

Exercise 8g

1. In each of the following, let x denote the unknown. Derive an equation involving x:

(a) When a certain number is increased by 7, the result is 18.
(b) When a number is decreased by 2 and the result multiplied by 3, the final result is 24.
(c) When 5 is subtracted from a certain number and the result multiplied by 7, the final result is 63.
(d) When a certain number is subtracted from 24 and the result divided by 5, the final result is 4.
(e) The sum of three consecutive numbers is 63.
(f) One number is bigger than the other number by 3 and the sum of these two numbers is 43.
(g) Six times of a certain number is 16 more than twice the number.
(h) The length of a rectangle is 5 m more than its width and the perimeter of the rectangle is 32 m.
(i) The length of a rectangle is twice its width and the perimeter is 54 m.

2. Peter has five times as much money as David. If Peter gives \$28 to David, both of them will have equal amounts of money. How much money did Peter have at the beginning?

3. There are a total of 225 pupils in Secondary one. If the number of pupils who pay their school fees through GIRO scheme is 14 times the number of pupils who do not, find the number of pupils who did not join the scheme.

4. \$4 800 is divided among three brothers A, B and C. A receives three times as much as B and C receives twice as much as B. If B receives \$$x$, form an equation in x.

5. Three wallets and two handbags cost \$450 and a handbag costs twice as much as a wallet. If a wallet costs \$$x$, form an equation in x.

Example 16

A man is now 3 times as old as his son. In 10 years' time, the sum of their ages will be 76. How old was the man when his son was born?

If 10 cats can catch 10 mice in 10 minutes, how many cats are required to catch 100 mice in 100 minutes?

Solution

Strategy: Use an equation

Let the present age of the son be x years old.
The man is now $3x$ years old.
In 10 years' time, the son will be $(x + 10)$ years old.
In 10 years' time, the man will be $(3x + 10)$ years old.

$$(x + 10) + (3x + 10) = 76$$
$$4x + 20 = 76$$
$$4x = 56$$
$$x = 14$$

Hence, the man is now $(3 \times 14) = 42$ years old.

$\therefore$ the man was $(42 - 14) = 28$ years old when his son was born.

Alternatively, we can **use models** to solve the above problem.

Age now son: $\boxed{}$ man: $\boxed{}\boxed{}\boxed{}$

Age when the son was born son: 0 years old man: $\boxed{}\boxed{}$

Age in 10 years' time from now son: $\boxed{}\boxed{\text{10 years}}$ + man: $\boxed{}\boxed{}\boxed{}\boxed{\text{10 years}}$ = 76 years

$\therefore$ 4 shaded parts = 76 – 20 = 56 years
1 shaded part = 56 ÷ 4 = 14 years
2 shaded parts = 2 × 14 = 28 years
$\therefore$ the man was 28 years old when the son was born.

Example 17

Find three consecutive odd numbers whose sum is 57.

Solution

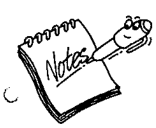
Strategy: Use an equation

Let the smallest of the three consecutive odd numbers be x.
The other odd numbers are $x + 2$ and $x + 4$.

$$\begin{aligned} x + (x + 2) + (x + 4) &= 57 \\ 3x + 6 &= 57 \\ 3x &= 51 \\ x &= 17 \end{aligned}$$

$\therefore$ the three consecutive odd numbers are 17, 19 and 21.

Alternatively, we can find the solution by **making a systematic list.** We shall start with 11, 13, 15, ... etc. Do you think it is necessary to start the list with 1, 3, 5, ...?



11, 13, 15, 17, 19, 21, 23, 25, 27, ...

Check: 17 + 19 + 21 = 57

The three circled odd numbers add up to 57.
$\therefore$ the three consecutive odd numbers are 17, 19 and 21.

Example 18

A woman buys 50 eggs for \$6.60. Some cost 12 cents each and the rest 14 cents each. How many of each kind of eggs has she bought?

Solution

Strategy: Use an equation

Let x be the number of eggs she buys at 12 cents each.
No. of eggs bought at 14 cents each = $(50 - x)$.
Cost of x eggs at 12 cents each = $12x$ cents.
Cost of $(50 - x)$ eggs at 14 cents each = $(50 - x) \times 14$ cents

$$12x + 14(50 - x) = 660$$
$$12x + 700 - 14x = 660$$
$$40 = 2x$$
$$x = 20$$

$\therefore$ no. of eggs bought a 12 cents each = 20.

$\therefore$ no. of eggs bought at 14 cents each = 50 – 20 = 30.

Check:
20(12¢) + 30(14¢)
= $2.40 + $4.20
= $6.60

Example 19

John walked for 45 minutes at the rate of 3 km/h and then ran for half an hour at a certain speed. At the end of that time he was 6 km away from the starting point. How fast did he run?

Solution

Strategy: Use an equation

Let the speed at which he ran be x km/h.

Distance = Speed × Time

$\therefore$ the distance he walked $= \left(\frac{45}{60} \times 3\right)$ km $= \frac{9}{4}$ km

The distance he ran $= \left(\frac{1}{2} \times x\right)$ km $= \frac{x}{2}$ km

$$\frac{9}{4} + \frac{x}{2} = 6$$
$$\frac{9 + 2x}{4} = 6$$

(Remember that when an equation is divided or multiplied by a number, every term must be divided or multiplied by that number.)

$$9 + 2x = 24$$
$$2x = 15$$
$$x = \frac{15}{2} = 7\frac{1}{2}$$

$\therefore$ John ran at a speed of $7\frac{1}{2}$ km/h.

Exercise 8h

1. What are the four consecutive numbers whose sum is 50?
2. If a number is trebled, it gives the same result as when 28 is added to it. What is the number?
3. When a number is added to another number 5 times as large, the result is 24. What is the first number?
4. Joe and Ahmad have 80 marbles altogether. Ahmad has 4 times as many marbles as Joe. How many marbles does each boy have?
5. Arasoo is 4 years older than Peter. Ali is 2 years younger than Peter. If the sum of their ages is 47, what are their respective ages?
6. Lilian and Susan share $30 between themselves. If Susan gets twice as much as Lilian, find each girl's share.

7. When loaded with bricks, a lorry weighs 11 600 kg. If the bricks weigh three times as heavy as the empty lorry, find the weight of the bricks.

8. When a number is added to two-thirds of itself, the result is 45. Find the number.

9. The numerator of a fraction is 5 less than the denominator. If 1 is added to both the numerator and the denominator the fraction would become $\frac{2}{3}$. Find the fraction.

10. The cost of mooncakes with double egg yolks cost 50 cents more than those with only a single egg yolk. Mrs Tan bought 6 mooncakes with double egg yolks and 5 with only a single egg yolk. If she paid \$36 for the 11 mooncakes, how much does a mooncake with double egg yolks cost?

11. Tom, Dick and Harry share \$256. Dick's share is four times as much as Tom's and Tom's share is one-third of Harry's. How much is each of their share?

☆12. Meng Kuang had saved a small sum of money from his weekly pocket money. After receiving a total of \$108 as hong bao money from his relatives during the Chinese New Year, he decided to donate one-fifth of his money to the Community Chest of Singapore. His other siblings also donated a total of \$148 to the Community Chest. If their total contribution to the Community Chest was \$200, how much did Meng Kuang save originally?

☆13. A group of boys had to choose between playing soccer and badminton. The number of boys choosing soccer was three times that of those choosing badminton. Asking 12 boys who chose soccer to play badminton would make the number of players for each game equal. Find the number who chose badminton originally.

Summary

To solve an equation, we can

1. add equal numbers to each side;

   e.g. if $x - 5 = 7$,
   then $(x - 5) + 5 = 7 + 5$

2. subtract equal numbers from each side;

   e.g. if $x + 4 = 15$
   then $(x + 4) - 4 = 15 - 4$

3. multiply each side by the same number;

   e.g. if $\frac{1}{3}x = 8$
   then $3\left(\frac{1}{3}x\right) = 3(8)$

4. divide each side by the same number, except 0.

   e.g. if $3x = 21$
   then $\frac{3x}{3} = \frac{21}{3}$

Review Questions 8

1. Solve the following equations:

   (a) $\frac{8-x}{3} = \frac{2x+3}{5}$ (b) $\frac{x+2}{6} = \frac{2x+7}{4}$ (c) $\frac{x}{2} = 5 + \frac{x}{3}$

   (d) $\frac{9}{4x} - 5 = 4$ (e) $\frac{2x+1}{6} - \frac{x-3}{4} = \frac{5}{12}$ (f) $\frac{x+3}{7} - \frac{2(x-4)}{3} = 1$

   (g) $\frac{3x-4}{5} - \frac{x+1}{4} = \frac{2x-4}{3}$ (h) $\frac{x}{3} + \frac{x-3}{4} - \frac{2x-7}{2} = 0$

2. State whether the following equations are identities. Give solutions to those that are equations.

(a) $3x - 5 = (x + 2) - (7 - 2x)$ (b) $7(x + 2) - 5(x - 6) = 12(x + 5)$

(c) $3(t - 4) + 5(t + 4) = 7(t - 3)$ (d) $6(a - 4) = 2(a - 10) - 4(1 - a)$

(e) $\frac{1}{2}(x - 3) - \frac{1}{3}(x + 6) = \frac{1}{6}(x - 21)$ (f) $\frac{1}{3}(x - 4) + \frac{1}{4}(x - 5) = \frac{1}{7}(x - 9)$

3. A man has a number of ducks costing \$10 each and three times as many chickens costing \$6 each. If the total cost of ducks and chickens is \$420, find the number of chickens the man has.

4. Three families, A, B, and C, share 480 kg of rice. B gets twice as much as A, and C gets half as much as B. How much does each family get?

5. A shopkeeper buys some eggs at 15 cents each. Six of them are broken while the rest are sold at 20 cents each. If he makes a profit of \$4.80, find how many eggs he bought.

6. Tiong Beng worked part time at a fast food restaurant during the school holidays. He was paid the normal hourly rate of \$2.80 per hour. Overtime rate is one and a half times the normal rate. He received \$173.60 for 54 hours of work. How many hours did he work overtime?

7. Peter has 25 sweets and Lilian has 55. How many sweets must Peter give Lilian so that Lilian will have 4 times as many sweets as Peter?

8. With a fixed amount of money \$M, Mr Phua can use it to pay the wages of a local worker for 40 days or to pay the wages of an unskilled foreign worker for 60 days. If he employs one local worker and one unskilled foreign worker for a renovation project, how many days can the \$M be enough to pay for these two workers?

9. Drawing pencils cost 8 cents each and coloured pencils cost 11 cents each. Two dozen assorted pencils cost \$2.16. How many coloured pencils are there?

10. A man travels from A to B at 4 km/h and from B to A at 6 km/h. The total journey takes 45 minutes. Find the distance travelled.

11. A number is 5 times another number. By adding 8 to each number, the first number becomes only 3 times the second. What are the two numbers?

12. How can the number 45 be divided into two parts so that 4 times one part is 9 less than 5 times the other?

*13. A man normally takes 5 hours to travel at a certain speed from city A to city B. One day, he increases his speed by 4 km/h and finds that the journey from A to B takes half an hour less than the normal time. Find his normal speed.

*14. The sum of the ages of Fandi and Ahmad is 38. Seven years ago, Fandi was three times as old as Ahmad. Find their present ages.

*15. Aramugum has enough money to buy 24 apples. If the price of each apple is reduced by 5 cents, he will be able to buy an extra 6 apples with the same sum of money. Find the original cost of each apple.

Problem Solving

1. A tank can be fully filled with water using a pipe that fills 20 litres in a minute. A bigger pipe that can fill 25 litres in a minute will take one minute less to fill the same tank. How many minutes does the smaller pipe take to fill the tank?

2. Mary is twice as old as John and half as old as Bob. In 22 years' time, Bob will be twice as old as John. How old is Mary?

3. Alice and Belinda start off simultaneously from two towns to meet one another. If Alice travels 2 km/h faster than Belinda, they would meet in 3 hours. If Belinda travels 1 km/h slower and Alice' speed is two-thirds of her previous speed, they would meet in 4 hours. How far apart are the two towns?

4. At a fast-food restaurant, for every three people who ordered a cheeseburger, there are five people who ordered an apple pie. The number of people who ordered the cheeseburger is 5 more than the people who ordered the apple pie. If the total number of people who ordered food is 1 678, how many people ordered apple pie?

5. Given that A, B, C and D are whole numbers such that $A + B = 8$, $B + C = 11$, $B + D = 13$ and $C + D = 14$, find the values of A, B, C and D.

6. Given that A, B, C and D are whole numbers such that $A \times B = 8$, $B \times C = 28$, $C \times D = 63$ and $B \times D = 36$, find the values of A, B, C and D.

Revision Exercise II No. 1

1. What is the sum of $4x$ dollars and $4x$ cents? Express your answer in cents.

2. Simplify $2x - \{[2x - 3(2y - 3x) - (5y - 2x) - 9y] - 3x\}$.

3. Simplify

(a) $15a \div 5$;	**(b)** $4c \times 7c$;
(c) $a \times 5 \times b$;	**(d)** $a \times 5 \times 2 + c$;
(e) $\frac{2}{3} \times 21a^2$;	**(f)** $5xy \times 0$;
(g) $2xy + 0$;	**(h)** $15x - 3 \times 5x$;
(i) $8a + 2 \times 6a$;	**(j)** $4x + 8x \div 2$.

4. Solve the following equations:

(a) $3x - (2x - 1) = 7$

(b) $8(7 - x) = (3x - 1)$

(c) $8 - \frac{3}{4}(x - 4) = \frac{1}{8}(x + 1)$

(d) $\frac{x-1}{2} - \frac{x-2}{3} = \frac{x-3}{4}$

5. (a) Find the value of $3x^2 + 14x - 7$ when $x = 5$.

(b) If $x = 2$ is a solution of the equation $x^2 - 7x + k = 0$, find the value of k.

6. A father is 36 years old and his son is 6 years old. In how many years' time will the father be twice as old as his son?

7. Evaluate each of the following showing each step clearly

(a) $(-3)^3 + (-2)^2 \times (4 - 7)$

(b) $2 \times (3 - 7)^3 - 4 \times 7$

(c) $5 \times (-4)^2 - 6 \times (18 - 23)^3$

8. Simplify each of the following and give your answer as a fraction in its simplest form.

(a) $3\frac{4}{5} \times \left(-1\frac{1}{2}\right) + 2\frac{1}{3} \div \left(-1\frac{1}{3}\right)$

(b) $2\frac{4}{9} - \frac{3}{2} \times \left(\frac{2}{3} - \frac{3}{2}\right)^2$

9. Copy and complete the following using one of the symbols >, = or <:

(a) $5 \ \square \ 2$	**(b)** $-5 \ \square \ -2$
(c) $\frac{1}{3} \ \square \ 0.33$	**(d)** $-7 \ \square \ 14 \div (-2)$

10. Evaluate the following:

(a) $(-100) - (350)$

(b) $(-13) \times (-2)$

(c) $(-64) \div (-4)$

(d) $(-5) + 7 + (-6) + 16$

Revision Exercise II No. 2

1. Simplify $2a - \{4[b - 3(a - 2 \times b - 2a)] - 10(3b - 5a)\}$.

2. Given that $a = 1$, $b = 2$, $c = 0$ and $d = -3$, evaluate

(a) $\frac{a^2bd}{3a - d}$;	**(b)** $\frac{d^2 + bc}{b + a}$;
(c) $a^2 + b^2 + d^2$;	**(d)** $a^3 + b^3 + d^3$.

3. Solve

(a) $\frac{x}{4} + \frac{1}{6} = \frac{x}{2} + \frac{1}{8}$;

(b) $\frac{3x + 2}{4} + \frac{x - 2}{3} = 2 + \frac{x - 5}{2}$.

4. I think of a number, halve it and then subtract 1 from it. The result is double the amount obtained by dividing the number by 3 and subtracting 4 from it. Find the number.

5. Ali earns \$480 a month and Ahmad earns \$720 a month. How many months must Ali work to earn as much as Ahmad does in 6 months?

6. Evaluate the following:

(a) $18 \times \{[11 + 3) \div 7] - 2\}$

(b) $[5 \times (-9)] - [(-9) \times (-3)]$

(c) $(-25) \div (-5) + 24 \div (-6)$

(d) $\frac{(-10)^2 \times 12}{8 \times (-2)}$

7. Evaluate

(a) $\left(1\frac{5}{11} + \frac{7}{22}\right) \div \left(\frac{7}{15} + \frac{2}{5}\right)$;

(b) $\dfrac{5\frac{3}{4} + 2\frac{2}{3} \times 1\frac{5}{16}}{\frac{2}{5} \div \frac{4}{15}}$;

(c) $\left(-5\frac{1}{2}\right) + \left[4\frac{1}{2} \times \left(-\frac{1}{12}\right)\right]$.

8. Estimate the value of

(a) $\dfrac{64.11}{1.62}$;

(b) 56.967 13 − 34.230 67;

(c) $\dfrac{7.14 \times 0.206}{0.050\,7 \times \sqrt{48.902}}$;

correct to 1 significant figure.

9. A car travels x km in t hours. Assuming the speed is constant, how many metres does it travel in one second?

10. Use your calculator to evaluate

(a) $4(25 - 4\pi)$;

(b) $\dfrac{\frac{2}{7} + \sqrt{15} \times \sqrt[3]{3}}{\sqrt{5} \div \frac{11}{2}}$;

(c) $\dfrac{10.92^3 \times \sqrt{7.42}}{5.68^2} + \sqrt[3]{\dfrac{5.16 \times 6.42}{1.56 \times 4.72}}$;

giving each answer correct to 3 decimal places.

Revision Exercise II No. 3

1. Remove the brackets and simpify
$2(p + r + s) + (2p - 3q - 4r - s)$
$- (4p - q - 5r + 2s)$.

2. Given that $a = 5$, $b = 6$ and $c = -1$, evaluate

(a) $3a + b + c$; (b) $\dfrac{ab}{2c}$;

(c) $2a + 3b - 4c$; (d) $ab - bc$.

3. Solve the following equations:

(a) $\frac{1}{3}x + 15 = 2x$

(b) $3(1.5x - 0.2) = 0.5x$

(c) $\dfrac{x+1}{5} + \dfrac{x-1}{5} = 6$

(d) $\dfrac{3x-5}{6} + 2 = \dfrac{5x-4}{5}$

4. A bookshelf can hold 45 books, each 6.3 cm thick. How many books can it hold if each book is 2.1 cm thick?

5. A mother is now three times as old as her daughter. If the sum of their ages five years ago was fifty, how old is the mother now?

6. Evaluate the following:

(a) $63 \div (-9)$

(b) $[7 \times (-2)] + [6 \times (-5)]$

(c) $(-8) \times (-15) - 22$

7. Use a calculator to evaluate the following giving your answers correct to 2 decimal places:

(a) $\sqrt{62.5^3 - 58.5^2}$ (b) $\dfrac{(36.27)^2 \div 21.68}{15.86 \div (2.66)^2}$

(c) $\dfrac{(4.62^2 + 2.68^2) \div 3.42}{\sqrt[3]{168.4^2 + 26.8^2}}$

8. Replace each $\square$ with '>' or '<'.

(a) $\frac{4}{7} \square \frac{3}{7}$ (b) $1\frac{2}{5} \square 2\frac{1}{5}$

(c) $-5 \square -4$ (d) $\frac{7}{12} \square \frac{7}{13}$

(e) $0.54 \square 0.539\,9$

(f) $-0.001 \square 0.000\,1$

9. Given that 23.78 × 583.5 = 13 875.6, find the value

(a) 0.237 8 × 58.35

(b) 13.875 6 ÷ 0.023 78

10. Evaluate each of the following, giving your answer as a fraction in its lowest term.

(a) $\dfrac{\frac{3}{4} - 3\frac{1}{2}}{2\frac{1}{3} \times \left(-\frac{3}{2}\right) + \frac{1}{4}}$

(b) $\left(\frac{-3}{5}\right) \times \left(2\frac{1}{2}\right) - 2 \times \left(-1\frac{1}{3}\right)^2$

Revision Exercise II No. 4

1. Round off

(a) the number 5.335 2 to 2 decimal places;

(b) the number 0.090 38 to 2 significant figures;

(c) the number 4 972 to the nearest hundred;

(d) the number 12 097 to the nearest ten.

2. Simplify

(a) $2x^2 - 5x - 2(1 - 2x + 3y^2)$;

(b) $(x - 3y + 4z) - (x - y - 2z)$.

3. Simplify
$\frac{1}{2}(x - y) + \frac{1}{3}(2x - 5y) - \frac{1}{5}(6x - 5y)$.

4. A student paid $5.40 for 30 pencils. Some pencils cost 10 cents each and others cost 20 cents each. How many pencils of the 10-cents type did he buy?

5. Find x when

(a) $9x = 15$; **(b)** $\frac{3}{4}x = 6$;
(c) $18 = 32 - x$; **(d)** $\frac{1}{2}x + 6 = 8$;
(e) $6x - 3x = 31 - 25$; **(f)** $1.2x = x + 1$.

6. A boy cycles $12x$ km in 5 hours. How far can he cycle in $3y$ hours at the same speed?

7. Estimate each of the following correct to 1 significant figure:

(a) $4\,980 \times 409$ **(b)** $2\,986 \times 304$
(c) $100\,523 \div 19$ **(d)** $199\,607 \div 51$

8. Find the approximate value of each of the following, giving your answer correct to 1 significant figure:

(a) $\frac{11.01 \times 0.661}{2\,199}$ **(b)** $\frac{83.9}{0.040\,7}$
(c) $\sqrt{\frac{16.01 \times 36.0}{3.999}}$ **(d)** $\sqrt[3]{\frac{9.06 \times 20.94}{7.025}}$

9. Given that $x = 2$, $y = -3$, $z = 4$ and $k = -1$, evaluate each of the following:

(a) $xy - 3zk + 2x$ **(b)** $y(2z - 3kx - xz)$
(c) $\frac{3x - 4z}{4y - 5k}$ **(d)** $\frac{4k}{3x - y + z}$

10. Peter is 24 years younger than his father. In 5 years' time, his father will be 3 times as old as Peter.

(a) How old is Peter now?
(b) How old will Peter's father be in 25 years' time?

Revision Exercise II No. 5

1. Given that $a = 3$, $b = 2$, $x = 0$ and $y = 1$, evaluate

(a) $a^2 - b^2$; **(b)** $ab + xy$;
(c) $ax - (b - y)^2$; **(d)** $ab^2 - xy^3$.

2. Simplify $\frac{7x - 1}{2} + \frac{2x + 3}{6} - \frac{5x - 4}{2}$.

3. Solve $\frac{6x}{5} - \frac{x - 1}{4} = x$.

4. Find x when

(a) $7x = 42$; **(b)** $6x = 16$;
(c) $2x + 19 = 41$; **(d)** $3x = 7 + 8$;
(e) $6x - 32 = 2x$; **(f)** $3.4x = x - 3$.

5. Simplify

(a) $2m \times 6n$; **(b)** $18xy \times \frac{1}{6}$;
(c) $7 \times 4a$; **(d)** $8a + 2a \div 4$.

6. **(a)** Round off 6.236 to 2 significant figures.
(b) Round off 2.594 74 to 3 decimal places.
(c) Write down, correct to the nearest whole number, the value of $\sqrt[3]{998}$.

7. Find the value of each of the following:

(a) $\frac{(-16) + (-20)}{(-9)}$ **(b)** $\frac{(-5) \times (11 - 15)}{8 \times [-19 - (-9)]}$
(c) $(-6) \times (-8) - 3 \times (-5) - (-6)^2$
(d) $\frac{3 - 5 - (-8)}{4 \times (-5)}$

8. Given that $x = 5$, $y = -4$, $z = -2$, $h = 0$ and $k = -1$, evaluate each of the following:

(a) $5x - 3yk + 4hz$ **(b)** $xy(2x - 4k + xhz)$
(c) $\frac{5x - 3z}{xhk - 5yk}$ **(d)** $\frac{2k}{xh - yz}$

9. **(a)** Ali is six times as old as his daughter Rohana. How old was Ali when Rohana was born if the sum of their ages will be 49 in 7 years' time?
(b) Peter is 8 years younger than Joanne. In 5 years' time, Joanne will be twice as old as Peter. How old will Peter be in 15 years' time?

10. Use your calculator to evaluate each of the following, giving your answer correct to 4 significant figures.

(a) $\frac{28.75^2 - \sqrt[3]{45.67}}{84.4 + \sqrt{75.6}}$ **(b)** $\frac{56.34^4}{384.7^2 - 46.5^3}$
(c) $46.9^2 - 15.3 \times \sqrt{47.4}$
(d) $\sqrt{5\,487} \div (48.3 - \sqrt[3]{784})$

Note: *Take* $\pi = \frac{22}{7}$ *unless otherwise stated for all the Specimen Papers.*

Mid-year Examination Specimen Paper 1

Part 1 (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. State whether each of the following statements is true or false:

(a) The first four prime numbers are 1, 2, 3 and 5. [1]

(b) Every prime number is a rational number. [1]

(c) 3.141 59 is an irrational number. [1]

(d) $\left(2\frac{3}{4} + 1.25\right)$ is an integer. [1]

2. (a) Find the HCF of 36, 48 and 60. [1]

(b) Find the LCM of 130, 195 and 325. [2]

3. Simplify the following and give each answer as a single fraction:

(a) $3\frac{1}{6} + 1\frac{2}{3} \div 1\frac{1}{2}$ [2]

(b) $\frac{14}{15} \times 1\frac{1}{54} \div 4\frac{5}{18}$ [2]

4. Evaluate each of the following:

(a) $14 + (-20) - (-30)$ [1]

(b) $(-2)^2 \times (-1)^5$ [1]

(c) $210 \div (-7) + 10$ [1]

(d) $327 \times 129 \times 0$ [1]

5. (a) Express 0.875 as a fraction in its lowest terms. [2]

(b) Express $1\frac{31}{80}$ as a decimal. [2]

6. (a) Express 32.748 7 correct to

(i) 2 decimal places; [1]

(ii) 3 significant figures. [1]

(b) Simplify $\frac{18.9 \times 6.3}{12.6 \times 0.108}$, giving your answer as a decimal. [2]

7. Express 4 624 in prime factors and hence, or otherwise, find the value of $\sqrt{4\,624}$. [3]

8. (a) Subtract $(10x^4 - 7x^3 - 2x^2 - 3x - 5)$ from $(3x^4 - 7x^3 - 10x^2 + 7x - 2)$. [2]

(b) Add $(-5x^2 + 12x + 17)$ to $(2x^3 + 7x^2 - 4x - 7)$. [2]

9. One-fifth of a plank is sawn off and three-eighths of the remaining piece is then thrown away. What fraction of the original plank remains? [3]

10. Lamp posts along one side of a street are 6 m apart. If they extend for one and a half kilometres, find the number of lamp posts along the street. [3]

11. Solve $\frac{3}{4}x + \frac{1}{2} = \frac{2}{3}x$. [4]

12. An apple costs x cents. An orange costs y cents more than an apple. Express the cost of 8 apples and 12 oranges in terms of x and y. [4]

13. Given that $a = \frac{1}{2}$, $b = \frac{1}{3}$, $c = -\frac{1}{4}$ and $d = 0$, find the value of $\frac{1}{a+b} - \frac{1}{bc - ad}$. [3]

14. Identify a rule for each of the following number sequences, and then complete it:

(a) 0.4, 0.5, 0.7, 1.0, 1.4, _____, _____ [1]

(b) 3, 4, 8, 17, 33, 58, _____, _____ [2]

Part II (50 marks) **Time: 1 h 15 min**

*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. (a) Simplify $5x - \{8x - [7 - (4x - 8 - 2x)]\} - 5$. [3]

(b) If $u = \frac{1}{2}h(a + b)$, find a when $u = 84$, $h = 7$ and $b = 16$. [3]

2. John is $1\frac{1}{3}$ times as heavy as Mary. If their total mass is 112 kg, find the mass of John. [4]

3. Consider the following pattern:

$$\frac{1}{1 \times 2} = 1 - \frac{1}{2}$$
$$\frac{1}{2 \times 3} = \frac{1}{2} - \frac{1}{3}$$
$$\frac{1}{3 \times 4} = \frac{1}{3} - \frac{1}{4}$$
$$\vdots \qquad \vdots$$
$$\frac{1}{342} = \frac{1}{a} - \frac{1}{b}$$

(a) Write down the 8th line in the pattern. [1]

(b) Using the above, find the value of $\frac{1}{50} - \frac{1}{51}$. [1]

(c) Find the value of a and of b. [2]

4. (a) The product of two numbers is $1\frac{1}{5}$. If one of the numbers is $\frac{1}{5}$, find the sum of the two numbers. [4]

(b) Solve the equation

$$\frac{4x + 1}{2} = \frac{2x - 1}{4} + \frac{x + 3}{8}.$$ [4]

Section B (28 marks)

5. Evaluate each of the following, giving your answer correct to 4 significant figures where necessary. (Take $\pi = 3.142$)

(a) $\left(\frac{2}{5}\right)^2 + 4\frac{3}{4} \div 1\frac{1}{4}$ **(b)** $\frac{15.76 + 3.58^3}{\sqrt{14.75 \div 0.03}}$

(c) $\frac{25\pi}{7.58 + 6.76^2}$ **(d)** $\frac{4}{26.7} - \frac{6}{\sqrt[3]{0.73}}$

[8]

6. (a) Estimate the value of $\frac{\sqrt{2\,505} \times 8.705}{4.98 \times 2.907}$ giving your answer as a whole number. [2]

(b) Given that $6.5 \times 232 = 1\,508$, find the value of

(i) 0.065×2.32 [2]

(ii) $1.508 \div 650$ [2]

showing your working clearly.

7. Simplify each of the following expressions

(a) $5h - 3[4k - 6m + 2\{3h - (2h + 3k)\}]$ [3]

(b) $2x + \frac{3y - 4x}{3} - \frac{2x - 4y}{5}$ [3]

8. (a) Solve the equation $\frac{3x + 5}{4} = 2x - 7$. [3]

(b) If $\frac{h}{k} = \frac{h - 2m}{g}$, find the value of g when $h = 5$, $k = 4$ and $m = -2$. [2]

(c) Subtract the sum of $3x^3 + 4x^2 - 7x + 5$ and $2x^3 - 4x - 4$ from $7x^3 - 4x^2 - 13$, giving your answer in ascending powers of x. [3]

Mid-year Examination Specimen Paper 2

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. (a) Find the LCM of 77, 132 and 198. [2]

(b) Find the smallest number which, when divided by 16, 20 or 24, leaves a remainder of 3. [2]

2. Evaluate **(a)** $2 \times 2\frac{2}{5} \times \left(3\frac{1}{4} + 1\frac{7}{16}\right)$, [2]

(b) $2 \times 2.41 \times (3.27 + 1.44)$. [2]

3. Simplify **(a)** $12a - 2(3a + 5) + 10$, [2]

(b) $3(2x - y) - 2(3x - y)$. [2]

4. Solve

(a) $4(y + 2) - (3y - 1) = 5 - (2y + 3)$; [2]

(b) $4(2x - 1) - 12 = 16 - 2x$. [2]

5. A father is 45 years old and his son is 9 years old. In how many years' time will the father be three times as old as the son? [4]

6. John has \$3.75 and David has twice as much money as John. How much money do they have altogether? [3]

7. Simplify **(a)** $4 - \left(1\frac{3}{4}\right)^2$; [2]

(b) $\frac{2}{7} + \frac{1}{2} \div \frac{3}{14}$. [2]

8. Express $\frac{7}{33}$ as

(a) a recurring decimal; [2]

(b) a decimal correct to 4 decimal places. [1]

9. (a) Estimate the value of $\frac{7.984 \times 9.017}{3.967 \times 0.304}$, correct to the nearest whole number. [2]

(b) Use your result in (a) to find the value of $\frac{7\,984 \times 90.17}{0.396\,7 \times 304}$. [1]

10. Write down the difference between $3y$ minutes and $25y$ seconds, giving your answer in seconds. [3]

11. Identify a rule for each of the following number pattern, and then complete it:

(a) 7, 9, 13, 21, 37, ____, ____ [2]

(b) $\frac{3}{5}, \frac{5}{8}, \frac{9}{13}, \frac{3}{4}$, ____, ____ [2]

12. On a certain morning, the temperature in London was –14°C, the temperature in Hong Kong was 8°C while the temperature in Singapore was 24°C.

(a) Find the difference in temperature between London and Singapore. [1]

(b) The temperature in Shanghai was mid-way between the temperature of London and Hong Kong. Find the temperature of Shanghai on that morning. [2]

13. Simplify $\frac{2x-5}{3} - \frac{2(3x+1)}{5} - 2x$. [3]

14. If $a = 2$, $b = 0$, $c = 1$ and $d = -3$, evaluate

(a) $ac - 2d^2 + 3bd$; [2]

(b) $(2ad)^2 - 2cd^2 + 2bc^3$. [2]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. Evaluate each of the following:

(a) $16 - 3 \times (-2)$ [1]

(b) $45 + 16 \div (-2)^3$ [1]

(c) $\{[(215 + 25) \div 5 \times 6] - 132 \div 8\} \times 4$ [2]

2. Evaluate each of the following, giving your answer correct to 4 significant figures: (Take $\pi = 3.142$)

(a) $3.26^3 - \sqrt{0.45}$ [2]

(b) $\frac{35\pi}{\sqrt[3]{4.87^5 + 9.76^4}}$ [2]

3. Solve (a) $\frac{5-x}{3} = 1 + \frac{1-x}{4}$, [4]

(b) $\frac{3}{x} + \frac{1}{3x} = \frac{1}{3}$. [4]

4. (a) The sum of two numbers is 35; half of the smaller number is equal to $\frac{1}{3}$ of the greater number. Find the numbers. [4]

(b) Simplify $\frac{a-1}{5} + \frac{2a-3}{4}$. [2]

Section B (28 marks)

5. John and Derek had \$24 and \$50 respectively. They spent the same amount of money on some books. How much money did each boy spend if Derek had 3 times as much money as John after buying the books? [4]

6. (a) If $h^2 = \frac{k}{2x} - \frac{g}{5y^2}$, find the value of x when $y = 3$, $h = -2$, $k = -1$ and $g = 4$. [3]

(b) A ribbon is 14 m long. Fourteen pieces, each of length 25 cm, are cut from it. The remaining piece is cut into equal lengths of 40 cm. How many pieces of length 40 cm are there? What is the length of the piece that is left over? [4]

7. The numerals –2, –1, 1, 2, 3, 4, 5 and 6 are written on eight separate cards, with one number on each card.

(a) List the pairs of cards that have a sum of 4. [2]

(b) List the pairs of cards that have a product of 2. [2]

(c) List the groups of three cards that have a sum of 10. [3]

8. **(a)** Simplify
$2x^2 - 5x - 2x(3 - 4x) + 2(3x - 1)$. [2]
(b) Subtract $2x^3 - 5 + 4x$ from the sum of $2x - 5x^2 + 3x^3$ and $7x^2 + 5x^3 - 4$. [3]

9. Consider the pattern

$$1 = 1 = 1^2$$
$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
$$\vdots \quad \vdots$$

(a) Write down the tenth line in the pattern. [1]
(b) Find the value of $1 + 3 + 5 + \ldots + 29$. [2]
(c) Given that $1 + 3 + 5 + \ldots + (2x - 1) = 13^2$, find the value of x. [2]

Mid-year Examination Specimen Paper 3

Part I (50 marks) Time: 1 h

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. Evaluate
$-20 + 2[-38 - 20 \times (-8) + 5 - (-10 - 20)]$. [3]

2. Find the HCF and LCM of $3^2 \times 5$, 3×5^2 and $2^3 \times 3 \times 5$. [4]

3. Express 3 136 as a product of prime factors and hence, find the value of $\sqrt{3\ 136}$. [3]

4. Solve the equation $\frac{5x}{6} - \frac{2}{3} = \frac{x}{2} + \frac{1}{6}$. [3]

5. Copy and complete the following statements with '<' or '>':
(a) 2 ____ 21 **(b)** -1 ____ 2
(c) -1 ____ -2 **(d)** $x + 1$ ____ $x + 2$
[4]

6. Find the value of a from the formula
$A = \frac{1}{2}h(a + b)$ if $A = 117$, $h = 13$ and $b = 11$. [3]

7. Find the exact value of $\frac{0.03 \times 0.49}{42}$, giving your answer as a decimal. [2]

8. Which of the following numbers is/are divisible by both 2 and 4?
102, 336, 3 306, 11 048 [4]

9. Simplify each of the following, giving your answer as a fraction in its lowest terms:
(a) $2\frac{2}{3} \times 1\frac{4}{5} - 1\frac{3}{10}$ [2]
(b) $3\frac{3}{4} - 1\frac{1}{2} \div 2\frac{2}{3}$ [2]

10. Identify a rule for each of the following number sequences, and then complete it.
(a) 3, 5, 8, 12, 17, 23, ____, ____ [2]
(b) $\frac{2}{3}$, 1, $1\frac{2}{3}$, $2\frac{2}{3}$, 4, $5\frac{2}{3}$, ____, ____ [2]

11. **(a)** Express $\frac{5}{8}$ as a decimal. [1]
(b) Change 0.86 into a fraction in its lowest terms. [1]
(c) Express 0.002 56 correct to 2 significant figures. [1]
(d) Express 2.445 7 correct to one decimal place. [1]

12. Simplify each of the following expressions:
(a) $x + 2(3x - y) - 5(y - x)$ [2]
(b) $-5(2x - z) + 2z - x$ [2]

13. Given that $a = 2$, $b = -1$ and $c = 5$, evaluate
(a) $3(c - b) + 2bc$; [2]
(b) $3a^2 - 2bc + b^3$. [2]

14. Ah Beng has \$38 and Ah Lian has \$20. If Ah Beng gives some money to Ah Lian, Ah Beng will then have one-third of what Ah Lian has. How much does Ah Beng give to Ah Lian? [4]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. Evaluate each of the following and give your answer correct to 4 significant figures:
(a) $2 \times 5.12 \div \sqrt{7.96}$ [2]
(b) $\sqrt[3]{42.6} + 8.75 \div 0.14$ [2]

2. (a) Given that $H = \dfrac{V(P-R)}{500}$, find the value of P when $V = 80$, $H = 3.2$ and $R = 65$. [3]

(b) Solve $\dfrac{x}{3} - \dfrac{3x-5}{2} = 6$. [4]

3. Mr Li's age is four times his son's. Six years ago, Mr Li was ten times as old as his son. How old are they now? [4]

4. (a) An apple costs a cents while a pear costs b cents more than an apple. Find, in terms of a and b, the cost of 5 apples and 7 pears. [4]

(b) Write the following using mathematical symbols:

(i) 7 is greater than 5 but less than 10. [1]

(ii) a lies between b and c where a, b and c are numbers. [2]

Section B (28 marks)

5. Copy and complete the following number patterns:

(a) 1, 2, 4, 7, 11, 16, 22, ____, ____ [2]

(b) 1, 2, 5, 10, 17, 26, 37, ____, ____ [2]

6. (a) Four girls, A, B, C and D, share a box of chocolates among themselves. A takes $\frac{1}{4}$ of the chocolates, B takes $\frac{3}{8}$ of the remainder and C takes $\frac{5}{9}$ of the left-overs. What fraction of the chocolates does D have? [4]

(b) What polynomial must be subtracted from $2x^5 + 3x^2 + 5x - 7$ to give $6x^5 - 4x^4 + 2x^2 - 8x$? [3]

7. If $37.56 \div 2.31 = 16.26$, find the value of

(a) $0.375\,6 \div 23.1$ [2]

(b) $1.626 \times 23\,100$ [2]

showing your working clearly.

8. (a) There are seven consecutive even numbers. If the largest number is x, write an algebraic expression for the middle term. If the middle term is 48, find the value of the smallest term. [4]

(b) Use the distributive property to evaluate 876×999. [2]

9. (a) Simplify $\dfrac{3x}{7y} \div \sqrt{\dfrac{81x^4}{49y^2}} + \dfrac{15}{x}$. [3]

(b) Solve the equation $\dfrac{3x-4}{7} = \dfrac{2}{3}(x+5)$. [4]

Mid-year Examination Specimen Paper 4

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. Arrange the following numbers in ascending order:

$0.\dot{1}\dot{7}$, $0.1\dot{7}$, 0.177, 0.178 [3]

2. Find the LCM and HCF of 135, 180 and 270. [4]

3. Express 5 832 as a product of prime factors and hence, find the cube root of 5 832. [3]

4. Simplify
$25 - (-3) + [12 \times (-3) + 15 - (-35)] \div 7$. [3]

5. Simplify (a) $1\frac{3}{4} - \frac{3}{8} \times \frac{2}{9}$, [2]

(b) $1\frac{1}{2} \div 2 - \frac{1}{2}$. [2]

6. If $x = 3$, $y = 2$ and $z = -1$, find the value of $\dfrac{2z^2 - 5x}{xz - y}$. [3]

7. Simplify $\dfrac{2x-1}{3} - \dfrac{2(x-2)}{5}$. [3]

8. Solve the following equations:

(a) $10x - 9 = 19 + 9x$ [2]

(b) $2\frac{1}{4}x = 18$ [2]

9. What polynomial must be added to $2x^3 - 5x^2 + 7x - 4$ to give $9 + 3x - 5x^2 + 6x^3$? [3]

10. Solve $2 - \frac{1}{2}(x-2) = \frac{3}{4}(x+3) - \frac{1}{4}(x-5)$. [4]

11. A man is 3 times as old as his son. Five years ago, he was 4 times as old as his son. Find their present ages. [4]

12. Write an algebraic expression for each of the following:
    (a) The distance a car travels in x hours if its speed is 70 km/h.
    (b) The number of 22-cent stamps you can buy with $\$x$.
    (c) The number of grammes in k kilogrammes. [3]

13. Simplify $25xy - 3x(5x - y) + 2[5(x - 4y) - 7(x - y)]$. [3]

14. Simplify $\dfrac{3\frac{2}{3} - 1\frac{4}{5}}{\left(1\frac{1}{2}\right)^2 + 2\frac{1}{4}}$. [3]

15. David cycles x km in 3 hours. If he maintains the same speed, how far can he cycle in $12y$ minutes? [3]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. Find the value of $\dfrac{(2.06 + 1.24)^2}{3.4^2 - 1.5^2}$, giving your answer correct to 2 decimal places. [3]

2. If $x = -1$, find the value of $3x^3 + 2x^2 + 5x + 9$. [3]

3. Solve the equation $\dfrac{3x - 1}{3} - \dfrac{x + 3}{5} = 8$. [4]

4. Arrange the following fractions in ascending order: $\dfrac{34}{51}, \dfrac{76}{95}, \dfrac{88}{121}, \dfrac{169}{273}$. [2]

5. The product of two numbers is $2\frac{2}{3}$. If one number is $\frac{3}{5}$, find their sum. [4]

6. A man's salary is \$1 200 a month. He spends $\frac{5}{8}$ of it on food and lodging, $\frac{1}{5}$ of the remainder on transport and saves the rest. Calculate the amount of money he saves. [6]

Section B (28 marks)

7. (a) Evaluate each of the following, giving your answer correct to 4 significant figures:
    (i) $\dfrac{\sqrt{54.6} \times 74.5^3}{46.7 - 0.8^4}$ [2]
    (ii) $\dfrac{84.5 + 7.5 \div 1.6}{\sqrt[3]{46.8} - \sqrt[4]{89.4}}$ [2]

    (b) Simplify $\dfrac{x - y}{2} - \dfrac{2x - 3y}{7}$. [3]

8. (a) Consider the number pattern

$$1^2 - 0^2 = 1$$
$$2^2 - 1^2 = 3$$
$$3^2 - 2^2 = 5$$
$$4^2 - 3^2 = 7$$
$$\vdots$$
$$x^2 - y^2 = 157$$

    (i) Write down the tenth line of the pattern.
    (ii) Find the value of $99^2 - 98^2$.
    (iii) Find the value of x and y in $x^2 - y^2 = 157$. [4]

    (b) Simplify $\sqrt[3]{64x^6y^9} - \frac{1}{2}[7 - (5 - 6x)]$. [3]

9. (a) If the value of $3x^3 + 2x^2 + xy + 7$ is equal to 40 when $x = -2$, find the value of y. [3]

    (b) $\frac{4}{7}$ of the passengers in an MRT train are men, $\frac{1}{3}$ of them are women and the rest are children. If there are 42 children, find
    (i) the total number of people in the train; [2]
    (ii) how many more men than women there are in the MRT train. [3]

10. (a) Copy and complete the following number sequence:

    5, 6, 10, 19, 35, 60, ____, ____ [2]

    (b) The sum of three consecutive odd numbers is 237. Find the largest of the three numbers. [4]

Mid-year Examination Specimen Paper 5

Part I (50 marks) Time: 1 h

*Answer **all** the questions. Calculators are **not** to be used in this section.*

1. Evaluate
   (a) $365 - 23 \times 10 - 32 \div (-4)$; [2]
   (b) $34 \times 6 + 5 - 4 \div 16$. [2]

2. If $a = \frac{1}{4}$, $b = \frac{-1}{3}$ and $c = \frac{3}{5}$, evaluate each of the following, giving your answer as a fraction in its lowest terms:
   (a) $3a - 2b + c$;
   (b) $2a - 3b \div 5c$. [4]

3. Find the HCF and LCM of $(2 \times 5 \times 7)$, $(3^3 \times 5)$ and $(2^2 \times 3^3 \times 5)$. [4]

4. Find the value of $x^2 - 4x + 2$ when x is
   (a) 0; **(b)** 1; **(c)** $\frac{1}{2}$; **(d)** -1. [4]

5. Simplify **(a)** $5a - b - (4a + 3b)$; [2]
   (b) $2(m - 2n) - 7m$. [2]

6. Evaluate $42 \div 6 \times 3 - 8 \times 5 - 70 \div (-5) \times 8 \div (-2)$. [3]

7. Simplify **(a)** $\frac{3}{5} + \frac{4}{7} - \frac{3}{35}$, [2]
   (b) $\frac{2}{5} + \frac{1}{3} \times \frac{2}{5}$. [2]

8. Solve the equation $2(x - 1) = 5 - (x + 2)$. [3]

9. Express 2 304 as a product of prime factors and hence, find the value of $\sqrt{2\,304}$. [3]

10. Arrange the fractions $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{7}$ and $\frac{8}{11}$ in descending order. [3]

11. Subtract $5x^2 - 3x - 2$ from $2x^3 - 3x^2 + 5x - 7$. [3]

12. Use the distributive law to simplify each of the following:
    (a) 567×99; [2]
    (b) $63 \times 7\frac{2}{5} - 17 \times 7.4 + \frac{37}{5} \times 24$. [2]

13. The cost of printing name cards is given by the equation $y = k + \frac{3\,000}{x}$ where y cents is the cost per card, x is the number of cards printed and k is a constant.
    (a) Given that $y = 10$ when $x = 600$, find the value of k.
    (b) Calculate the cost per card if 750 cards are printed.
    (c) How many cards will be printed if the cost per card is to be 7.5 cents? [4]

14. **(a)** List the whole numbers between $\sqrt[3]{65}$ and $\sqrt{65}$. [2]
    (b) What is the value of the number that is mid-way between -24 and 6? [1]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used in this section.*

Section A (22 marks)

1. Evaluate each of the following:
   (a) $\frac{2}{14.5} - \frac{7.2}{4.5^3}$ [2]
   (b) $\sqrt{54.89} \div \frac{5}{1.27^2} - 6.89$ [2]

2. Solve the equation:
   $3(m^2 - 2m) = 7 - m^2 + m(4m - 8)$. [3]

3. The table below shows a man's deposits and withdrawals in dollars. If he had \$520 in his savings account initially, calculate his balance after his withdrawals and last deposit of \$75. [4]

Deposits (in \$)	Withdrawals (in \$)
65	44
30	134
20	25
40	55
60	50
75	

4. If $x = \dfrac{5y}{z - 4y}$, find the value of y when $x = 3$ and $z = 2$. [4]

5. (a) Subtract $2\frac{1}{2}$ from the sum of $3\frac{1}{6}$ and $4\frac{5}{8}$. [3]

(b) Subtract the sum of $(2x^2 + 5x - 3)$ and $(4x^3 + 3x - 9)$ from the product of $2x$ and $(3x^2 - 5x - 4)$. [4]

Section B (28 marks)

6. (a) The sum of three consecutive odd numbers is 141, find the largest of the three numbers. [4]

(b) Five teachers took a group of students for a movie. Each adult ticket cost \$7.20 and students' tickets were sold at half price. If the total cost for the group was \$212.40, calculate the number of children in the group. [3]

7. (a) Solve the equation $x - 2 = \dfrac{x - 4}{3}$. [4]

(b) Copy and complete the following number sequence:

7, 4, 8, 6, 11, 10, 16, 16, 23, 24, _____, _____ [2]

8. (a) Similar bars of soap are sold in packs of 3 for \$2 or packs of 8 for \$5.20. Find the difference in the price for 48 bars of soap. [3]

(b) Mr Tan gave \$240 to his wife, $\frac{2}{5}$ of the remainder of his money to his son and kept the rest. If he had \$195 left, how much money did he have originally? [3]

(c) The temperature in Arrowtown for six days are −4°C, −12°C, −18°C, 2°C, 4°C and −8°C. Find the average temperature for these six days. [2]

9. (a) The result of adding 90 to a number is the same as multiplying that number by 6. Find the number. [3]

(b) John is 4 years older than David and Joe is 2 years younger than David. If the sum of their ages is 41, how old will John be in 8 years' time? [4]

CHAPTER 9

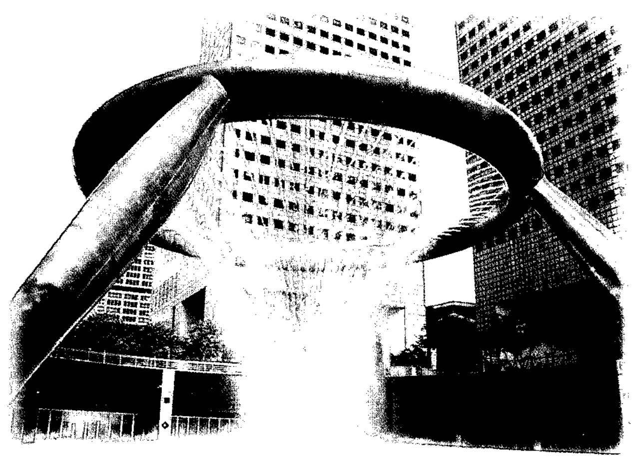
Perimeter and Area of Simple Geometrical Figures

In this chapter, you will learn how to

- △ find the perimeter and area of simple geometrical figures;
- △ solve problems involving these figures and figures related to them.

Preliminary Problem

The picture shows the famous "Fountain of Wealth', the world's largest fountain. Located at Suntec City in Singapore, it has attracted many visitors from all over the world. The bronze fountain has a circular ring of perimeter 66 m and a base area of 1 683 m^2.



Perimeter

The perimeter of a closed plane figure is the distance to go along one round of the plane.

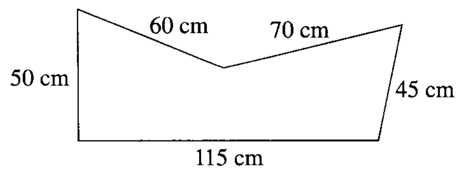


Fig. 9.1

In Fig. 9.1, the perimeter of the closed figure = (50 + 60 + 70 + 45 + 115) cm
= 340 cm

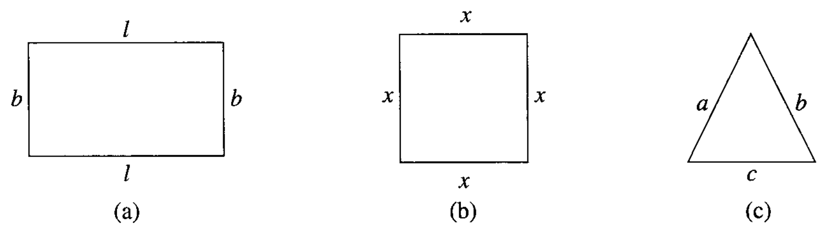


Fig. 9.2

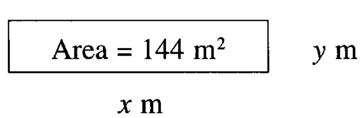
The perimeter of the rectangle in Fig. 9.2(a) is $\boldsymbol{P = (2l + 2b)}$ **units**
$\boldsymbol{= 2(l + b)}$ **units**

The perimeter of the square in Fig. 9.2(b) is $\boldsymbol{P = 2(x + x)}$ **units**
$\boldsymbol{= 4x}$ **units**

The perimeter of the triangle in Fig. 9.2(c) is $\boldsymbol{P = (a + b + c)}$ **units**

In-Class Activity

Adam intends to build a house of floor area 144 m^2.



Complete the following table to get some possible different perimeters, but the same area, of the floor:

x (m)	1	2	4	6	8	12	16	18	24	36	72	144
y (m)	144	72	36	24	18	12	9	8	6	4	2	1
Perimeter (m)	290	148	80	60	52	48	50	52	60	80	148	290

(a) Which design do you think is the cheapest to build? What is its perimeter?
(b) Which is the most expensive design to build?

Down Memory Lane

The metric system of units was first introduced in the nineteenth century. It is essentially a simple system based on the decimal system. As such, it allows easy conversion from one unit to another. The metric system of units involves measurements of length, area, mass, capacity and volume; all these units being related through the decimal notation.

Units of Length or Distance

We often use the following units to measure lengths or distances.

Metre (m): The basic unit of length is the metre.
This is normally used to measure distance between two places within a small compound. For example, the distance between your school gate and the school hall is measured in metres.

Centimetre (cm): This is a unit used to measure the length of small objects or the distance between two neighbouring points. For example, the length of your desk is measured in centimetres.

Millimetre (mm): This is normally used for measuring small lengths or thickness. For example, the thickness of a page of this book is given in millimetres.

Kilometre (km): This is used to measure the distance between two places far away from each other. For example, the distance between Hongkong and Singapore is measured in kilometres.

The above units are related as shown below:

1 centimetre (cm) = 10 millimetres (mm)
1 metre (m) = 100 centimetres (cm)
1 kilometre (km) = 1 000 metres (m)

In-Class Activity

Work with a partner.

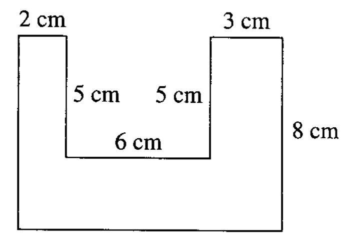
Use a measuring tape to measure and then record the perimeter of your
(a) classroom blackboard; (b) classroom floor.

Example

Find the perimeter of the given figure.

Solution

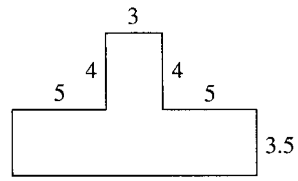
Perimeter of the figure
= [2 + 2(5) + 6 + 3 + 2(8) + (2 + 6 + 3)] cm
= (2 + 10 + 6 + 3 + 16 + 11) cm
= 48 cm

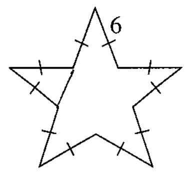


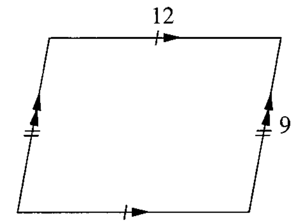
Exercise 9a

1. Find the perimeter of each of the following geometrical figures:
   (a) A triangle of sides 8 cm, 9 cm and10 cm.
   (b) A rectangle with length 9 cm and breadth 7 cm.
   (c) A square of side 7 cm.

2. Find the perimeter of each of the following figures. All measurements are in cm:

(a) 

(b) 

(c) 

3. A piece of wire is bent to form a square of side 8 cm. It is then reshaped to form a rectangle of length 10 cm and breadth x cm. Find x.

4. A boy is asked to run 15 times round the edge of a rectangular field measuring 30 m by 25 m. Find the total distance the boy ran.

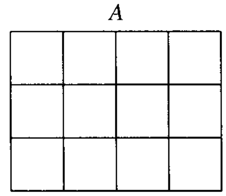
5. The length of a rectangular plot of land is twice its breadth. If its perimeter is 102 m, calculate its breadth.

6. Find, in cm, the perimeter of a rectangle measuring a m by b cm.

Area of Simple Figures

Which is bigger — a football field or a basketball court? The football field is bigger because it covers a larger surface than the basketball court. In other words, the football field has a larger area.

A



B

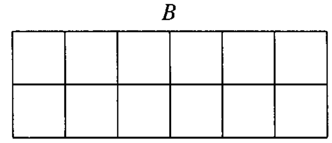


Fig. *A* and Fig. *B* consist of squares of the same size. Count the number of squares in each. What do you notice? Can we say that *A* and *B* cover the same amount of space, i.e., they have the same area?

Area is the measure of the amount of surface covered.

Units of Area

We often use the following units to measure area.

Square centimetre (cm²): We usually use unit squares to compare areas. A square of side 1 cm is used as a standard unit. We call this unit area 1 square centimeter (1 cm²). See Fig. 9.3.

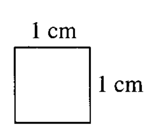


Fig. 9.3

Square millimetre (mm²): In Fig. 9.4, each small square has an area of 1 square millimetre (1 mm²).

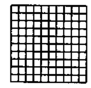


Fig. 9.4

$$1 \text{ cm} = 10 \text{ mm}$$
$$\therefore \quad 1 \text{ cm}^2 = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$$

Square millimetres are used to measure the areas of very small shapes.

Square metre (m²): The square metre is used to measure the area of large surfaces such as the floor area of a flat.

$$1 \text{ m} = 100 \text{ cm}$$
$$\therefore \quad 1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$$

Hectare (ha): The hectare is used to measure large land areas such as farms.

$$1 \text{ ha} = 10\,000 \text{ m}^2$$

Square km (km²): The square kilometre is used to measure the area of a very large surface such as the area of a country.

$$1 \text{ km} = 1\,000 \text{ m}$$
$$\therefore \quad 1 \text{ km}^2 = 1\,000 \text{ m} \times 1\,000 \text{ m} = 1\,000\,000 \text{ m}^2 = 100 \text{ ha}$$

Library Corner

The British used feet, inches, yards, furlongs, miles, etc. to measure length, and acre to measure area.

Find out what these units are and compare them with the SI units.

It's a Fact

The largest freshwater lake in the world is Lake Superior, one of the Great Lakes of North America. It covers an area of 82 350 km², roughly 130 times the size of Singapore.

Example 2

Express (a) *975 cm² in m²;* (b) *2.65 km² in m²;* (c) *48 000 m² in ha;* (d) *5 mm² in cm².*

Solution

(a)
$$975 \text{ cm}^2 = \underline{\qquad} \text{ m}^2$$
$$1 \text{ cm} = \frac{1}{100} \text{ m}$$
$$1 \text{ cm}^2 = \frac{1}{100} \text{ m} \times \frac{1}{100} \text{ m} = \frac{1}{10\,000} \text{ m}^2$$
$$\therefore \quad 975 \text{ cm}^2 = 975 \times \frac{1}{10\,000} \text{ m}^2 = 0.097\,5 \text{ m}^2$$

(b)
$$2.65 \text{ km}^2 = \underline{\qquad} \text{ m}^2$$
$$1 \text{ km} = 1\,000 \text{ m}$$
$$1 \text{ km}^2 = 1\,000 \times 1\,000 \text{ m} = 1\,000\,000 \text{ m}^2$$
$$\therefore \quad 2.65 \text{ km}^2 = 2.65 \times 1\,000\,000 \text{ m}^2 = 2\,650\,000 \text{ m}^2$$

(c)
$$48\,000 \text{ m}^2 = \underline{\qquad} \text{ ha}$$
$$10\,000 \text{ m}^2 = 1 \text{ ha}$$
$$1 \text{ m}^2 = \frac{1}{10\,000} \text{ ha}$$
$$\therefore \quad 48\,000 \text{ m}^2 = 48\,000 \times \frac{1}{10\,000} \text{ ha} = 4.8 \text{ ha}$$

(d)
$$5 \text{ mm}^2 = \underline{\qquad} \text{ cm}^2$$
$$1 \text{ mm} = \frac{1}{10} \text{ cm}$$
$$1 \text{ mm}^2 = \frac{1}{10} \text{ cm} \times \frac{1}{10} \text{ cm} = \frac{1}{100} \text{ cm}^2$$
$$\therefore \quad 5 \text{ mm}^2 = 5 \times \frac{1}{100} \text{ cm}^2 = 0.05 \text{ cm}^2$$

Area of a Rectangle

Consider a rectangle of length l units and breadth b units (see Fig. 9.5). The rectangle is made up of b rows, each with l unit squares.

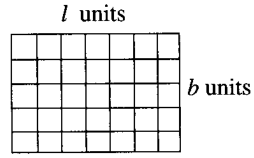


Fig. 9.5

No. of unit squares in the rectangle = $l \times b$

$\therefore$ area of rectangle = $(l \times b)$ unit^2

Problems

You are given 4 rectangular pieces of wood. Two of these measure 4 cm by 3 cm while the other two measure 13 cm by 1 cm. Use these 4 pieces of wood to enclose an area as large as you possibly can.

In general,

area of a rectangle = length × breadth

Hence, length = $\dfrac{\text{area}}{\text{breadth}}$, breadth = $\dfrac{\text{area}}{\text{length}}$

Problem Solving — Draw a diagram

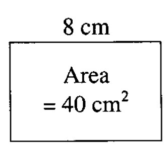
Many problems can be made easier by **drawing a diagram**.

Example 3

The area of a rectangle is 40 cm² and one of its sides is 8 cm long. Find the breadth and the perimeter of the rectangle.

Solution

Draw a simple diagram like this



Breadth of the rectangle = $\dfrac{40 \text{ cm}^2}{8 \text{ cm}} = 5$ cm

Perimeter of the rectangle = $2(8 + 5)$ cm = 26 cm

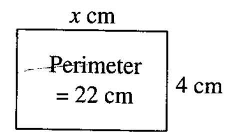
Example 4

The perimeter of a rectangle is 22 cm and its breadth is 4 cm. Find its area.

Solution

We can **draw a diagram** and then **form an equation** to solve the problem.

Let the length of the rectangle be x cm.



Then $2(x + 4) = 22$

$$2x + 8 = 22$$
$$2x = 14$$
$$x = 7$$

$\therefore$ the area of the rectangle = (7×4) cm^2 = 28 cm^2

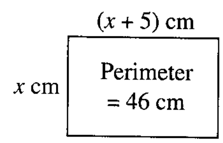
Example 5

The width of a rectangle is 5 cm less than its length, and its perimeter is 46 cm. Find its width and its area.

Solution

Let the width of the rectangle be x cm.

Then the length is $(x + 5)$ cm.



Perimeter = $2[x + (x + 5)]$ cm = 46 cm

$$4x + 10 = 46$$
$$\therefore \quad 4x = 36$$
$$x = 9$$

$\therefore$ Its width is 9 cm and its length is 14 cm.

$\therefore$ Its area = (9×14) cm^2

= 126 cm^2

Example 6

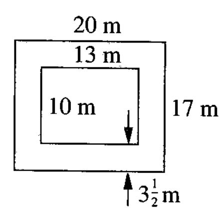
A rectangular field is 13 m long and 10 m wide. It has a cement path $3\frac{1}{2}$ m wide around it. What is the area of the cement path?

Solution

Area of the field and cement path = (20×17) m^2 = 340 m^2

Area of the field = (13×10) m^2 = 130 m^2

$\therefore$ area of cement path = $(340 - 130)$ m^2 = 210 m^2



Exercise 9b

1. Copy and fill in the missing numbers:
   - **(a)** 8.5 cm^2 = ______ m^2
   - **(b)** 2.5 mm^2 = ______ cm^2
   - **(c)** 6.3 m^2 = ______ cm^2
   - **(d)** 40.6 cm^2 = ______ mm^2
   - **(e)** 44.4 km^2 = ______ ha
   - **(f)** 3.1 ha = ______ m^2
   - **(g)** 53.7 m^2 = ______ km^2
   - **(h)** 0.28 km^2 = ______ m^2
   - **(i)** 53 200 mm^2 = ______ m^2
   - **(j)** 69 450 cm^2 = ______ m^2
   - **(k)** 3.4 ha = ______ km^2
   - **(l)** 462 m^2 = ______ ha

2. Copy and complete the table below for each given rectangle:

	Length	Breadth	Perimeter	Area
(a)	6 m	4 m		
(b)	8 m			48 m^2
(c)		2.2 m		8.8 m^2
(d)	4.5 m		23 m	
(e)		26 mm	98 mm	

3. Find the number of 15-centimetre square tiles required to cover a rectangular floor 5.4 m long and 4.05 m wide.

4. Find the area, in square centimetres, of a rectangular strip of board 3.28 m long and 75 mm wide.

5. A square cardboard of side 20 m has a 4 m wide border round three of its sides. Find the area of the border.

6. A paper box without a lid is 25 cm long. 16 cm wide and 5 cm deep. How many square centimetres of paper have been used to make the box?

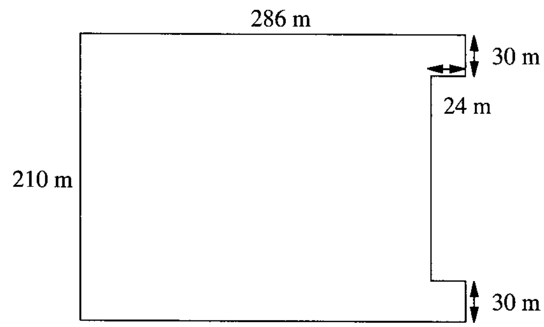
7. Find the cost required to carpet a hall 8 m by 5.5 m if a rectangular section 2 m by $1\frac{1}{2}$ m is taken out to provide for the fire-place and the carpet costs $52.50 per m^2.

8. A swimming pool 25 m by 10 m has a concrete border all round. Find the area of the concrete border if it is 2.5 m wide at the sides and 5 m at the ends.

9. The perimeter of a square is 36 cm. Find its area.

⋆10. Find the total area of cardboard used in making a match box, complete with the sliding portion, 4 cm long, 2.5 cm wide and 1.2 cm deep (ignore the thickness of the cardboard).

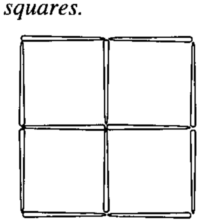
11. Find, in hectares, the area of the figure shown below. Give your answer correct to 2 decimal places.



12. The length of a rectangle is 8 cm more than its width. If its perimeter is 56 cm, find its length and its area.

Problems

12 toothpicks are arranged as shown below. Remove only 2 toothpicks so as to leave only 2 squares.



Area of a Parallelogram

A **parallelogram** is a quadrilateral in which the opposite pairs of sides are parallel.

We can obtain a rectangle from a parallelogram. To do so, draw on a piece of paper a parallelogram $ABCD$ as shown in Fig. 9.6.

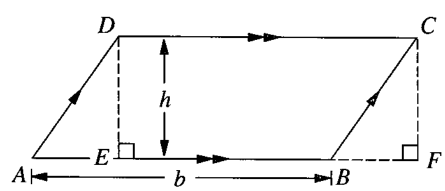


Fig. 9.6

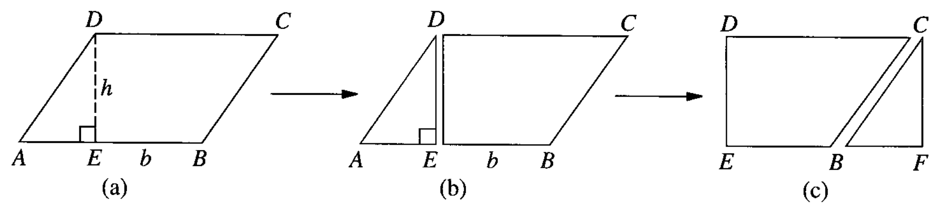


Fig. 9.7

Cut off $\triangle AED$ and place it in the position BFC (Fig. 9.7(c)). A rectangle $EFCD$ is obtained. Do you agree that the parallelogram $ABCD$ and the rectangle $EFCD$ have the same area?

Area of the parallelogram $ABCD$ = Area of the rectangle $EFCD$
$= DC \times DE$

But $DC = AB$ (opposite sides of a parallelogram)

$\therefore$ area of parallelogram $ABCD = AB \times DE$
$= b \times h$
$=$ base $\times$ height

In general,

area of a parallelogram = base × height = $b \times h$

Area of a Triangle

Look at the square $ABCD$, the rectangle $PQRS$ and the parallelogram $HIJK$ in Fig. 9.8.

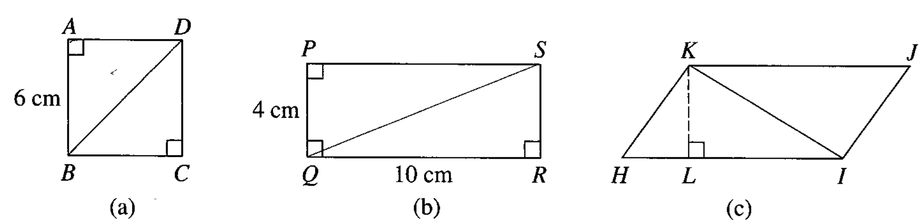


Fig. 9.8

The square $ABCD$ in Fig. 9.8(a) is cut into two halves by the diagonal BD. Similarly, the diagonal QS cuts the rectangle $PQRS$ in Fig. 9.8(b) into two equal right-angled triangles PQS and SQR.

Area of square $ABCD = BC \times CD$

$\therefore$ area of $\triangle BCD = \frac{1}{2} \times BC \times CD$

Since BC is the base and CD is the height of $\triangle BCD$,

$\therefore$ area of $\triangle BCD = \frac{1}{2} \times$ base $\times$ height.

How would you deduce that in Fig. 9.8(b), area of $\triangle QRS = \frac{1}{2} \times QR \times RS = \frac{1}{2} \times$ base $\times$ height?
If QS is taken as the base, where should the height of $\triangle QRS$ be?

The diagonal KI cuts the parallelogram $HIJK$ in Fig. 9.8(c) into two identical triangles which are not right-angled.

Area of $HIJK = HI \times KL$

$\therefore$ area of $\triangle HIK = \frac{1}{2} \times HI \times KL$

HI is the base and KL is the height of $\triangle HIK$.

$\therefore$ area of $\triangle HIK = \frac{1}{2} \times$ base $\times$ height

In general

$$\textbf{area of a triangle} = \frac{1}{2} \times \textbf{base} \times \textbf{height}$$
$$= \frac{1}{2} bh$$

Area of a Trapezium

A **trapezium** is a quadrilateral with one pair of parallel sides. Fig. 9.9 shows a trapezium $ABCD$ in which AD is parallel to BC with a height of AH. The trapezium is divided into $\triangle ABC$ and $\triangle ACD$.

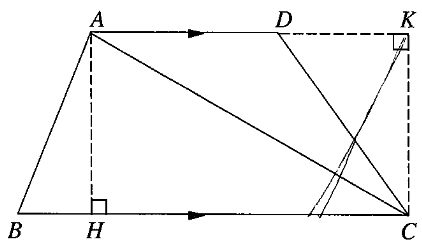


Fig 9.9

Area of trapezium $ABCD$ = area of $\triangle ABC$ + area of $\triangle ACD$

$$= \left(\frac{1}{2} \times BC \times AH\right) + \left(\frac{1}{2} \times AD \times CK\right)$$

Note: $AH = CK$

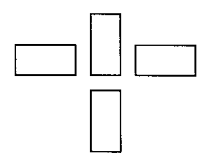
Area of trapezium $ABCD = \frac{1}{2} AH\,(BC + AD)$

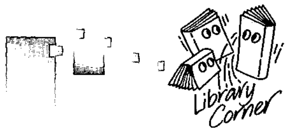
$= \frac{1}{2} \times$ height $\times$ sum of parallel sides

In general,

$$\textbf{area of a trapezium} = \frac{1}{2} \times \textbf{height} \times \textbf{sum of parallel sides}$$

Four rectangular cards of identical size are arranged as shown below. You are to move only one card so as to form a square.

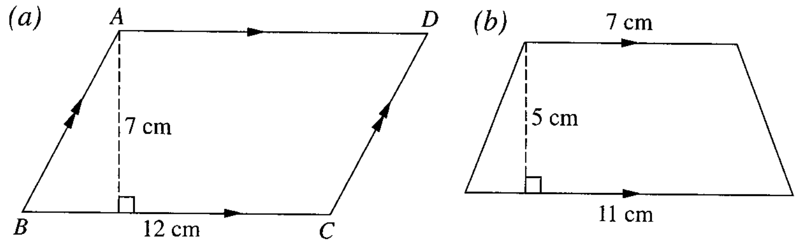




Due to land reclamation, the area and perimeter of Singapore have increased in size. Find out the area of Singapore and its perimeter in 1970, 1980, 1990 and today.

Example 7

Find the areas of the following figures:



Solution

(a) Area of parallelogram $ABCD$ = base × height

$= (12 \times 7)\text{ cm}^2 = 84\text{ cm}^2$

(b) Area of trapezium $= \frac{1}{2} \times \text{height} \times \text{sum of parallel sides}$

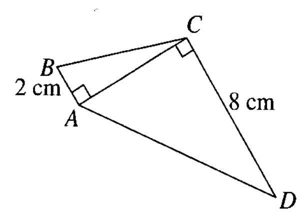
$= \left[\frac{1}{2} \times 5 \times (7 + 11)\right]\text{cm}^2 = 45\text{ cm}^2$

Example 8

In the figure below, the sides AB and DC of the quadrilateral ABCD are both perpendicular to the diagonal AC. Given AB = 2 cm, DC = 8 cm and the area of △ABC = 6 cm², calculate

(a) the length of AC;

(b) the area of the quadrilateral ABCD.



Solution

(a) Area of $\triangle ABC = \frac{1}{2} \times AB \times AC$

$\therefore \quad \frac{1}{2} \times AB \times AC = 6$

$\frac{1}{2} \times 2 \times AC = 6$

$\therefore \quad AC = 6\text{ cm}$

(b) Area of $\triangle ACD = \frac{1}{2} \times AC \times CD$

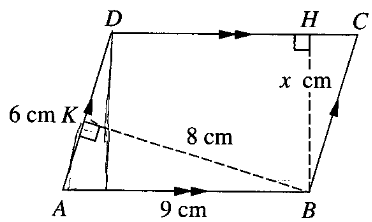
$= \left(\frac{1}{2} \times 6 \times 8\right)\text{cm}^2 = 24\text{ cm}^2$

$\therefore$ the area of the quadrilateral $ABCD = (6 + 24)\text{ cm}^2 = 30\text{ cm}^2$

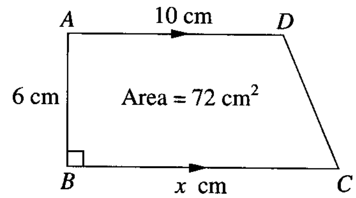
Example 9

Find the value of x in the following figures:

(a)



(b)



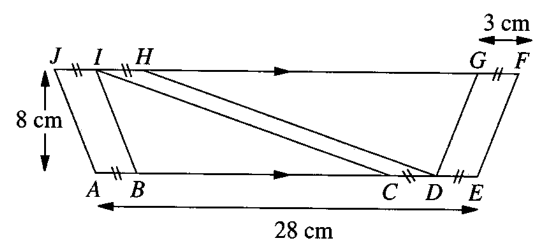
Solution

(a) Area of parallelogram $ABCD$
$= AB \times BH = AD \times BK$
$9 \times x = 6 \times 8$
$\therefore \quad x = \frac{6 \times 8}{9} = 5\frac{1}{3}$

(b) Area of trapezium $ABCD = \frac{1}{2} \times AB \times (AD + BC)$
$72 = \left[\frac{1}{2} \times 6 \times (10 + x)\right]$
$24 = 10 + x$
$\therefore \quad x = 14$

Example 10

Find the total area of the shaded parts in the diagram.



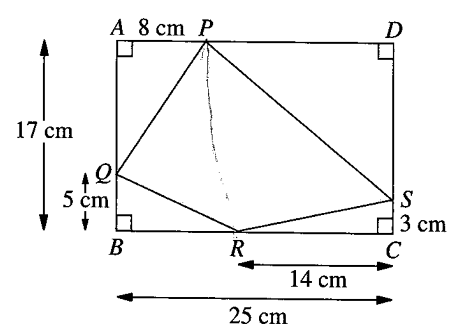
Solution

The total area of the shaded parts is made up of 3 parallelograms of the same base length (3 cm) and of the same height (8 cm).

$\therefore$ area of shaded region = $[3(3 \times 8)]$ cm^2 = 72 cm^2

Example 11

In the figure on the right, AB = 17 cm, BC = 25 cm, AP = 8 cm, BQ = 5 cm, CR = 14 cm and CS = 3 cm. Find the area of the shaded region.



Solution

Area of shaded region $PQRS$ = area of $ABCD$ – area of triangles ($APQ + PDS + CSR + BQR$)

$$\text{Area of } ABCD = (17 \times 25)\text{ cm}^2 = 425\text{ cm}^2$$

$$\text{Area of } \triangle APQ = \frac{1}{2} \times AP \times AQ = \left[\frac{1}{2} \times 8 \times (17-5)\right]\text{cm}^2 = 48\text{ cm}^2$$

$$\text{Area of } \triangle PDS = \frac{1}{2} \times PD \times DS = \left[\frac{1}{2} \times (25-8) \times (17-3)\right]\text{cm}^2 = 119\text{ cm}^2$$

$$\text{Area of } \triangle SRC = \frac{1}{2} \times RC \times SC = \left(\frac{1}{2} \times 14 \times 3\right)\text{cm}^2 = 21\text{ cm}^2$$

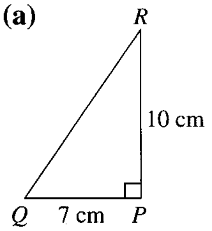
$$\text{Area of } \triangle BRQ = \frac{1}{2} \times BR \times BQ = \left[\frac{1}{2} \times (25-14) \times 5\right]\text{cm}^2 = 27.5\text{ cm}^2$$

$$\therefore \text{ Area of } PQRS = (425 - 48 - 119 - 21 - 27.5)\text{ cm}^2 = 209.5\text{ cm}^2$$

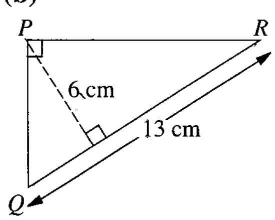
Exercise 9c

1. Find the area of the triangle PQR in the following cases:

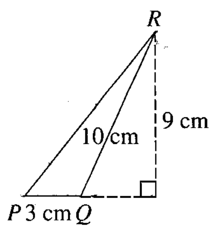
(a)



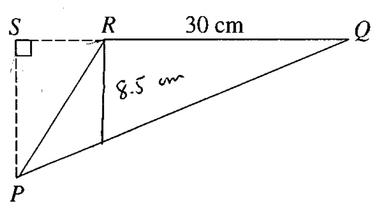
(b)



(c)



2. In the diagram below, the area of $\triangle PQR$ is 255 cm² and the length of QR is 30 cm. Find the length of PS.

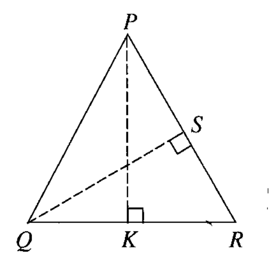


3. For questions (a) to (c), refer to the figure in which QS is perpendicular to PR and PK is perpendicular to QR.

(a) Find the area of $\triangle PQR$ if $PR = 17$ cm and $QS = 12$ cm.

(b) Find the area of $\triangle PQS$ if $QS = 7$ cm, $PR = 14$ cm and $SR = 9$ cm.

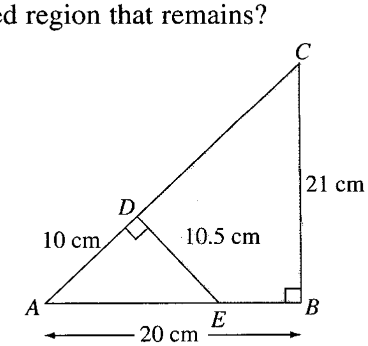
(c) Find QS if $PR = 14$ cm and the area of $\triangle PQR = 147$ cm².



4. Copy and complete the following table for each parallelogram:

	(a)	(b)	(c)
Base (cm)	12		7.8
Height (cm)	7	6	
Area (cm²)		42	42.9

5. In the diagram, $AB = 20$ cm, $BC = 21$ cm, $AD = 10$ cm and $DE = 10.5$ cm. Angles ABC and ADE are right angles. If $\triangle ADE$ is removed from $\triangle ABC$, what is the area of the shaded region that remains?



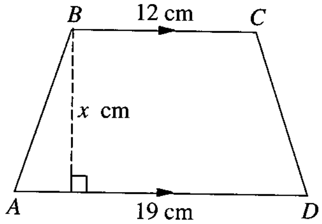
6. Copy and complete the following table for each trapezium:

	(a)	(b)	(c)
Height (cm)	6	14	8
Parallel side 1 (cm)	7	8	
Parallel side 2 (cm)	11		5
Area (cm²)		126	72

∗7. What is the cost of spraying insecticide on a field measuring 2 000 m by 3 200 m if the cost is $22 per hectare? (1 ha = 10 000 m²)

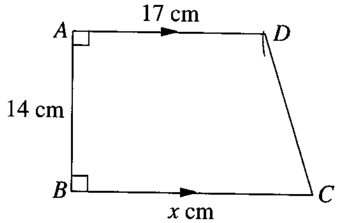
∗8. Find the unknowns, marked x, in the following figures:

(a)



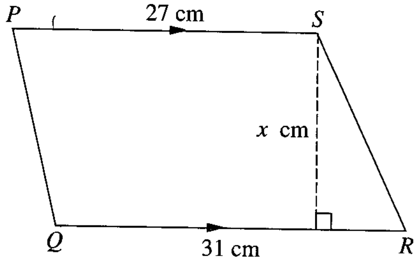
Area of $ABCD$ = 124 cm²

(b)



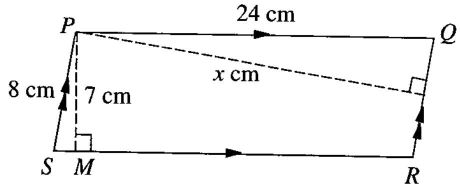
Area of $ABCD$ = 280 cm²

(c)



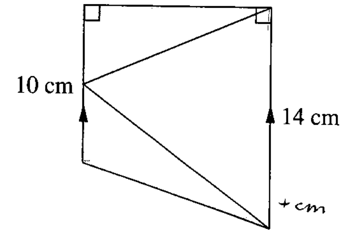
Area of $PQRS$ = 348 cm²

(d)

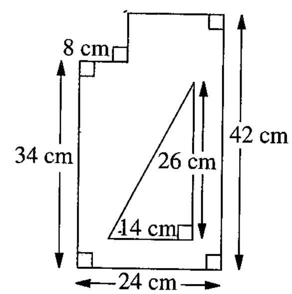


9. Find the areas of the following shaded parts:

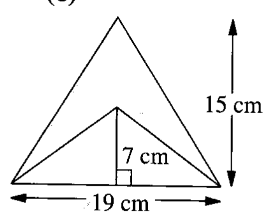
(a)



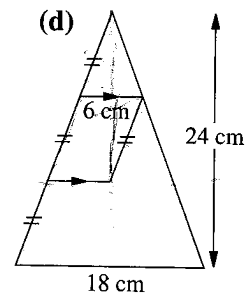
(b)



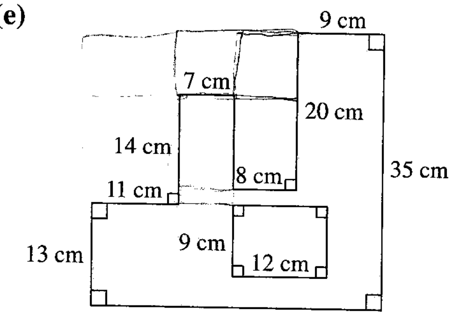
(c)



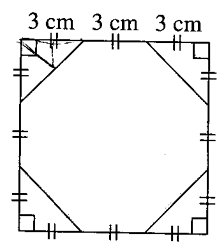
(d)



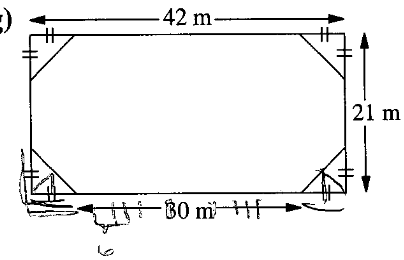
∗(e)



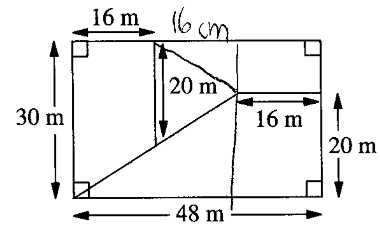
(f)



(g)



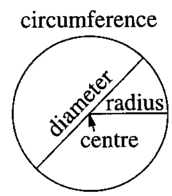
(h)



Perimeter of a Circle

The circumference of the Earth's equator is approximately 40 000 km. An imaginary belt of 40 000 km will fit the equator nicely. If we increase the length of the belt by 1 m, will it be possible for a cat to squeeze through? How far above the surface of the equator will the belt be?

A circle consists of points that are all equidistant from a particular point called the **centre**. The **perimeter** of a circle, or the length of its boundary, is called the **circumference**. The distance from the centre of a circle to any point on its circumference is called the **radius**. The **diameter** of the circle is twice the length of its radius.



The diameter of a tin can be found by placing it between two books as shown in Fig. 9.10.

To find the circumference of the tin can, simply wrap a strip of paper round the top as shown in Fig. 9.11. Then measure the length of the strip of paper to get the circumference.

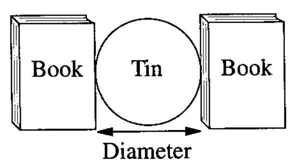


Fig 9.10

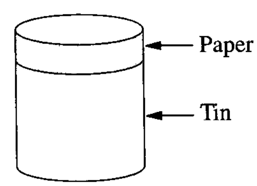


Fig 9.11

The table below shows the circumferences and diameters of several tin cans found by the above methods.

Tin Can	*Circumference (c)*	*Diameter (d)*	$\frac{c}{d}$
A	48.5 cm	15.5 cm	
B	40.0 cm	12.7 cm	
C	31.3 cm	9.9 cm	
D	26.1 cm	8.3 cm	

In each case, find the value of $\frac{c}{d}$ correct to two decimal places. What do you notice?

As a matter of fact, the ratio $\frac{c}{d}$ is the same for all circles. This ratio $\frac{c}{d} = \frac{\text{circumference}}{\text{diameter}}$ is called **pi** and is denoted by the symbol π. Usually π is taken to be approximately equal to 3.14, $\frac{22}{7}$ or 3.142.

To find the circumference of a circle, we use $\frac{\text{circumference}}{\text{diameter}} = \frac{c}{d} = \pi$.

Therefore $c = \pi d$.

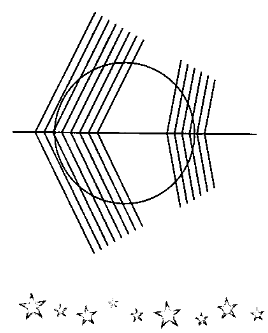
Since $d = 2r$ where r denotes the radius of the circle, $c = 2\pi r$.

Hence,

circumference of a circle, $c = \pi d$ or $2\pi r$, where d = diameter and r = radius.

Area of a Circle

Is the circle perfectly round?



Look at the following figures:

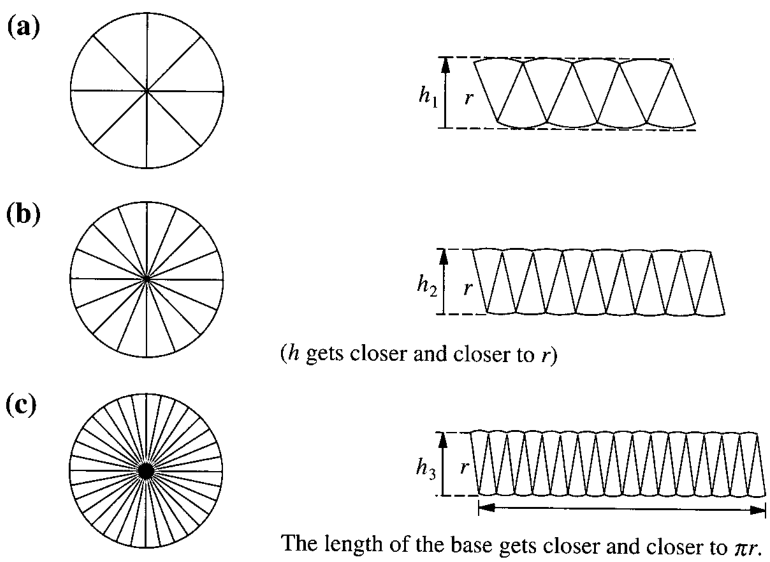


Fig. 9.12

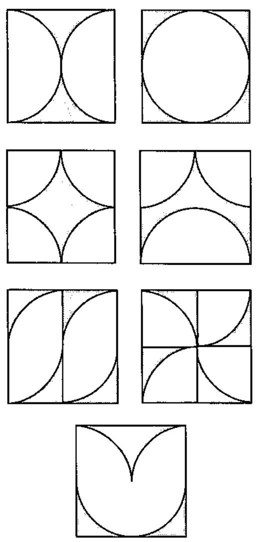
In Fig. 9.12(a), a circle is divided into 8 equal parts and rearranged as shown. In Fig. 9.12(b) and (c), the circles are divided into 16 and 32 equal parts respectively. In each case, the parts are rearranged in a straight line as shown. Notice that the figures resulting from the rearrangements of the parts tend to look like parallelograms. As the number of equal parts increases, the area of the resulting figure, which is the same as the area of the **original circle**, will be closer and closer to the area of a parallelogram. Notice also that the height h of the parallelogram gets closer and closer to r, the radius of the circle, and the length of the base gets closer and closer to πr, which is half of the circumference of the circle.

$\therefore$ area of the parallelogram = base × height

$= \pi r \times r = \pi r^2$

Area of a circle = πr^2, where r = radius.

Which of the following shaded figures has the greatest area? The squares are of the same length and the curved lines are all arcs of circles.



Example 12

A circle has a radius of 7 m. Find its area and circumference. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution

Area of circle $= \pi r^2 = \left(\frac{22}{7} \times 7 \times 7\right)\text{m}^2$
$= 154 \text{ m}^2$

Circumference of circle $= 2\pi r = \left(2 \times \frac{22}{7} \times 7\right)\text{m}$
$= 44 \text{ m}$

Example 13

The area of a circle is 78.5 cm². Calculate the circumference of the circle. (Take $\pi = 3.14$)

Solution

Area of circle $= \pi r^2$
$78.5 = 3.14r^2$
$r^2 = \frac{78.5}{3.14} = 25$
$r = \sqrt{25} = 5$

$\therefore$ circumference of circle $= 2\pi r$
$= 2(3.14)(5)$ cm
$= 31.4$ cm

Example 14

The diameter of the wheel of a car is 0.35 m. Find the number of revolutions made by the wheel per minute when the car is travelling at 33 km/h. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution

In 60 minutes, the car travels $(33 \times 1\,000)$ m.

In 1 minute, the car travels $\frac{33 \times 1\,000}{60}$ m.

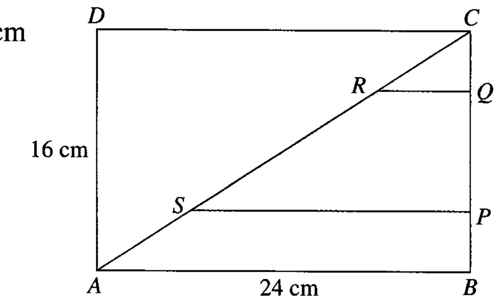
Number of revolutions made per minute $= \frac{\text{distance travelled}}{\text{circumference of wheel}}$
$= \frac{33 \times 1\,000}{60} \times \frac{1}{\pi d}$
$= \frac{33 \times 1\,000}{60} \times \frac{7}{22} \times \frac{100}{35}$
$= 500$

Example 15

In the figure, ABCD is a rectangle of length 24 cm and breadth 16 cm. Given that $CQ = PB = \frac{1}{2}PQ$, calculate the area of the trapezium PQRS.

Solution

Strategy 1: Use an equation



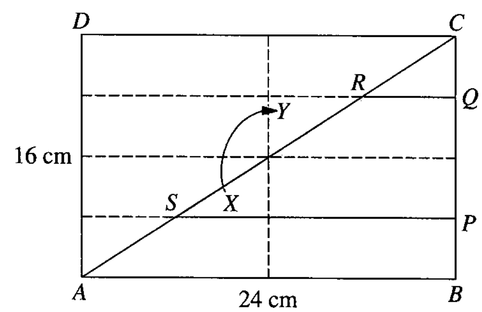
$PQ = 8$ cm, $RQ = \frac{1}{4}AB = 6$ cm and $PS = \frac{3}{4}AB = 18$ cm

Using the formula $\frac{1}{2} \times$ height $\times$ sum of parallel sides = area of trapezium,

area of $PQRS = \frac{1}{2} \times 8 \times (6 + 18)$

$= 96$ cm^2

Strategy 2: Draw a diagram



Divide the rectangle into 8 equal parts as shown. If we move the shaded triangle X onto Y, the total shaded area is equal to $\frac{1}{4}$ of the big rectangle.

$\therefore$ area of $PQRS = \frac{1}{4} \times 16 \times 24$

$= 96$ cm^2

Exercise 9d

Take π to be $\frac{22}{7}$ for this exercise unless otherwise stated.

1. Copy and complete the following table below for each circle:

	(a)	**(b)**	**(c)**	**(d)**
Radius	10 m			
Diameter				3.6 m
Circumference		176 mm		
Area			616 cm^2	

2. Calculate the circumference and area of each circle, given its diameter:

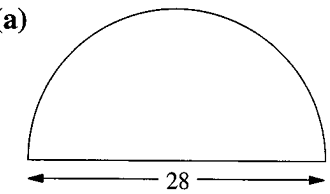
(a) 70 mm **(b)** 28 cm **(c)** 35 cm **(d)** $\frac{14}{3}$ cm

3. Calculate the circumference and area of each circle, given its radius (take $\pi = 3.14$), giving your answer correct to 2 decimal places:

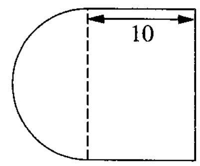
(a) 3.5 cm **(b)** 13.8 m **(c)** 0.37 m **(d)** 5.25 cm

★4. Find the perimeter and area of each of the following figures. All dimensions are given in cm and the circular portions are semicircles.

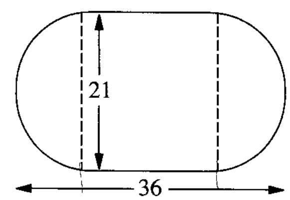
(a)



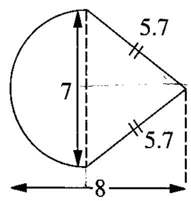
(b)



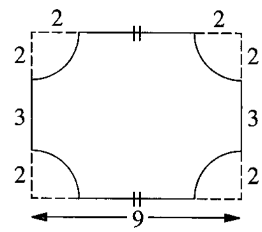
(c)



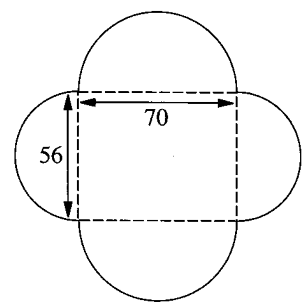
★(d)



(e)



(f)



5. Two wire circles of diameters 12 cm and 8 cm are cut and then joined to make one large circle. Find the diameter of this larger circle.

6. As many 8-cm diameter discs as possible are cut from a sheet of rectangular cardboard measuring 170 cm by 90 cm. Find the area of the sheet that is left.

7. If the minute hand of a big clock is 1.12 m long, find the rate at which its tip is moving in centimetres per minute.

8. Find the speed of a point on the rim of a 24-cm diameter fly-wheel which is turning at 2 800 revolutions per minute. Give your answer in metres per second.

★9. A lorry travels at 50 km/h. Given that the diameter of its wheel is 88 cm, find how many revolutions per minute the wheel is turning. Give your answer to the nearest whole number.

10. Find the difference between the perimeter of a square of area 1 m² and the circumference of a circle of the same area.

Summary

1. For a rectangle with length l units and breadth b units, the perimeter = $2(l + b)$ units and the area = $(l \times b)$ units^2.

2. Area of a parallelogram = base × height

3. Area of a triangle = $\frac{1}{2}$ × base × height

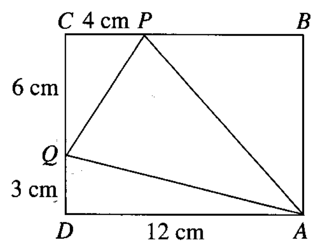
4. Area of a trapezium = $\frac{1}{2}$ × height × sum of parallel sides

5. For a circle with radius r units, the circumference = $2\pi r$ units and the area = πr^2 unit^2.

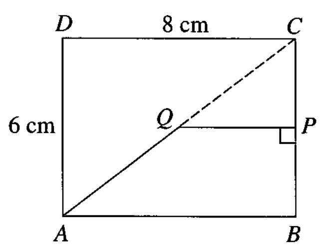
Review Questions 9

Take π to be $\frac{22}{7}$ for this exercise.

1. $\triangle APQ$ is enclosed within the rectangle $ABCD$ as shown in the figure below. Calculate the area of $\triangle APQ$.



2. In the figure, $ABCD$ is a rectangle of length 8 cm and breadth 6 cm. If $BP = CP$, calculate the area of trapezium $ABPQ$, where AQC is a diagonal of the rectangle.



3. A piece of wire 48 cm long is bent to form a rectangle whose length is twice its width. Calculate its area.

4. The length of a rectangle is 4 cm longer than its width and its perimeter is 44 cm. Find the length and area of the rectangle.

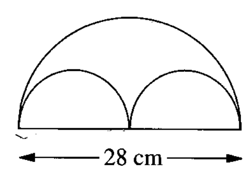
5. A rectangular driveway 12 m long and $4\frac{1}{2}$ m wide is to be covered by similar square tiles of side 25 cm each. Find the number of tiles needed to cover the driveway.

6. The area of a trapezium is 36 cm² and the perpendicular distance between its parallel sides is 6 cm. If the lengths of these parallel sides are x cm and y cm, find the value of $(x + y)$. Given further that x is twice as big as y, find the values of x and y.

7. A bucket of water is brought up from a well 9.68 m deep by a rope which winds round a drum 22 cm in diameter. How many turns of the handle are required to bring up a bucket from the bottom of the well?

8. A racing track is a circular ring with inner diameter 140 m and track 7 m wide. How much further does a motorist on the outside rim travel, when he goes round the circuit once, than another who goes round the circuit on the inside rim?

∗9. The diagram shows three semicircles. Calculate the perimeter and area of the shaded region.



Problem Solving

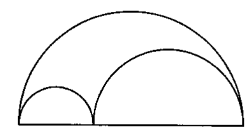
$\left(\text{Take } \pi = \frac{22}{7}\right)$

1. A single turn of wire wound onto a 5-cm diameter transformer has a mass of 5.5 g. What is its length if the mass of the complete coil of the wire is $1\frac{3}{4}$ kg?

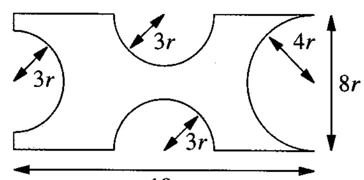
2. A goat, tethered by a rope 1.5 m long, eats a square metre of grass in 14 minutes. Find the time taken if it is to eat all the grass within its reach.

3. A metal disc of radius 6 cm costs 66 cents. Find the cost of 3 square metres of the metal.

4. In the figure below, AB is the diameter of the big semicircle. AK and BK are the diameters of the two smaller semicircles. Given $AK = x$ cm and $BK = y$ cm, find, in terms of x and y, the area of the shaded region enclosed by the three semicircles.

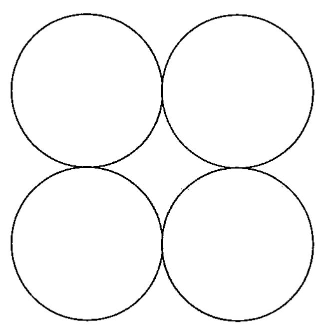


5. The figure shows a rectangular cardboard with 4 semicircles being cut off. Find the area of the remaining cardboard in terms of r.



6. Kumar walks round a rectangular field the length of which is twice its width. He then walks round another rectangular field half as wide but having the same perimeter as the first field. If the difference in area between the two fields is 432 m^2, find the length of the second field.

7. The diagram shows 4 circles of equal radius touching each other. If the radius of each circle is 12 cm, calculate the area of the shaded region.



CHAPTER 10

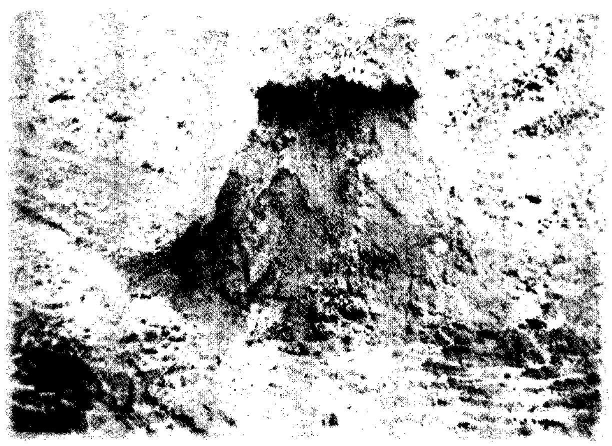
Volume and Surface Area

In this chapter, you will learn how to

- find the volume and surface area of cubes, cuboids, prisms and cylinders;
- solve problems involving volumes made up of the above solids;
- solve problems involving density.

Preliminary Problem

The picture shows the land being cleared for the construction of infrastructure for a new township. The contractor has deliberately left some heaps of soil behind. Do you know that the purpose of this is to estimate the volume of soil taken from the site?



Volume

Fig. 10.1 shows a piece of wood, a brick and a matchbox.

The volume of an object is the amount of space it occupies. The object that occupies more space is said to have a greater volume.

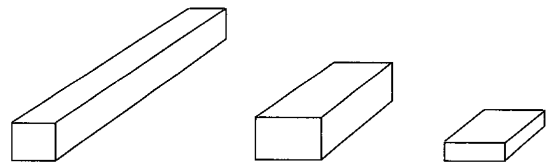


Fig. 10.1

Which one of them occupies the least amount of space? Obviously, the matchbox occupies the least space. But which of the other two, the piece of wood or the brick, occupies more space? To answer this question, we first have to make some measurements and then obtain the volume of each object.

Units of Volume

The British System of measure uses pints, gallons, quarts and barrels as units for volume. Find out what these units are and compare them with the SI units.

As with the case of the area of a plane figure, we compare the volume of an object with a standard unit. A standard unit for volume is a cube with side 1 cm (see Fig. 10.2). We call this 1 cubic centimetre, written as 1 cm^3.

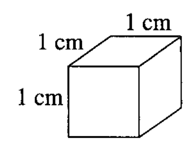


Fig. 10.2

Similarly, a cube with side 1 mm will have a volume of 1 mm^3 and that with side 1 m will have a volume of 1 m^3.

Volume of a Cuboid

Fig. 10.3 shows a rectangular block with dimensions 5 cm by 1 cm by 1 cm. The block contains 5 unit cubes, each of volume 1 cm^3. So the volume of the block is $(5 \times 1 \times 1)$ cm^3 = 5 cm^3.

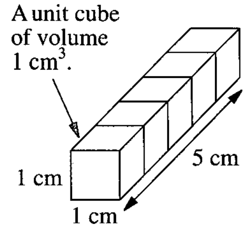


Fig. 10.3

Fig. 10.4 shows a rectangular block with dimensions 5 cm by 2 cm by 1 cm. It contains 10 unit cubes. Hence, the volume of the block is $(5 \times 2 \times 1)\ \text{cm}^3 = 10\ \text{cm}^3$.

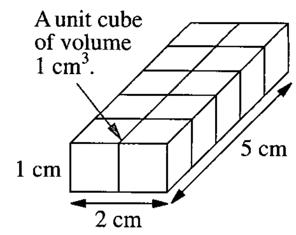


Fig. 10.4

The rectangular block in Fig. 10.5 has dimensions 5 cm by 2 cm by 4 cm. It contains 4 layers of the block shown by Fig. 10.4. Hence, it is made of (4×10) unit cubes = 40 unit cubes and its volume is $(5 \times 2 \times 4)\ \text{cm}^3 = 40\ \text{cm}^3$.

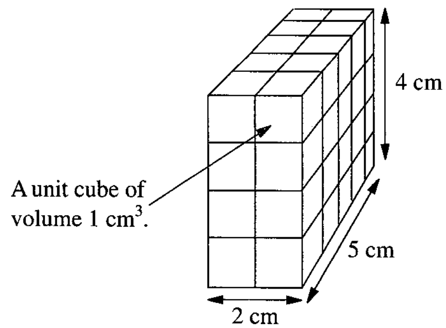
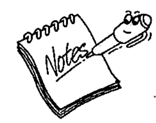


Fig. 10.5



A prism is a solid figure with a flat base and parallel upright edges. A glass prism breaks up white light into different colours.

NB: Each of the rectangular blocks in Figs. 10.3, 10.4 and 10.5 is called a **rectangular prism** or a **cuboid**.

From the above discussion, we see that we can find the volume of a cuboid by multiplying together the length, width and height, which must all be measured in the same units. That is, the volume, V, of a cuboid L units long, W units wide and H units high is given by

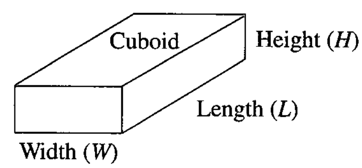


Fig. 10.6

$$V = (\underbrace{L \times W}_{\text{Area of the base}} \times H)\ \text{unit}^3$$

A cube can be considered as a special cuboid whose length, width and height are equal, i.e., $L = W = H$. The volume of a cube whose side is L units long is given by

$$\begin{aligned} V &= (L \times L \times L)\ \text{unit}^3 \\ &= L^3\ \text{unit}^3 \end{aligned}$$

Surface Area of a Cuboid

If we unfold a cardboard cuboid, we get a *net* of the cuboid as shown in Fig. 10.7. This net will help us find the total surface area of the cuboid.

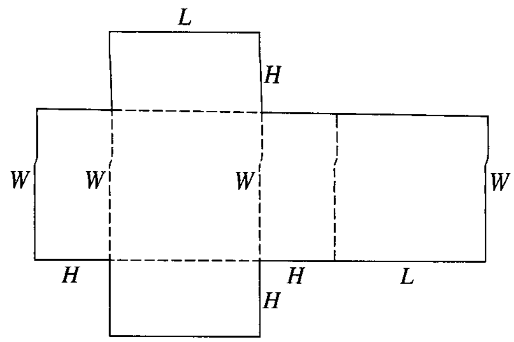


Fig. 10.7

Which area is larger and by how much — a half-km square or a half square km?

There are several different ways of unfolding the same cardboard cuboid to obtain different nets of the same solid. Fig. 10.8 shows another net of the same cuboid. Can you draw another net for the same cuboid?

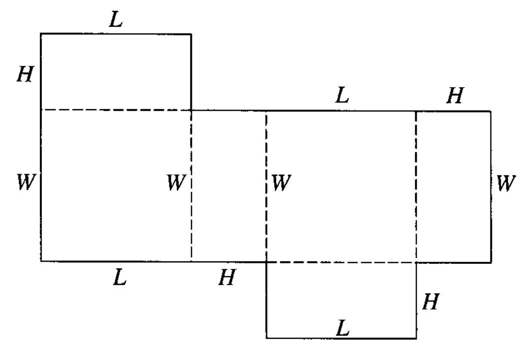


Fig. 10.8

From Fig. 10.7, the surface area of a cuboid of length L units, width W units and height H units
$= [2(L \times W) + 2(L \times H) + 2(W \times H)]$ units^2
$= 2(L \times W + L \times H + W \times H)$ units^2.

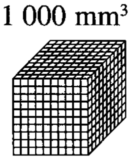
In the case of a cube where the length, width and height are all equal, i.e. $L = W = H$, the total surface area
$= 2(L \times L + L \times L + L \times L)$
$= 2(L^2 + L^2 + L^2)$ units^2
$= 6L^2$ units^2.

Example 7

Express (a) 1 cm^3 in mm^3 and (b) 1 m^3 in cm^3.

Solution

(a) Since 1 cm = 10 mm, a cube with side 10 mm has a volume 1 cm^3.
i.e., 1 cm^3 = $(10 \times 10 \times 10)$ mm^3
= 1 000 mm^3



(b) Similarly, since 1 m = 100 cm.
1 m^3 = $(100 \times 100 \times 100)$ cm^3
= 1 000 000 cm^3

Example 2

The figure below shows a rectangular prism 9 cm long and 7 cm wide. Given that the volume of the prism is 315 cm³, find

(a) the height of the prism;
(b) its surface area.

Solution

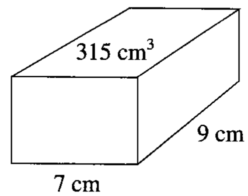
(a) $L = 9$, $W = 7$, $H = ?$

Volume of the prism $= L \times W \times H$

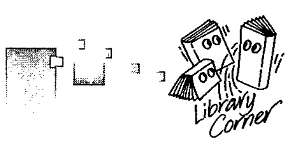
$$315 = 9 \times 7 \times H$$

$$H = \frac{315}{9 \times 7} = 5$$

$\therefore$ the height of the prism is 5 cm.



(b) The surface area of the prism $= 2(9 \times 7 + 9 \times 5 + 7 \times 5)$ cm²
$= 286$ cm²



Find out the names and capacities of the largest and the smallest reservoirs in Singapore. Also, find out the average daily consumption of water in Singapore in 1999.

Volume of Fluids

The volume of fluids, or liquids, is measured using special units. These units are the millilitre (m*l*), the litre (*l*) and the kilolitre (k*l*). Normally, we buy milk and petrol by the litre and we take medicine by the millilitre.

$$1 \text{ m}l = 1 \text{ cm}^3$$
$$1 \text{ litre} = 1\,000 \text{ m}l = 1\,000 \text{ cm}^3$$
$$1 \text{ kilolitre} = 1\,000 \text{ litres} = 1 \text{ m}^3$$

Example 3

A container is in the form of a cuboid 20 cm long, 3 cm wide and 14 cm high. Find the volume of the liquid, in litres, that the container can hold (i.e., the capacity of the container).

Solution

The volume of the container $= (20 \times 3 \times 14)$ cm³ $= 840$ cm³

$1\,000$ cm³ $= 1$ litre

$\therefore$ the volume of the liquid $= \frac{840}{1\,000}$ litre $= 0.84$ litre

Example 4

Express (a) 3 600 000 mm³ in (i) cm³ and (ii) ml;
(b) 0.7 m³ in (i) cm³ and (ii) litres.

Solution

(a) (i)
$$10\text{ mm} = 1\text{ cm}$$
$$1\,000\text{ mm}^3 = 1\text{ cm}^3$$
$$3\,600\,000\text{ mm}^3 = \frac{3\,600\,000}{1\,000}\text{ cm}^3$$
$$= 3\,600\text{ cm}^3$$

(ii)
$$1\text{ cm}^3 = 1\text{ m}l$$
$$3\,600\text{ cm}^3 = 3\,600\text{ m}l$$

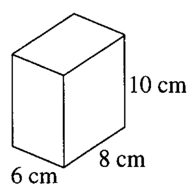
(b) (i)
$$1\text{ m} = 100\text{ cm}$$
$$1\text{ m}^3 = 1\,000\,000\text{ cm}^3$$
$$0.7\text{ m}^3 = (0.7 \times 1\,000\,000)\text{ cm}^3$$
$$= 700\,000\text{ cm}^3$$

(ii)
$$1\,000\text{ cm}^3 = 1\text{ litre}$$
$$700\,000\text{ cm}^3 = \frac{700\,000}{1\,000}\text{ litres}$$
$$= 700\text{ litres}$$

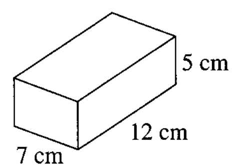
Exercise 10a

1. Find the volume and surface area of the following cuboids, and draw its net:

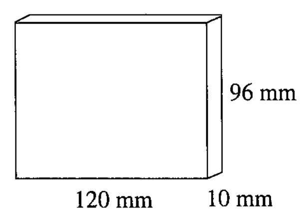
(a)



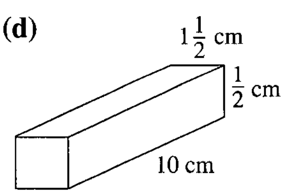
(b)



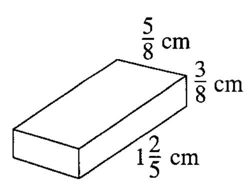
(c)



(d)



(e)



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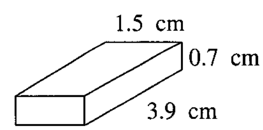


2. Copy and complete the following table for each cuboid:

	Length	*Width*	*Height*	*Volume*	*Surface Area*
(a)	24 mm	18 mm	5 mm		
(b)	5 cm	3 cm		120 cm³	
(c)		6 cm	$3\frac{1}{2}$ cm	52.5 cm³	
(d)	12 m		6 m	576 m³	
(e)	$2\frac{1}{4}$ cm	8 cm		$58\frac{1}{2}$ cm³	
(f)	9 cm	12 cm			426 cm²

3. Find the capacity of each of the following rectangular tanks, giving your answer in litres:

(a) Height = 3.6 m, length = 5.5 m, width = 3.5 m.

(b) Height = 2.7 m, length = 4.75 m, width = 2.6 m.

(c) Height = 0.15 m, length = 0.24 m, width = 0.19 m.
(d) Height = 38 cm, length = 52 cm, width = 18 cm.

4. Find the total surface area of a solid cube of volume 64 cm^3.

5. A man sells sugarcane juice in 200 ml cups. How many cups of sugarcane juice can he dispense from his big rectangular tank of length 65 cm, width 40 cm and height 54 cm?

***6.** An open water tank with length 20 cm and width 15 cm holds 4.8 litres of water. Calculate the height of the water level in the tank and the total surface area of the cuboid in contact with the water.

7. A rectangular tank measures 4 m long, 2 m wide and 4.8 m high. Initially it is half filled with water. Find the depth of water in the tank after 4 000 litres more of water are added to it.

8. A rectangular water tank of length 60 cm and width 40 cm contains water up to a depth of 30 cm. A piece of ice measuring 20 cm by 15 cm by 12 cm is dropped into the tank of water. Calculate the new depth of water when the ice melts completely, assuming its volume decreases by $\frac{1}{10}$.

9. It took two and a half years and 2.85 million m^3 of earth to fill the disused Sin Seng quarry at Rifle Range Road.
(a) If each truck can carry a maximum load of 6.25 m^3 of earth per trip, how many trips are needed to fill the entire quarry?
(b) If the cost of transport, material and administration for each truck load is $55, how much would it cost to fill the quarry?
The quarry site now provides an area of approximately 3 hectares for future development. Calculate the cost of one m^2 of the land. (1 hectare = 10 000 m^2)

10. In November 1998, the government announced in Parliament a $10.5 billion package to help the country overcome the Asian economic crisis. If the $10.5 billion is to be issued in $2 notes, what will be the volume of all the $2 notes, assuming that a $2-note has a length of 13.3 cm, a width of 6.4 cm and a thickness of 0.15 mm. Give your answer in m^3.

Density

Which is heavier, 1 kg of iron or 1 kg of feathers?

The density of a substance is defined as the mass of one unit volume of the substance.

$$\textbf{Density} = \frac{\textbf{mass}}{\textbf{volume}}$$

We usually express density in g/cm^3 or kg/m^3.

If 1 cm^3 of a certain substance weighs 3.5 g, we say that the density of the substance is 3.5 g per cm^3 or 3.5 g/cm^3. Similarly, if the mass of 1 m^3 of a substance is 500 kg, then the density of the substance is 500 kg/m^3.

Example 5

If 15 cm^3 of a solid weighs 42 g, find the density of the solid.

Solution

15 cm^3 of the solid weighs 42 g.

1 cm^3 of the solid weighs $\frac{42}{15}$ g = 2.8 g.

$\therefore$ the density of the solid is 2.8 g/cm^3.

It's a Fact

Mercury is the liquid with the greatest density. Its density is 13.6 g/cm^3, while that of water at 4°C is only 1 g/cm^3.

Example 6

The density of a substance is 2.5 g/cm^3. If the substance has a volume of 32 cm^3, find its mass.

Solution

Mass = density × volume
= (2.5 × 32) g = 80 g

$\therefore$ the mass of the substance is 80 g.

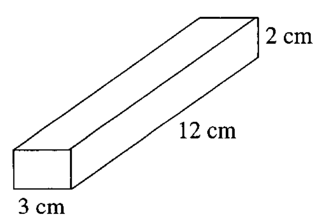
Just For Fun

If the population of the world is 5×10^9, what is the length of the edge of a cubical box that could hold this many people assuming that the volume of an average person is 5.4×10^{-2} m^3?

Example 7

The diagram shows a rectangular solid weighing 360 g. Find (a) its volume, and (b) its density in (i) g/cm^3 and (ii) kg/m^3.

Solution



(a) Volume of cuboid = $L \times W \times H$
= (12 × 3 × 2) cm^3 = 72 cm^3

$\therefore$ the volume of the solid is 72 cm^3.

(b) (i) Density = $\frac{\text{mass}}{\text{volume}} = \frac{360}{72}$ g/cm^3 = 5 g/cm^3

(ii) 5 g = $\frac{5}{1\,000}$ kg

$1 \text{ cm}^3 = \left(\frac{1}{100} \times \frac{1}{100} \times \frac{1}{100}\right) \text{m}^3 = \frac{1}{1\,000\,000} \text{ m}^3$

$\text{Density} = \left(\frac{5}{1\,000} \div \frac{1}{1\,000\,000}\right) \text{kg/m}^3 = 5\,000 \text{ kg/m}^3$

$\therefore$ the density of the solid is 5 g/cm^3 or 5 000 kg/m^3.

Exercise 10b

1. Find the density of a metal if 25 g of it has a volume of 8 cm^3.

2. Calculate the density of a solid if 40 cm^3 of it weighs 96.4 g.

3. If 12 cm^3 of a liquid weighs 15.6 g, find the density of the liquid.

4. Calculate the mass of a piece of solid of volume 26 cm^3 and density 2.8 g/cm^3.

5. Calculate the volume of a piece of cork of mass 105 g and density 0.84 g/cm^3.

6. Calculate the volume of a liquid of mass 3.4 kg and density 13.6 g/cm^3.

7. A rectangular block, 12 cm by 8 cm by 7 cm, has a density of 2.8 g/cm^3. Find
   (a) its volume; **(b)** its mass.

8. A rectangular block, 14 cm by 22 cm by 4 cm, has a mass of 9.4 kg. Find
   (a) its volume; **(b)** its density.

Right Prisms

For Your Information

In general, a right prism is a solid which has two parallel planes of the same shape and size. Also, its lateral surface are perpendicular to its parallel ends.

Cut out a large number of identical triangles from a piece of cardboard and pile them up as shown in Fig. 10.8. A solid is formed. This solid is called a triangular prism. The two parallel planes, PQR and $P'Q'R'$, are triangular in shape and the *triangular prism* takes its name from these planes.

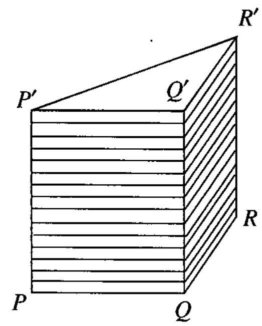


Fig. 10.8

A cuboid is a prism with rectangular planes (see Fig. 10.9). Hence, it is called a rectangular prism. Notice that, in Fig. 10.8, the other three surfaces, which are called the lateral surfaces of the triangular prism, are all rectangular, and that PP', QQ' and RR' are all perpendicular to the planes PQR and $P'Q'R'$. Similarly, in Fig. 10.9, the four lateral surfaces of the cuboid are rectangular and AA', BB', CC' and DD' are perpendicular to the planes $ABCD$ and $A'B'C'D'$. These prisms are called **right prisms**.

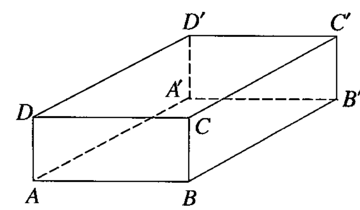


Fig. 10.9

Fig. 10.10 and Fig. 10.11 show a right pentagonal and a hexagonal prism respectively.

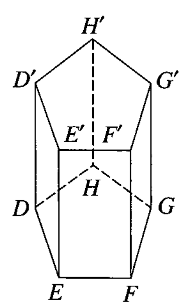


Fig. 10.10

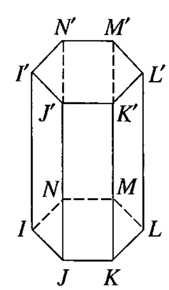


Fig. 10.11

A right prism has a **uniform cross-section**, i.e., the cross-section of the prism is identical to the two parallel ends (see Fig. 10.12).

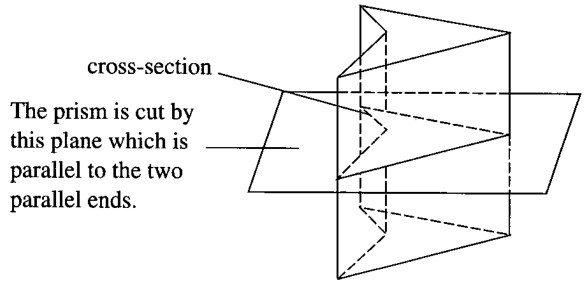


Fig. 10.12

Volume of a Prism

The three prisms shown in Fig. 10.13 are obtained by stacking up a large number of respective identical shapes cut out from cardboards.

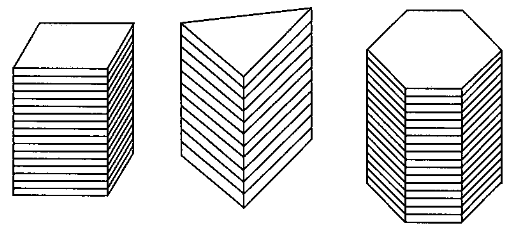


Fig. 10.13

The volume of the right rectangular prism or cuboid
= area of base × height
= area of an identical cardboard × height of rectangular stack
= area of rectangular cross-section × distance between parallel rectangular ends

The volume of a right triangular prism
= area of triangular cross-section × distance between parallel triangular ends

Try to obtain a similar formula as the ones above for the volume of a hexagonal prism.

In general, for a right prism, the volume is given by

volume = area of cross-section × distance between parallel ends
= base area × height

Surface Area of a Prism

Fig. 10.14 shows a right prism whose base is a polygon.

Let A denote the surface area of the prism. Suppose the height of the prism is H and the lengths of the sides of the base are L_1, L_2, L_3, L_4, L_5 and L_6.

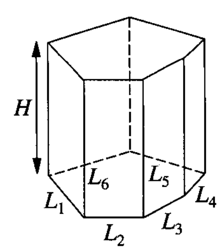
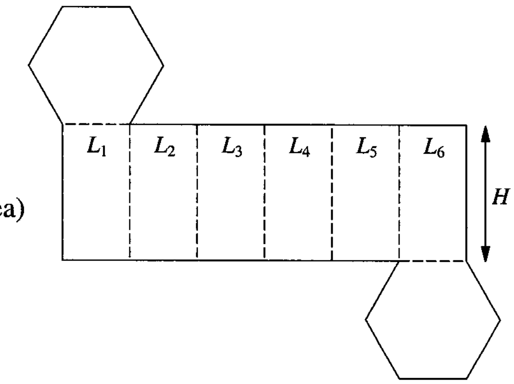


Fig. 10.14

A net of the prism is shown on the right.



The dotted lines indicate the folds.

The area, A, is given by

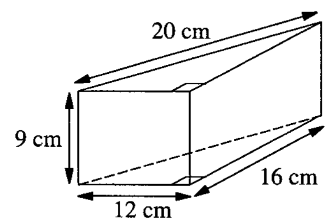
$$A = L_1H + L_2H + L_3H + L_4H + L_5H + L_6H + 2(\text{base area})$$
$$= (L_1 + L_2 + L_3 + L_4 + L_5 + L_6)H + 2(\text{base area})$$
$$= \text{perimeter of base} \times \text{height} + 2(\text{base area})$$

In general,

surface area of a right prism = perimeter of the base × height + 2(base area)

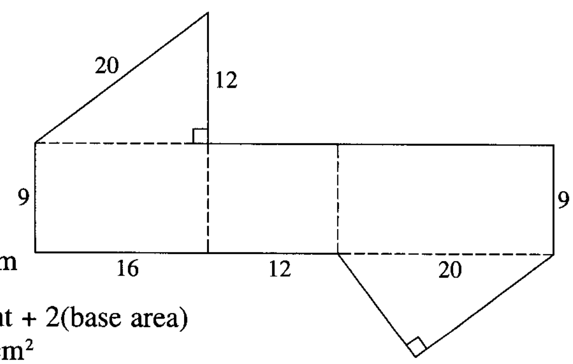
Example 8

Draw a net of the right prism shown on the right and then find its volume and surface area.



Solution

Here is a net of the right prism.



Area of the base $= \left(\frac{1}{2} \times 12 \times 16\right)$ cm^2 = 96 cm^2

Volume of the solid = area of the base × height
= (96 × 9) cm^3 = 864 cm^3

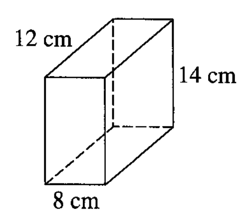
Perimeter of the base = (12 + 16 + 20) cm = 48 cm

Total surface area = perimeter of the base × height + 2(base area)
= [48 × 9 + 2(96)] cm^2 = 624 cm^2

Example 9

Find the volume and surface area of the right prism shown.

Solution

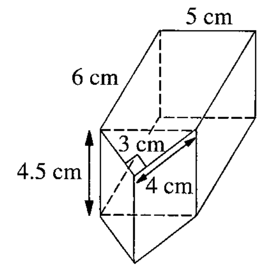


Volume of the right prism = (12 × 8 × 14) cm^3
= 1 344 cm^3

Surface area of the right prism
= 2(12 × 8 + 8 × 14 + 14 × 12) cm^2
= 2(96 + 112 + 168) cm^2 = 752 cm^2

Example 10

Find the volume and surface area of the right pentagonal prism shown.



Solution

Area of the base = $\left[(6 \times 5) + \left(\frac{1}{2} \times 3 \times 4\right)\right]$ cm^2 = 36 cm^2

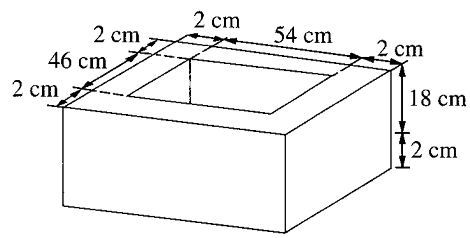
$\therefore$ volume of the prism = (36×4.5) cm^3
= 162 cm^3

Area of the lateral surfaces = perimeter of the base × height
= $[(6 + 5 + 6 + 4 + 3) \times 4.5]$ cm^2
= 108 cm^2

$\therefore$ total surface area of the prism = $[108 + 2(36)]$ cm^2
= 180 cm^2

Example 11

Find the volume of wood used in making an open rectangular box 2 cm thick, given that its internal dimensions are 54 cm long, 46 cm wide and 18 cm deep.



Solution

External length = (54 + 2 + 2) cm = 58 cm

External breadth = (46 + 2 + 2) cm = 50 cm

External height = (18 + 2) cm = 20 cm

External volume = $(58 \times 50 \times 20)$ cm^3 = 58 000 cm^3

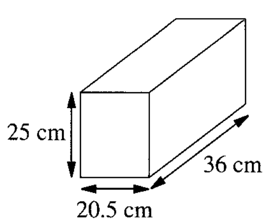
Internal volume = $(54 \times 46 \times 18)$ cm^3 = 44 712 cm^3

$\therefore$ volume of wood used = (58 000 − 44 712) cm^3
= 13 288 cm^3

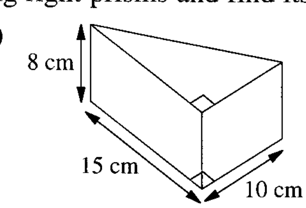
Exercise 10c

1. Draw a net of each of the following right prisms and find its volume.

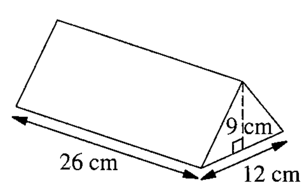
(a)



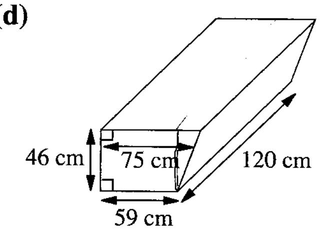
(b)



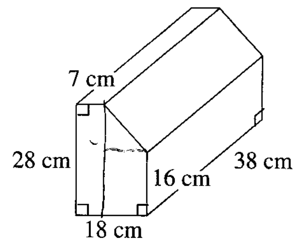
(c)



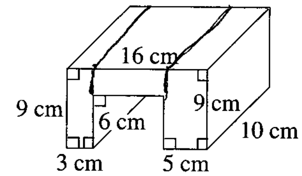
(d)



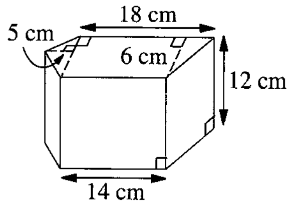
(e)



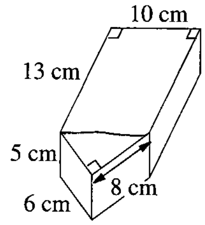
(f)



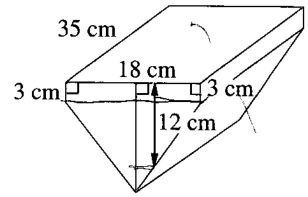
(g)



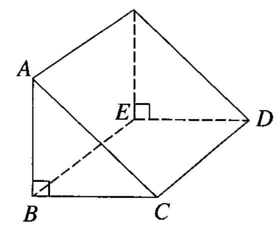
(h)



(i)



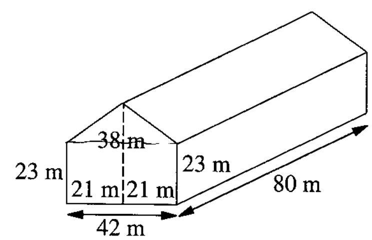
2. The figure shows a right prism standing on a horizontal, rectangular base $BCDE$. The triangle ABC is a vertical cross-section of the solid prism.



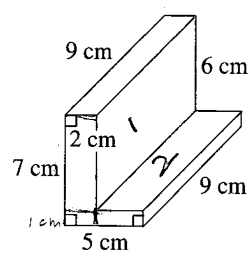
Copy and complete the table below:

	AB	BC	CD	*Area of* $\triangle ABC$	*Volume of prism*
(a)	3 cm	4 cm	7 cm		
(b)	9 cm		11 cm	63 cm^2	
(c)		15 cm	300 cm		72 000 cm^3
(d)	24.6 cm	7.8 cm			38 376 cm^3

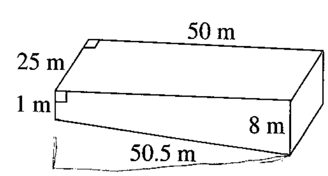
***3.** Not taking into consideration the thickness of the walls and roof, find the air space in the hall with the dimensions given in the figure:



***4.** Find the volume and the surface area of the solid, which is in the shape of a right prism, as shown:



***5.** A swimming pool is 50 m long and 25 m wide. It is 1 m deep at the shallow end and 8 m deep at the other end. Find the volume of water in the pool when it is full as well as the total area of the pool which is in contact with the water (refer to figure).

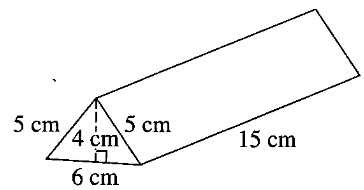


6. Find the volume and surface area of a right prism of height 20 cm whose base is a square of side 15 cm.

7. $4\frac{1}{2}$ litres of oil are poured into a rectangular container whose cross-section is a square of side 12 cm. What is the depth of the oil in the container?

8. A closed box is 135 cm long, 80 cm wide and 60 cm deep internally. It is to be lined on its sides and bottom with cedar veneer of negligible thickness. Find, in square metres, the area of veneer needed.

9. The parallel ends of a right prism, 15 cm long, are isosceles triangles with measurements shown below. Find
   (a) the volume;
   (b) the surface area of the prism.



10. The internal dimensions of an open concrete tank are 1.8 m long, 0.8 m wide and 1.2 m high. Find the capacity of the tank in litres. If the concrete is 0.1 m thick, find also, in cubic metres, the volume of concrete used.

11. A trough, in the form of an open rectangular box, is 1.85 m long, 45 cm wide and 28 cm deep externally. If the trough is made of wood 2.5 cm thick, find, in cubic centimetres, the volume of wood required.

12. A tin which is 12 cm long, 9 cm wide and 4 cm deep holds 120 g of tea. If 1 kg of the same tea is packed into a tin which has a 12-cm square base, how tall will the tin have to be?

★13. The cross-section of a drain is a rectangle 30 cm wide. If water 3.5 cm deep flows along the drain at a rate of 22 cm per second, how many litres of water will flow through each minute?

Cylinders

Formally, the cylinder shown here is called a right circular cylinder. In this book, we use 'cylinder' to represent a 'right circular cylinder'.

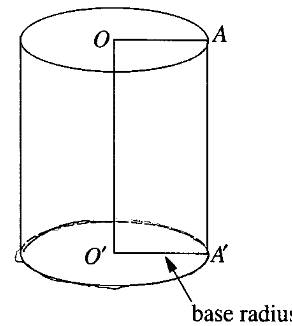
We can form a cylindrical solid by vertically stacking up a pile of 50-cent coins as shown in the figure. This solid is called a right circular prism or simply a right cylinder. Its cross-sectional area is a circle. Steel pipes, oil drums and many tin containers for liquids and preserved food are all common examples of cylinders. Can you name other objects which are cylindrical in shape?

a stack of 50-cent coins

Volume of a Cylinder

Since a cylinder is a right prism with uniform cross-section, we can find its volume by applying the same method used in finding the volume of a right prism,

i.e., **volume of a cylinder = base area × height**



Thus, the volume of a cylinder of base radius r and height h is given by

$$\textbf{volume} = \pi r^2 h$$

Surface Area of a Cylinder

Fig. 10.15(a) shows two equal circles of radius r and a rectangle of height h. It is a net of a cylinder. To form the cylinder shown in Fig. 10.15(b), we roll up the rectangle and bring the two edges AB and CD together. The two equal circles formed will become the top and base circles of the cylinder. Obviously, the length of the rectangle is equal to $2\pi r$, the circumference of each circle.

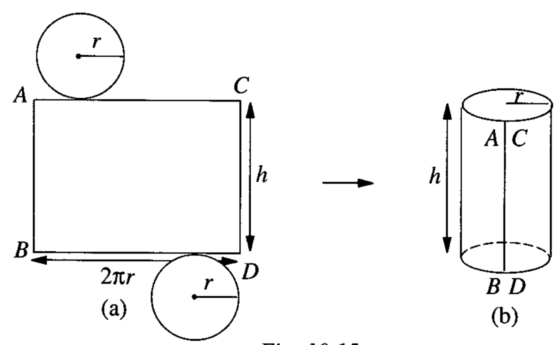


Fig. 10.15

$\therefore$ the area of the curved surface of the cylinder
= area of rectangle $ABCD = 2\pi rh$

Surface area of a solid cylinder = the area of curved surface + 2 × the area of the base circle
$$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

Problems

A man wishes to take 4 litres of water out of a big tank of water. But he has only one 5-litre and one 3-litre jar. How can he do it?

Example 12

The diameter of the base of a right circular cylinder is 14 cm and its height is 10 cm. Find the volume and surface area of the solid cylinder. $\left(\textit{Take } \pi = \frac{22}{7}\right)$

Solution

$r = \frac{14}{2} = 7$, $h = 10$

Volume $= \pi r^2 h$

$= \left(\frac{22}{7} \times 7^2 \times 10\right)$ cm^3

$= 1\ 540$ cm^3

$\therefore$ the volume is $1\ 540$ cm^3.

Surface area $= 2\pi r(h + r)$

$$= \left[2 \times \frac{22}{7} \times 7 \times (10 + 7)\right] \text{cm}^2$$

$= 748 \text{ cm}^2$

$\therefore$ the surface area is 748 cm^2.

It's Fact

The standard unit of measure of weight, or size, of precious stones like diamond is the "carat". A carat is equal to 0.2 gram or 200 milligrams (mg). A carat is further divided into 100 points. Thus, a 20-point diamond has a weight of 40 mg.

However, the carat is a measure of purity as far as gold jewellery is concerned. 24 carat gold refers to pure gold and 18 carat gold is $\frac{18}{24} \times 100\%$, i.e., 75% pure gold. If you are given a choice of a 12 carat diamond or 500 g of 12 carat gold bar, which would you choose?

Example 13

Find the mass, in kg, of a cylindrical metal bar 1.2 m long and 1.4 cm in radius. (The density of the metal is 7.5 g/cm³.)

Solution

$r = 1.4$ cm, $h = 1.2$ m $= 120$ cm

Volume $= \pi r^2 h$

$$= \left(\frac{22}{7} \times 1.4 \times 1.4 \times 120\right) \text{cm}^3$$

$= 739.2 \text{ cm}^3$

Mass = density × volume

$= (7.5 \times 739.2)$ g

$= 5\,544$ g

$= 5.544$ kg

$\therefore$ the mass of the bar is 5.544 kg.

Example 14

If water flows through a 56-mm diameter pipe at the rate of 3 m/s, what volume of water, in litres, is discharged per minute? $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution

$r = 28$ mm $= 2.8$ cm, $h = 3$ m $= 300$ cm

Volume of water discharged per second $= \pi r^2 h$

$$= \left(\frac{22}{7} \times 2.8 \times 2.8 \times 300\right) \text{cm}^3$$

$= 7\,392 \text{ cm}^3$

Volume of water discharged per minute $= (7\,392 \times 60) \text{ cm}^3$

$= 443\,520 \text{ cm}^3$

$= 443.5$ litres (correct to 1 decimal place)

$\therefore$ 443.5 litres of water are discharged per minute.

Example 15

A circular metal sheet 30 cm in diameter and 0.25 cm thick is melted and then recast into a cylindrical bar of diameter 5 cm. Find the length of the bar.

Solution

Volume of circular sheet = $\left(\pi \times 15 \times 15 \times \frac{1}{4}\right)$ cm^3

Let the length of the bar be x cm.

Volume of bar = $\left(\pi \times \frac{5}{2} \times \frac{5}{2} \times x\right)$ cm^3

Volume of bar = Volume of circular sheet

$$\pi \times \frac{5}{2} \times \frac{5}{2} \times x = \pi \times 15 \times 15 \times \frac{1}{4}$$

$$x = \frac{\pi \times 15 \times 15 \times \frac{1}{4}}{\pi \times \frac{5}{2} \times \frac{5}{2}} = 9$$

$\therefore$ the bar is 9 cm long.

In this example, no numerical value is used for π.

Hollow Cylinders

Imagine a solid cylinder of radius R and height h. Suppose another cylinder of smaller radius r (i.e. $r < R$) but of the same height h is scooped out from it. This results in a tube, or a hollow cylinder, as shown in Fig. 10.16. The volume of the hollow cylinder is the difference between the volumes of the two solids.

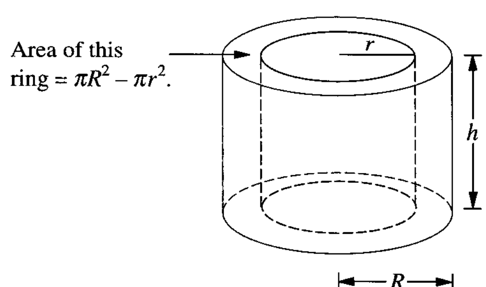


Fig. 10.16

$\therefore$ **Volume of a hollow cylinder**
$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$

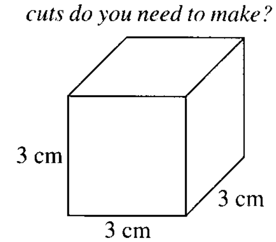
NB: An open cylinder refers to one with a base but without a lid. A closed cylinder refers to one with a base and a lid.

Notes

Do not substitute the value of π until it is clear that such a substitution is necessary.

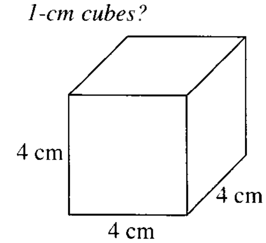
Problems

A cube of side 3 cm is painted green on all its 6 faces. It is to be cut into 27 1-cm cubes. How many cuts do you need to make?



How many small cubes have
(1) no painted faces at all;
(2) 1 face painted green;
(3) 2 faces painted green;
(4) 3 faces painted green;
(5) 4 faces painted green?

You are now given a 4-cm cube which is also painted green on all its faces. How many cuts do you need to make to reduce it to 64 1-cm cubes?



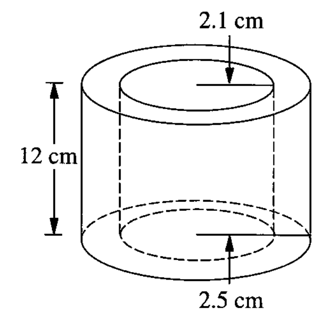
How many small cubes have
(1) 4 faces painted green;
(2) 3 faces painted green;
(3) 2 faces painted green;
(4) 1 face painted green;
(5) no painted faces at all?

Example 16

The figure on the right shows a section of a steel pipe. Given the internal radius of the pipe is 2.1 cm, the external radius is 2.5 cm and the length of the pipe is 12 cm, find

(a) the volume of steel used; *(b) its total surface area. (Take $\pi = 3.14$)*

Solution



(a) $R = 2.5$, $r = 2.1$, $h = 12$

The cross-section of the pipe is a ring.

Area of ring $= [\pi(2.5)^2 - \pi(2.1)^2]$ cm^2
$= 1.84\pi$ cm^2

Volume of pipe $= (18.4\pi \times 12)$ cm^3
$= (1.84 \times 3.14 \times 12)$ cm^3
$= 69.3$ cm^3 (correct to 1 decimal place)

$\therefore$ the volume of steel used = 69.3 cm^3.

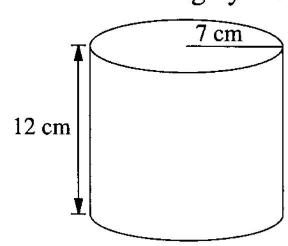
(b) Total surface area of pipe
= areas of internal and external curved surfaces + area of 2 rings
$= [(2\pi \times 2.1 \times 12) + (2\pi \times 2.5 \times 12) + (2 \times 1.84\pi)]$ cm^2
$= (50.4\pi + 60\pi + 3.68\pi)$ cm^2
$= 358.2$ cm^2 (correct to 1 decimal place)

Exercise 10d

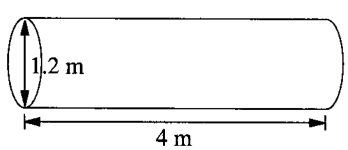
In this exercise, take π to be $\frac{22}{7}$ unless otherwise stated.

1. Find the volume and total surface area of each of the following cylindrical solids:

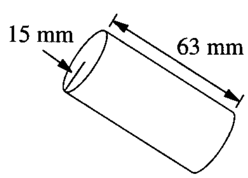
(a)



(b)



(c)



2. Find the diameters of the cylinders given the following:

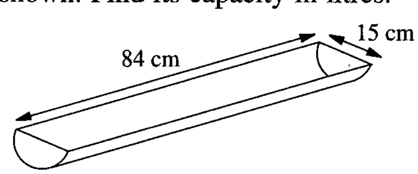
(a) volume 704 cm^3, height 14 cm;
(b) volume 12 320 cm^3, height 20 cm.

3. Find the heights of the cylinders given the following:

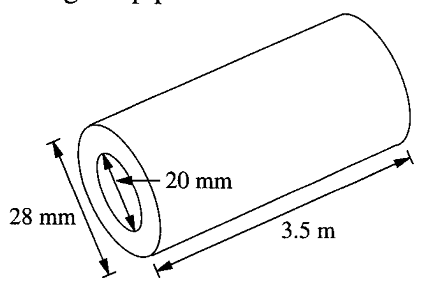
(a) volume 528 cm^3, diameter 4 cm;
(b) volume 1 056 m^3, radius 4 m.

4. A cylindrical can of radius 5 cm and height 8 cm is used to pour water into a larger cylinder of radius 20 cm and height 2 m. How many times must this be done to fill the larger cylinder?

5. The diagram shows a drinking trough in the shape of a half-cylinder with dimensions as shown. Find its capacity in litres.



6. The diagram shows a metal pipe with an outer diameter of 28 mm and an inner diameter of 20 mm. Its length is 3.5 m. Find the volume, in cm³, of the metal used in making the pipe.



7. In a toy factory, 200 wooden solid cylinders 7 cm long and 35 mm in diameter have to be painted. What is the total surface area, in cm², that needs to be painted?

8. 500 cylindrical cans, without top lids and each 14 cm high with diameter 8 cm, are to be made from a sheet of metal. Find, in m², the total area that needs to be painted externally. (Take $\pi = 3.14$)
Correct your answer to one decimal place.

9. A railway tunnel 147 m long is to be bored with a circular cross section of radius 5 m. What volume of soil has to be excavated? If the soil is to be taken away in wagons of capacity 75 m³ each, how many wagons are needed?

10. A beer cask has a height of 63 cm and a diameter of 50 cm. Find its capacity in litres. How many glasses full of beer can it serve if the capacity of each glass is 0.6 litre?

11. A cylindrical solid, whose base radius and height are 10 cm and 14 cm respectively, has a density of 8.6 g/cm³. Find
(a) its volume; **(b)** its mass.

12. A cylindrical solid with a base radius of 7 cm and a height of 20 cm has a mass of 2.6 kg. Find
(a) its volume; **(b)** its density.

13. Assuming that a $1 coin is cylindrical with a diameter of 2.24 cm and a thickness of 2.5 mm, find the volume of the coin, giving your answer in cm³. If the density of the coin is 5.4 g/cm³, find its weight, correct to 2 decimal places.
The Singapore government announced a $10.5 billion recovery package in Parliament in 1998 to help the country overcome the Asian economic crisis. If the $10.5 billion is to be given out in $1 coins, what is the volume of all the coins? (Give your answer in m³, correct to 1 decimal place.) Also find the weight of all the $10.5 billion coins, giving your answer in tonnes, correct to the nearest tonnes.

Summary

1. The volume of an object is the amount of space it occupies. A standard unit for volume is 1 cm³, which is the volume of a cube of side 1 cm.

2. **(a)** Volume of a cuboid L units long, W units wide and H units high = $(L \times W \times H)$ unit³.
(b) Volume of a cube with side L units long = L^3 unit³.
(c) Volume of a right prism = base area × height.

3. **(a)** Surface area of a cuboid, L units long, W units wide and H units high $= 2(L \times W + L \times H + W \times H)$ unit².
(b) Surface area of a cube with side L units long = $6L^2$ unit².

4. For a cylinder of base radius r and height h, curved surface area = $2\pi rh$, total surface area = $2\pi rh + 2\pi r^2$ or $2\pi r(h + r)$ and volume = $\pi r^2 h$.

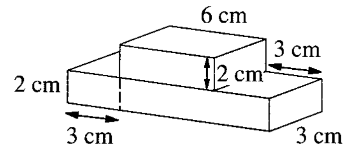
5. The volume of a hollow cylinder with external radius R, internal radius r and height h is given by $V = h(\pi R^2 - \pi r^2)$.

Review Questions 10

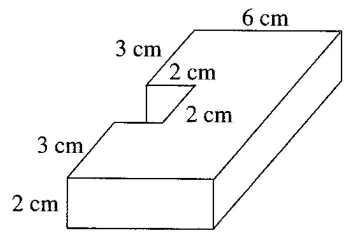
Take π to be $\frac{22}{7}$, where necessary, for the following questions:

*1. **(a)** Find the volume and surface area of each of the following right prisms:

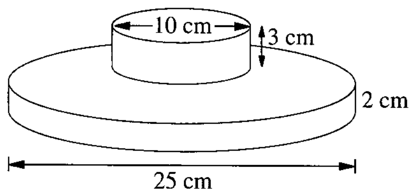
(i)



(ii)

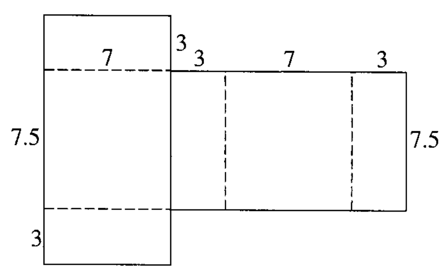


(b) Find the volume and surface area of the following solid, which is made up of two cylinders:

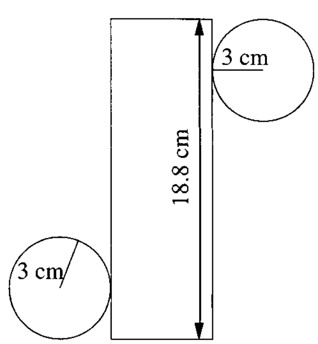


2. The following shows the nets of certain solids. State the name of each of the solids formed and draw a sketch of the solid.

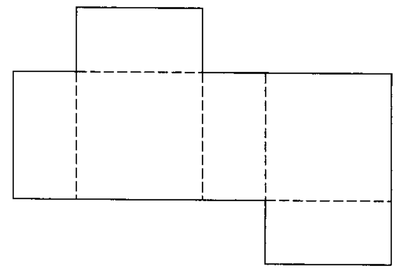
(a)



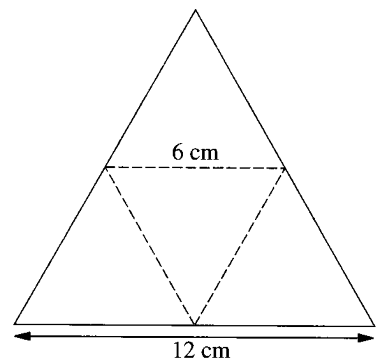
(b)



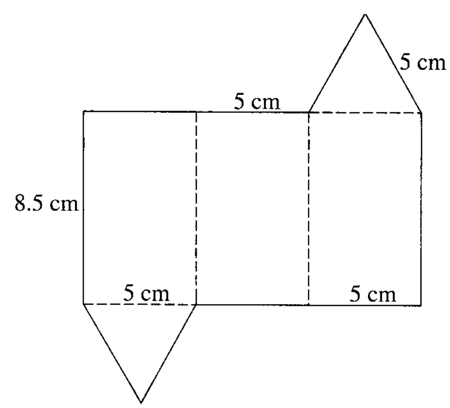
(c)



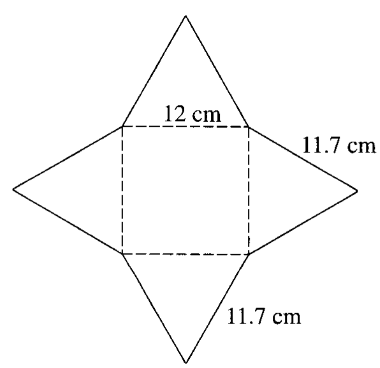
(d)



(e)



(f)



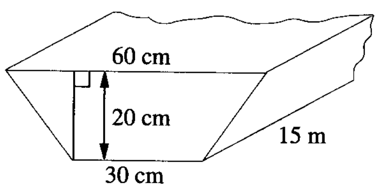
3. A room 8 m long and 5.5 m wide contains 123.2 m^3 of air. Find the height of the room.

4. A brick measures 18 cm by 9 cm by 6 cm. Find the number of bricks that will be needed to build a wall 4.5 m wide, 18 cm thick and 3.6 m high.

5. A water tank, 0.8 m long, 0.8 m wide and 2.4 m deep is half-full of water. How many times can a watering-can be filled if its capacity is approximately 12 litres?

6. How many matchboxes, each 80 mm by 75 mm by 18 mm, can be packed into a box 72 cm by 60 cm by 45 cm internally?

7. The following figure shows a trough 15 m long. Its cross-section is a trapezium. Find the amount of water that the trough can hold in litres.



8. A slab of marble is 2.4 m long, 28 cm wide and 5 cm thick. If the density of the marble is 3.1 g/cm^3, what is its mass?

9. A rectangular wooden beam is 24 cm by 16 cm in cross-section and 6 m long. Find the mass of the beam if the wood has a density of 750 kg/m^3.

10. Find the mass of the water that has fallen onto a flat roof 10.4 m long and 6.5 m wide, when 25 mm of rain is recorded. (The mass of 1 cm^3 of water is 1 g.)

11. How many cubic metres of concrete are needed to surround a rectangular pond $4\frac{1}{4}$ m by 4 m with a border $\frac{1}{4}$ m wide and 18 cm thick?

12. 132 litres of oil is poured into a cylindrical drum, 40 cm in diameter. What is the depth of the oil in the drum?

13. Ten open cylindrical containers are to be painted on the outside, including the base. Each container has a radius of 30 cm and a height of 28 cm. Given that 150 g of paint is needed to paint an area of 1 m^2, find the amount of paint required to paint the ten cylinders. Give your answer in kg.

14. A cylindrical barrel 70 cm in diameter and 80 cm in height is filled with water. A leak at the bottom drains away 0.2 litres of water every minute. How long will it take for the water level to drop by 6 cm?

15. The Singapore Expo has an exhibition area of 60 000 m^2, making it the largest exhibition centre in the region. If the average height of the exhibition centre is 4.85 m, find the volume of air in the centre. If the density of air is approximately 1.26 kg/m^3, find the mass of air contained in the centre.

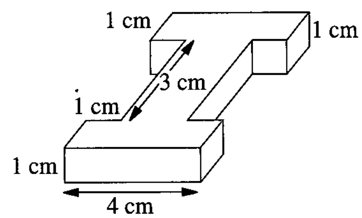
16. It took two and half years and 2.85 million m^3 of earth to fill the disused Sin Seng quarry in Rifle Range Road.
    (a) If each truck can carry 5.75 m^3 of earth per trip, how many trips are needed to fill the quarry?
    (b) Taking one year to be 365 days, find the number of truck loads ferried per day for the above project, giving your answer correct to the nearest whole number.

Problem Solving

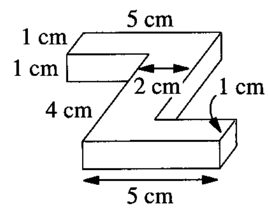
Take π to be $\frac{22}{7}$, where necessary, for the following questions.

1. **(a)** Calculate the volume and surface area of each of the following right prisms:

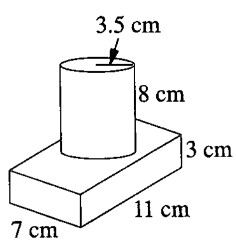
(i)



(ii)



(b) Calculate the volume and surface area of the following solid, which is made up of a cylinder and a right prism:

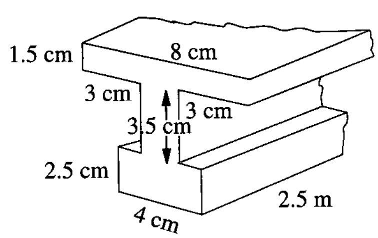


2. The figure shows the dimensions of the cross-section of a girder which is 2.5 m long. Find

(a) the volume of the girder;

(b) the surface area;

(c) its weight if the material weighs 7.8 g per cm^3.

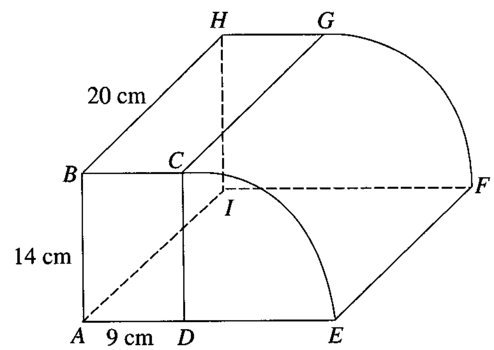


3. The figure shows a closed container of uniform cross-section. The cross-section consists of rectangle $ADCB$ and a quadrant DEC of a circle, centre D. Given AB = 14 cm, AD = 9 cm and $BH = CG = EF = AI$ = 20 cm, calculate

(a) the area of the cross-section $ADECB$;

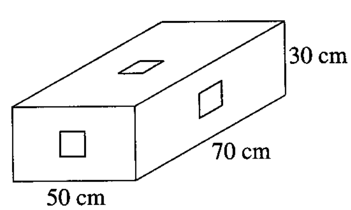
(b) the volume of the container;

(c) the area of the surface $BCEFGH$.



4. A section of a metal pipe has internal diameter 4.2 cm and external diameter 5.0 cm. If the length of the metal used for the pipe is 8.9 cm, calculate the volume of the metal used for making the pipe. If the metal costs \$8 per kg and 1 m^3 of the metal has a mass of 2 700 kg, find the cost of the pipe.

5. A cuboid of dimension 70 cm by 50 cm by 30 cm has "square holes" measuring 10 cm by 10 cm in the centre of three faces of the cuboid, as shown. Calculate the volume and the surface area of the remaining solid.



CHAPTER 11

Ratio, Rate and Proportion

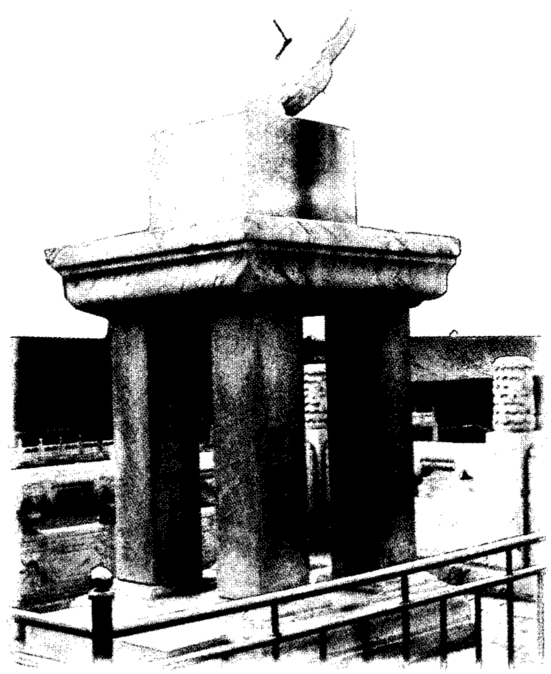
In this chapter, you will learn how to

- △ find the ratio of two or more quantities;
- △ recognize and use common measures of rate;
- △ solve problems involving rate;
- △ use direct and inverse proportions;
- △ solve problems involving ratios and proportions.

Preliminary Problem

The picture shows an ancient sundial used in China for measuring the time of the day.

The measurement of time is essential to calculate the speed of a moving object. It is also needed in many other situations such as measuring the rate of one's heart beat, the rate of change of force in the measure of power and the rate at which your body digests food.



Ratio

In a secondary one class of 45 pupils, 15 of the pupils are girls. We can compare the number of boys and the number of girls in the class using two different ways:

(1) There are 15 more boys than girls in the class. Here, we are comparing the number of boys and the number of girls in the class by finding their difference.

(2) The number of boys in the class is twice that of girls. Here, we are comparing the number of boys and the number of girls by finding a fraction consisting of the number of boys over the number of girls. The fraction is thus $\frac{30}{15}$.

The fraction obtained in (2) is an example of a **ratio** which is used to compare two quantities of the same kind.

A ratio may be written with two dots in between the numbers. In our example, the boy-girl ratio in the class is expressed as 30 : 15, or $\frac{30}{15}$.

30 : 15 means 'the ratio of 30 to 15'.

In general, the ratio of a to b, where a and b represent two quantities and b is not zero, is written as $\boldsymbol{a : b}$, or $\frac{a}{b}$.

In-Class Activity

If the ratio of the length of Fatimah's hair to that of Fandi's hair is 3 : 1, can we make the following conclusions?

1. Fatimah's hair is 3 times as long as Fandi's hair;
2. Fatimah's hair is very long;
3. Fatimah is 3 times as old as Fandi;
4. Fatimah is 3 times taller than Fandi.

Notes

Expressing a ratio in its simplest form is the same as reducing it to its lowest term. Thus 25 : 15 in its simplest terms is 5 : 3.

We usually express a ratio in its simplest form.

Equivalent Ratios

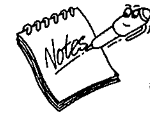
We know that $\frac{30}{15} = \frac{2}{1}$, thus 30 : 15 = 2 : 1. We say that 30 : 15 and 2 : 1 are **equivalent ratios**.

NB: The **order** in which the ratio is expressed is important.
Using the previous example of the class of 45 pupils, the boy-girl ratio is 30 : 15 = 2 : 1, or $\frac{2}{1}$, while the girl-boy ratio is 15 : 30 = 1 : 2, or $\frac{1}{2}$.

A ratio has **no units**. It is merely a number which indicates how many times one quantity is as great as the other or what fraction one quantity is of another. For example, the boy-girl ratio of 2 : 1 indicates that the number of boys is twice that of girls, and the girl-boy ratio of 1 : 2, or $\frac{1}{2}$, indicates that there are half as many girls as there are boys.

Example 1

Find the ratio of (a) 50 g to 200 g and (b) 700 g to 1 kg.



Divide both terms of the ratio by the HCF of the terms, i.e., 50.

Solution

(a) The ratio of 50 g to 200 g can be found using two different methods.

Method 1 $50 : 200 = \frac{50}{200} = \frac{1}{4} = 1 : 4$

Method 2 $50 : 200 = \frac{50}{50} : \frac{200}{50} = 1 : 4$

$\therefore$ the ratio of 50 g to 200 g is 1 : 4.

(b) 700 g and 1 kg are of different units and thus we have to express them in the same units first.

It is easier to express 1 kg as 1 000 g.

$\therefore$ the ratio of 700 g to 1 kg is 700 : 1 000 or 7 : 10.

Ratios can be used to compare more than two quantities. For example, three men, *A*, *B* and *C*, share the profit of a business. They receive \$4 000, \$3 000 and \$1 000 respectively. The ratio of their share of the profit is then 4 000 : 3 000 : 1 000 or 4 : 3 : 1.

Exercise 11a

1. Copy and complete the following equivalent ratios:

(a) $2 : 3 = \square : 9$ **(b)** $\square : 8 = 12 : 32$

(c) $6 : 24 = 3 : \square$ **(d)** $12 : \square = 36 : 21$

2. Express each of the following ratios in its simplest form:

(a) 6 : 10 **(b)** 44 : 8

(c) 3.6 : 4.5 **(d)** 0.4 : 20

(e) $1\frac{1}{2} : 2$ **(f)** 32 : 40 : 24

(g) $1\frac{1}{3} : \frac{2}{3} : \frac{1}{6}$ **(h)** 1.2 : 2 : 2.8

(i) $1.4 : \frac{2}{5} : \frac{1}{3}$ **(j)** $6\frac{2}{5} : 9.6 : 16$

3. Express each of the following as a ratio of the first quantity to the second, in its lowest term, **(i)** in the form a : b, **(ii)** as a fraction:

(a) 25 cents, 80 cents
(b) 210°, 360°
(c) 250 cm, 1 m
(d) 80 cents, \$1.20
(e) 1 kg 250 g, 3 kg
(f) 3 min 30 s, 1 h

4. In a carpark, the ratio of red cars to green is 5 : 6, while that of green cars to blue is 3 : 10. Find the ratio of red cars to blue cars.

5. A school has an enrolment of 630 local students and 120 foreign students. Find the ratio of foreign students to local students.

6. A man earns $1 200 and spends $450 per month. Find the ratio of **(a)** his income to his expenditure and **(b)** his savings to his income.

7. Three people, A, B and C, share $416 among themselves. A receives $169 and B receives $156. Find the ratio in which the sum of money is shared.

8. The interior angles of a quadrilateral are 40°, 60°, 120° and 140°. Find the ratio of these angles according to the order given.

9. The sides of two squares are 4 cm and 6 cm. Find the ratio of **(a)** their areas and **(b)** their perimeters.

10. The table below shows how 117 people travel to work.

Taxi	*MRT Train*	*Bus*	*Car*
9	21	72	15

Find the ratio of people using the four different modes of transport.

Increase and Decrease in Ratio

If the number of teachers in a school is increased from 45 to 55, then the ratio no. of present staff : no. of previous staff = 55 : 45 = 11 : 9.

$$\frac{\text{no. of present staff}}{\text{no. of previous staff}} = \frac{55}{45} = \frac{11}{9}$$

$\frac{11}{9}$ is an improper fraction.

We say that the number of teachers has been **increased in the ratio** 11 : 9, or $\frac{11}{9}$. In other words, the number of present staff is $\frac{11}{9}$ times that of previous staff. Hence, we have no. of present staff = $\frac{11}{9}$ × no. of previous staff.

Notice that when a number x is multiplied by an improper fraction, its value is increased.

Example 2

Increase $20 in the ratio 6 : 5; what is the result?

Solution

The new value is $20 × $\frac{6}{5}$ = $24.

A newspaper agent orders 84 copies of newspapers everyday. During the holidays, he decreases his order to 63 copies. The ratio no. of copies ordered during the holidays : usual no. of copies ordered = 63 : 84 = 3 : 4, or $\frac{3}{4}$.

$\frac{3}{4}$ is a proper fraction.

We say that the number of copies ordered per day has been **decreased in the ratio** 3 : 4, or $\frac{3}{4}$, during the holidays. In other words, the number of copies ordered per day during the holidays is $\frac{3}{4}$ of the usual number of copies ordered.

i.e., new no. of copies ordered during the holidays = $\frac{3}{4}$ × usual no. of copies ordered

To decrease a number x, we multiply it by a proper fraction.

Example 3

Find the result of decreasing 56 m in the ratio 7 : 8.

Solution

The new length is $\left(56 \times \frac{7}{8}\right)$ m = 49 m.

Example 4

In what ratio must 40 m^3 be decreased to become 24 m^3?

Solution

$$\begin{aligned}\text{The required ratio} &= \text{new value} : \text{old value}\\ &= 24\ \text{m}^3 : 40\ \text{m}^3\\ &= 24 : 40 = 3 : 5\end{aligned}$$

Exercise 11b

1. Increase 96 in the ratio 7 : 4; what is the result?

2. Decrease $288 in the ratio 2 : 9; what is the result?

3. Find the result of increasing or decreasing the quantities in the given ratios:
   - **(a)** 40 kg, 5 : 8
   - **(b)** 56 m, 8 : 7
   - **(c)** 35 hectares, 2.5 : 1
   - **(d)** 2.5 cm^2, 2 : 5

4. **(a)** In what ratio must 35 be increased to become 49?
   (b) In what ratio must 72 kg be increased to become 96 kg?

5. **(a)** In what ratio must 105 be decreased to become 75?
   (b) In what ratio must 144 kg be decreased to become 108 kg?

6. The price of petrol drops from $1.20 per litre to 95 cents per litre. Find the ratio in which the price decreases.

7. Two sums of money are in the ratio 5 : 8. The smaller amount is $65. Find the larger amount.

8. A photograph measuring 5.5 cm by 9 cm is enlarged in the ratio 7 : 5. Find the dimensions of the enlarged photograph.

9. The cost of mutton has increased in the ratio 9 : 7. If the original price was $5.60 per kg, what is the new price?

10. Due to import duty, the price of a car increases in the ratio 11 : 8. What is the new price of a car which originally cost $25 600?

Rate

Just For Fun

If it takes four minutes to boil one egg, how long will it take to boil three eggs?

Let us consider the following questions:

A : If one dozen eggs cost \$1.80, what is the cost of 30 eggs?

B : If eight tins of a certain brand of tonic food beverage cost \$26.40, what is the cost of 18 tins?

C : A car travels 570 km on 60 litres of petrol. If the car travels 190 km, what will the petrol consumption be in litres?

D : A boy works 5 hours and is paid \$12.50. How much will he be paid if he works 12 hours?

To answer each of the above questions, we must first find

A : The cost of 1 egg = $\$\frac{1.80 \leftarrow \text{dollars}}{12 \leftarrow \text{eggs}}$ or $\frac{180 \leftarrow \text{cents}}{12 \leftarrow \text{eggs}}$

B : The cost of 1 tin = $\$\frac{26.40 \leftarrow \text{dollars}}{8 \leftarrow \text{tins}}$

C : The petrol consumption for 1 km = $\frac{60 \leftarrow \text{litres}}{570 \leftarrow \text{km}}$

D : The pay for 1 hour = $\$\frac{12.50 \leftarrow \text{dollars}}{5 \leftarrow \text{hours}}$

Each of above results is different from a ratio in that it involves two quantities of different kinds. Each of them is called a **rate**.

A : The rate = $\$\frac{1.80}{12} = \$0.15$ per egg

$\therefore$ the cost of 30 eggs = $\$0.15 \times 30 = \$4.50$.

B : The rate = $\$\frac{26.40}{8} = \$3.30$ per tin

$\therefore$ the cost of 18 tins = $\$3.30 \times 18 = \$59.40$.

C : The rate = $\frac{60}{570} = \frac{2}{19}$ litre per km

$\therefore$ if the car travels 190 km, the petrol consumption = $\frac{2}{19} \times 190$ = 20 litres.

D : The rate = $\$\frac{12.50}{5} = \$2.50$ per hour

$\therefore$ he will be paid $\$2.50 \times 12 = \$30.00$ for working 12 hours.

NB: We normally use the word "per" or the symbol "/" to denote a rate. Thus we have \$2.50 per hour or \$2.50/hour.

Problems

Understand the problem by asking the questions:

1. *How far can a car travel on 1 litre of petrol?*
2. *How much petrol is needed to travel 1 km?*
3. *How many litres of petrol are required to travel 260 km?*

Example 5

How far can a car travel on 15 litres of petrol if it can travel 91 km on 7 litres of petrol? How much does the owner of the car spend on petrol, which costs $1.10 per litre, when he travels 260 km?

Solution

Distance travelled on 1 litre of petrol = $\frac{91}{7}$ km = 13 km.

$\therefore$ distance travelled on 15 litres of petrol = (13 × 15) km = 195 km.

Consumption of petrol per km = $\frac{7}{91}$ litre = $\frac{1}{13}$ litre.

Consumption of petrol for 260 km = $\left(\frac{1}{13} \times 260\right)$ litres = 20 litres.

$\therefore$ the owner spends $1.10 × 20 = $22 on petrol when he travels 260 km.

Exercise 11c

1. Copy and complete the following:
   (a) If a typist types 900 words in 1 hour, her rate of typing is ______ words per minute.
   (b) If a man pays $600 rent for 3 months, the rental rate is ______ dollars per month.
   (c) If $14.06 is charged for 74 units of electricity, the rate is ______ cents per unit.
   (d) A machine is used to stamp bottle caps. If 150 bottle caps can be stamped in 30 seconds, the rate is ______ caps per second.

2. A man earns $250 in a five-day week. What is his pay for 3 days?

3. A car uses 40 litres of petrol to travel 340 km. How far can it travel if it has only 32 litres of petrol?

4. A machine stamps 720 bottle caps in 2 minutes. How many bottle caps can it stamp in 40 seconds?

5. A wire 22 cm long has a mass of 374 g. What is the mass of 13 cm of this wire?

6. A shopkeeper buys 72 articles for $82.80. How much will he have to pay if he buys 150 such articles?

7. 40 cm of a certain type of piping cost $2.00. What is the cost of 1 km of such piping?

8. The cost of a long-distance call lasting 4 minutes and 20 seconds was $23.40. At this rate, what was the cost of a call lasting 6 minutes 30 seconds?

9. 250 cm^3 of a liquid weighs 125 g. Find the weight of 1 000 cm^3 of the liquid.

10. 200 g of fertilizer is required for a land area of 8 m^2. At this rate,
    (a) how many grams of fertilizer are needed for a land area of 1 m^2?
    (b) how many grams of fertilizer are required for a land area of 14 m^2?
    (c) for what land area will 450 g of fertilizer be sufficient?

*11. A cook uses fifteen 2-kg bottles of cooking-oil over a 4-week period. If he decides to buy 5-kg tins of oil instead, how many tins of cooking oil will he use over a 10-week period if the rate of using it remains unchanged?

☆**12.** In a certain company, the amount of travelling expenses an employee may claim is calculated as follows:

If the distance travelled exceeds 20 km, claimable amount = 20 × rate + (number of km – 20) × \$0.70.

Otherwise,

claimable amount = number of km × rate, where the rate in both instances is \$0.55 per km if the engine capacity of the employee's car exceeds 1 000 cc, otherwise it is \$0.50 per km.

Find the travelling expenses allowed in each case:

(a) **(i)** 18 km;
(ii) 28 km travelled in a 1 298 cc car;

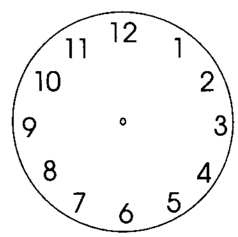
(b) **(i)** 16 km;
(ii) 25 km travelled in a 998 cc car.

For Your Information

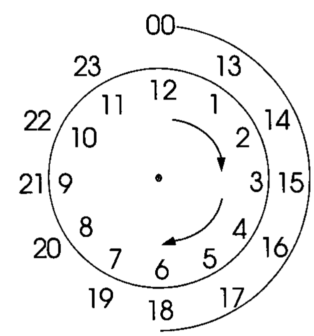
a.m. stands for "ante meridiem" (Latin word) meaning "before midday". p.m. stands for "post meridiem" meaning "after midday".

Time

To record the time of the day, we can either use the 12-hour clock or the 24-hour clock. In the 12-hour clock, morning (from midnight to just before noon) is denoted by a.m.; afternoon, evening and night are denoted by p.m. In the 24-hour clock, four digits are used to indicate time. The first two digits denote hours and the last two denote minutes.



12-hour clock



24-hour clock

The table below shows some examples.

Time	*12-hour clock*	*24-hour clock*
2 o'clock early morning	2.00 a.m.	02 00
5 to 11 in the morning	10.55 a.m.	10 55
Noon	12.00 p.m.	12 00
Half past 12 early afternoon	12.30 p.m.	12 30
Quarter to 3 in the afternoon	2.45 p.m.	14 45
5 past 8 in the evening	8.05 p.m.	20 05
One minute to midnight	11.59 p.m.	23 59
Midnight	12.00 a.m.	00 00
One minute past midnight	12.01 a.m.	00 01

Example 6

A journey starting at 08 40 takes $4\frac{3}{4}$ hours. Find the time the journey ends.

Solution

	h	min	
Starting time	08	40	$\left(4\frac{3}{4}\text{ h}\right)$ 40 + 45 = 60 + 25 = 1 h 25 min
Journey time	04	45	
Arrival time	13	25	
	(8 + 4 + 1)		

∴ the journey ends at 13 25, or 1.25 p.m.

Just for fun

A cashier of a bank is given one million one-cent coins to count. How long will she take if she can count five coins in one second?

Example 7

A car leaves Singapore at 21 15 on Wednesday and arrives in Kuala Lumpur $5\frac{1}{2}$ hours later. At what time and day does the car arrive in Kuala Lumpur?

Solution

	h	min		
Starting time	21	15	$\left(5\frac{1}{2}\text{ h}\right)$	
Journey time	05	30		
Arrival time	26	45 →	02	45
	−24			

∴ the car arrives in Kuala Lumpur at 02 45, or 2.45 a.m., on Thursday.

Notes

26 indicates that the car arrives in Kuala Lumpur the next day.

Example 8

A train left Singapore at 07 35 and arrived in Seremban at 13 05. How long was the train journey?

Solution

	h	min
	12	65
Arrival time	$\cancel{13}$	$\cancel{05}$
Departure time	07	35
	5	30

∴ the train journey was 5 h 30 min long.

Notes

One hour is converted to 60 minutes.

24 h is added so that the arrival time is measured from 00 00 on Saturday.

Example 9

A bus leaves Town A on Saturday night and is supposed to arrive at Town B at 08 17 on Sunday morning. If the estimated journey time is 10 h 43 min, at what time should the bus leave Town A?

Solution

	h	min	
	+24 → 31	77	
Arrival time 08 17 (Sunday) =	$\cancel{32}$	$\cancel{17}$	(Saturday)
Journey time	10	43	
Departure time	21	34	(Saturday)

∴ the bus should leave Town *A* at 21 34, or 9.34 p.m., on Saturday.

Exercise 11d

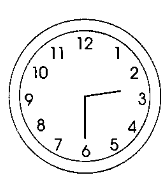
1. Convert the following times to 24-hour clock notation:

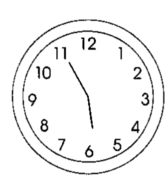
(a) 8.00 a.m. **(b)** 2 p.m.
(c) 5.30 p.m. **(d)** 9.42 p.m.
(e) noon **(f)** 12.45 a.m.
(g) midnight **(h)** 2.42 a.m.

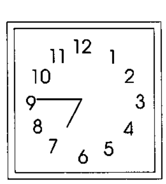
2. Convert the following times to 12-hour clock notation:

(a) 03 30 **(b)** 15 00
(c) 23 12 **(d)** 19 15
(e) 09 23 **(f)** 12 00
(g) 00 05 **(h)** 24 00

3. Write down, using the 24-hour clock notation, the times shown:

(a)

a.m.

(b)

p.m.

(c)

a.m.

4. Copy and complete the following table:

	Departure time	*Journey time*	*Arrival time*
(a)	15 45	5 hours	
(b)	02 40	55 minutes	
(c)	08 45	$9\frac{3}{4}$ hours	
(d)	22 35	8 hours	
(e)	15 45		17 50
(f)	11 50		15 15
(g)	09 48		22 16
(h)	20 35 (Tue)		07 15 (Wed)
(i)		$1\frac{1}{4}$ hours	23 50
(j)		$17\frac{3}{4}$ hours	12 45 (Fri)

5. A train left a station at 8.35 a.m. and arrived at its destination at 3.12 p.m. How long did the journey take?

6. An overnight train left at 21 55 on a journey that took 9 h 18 min. Find the time at which it arrived at its destination.

7. A car arrived at a town at 15 06 after travelling for $4\frac{1}{4}$ hours. Find the time the car started its journey.

8. According to a timetable, a coach was due to leave a station at 22 55 and arrive at its destination at 06 05 the next day. How long would the journey take? If the train actually arrived 35 minutes early, at what time did it arrive?

☆9. Lessons in a certain school start at 7.45 a.m. and end at 3.45 p.m., with an hour's break at lunchtime and 20 minutes morning recess. If there are altogether 8 lessons of equal length, how long is each lesson?

☆**10.** Shown below is the schedule of the arrival and departure times of a long-distance express overnight coach.

Destination	*Arrival*	*Departure*
Singapore	—	21 30
Johor Baru	22 15	22 30
Seremban	02 25	02 30
Kuala Lumpur	03 50	04 20
Ipoh	07 50	08 00
Taiping	09 20	09 30
Butterworth	10 45	—

Find the time taken for the coach to travel from

(a) Singapore to Seremban;
(b) Johor Baru to Ipoh;
(c) Seremban to Taiping;
(d) Kuala Lumpur to Butterworth;
(e) Singapore to Butterworth.

Average Speed

It's a Fact

If a cyclist is equipped with a speedometer, which gives his speed at a particular instant, the readings from the speedometer will change from time to time.

Two cyclists, A and B, travel 90 km, in a race, in 5 hours and $4\frac{1}{2}$ hours respectively. Which cyclist travels faster?

Cyclist B travels faster since he takes less time to complete the race.

We can also find the **speed** at which each cyclist travels to find out who travels faster. As you can see, speed is a special kind of rate.

$$\text{Cyclist } A\text{'s speed} = \frac{90 \text{ km}}{5 \text{ h}} = 18 \text{ km/h}$$

$$\text{Cyclist } B\text{'s speed} = \frac{90 \text{ km}}{4\frac{1}{2} \text{ h}} = 20 \text{ km/h}$$

Cyclist B travels faster since he travels at a greater speed.

When calculating the speed of each cyclist, we assume that one travels at the same speed all the time. In reality, each cyclist will have difficulty cycling at the same speed all the time. For example, he may slow down when he is cycling up a slope or he may speed up when he is going down a slope. Thus, the speed calculated for each cyclist is not his exact speed at a particular instant. Instead, it is his average speed. For example, the average speed of cyclist A is 18 km/h. This means that on the average, he travels 18 km every hour. The average speed can be obtained by using the formula:

$$\textbf{Average speed} = \frac{\textbf{Total distance travelled}}{\textbf{Total time taken}}$$

NB: We can also express average speed in m/s.

The highest speed limit for cars on Singapore roads is 90 km/h. How many demerit points will a motorist be awarded if he is caught speeding on the expressway at

(a) 100 km/h;
(b) 120 km/h;
(c) 160 km/h?

Example 10

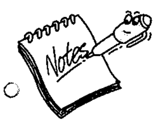
A car travelled 510 km in 6 hours. Find the average speed of the car for the whole journey.

Solution

Distance travelled = 510 km

Time taken = 6 hours

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}} = \frac{510\text{ km}}{6\text{ h}} = 85\text{ km/h}$$



Convert km to m and hour to seconds.

Example 11

A cyclist is travelling at an average speed of 18 km/h.

(a) Express his average speed in m/s.
(b) Find the distance he travels in 3 hours.
(c) Find how far he travels in 25 seconds.

Solution

(a) 18 km = 18 × 1 000 m, 1 h = (60 × 60) s

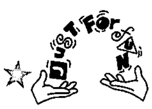
$$\therefore\quad 18\text{ km/h} = \frac{18\text{ km}}{1\text{h}} = \frac{(18 \times 1\,000)\text{ m}}{(60 \times 60)\text{ s}} = 5\text{ m/s}$$

(b) In 3 hours, the cyclist travels $(\underbrace{18}_{\text{Average speed in km/h}} \times \underbrace{3}_{\text{Time taken in hours}})$ km = 54 km

(c) In 25 seconds, he travels $(\underbrace{5}_{\text{Average speed in m/s}} \times \underbrace{25}_{\text{Time taken in seconds}})$ m = 125 m

In general,

$$\textbf{Distance travelled} = \textbf{Average speed} \times \textbf{Time taken}$$



A sports car leaves Singapore for Kuala Lumpur at the same time as a bus, which leaves Kuala Lumpur for Singapore. They travel along the same road, the sports car at 110 km/h and the bus at 55 km/h. Which vehicle is further away from Singapore when they meet?

Example 12

A train travels at an average speed of 15 m/s.

(a) Express its average speed in km/h.
(b) Find the time taken by the train to travel 750 m.
(c) If the train sets off from Station A at 8.00 a.m., find the arrival time of the train at Station B which is 36 km away.

It's Fact

1. The cheetah is the fastest land animal. It can acquire a speed of 110 km/h in a matter of seconds.
2. The men's world record for the 100 m sprint is approximately 36.7 km/h.
3. The speed of sound is about 34 times faster than the speed of the fastest 100 m human sprinter.

Solution

(a) $15 \text{ m} = \frac{15}{1\,000} \text{ km}$, $1 \text{ h} = 3\,600 \text{ s}$

In 1 second, the train travels $\frac{15}{1\,000}$ km.

In 1 hour, the train travels $\left(\frac{15}{1\,000} \times 3\,600\right)$ km = 54 km.

$\therefore$ its average speed is 54 km/h.

(b) $15 \text{ m/s} = \frac{15 \text{ m}}{1 \text{ s}}$

In 1 second, the train travels 15 m.

The time taken to travel 1 m is $\frac{1}{15}$ s.

$\therefore$ the time taken by the train to travel 750 m $= \frac{1}{15} \times 750$ s

$= \frac{750}{15}$ s — Distance travelled in m. / Average speed in m/s.

$= 50$ s

(c) Similarly in 1 hour, the train travels 54 km.

The time taken to travel 1 km is $\frac{1}{54}$ h.

$\therefore$ the time taken by the train to travel 36 km $= \frac{1}{54} \times 36$ h

$= \frac{36}{54}$ h — Distance travelled in km. / Average speed in km/h.

$= \frac{2}{3}$ h

$= \frac{2}{3} \times 60 \text{ min} = 40 \text{ min}$

$\therefore$ the arrival time of the train at Station B is 8.40 a.m.

In general,

$$\textbf{Time taken} = \frac{\textbf{Distance travelled}}{\textbf{Average speed}}$$

Exercise 11e

1. Copy and complete the following. The first one has been done for you.

	Distance travelled	*Time taken*	*Average speed*
(a)	180 km	$1\frac{1}{2}$ h	120 km/h
(b)	200 m	25 s	
(c)	400 m	1 min	
(d)		$5\frac{1}{2}$ h	80 km/h
(e)		$\frac{1}{3}$ min	25 m/s
(f)	100 m		20 m/s

2. Express the following in m/s:

(a) 18 km/h **(b)** 72 km/h
(c) 90 km/h

3. Express the following in km/h:

(a) 10 m/s **(b)** 35 m/s
(c) $\frac{1}{2}$ km/s

4. How long will a man take to run, once, round a circular track of radius 28 m at an average speed of 8 m/s? $\left(\text{Take } \pi = \frac{22}{7}\right)$

5. A cyclist begins on a 24-km journey at 09 23. When will he complete his journey if he travels at an average speed of 16 km/h?

6. A train leaves Town X at 12 57 and arrives at Town Y 45 minutes later.

(a) At what time does the train arrive in Town Y?
(b) What is the average speed of the train, in km/h, if the distance between the two towns is 84 km?

7. A car travels at an average speed of 24 km/h. Find, in metres, the distance travelled by the car in 12 seconds.

8. A car travelled on a B class road for 20 minutes at an average speed of 57 km/h. It then travelled a distance of 55 km in 30 minutes on an expressway. Find

(a) the distance the car travelled on the B class road;
(b) the average speed, in km/h, of the car when it travelled on the expressway.

9. A man cycles for two hours at an average speed of 16 km/h and then walks for 3 hours at an average speed of 6 km/h. Find his average speed for the whole journey.

10. A train travels 68 km at an average speed of 51 km/h. It then travels another 20 km at an average speed of 40 km/h before reaching its destination. Calculate the average speed for the whole journey.

☆**11.** Two points, X and Y, are 120 m apart. M is the mid-point of X and Y. An object travels from X to M in 12 seconds and then from M to Y at an average speed of 15 m/s. Calculate

(a) the average speed of the object from X to M;
(b) the time taken to travel from M to Y;
(c) the average speed for the whole journey from X to Y.

☆**12.** Three points, L, M and N, lie on a straight line with LN = 160 m. An object travels from L to M at an average speed of 10 m/s in 6 seconds and then from M to N at an average speed of 25 m/s. Calculate

(a) the distance from L to M;
(b) the time taken to travel from M to N;
(c) the average speed for the whole journey from L to N.