PREFACE

New Syllabus Mathematics is a series of four books. These books follow the Mathematics Syllabus for Secondary Schools, implemented from 2001 by the Ministry of Education, Singapore. The whole series covers the complete syllabus for the Singapore-Cambridge GCE 'O' Level Mathematics.

The fifth edition of New Syllabus Mathematics 2 retains the goals and objectives of the previous edition, but has been revised to meet the requests of users of the fourth edition and to keep materials up-to-date as well as to give students a better understanding of the contents.

All topics are comprehensively dealt with to give students a firm grounding in the subject. Explanations of concepts and principles are concise and written in clear language with supportive illustrations and examples. Examples and exercises have been carefully graded to aid students in progressing within, as well as up, each level. Those exercises marked with a ✶ are either tricky or involve more calculations. "Problem Solving" and "Exploration", placed at the end of the chapter, contain more difficult and challenging questions requiring students to apply their knowledge and experience in solving them.

Numerous revision exercises are provided at appropriate intervals to enable students to recapitulate what they have learnt. In addition, there are mid-year and final-year examination specimen papers.

Important features which have been retained in this edition to facilitate learning are:

- an interesting introduction at the beginning of each chapter complete with photographs or graphics
- brief specific instructional objectives for each chapter
- in-class activities (investigation / discussion / problem solving)
- activities and interesting information in the marginal text (clip-notes, "Down Memory Lane", "Back In Time", "Investigate", "Check This Out!", "It's A Fact", "Just For Fun", "Are You Game Enough?", "For Your Information", "Library Corner" and "Problems")

Problem-solving heuristics are subsequently introduced at appropriate sections of the book to reinforce problem-solving skills. In addition, questions which call for problem-solving skills are also set in the margin for students to do at their own pace and time.

Ample opportunities are also provided for mathematical investigative and communicative activities.

It is hoped that these features will help students learn mathematics with more zest and excel in the subject.

CONTENTS

CHAPTER 1

Arithmetic Problems and Standard Form

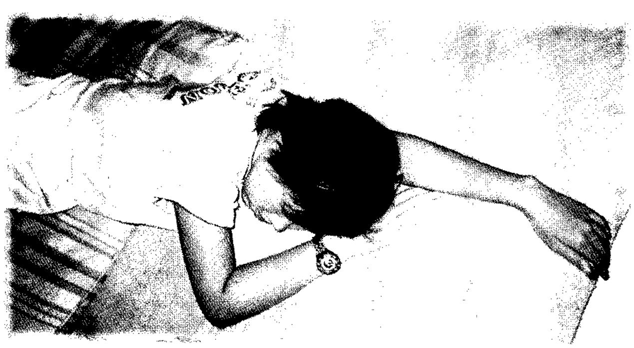
In this chapter, you will learn how to

- △ solve problems involving ratio and proportion;
- △ solve more difficult problems involving percentages and other financial problems;
- △ use the standard form to express very large or very small numbers.

Preliminary Problem

The rate at which our body uses energy depends largely on the type of activities we are doing. The following are approximate rates at which our body uses energy.

Can you think of other instances when our body uses energy within approximately an hour?



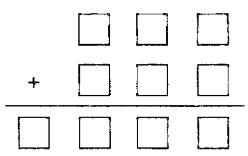
Resting: 60 calories/hour

Running: 900 calories/hour

Rate, Ratio, Proportion and Speed

A wide variety of arithmetic problems involve rate, ratio, proportion and speed. Rate, ratio, proportion and speed have already been dealt with in Secondary 1. The following examples serve as a revision.

Use all the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 only once so as to make the addition below correct.



(There are a few possible answers.)

Example 1

Five workers, each working 8 hours per day, can dig a rectangular trench 600 m long, 0.9 m wide and 1 m deep in 3 days. Find the number of workers needed to dig a trench 1.2 km long, 1.1 m wide and 1.8 m deep in 3 days. Calculate the labour cost given that each worker is paid \$5 per hour. (Assume all the workers worked 8 hours per day for 3 days.)

Solution

Let x be the number of workers needed to dig the trench.

$$\frac{x}{5} = \frac{1\,200 \times 1.1 \times 1.8}{600 \times 0.9 \times 1} \quad \text{(Direct proportion)}$$

$$x = \frac{1\,200 \times 11 \times 18}{600 \times 9 \times 10} \times 5$$

$$= 22$$

$\therefore$ 22 workers are needed for the job.

The total number of working hours for 22 workers = $22 \times 3 \times 8$

$\therefore$ the labour cost = $22 \times 3 \times 8 \times \$5$

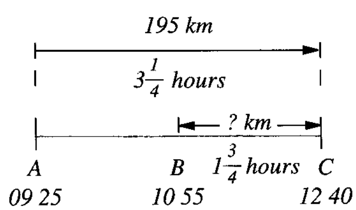
= \$2 640

Example 2

A man leaves Town A at 09 25 in a car, passes through Town B at 10 55 and arrives in Town C at 12 40. Given that the distance from Town A to Town C is 195 km and that the man travels at a constant speed throughout, find the distance from Town B to Town C.

In air and sea navigation, the speed of an aircraft or a ship is measured in 'knots'. Find out more about the unit 'knot'.

Solution



Time taken to travel from Town A to Town C = 3 h 15 min

$= 3\frac{1}{4}$ h

	12	40
−	09	25
	3	15

Distance between Town A and Town C = 195 km

$$\therefore \quad \text{speed} = \frac{195}{3\frac{1}{4}} \text{ km/h} = \left(195 \times \frac{4}{13}\right) \text{ km/h} = 60 \text{ km/h}$$

Time taken to travel from Town B to Town C = 1 h 45 min $= 1\frac{3}{4}$ h

$$\begin{array}{rr} 12 & 40 \\ -\ 10 & 55 \\ \hline 1 & 45 \\ \hline \end{array}$$

$$\therefore \quad \text{distance from Town } B \text{ to Town } C = \left(60 \times 1\frac{3}{4}\right) \text{km} = \left(60 \times \frac{7}{4}\right) \text{km} = 105 \text{ km}$$

It's Fact

The growth rate of human hair varies from person to person. On the average, a human hair grows at a rate of 0.35 mm per day. If the length of a strand of hair is 6 cm, how long will it take the same strand of hair to grow to a length of 26 cm?

Alternatively, let the distance between Town B and Town C be x km.

Since the man travels at a constant speed throughout,

$$\frac{x}{1\frac{3}{4}} = \frac{195}{3\frac{1}{4}} \quad \text{or} \quad x = 195 \times \frac{4}{13} \times \frac{7}{4} = 105$$

$\therefore$ distance from Town B to Town C is 105 km.

Example 3

Three boys, Ali, Bala and Paul, aged 14, 16 and 18 respectively, are to share a sum of money in the ratio of their ages. Given that Ali receives $1 050, calculate how much each of the other two boys receives and the total amount shared.

Solution

Ali	*Bala*	*Paul*
14	16	18

Ali receives 14 shares which is equivalent to $1 050.

$\therefore$ 1 share = $1 050 ÷ 14 = $75

Hence, Bala will receive $75 × 16 = $1 200

Paul will receive $75 × 18 = $1 350

The total amount shared by the three boys = $1 050 + $1 200 + $1 350
= $3 600

Alternatively,

$$14 : 16 : 18 = 7 : 8 : 9$$

Let $\$x$ be the total amount shared.

$$\text{Ali's share} = \frac{7}{7+8+9} \text{ of } \$x$$

$$1\,050 = \frac{7}{24}x \quad (\text{Ali's share} = \$1\,050)$$

$$x = \frac{24 \times 1\,050}{7}$$

$$= 3\,600$$

$\therefore$ The total amount shared = \$3 600

$$\text{Bala's share} = \frac{8}{24} \times \$3\,600$$

$$= \$1\,200$$

$$\text{Paul's share} = \$3\,600 - \$1\,050 - \$\,1\,200$$

$$= \$1\,350$$

Nine hens lay nine eggs in nine days. How many eggs will three hens lay in three days?

Exercise 1a

1. Given that the angles of a quadrilateral are in the ratio 1 : 2 : 4 : 5, find the angles.

2. A journey takes 6 hours if John travels at 30 km/h. How long will it take if John travels at 45 km/h?

3. The cost of petrol for a 240-km journey for a car which runs 12 km on each litre of petrol is \$24. What would be the cost of petrol for a 500-km journey for a van which runs 11 km on each litre of petrol?

4. Find the speed in m/s of a point on a bicycle rim of diameter 88 cm, making 200 revolutions per minute.

5. In producing a certain article, the cost of overheads, labour and materials are in the ratio 2 : 7 : 5. If the cost of labour in producing an article is \$28, find
   (a) the total cost of producing an article,
   (b) the cost of materials for each article.

6. Eight tins of paint are required for a surface of an area 50 m^2 if 4 coats of paint are applied. Find the number of tins of paint needed for an area of 70 m^2 if 5 coats of paint are applied.

7. The Singapore Navy acquired 4 Swedish-build Sjoormen class submarine to beef up its defence capabilities in the late 1990s. The submarine is capable of attaining a top speed of 16 knots (nautical miles per hour) when submerged. Convert 16 knots to
   (a) km/h (b) m/s.
   (Take 1 nautical mile to be 1 853 m.)

8. A scouts' camp has enough food for 36 days. How long will the food last if each scout's ration is reduced in the ratio 4 : 5?

9. In a factory, 25 men working 26 hours can produce 1 300 radios. How many hours must the same group of men work to produce 450 radios?

10. Twelve women working 7 hours a day can finish a piece of work in 8 days. How many hours a day must 16 women work in order to finish the job in 14 days?

11. When petrol cost \$1.25 per litre, the cost of filling a tank was \$37.50. Given that the cost of petrol has now gone up by 7 cents per litre, calculate the new cost of filling the tank.

12. A packet of sweets was divided into two portions in the ratio 3 : 5. If the smaller portion had 18 sweets, what was the total number of sweets in the packet?

✫**13.** **(a)** An aircraft took off from City A to travel to City B at 11 00 on a certain day. Given that the aircraft flew at an average speed of 900 km/h and took 6 hours to arrive in City B, find

(i) its arrival time in City B,

(ii) the distance between City A and City B.

(b) The next day, the aircraft started its return journey at 19 00 and flew at an average speed of 1 080 km/h. Find

(i) the time taken for the return journey,

(ii) the arrival time of the aircraft in City A.

***14.** A man starts cycling from home to Town X at 11 00. He cycles at a speed of $8\frac{1}{2}$ km/h until he stops to rest at 13 00. He rests for half an hour before continuing to cycle for $2\frac{1}{2}$ hours at 10 km/h to reach Town X.

(a) At what time did he arrive in Town X?

(b) Find the distance between his home and Town X.

(c) Calculate the man's average speed for the journey from his home to Town X.

15. Brushing your teeth with the tap running wastes 44.5 litres of water per brush. How much water was wasted by a boy who has the habit of brushing his teeth twice a day with the tap running for a year with 366 days? How much more money must his parents pay per year for his wasteful habit, if each litre of water costs 0.112 cents?

Percentages

Percentages often have to do with problems involving money in the form of interest rate, percentage profit, rate of discount and so on.

It's a Fact

The Pacific Ocean, the Atlantic Ocean and the Indian Ocean cover 32%, 16% and 14% of the Earth's surface area respectively.

Example 4

(a) In a school library, 28% of the books are classified as fiction and the remainder as non-fiction. Given that there are 1 980 more non-fiction books than fiction books, find the number of books in each category and the total number of books in the library.

(b) The library decides to increase the number of non-fiction books by $7\frac{1}{2}$% at an average cost of \$15.00 per book and the number of fiction books by 5% at \$4.50 per book. Calculate

(i) the total cost of the new books,

(ii) the percentage increase in the total number of books in the library.

Solution

(a) Percentage of fiction books = 28%

Percentage of non-fiction books = 100% – 28% = 72%

∴ there are (72 – 28)% = 44% more non-fiction books than fiction books.

After months of constant practice, Haron manages to improve his time for the 1 500 m race by 10%. What is the percentage change in his speed?

Let x be the total number of books in the library.

$$44\% \text{ of } x = 1\,980$$

$$\frac{44}{100} \times x = 1\,980$$

$$x = \frac{1\,980 \times 100}{44}$$

$$= 4\,500$$

$\therefore$ the total number of books in the library is 4 500.

$\therefore$ number of fiction books $= 28\%$ of 4 500

$$= \frac{28}{100} \times 4\,500$$

$$= 1\,260$$

$\therefore$ number of non-fiction books $= 4\,500 - 1\,260$

$$= 3\,240$$

(b) (i) The number of new non-fiction books $= 7\frac{1}{2}\% \times 3\,240$

$$= \frac{15}{2} \times \frac{1}{100} \times 3\,240$$

$$= 243$$

$\therefore$ the cost of the new non-fiction books $= 243 \times \$15$

$$= \$3\,645$$

The number of new fiction books $= 5\% \times 1\,260$

$$= 5 \times \frac{1}{100} \times 1\,260$$

$$= 63$$

$\therefore$ the cost of the new fiction books $= 63 \times \$4.50$

$$= \$283.50$$

$\therefore$ the total cost of the new books $= \$3\,645 + \$283.50$

$$= \$3\,928.50$$

(ii) The total number of new books $= 243 + 63 = 306$

Percentage increase in the total number of books in the library

$$= \frac{306}{4\,500} \times 100\% = 6.8\%$$

Example 5

Mr Li, Mr Chen and Mrs Deng invest \$800 000, \$900 000 and \$1 100 000 respectively in a business. In a particular year, the gross profit is \$250 000 and the expenses amount to 20% of the gross profit. Mr Li, being the manager, gets 16% of the net profit and the remainder is shared among the three in proportion to their investments. Find the profit each receives.

Solution

Net profit = 80% of the gross profit

$= 80\% \times \$250\,000 = \$200\,000$

As the manager, Mr Li receives 16% of the net profit $= \frac{16}{100} \times \$200\,000$
$= \$32\,000$

The ratio of the investments of Li : Chen : Deng = \$800 000 : \$900 000 : \$1 100 000
= 8 : 9 : 11

$$\text{The profit Mr Li receives from the investment} = \frac{8}{8+9+11} \times (\$200\,000 - \$32\,000)$$
$$= \frac{8}{28} \times \$168\,000 = \$48\,000$$

$\therefore$ total profit Mr Li receives = \$32 000 + \$48 000 = \$80 000

The profit Mr Chen receives $= \frac{9}{28} \times \$168\,000 = \$54\,000$

The profit Mrs Deng receives $= \frac{11}{28} \times \$168\,000 = \$66\,000$

Exercise 1b

1. **(a)** The total population of a region in a certain country is 15 500. A village with a population of 620 is situated in this region. Express the population of the village as a percentage of the total population of the region.
   (b) Another village in the region has a population of 992. If a grant of \$65 000 is to be shared between the two villages in proportion to their population, calculate how much each village receives.

2. The table shows the production costs of a certain article in 1999 and 2000:

Year	*Overheads*	*Labour*	*Materials*
1999	\$40	\$260	\$150
2000	\$46	Increased by 15% over 1999	Increased by 30% over 1999

   (a) Calculate the total cost of producing an article in **(i)** 1999, **(ii)** 2000.
   (b) Express the increase in the total cost as a percentage of 1999's cost.

3. The revenue department of a certain town raises money by collecting from the owner of each house a yearly tax of 0.6% of the value of that house.
   (a) Calculate the tax to be paid by the owner of a house valued at \$98 000.
   (b) Given that the total value of all the houses in the town is \$5 400 000 000, calculate the total amount of money to be collected by the department.
   (c) The sum of money collected is to be spent on four services as follows:

Maintenance	*Education*	*Police*	*Libraries*
14%	62%	18%	6%

   Calculate the amount to be spent on each of the four services.

4. **(a)** A garrison has enough food to last for 24 days. How much longer will the food last if each individual ration can be reduced by 20%?
   (b) By how much will each individual ration have to be reduced if the food is to last 40 days?

5. The rice consumption in a certain city was 80 000 tonnes in 1995. By 2000, the rice consumption had increased by 24%. If the consumption of rice in the city continues to increase at the same rate, i.e., 24% every 5 years, find the consumption of rice in the city in 2005.

6. Three persons, *A*, *B* and *C*, enter into a business together. They contribute $50 000, $60 000 and $100 000 respectively. In a particular year, the gross profit is $224 000 and the expenses amount to 25% of the gross profit. *C*, being the manager, gets 10% of the net profit and the remainder is shared among *A*, *B* and *C* in proportion to their contributions. Find the profit each receives and express *C*'s profit as a percentage of his contribution.

7. An association of insurance agents helps children from low-income homes at a primary school by paying each child's school fees of $10 per month, giving each child $2 in pocket money a day and paying the fees for two enrichment programmes costing between $35 and $70 each a year. The principal of the school estimates that 30% of 680 pupils in her school come from needy families where the family's net income per month is not more than $530 for one- or two-child families or $625 for families with three or more children.

(a) Estimate the maximum amount the programme will cost the association a year, given that school fees are payable for 12 months in a year and each child gets the daily pocket money only for 40 5-day weeks.

(b) The Chan family has a net income of $520 per month. It has two children studying in this particular primary school. Express the amount the family receives from the association as a percentage of the family's annual net income, given that each enrichment programme costs $50 for each child and that the children attend 2 enrichment programmes each.

8. The Singapore Assault Rifle 21 (SAR 21) developed locally will be the standard issue for Singapore soldiers in the 21 century, replacing the M-16. The SAR 21 boasts many advanced features, one of which is a laser aiming device. The overall length of the SAR 21 is 80.5 cm while the M-16 has an overall length of 99 cm. The weight of the SAR 21 is 3.98 kg while the M-16 has a weight of 4.06 kg. Calculate

(a) the percentage difference in the overall length of the SAR 21 as compared to the M-16,

(b) the percentage difference in the weight of the M-16 as compared with the SAR 21.

9. The table below shows the number of Excellent Service Awards (EXSA) given to individuals and companies from 1996 to 1999 in Singapore.

Year	*Individual Award*	*Company Award*
1996	935	32
1997	1764	40
1998	1985	52
1999	2492	63

(a) Draw a bar graph to illustrate the number of individual awards given out in the four-year period.

(b) Draw a line graph to illustrate the number of company awards given out in the four-year period.

(c) Calculate the percentage increase in the number of individual awards given from 1996 to 1997.

(d) Which year shows the greatest percentage increase in the number of company awards as compared to the previous year?

The examples below deal with finance-related problems we encounter in our everyday lives.

Example 6

(a) Peter and Jane earn a combined monthly income of \$3 000. They have to meet the following expenses each month:

food \$640	*electricity, water and gas \$110*
telephone \$40	*hire purchase payments \$180*
housing loan \$360	*insurance \$75*
car maintenance \$165	*others \$350*

(i) Calculate their total monthly expenses.

(ii) Express their monthly savings as a percentage of their income.

(b) Given that their combined monthly income increases by 8% and their monthly expenses on food, electricity, water and gas, and car maintenance increase by 3%, 6% and 9% respectively, calculate

(i) their new monthly savings,

(ii) their percentage increase or decrease in their monthly savings.

When a person borrows money from a bank, he has to pay interest. There are several types of interest that a bank charges. Find out what the following are:
(1) prime rate
(2) yearly declining rate
(3) monthly declining rate
(4) daily interest
(5) flat rate
(6) overdraft rate of 2% above prime

Solution

(a) (i) Total monthly expenses

$= \$(640 + 110 + 40 + 180 + 360 + 75 + 165 + 350)$

$= \$1\ 920$

(ii) Monthly savings $= \$(3\ 000 - 1\ 920) = \$1\ 080$

$\therefore$ percentage of monthly savings $= \frac{1\ 080}{3\ 000} \times 100\% = 36\%$

(b) Increase in their monthly income $= \frac{8}{100} \times \$3\ 000 = \$240$

Increase in their monthly expenses

$= \frac{3}{100} \times \$640 + \frac{6}{100} \times \$110 + \frac{9}{100} \times \$165$

$= (19.20 + 6.60 + 14.85)$

$= \$40.65$

(i) Their new monthly savings $= \$1\ 080 + \$240 - \$40.65$

$= \$1\ 279.35$

(ii) Percentage increase in their monthly savings

$= \frac{1\ 279.35 - 1\ 080}{1\ 080} \times 100\%$

$= 18.46\%$

Example 7

A television set with a cash price of $998 is offered for sale at three different shops as follows:

Shop A : 10% deposit + 36 monthly payments of $29.65
Shop B : 15% deposit + 24 monthly payments of $42.80
Shop C : 20% deposit + 12 monthly payments of $79.80

(a) Calculate the price of the television set offered by each shop.
(b) What is the difference between the highest and lowest price offered?
Express this difference as a percentage of the lowest price offered.

Solution

(a) Shop A: The price = 10% × $998 + 36 × $29.65
= $1 167.20
Shop B: The price = 15% × $998 + 24 × $42.80
= $1 176.90
Shop C: The price = 20% × $998 + 12 × $79.80
= $1 157.20

(b) The difference between the highest and the lowest price offered
= $1 176.90 – $1 157.20
= $19.70

$\therefore$ percentage of the difference $= \frac{19.70}{1\ 157.20} \times 100\%$
$= 1.7\%$

Credit cards are now a common sight in Singapore. Find out what percentage a shop has to pay a credit card company. What is the interest charged for purchases by the credit card company on a cardholder who fails to pay in full the amount due? What interest rate is charged by the credit card company to cardholders who use their credit extension through an Automated Teller Machine (ATM)?

Example 8

Muthu has a savings account which earns him 1.25% per annum simple interest for minimum monthly balances up to $3 000 and 1.5% per annum simple interest for balances over $3 000. The interest is computed half-yearly. Calculate the total interest Muthu receives if the minimum monthly balances in the six months up to June were $2 925, $2 865, $2 760, $3 150, $3 480 and $4 010.

Solution

Total minimum monthly balances up to $3 000 = $(2 925 + 2 865 + 2 760)
= $8 550

Total minimum monthly balances over $3 000 = $(3 150 + 3 480 + 4 010)
= $10 640

$\therefore$ the interest Muthu receives
= $(8 550 × 0.0125 + 10 640 × 0.015) ÷ 12
= $22.21 (correct to the nearest cent)

A man was born in the year 30 BC. How old was he on his birthday in AD 30?

Example 9

A shopkeeper paid \$120 each for 3 handbags. He sold the first handbag making a profit of 25% of the cost price. He sold the second handbag, which was damaged, at a loss of 15% of the cost price.

(a) (i) Calculate the selling price of the first and second handbags.
(ii) Given that he sold the third handbag for \$153, calculate his total profit, expressing it as a percentage of his total cost.

(b) At what price must he sell the third handbag so that his total profit is 17.5% of his total cost?

Solution

(a) (i) Selling price of the first handbag = 125% × \$120

$= \frac{125}{100} \times \$120 = \$150$

Selling price of the second handbag = 85% × \$120

$= \frac{85}{100} \times \$120 = \$102$

(ii) Total amount of money received from selling the 3 handbags
= \$150 + \$102 + \$153 = \$405
His total cost = \$120 × 3 = \$360
His total profit = \$405 − \$360 = \$45

$\therefore$ percentage profit $= \frac{45}{360} \times 100\% = 12.5\%$

(b) If the total profit is 17.5% of his total cost, the total selling price

$= 117.5\% \times \$360 = \frac{235}{2} \times \frac{1}{100} \times \$360 = \$423$

$\therefore$ he must sell the third handbag for \$(423 − 150 − 102)
= \$171

Exercise 1c

1. A credit card company charges 2% interest per month on accounts not paid in previous months. If Norman pays \$50 of his August account of \$990, find the interest charges which will be added to his September account.

2. On 2 May 2000, a man borrowed a sum of money from a bank at 6.5% p.a. simple interest and on 2 September 2000, the interest amounted to \$137.50. Calculate the amount of money he borrowed.

3. A fish merchant bought a container of fish for \$36 000. He sold $\frac{1}{2}$ of the fish at a profit of 20% and $\frac{1}{6}$ of it at a loss of 10%. At what price must he sell the remaining fish in order to make a profit of 15% on the whole?

4. Coffee powder costing \$12.75 per kg is mixed with coffee powder costing \$8.95 per kg in the ratio 5 : 7 by weight. The mixture is sold at \$12 per kg. Find the gain percent.

5. An unmarried man pays annual income tax to the government at the following rate:

Income	*Rate*
On the first $7 500	2%
On the next $12 500	5%
On the next $15 000	8%
On the next $15 000	12%

A bachelor earns $3 185 per month. Find the amount of tax he pays in one year.

6. A man borrowed $4 800 from a finance company and agreed to pay interest at 5.5% per annum on the amount owed at the beginning of each year. By the end of the first year, he had repaid $2 264. By the end of the second year, he had repaid $1 354. How much must he pay the company to clear the debt by the end of the third year?

7. A second-hand car costing $24 000 may be paid for by cash or hire purchase. Hire purchase requires an initial payment of 30% of the cash price plus a monthly payment of $784 for the next 2 years. Calculate the extra cost of the car if it is paid for by hire purchase.

8. The annual income tax to be paid on a man's salary is computed as follows:
the first $7 500 is taxed at 2%,
the next $12 500 is taxed at 5%,
the next $15 000 is taxed at 8%,
and the remainder of his salary is taxed at 12%.
   (a) How much income tax must a man who earns $36 000 per year pay? Express his income tax as a percentage of his salary.
   (b) Given that his salary is increased by 12.5%, calculate the amount of income tax that he will have to pay.

9. A shopkeeper marks up his price to make a 30% profit but allows 5% discount on cash terms. If he sells an article for $247 on cash terms, find his actual profit and express it as a percentage of the cost price of the article.

10. In a certain year, a tourist changed US$2 200 into Singapore dollars at the rate of US$1 = S$1.78. He spent S$2 285 and changed the remaining money back to American dollars at the rate of US$1 = S$1.75. How many American dollars did he get?

11. Mr Lee bought a second hand car for $25 480 and made a down payment of $10 000. He arranged to pay the balance at the end of two years with compound interest at 4.75%. How much did he pay at the stipulated time?

12. John has $5 600 to invest. He could put his money in fixed deposit earning 3% simple interest annually or invest in a unit trust with a return equivalent to 4.5% compound interest per annum. At the end of two years, he would get $180 more by investing in the unit trust. Is this correct?

13. In 1999, 460 000 Singaporeans used their Central Provident Fund (CPF) savings to make investments in shares and other financial instruments like gold investment, endowment insurance, fixed deposits, etc. A total of 119 000 investors either broke even or made more money than that they would have made if they left their money in CPF to earn interest. Of the 119 000 investors, 77 000 made less than $5 000 and 7 800 of them just broke even.
   (a) Express 119 000 as a percentage of 460 000.
   (b) Assuming that each of the 77 000 investors made an average of $2 500, and the remaining investors who made money had made an average of $7 500 each, calculate the total amount of money made by the 119 000 investors.

14. The Singapore airforce took delivery of the first batch of F-16 fighter planes that it bought in November 1999. The US$350 million package includes 23 new version F-16C and F-16D fighter planes and other items like training and spare parts. Calculate the average cost of each F-16 fighter plane in Singapore dollars taking US$1 to be equivalent to S$1.692.

Powers of 10 and the Standard Form

Powers of 10

We learnt in Book 1 that $10^3 = 10 \times 10 \times 10$ and $10^4 = 10 \times 10 \times 10 \times 10$.

We have,
$$\begin{aligned} 10^3 \times 10^4 &= (10 \times 10 \times 10) \times (10 \times 10 \times 10 \times 10) \\ &= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \\ &= 10^7. \end{aligned}$$

Similarly,
$$\begin{aligned} 10^2 \times 10^6 &= (10 \times 10) \times (10 \times 10 \times 10 \times 10 \times 10 \times 10) \\ &= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \\ &= 10^8. \end{aligned}$$

Consider any positive integers m and n,

we have,
$$\begin{aligned} 10^m \times 10^n &= \underbrace{(10 \times 10 \times \ldots \times 10)}_{m \text{ factors}} \times \underbrace{(10 \times 10 \times 10 \times \ldots \times 10)}_{n \text{ factors}} \\ &= \underbrace{10 \times 10 \times 10 \times 10 \times \ldots \times 10 \times 10 \times 10}_{m+n \text{ factors}} \\ &= 10^{m+n}. \end{aligned}$$

We conclude that

$$\mathbf{10^m \times 10^n = 10^{m+n}.}$$

We have,
$$\begin{aligned} 10^8 \div 10^3 &= \frac{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10} \\ &= 10 \times 10 \times 10 \times 10 \times 10 \\ &= 10^5. \end{aligned}$$

Similarly,
$$\begin{aligned} 10^9 \div 10^2 &= \frac{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10} \\ &= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \\ &= 10^7. \end{aligned}$$

Consider any positive integers m and n where $m > n$,

we have,
$$\begin{aligned} 10^m \div 10^n &= \frac{10 \times 10 \times \ldots \times 10 \quad (m \text{ factors})}{10 \times \ldots \times 10 \quad (n \text{ factors})} \\ &= 10^{m-n}. \end{aligned}$$

We conclude that

$$\mathbf{10^m \div 10^n = 10^{m-n}.}$$

Consider the following table of values for numbers in powers of 10.

10 000	=	10^4
1 000	=	10^3
100	=	10^2
10	=	10^1
1	=	$10^?$
$0.1 = \frac{1}{10} = \frac{1}{10^1}$	=	$10^?$
$0.01 = \frac{1}{100} = \frac{1}{10^2}$	=	$10^?$
$0.001 = \frac{1}{1\,000} = \frac{1}{10^3}$	=	$10^?$
$0.0001 = \frac{1}{10\,000} = \frac{1}{10^4}$	=	$10^?$
$0.00001 = \frac{1}{100\,000} = \frac{1}{10^5}$	=	$10^?$

For the pattern of powers of 10 to continue, we notice that $1 = 10^0$, $0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$, $0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$, $0.001 = \frac{1}{1\,000} = \frac{1}{10^3} = 10^{-3}$, $0.0001 = \frac{1}{10\,000} = \frac{1}{10^4} = 10^{-4}$ and so on. Thus $10^{-x} = \frac{1}{10^x}$.

Thus the laws of powers of 10: $10^m \div 10^n = 10^{m-n}$ also holds true for $m < n$.

In particular, $10^3 \div 10^3 = 10^{3-3} = 10^0$ and

$$10^3 \div 10^3 = \frac{10 \times 10 \times 10}{10 \times 10 \times 10} = 1.$$

Hence we conclude that $10^0 = 1$ and that $10^m \div 10^n = 10^{m-n}$ also holds true for $m \leqslant n$.

Example 10

Express the following in the form 10^n, where n is an integer.

(a) $10^{10} \div 10^7$ *(b)* $10^6 \div 10^{13}$ *(c)* $10^{-3} \div 10^7$

(d) $\frac{10^9}{10^{-7}}$ *(e)* $\frac{10^{-15}}{10^{-4}}$ *(f)* $\frac{10^{-6}}{10^{-17}}$

Solution

(a) $10^{10} \div 10^7 = 10^{10-7} = 10^3$

(b) $10^6 \div 10^{13} = 10^{6-13} = 10^{-7}$

(c) $10^{-3} \div 10^7 = 10^{-3-7} = 10^{-10}$

(d) $\frac{10^9}{10^{-7}} = 10^{9-(-7)} = 10^{9+7} = 10^{16}$

(e) $\frac{10^{-15}}{10^{-4}} = 10^{-15-(-4)} = 10^{-15+4} = 10^{-11}$

(f) $\frac{10^{-6}}{10^{-17}} = 10^{-6-(-17)} = 10^{-6+17} = 10^{11}$

The Standard Form

Many measurements in modern scientific fields involve very large and very small numbers. *For example*, the speed of light is approximately 300 000 000 m/s and the wavelength of violet light is approximately 0.000 038 cm.

These can conveniently be written as follows:

Speed of light: $3.0 \times 100\ 000\ 000$ m/s or 3.0×10^8 m/s (correct to two significant figures).

Wavelength of violet light: $3.8 \times 0.000\ 01$ cm or 3.8×10^{-5} cm (correct to two significant figures).

The number of significant figures for a particular number becomes definite when it is expressed in this form. Such a number is said to be expressed in the **standard form** or in **scientific notation**. It is always written as

$$A \times 10^n, \text{ where } 1 \leqslant A < 10 \text{ and } n \text{ is an integer.}$$

It's Fact

In astronomy, the distance between stars is measured in light years. This is the distance travelled by light in a year. One light year $= 3 \times 10^5 \times 60 \times 60 \times 24 \times 365$ km; this is approximately 9 460 800 000 000 km. How long would it take for light to travel from the sun to the Earth if their distance apart is 1.5×10^8 km?

In-Class Activity

Consider each of the following and determine whether it would be appropriate or convenient to express them in the standard form.

(a) the monthly salary of a teacher
(b) the number of grains of sand on a beach
(c) the number of parents attending a school concert
(d) the mass of a hydrogen atom
(e) the number of red blood cells in 1 cm³ of blood plasma
(f) the number of bullets used in the Second World War
(g) the mass of dissolved gold in 2 000 m³ of sea water
(h) the number of oxygen molecules in your classroom
(i) the number of salt particles in a 50-kg bag of salt

It's Fact

On the average, a human heart beats 75 times a minute. 4 500 times an hour, 108 000 times a day, 39 420 000 times a year and 3 153 600 000 times for a man who lives up to 80 years old.

Example 11

Evaluate each of the following and express your answers in the standard form.

(a) $2.43 \times 10^3 + 3.24 \times 10^4$
(b) $4.23 \times 10^7 - 8.6 \times 10^6$
(c) $(2.8 \times 10^5) \times (7.6 \times 10^{-2})$
(d) $(8.58 \times 10^7) \div (3.25 \times 10^4)$

Library Corner

Find out the mass and volume of:
(a) the Earth,
(b) the Moon and
(c) the Sun,
giving your answers in standard form.

Solution

Before you do the actual computation, it is advisable to do a mental approximation to check the accuracy of your answer.

(a) **Mentally:** $2.43 \times 10^3 \approx 2 \times 10^3$ and $3.24 \times 10^4 = 3.24 \times 10^1 \times 10^3 \approx 32 \times 10^3$

$\therefore \quad 2.43 \times 10^3 + 3.24 \times 10^4 \approx 2 \times 10^3 + 32 \times 10^3 = 34 \times 10^3 = 3.4 \times 10^4$

Actual Computation: $2.43 \times 10^3 + 3.24 \times 10^4$

$= 2.43 \times 10^3 + 3.24 \times 10^1 \times 10^3$
$= 2.43 \times 10^3 + 32.4 \times 10^3$
$= (2.43 + 32.4) \times 10^3$
$= 34.83 \times 10^3$
$= 3.483 \times 10^4$

It's a Fact

Do you know that an electron weighs approximately 9.11×10^{-28} g; a hydrogen atom is approximately 2×10^{-9} cm in diameter and the star, Alpha Centauri, is approximately 4.07×10^{13} km away from our Earth?

(b) **Mentally:** $4.23 \times 10^7 \approx 42 \times 10^6$ and $8.6 \times 10^6 \approx 9 \times 10^6$

$\therefore \quad 4.23 \times 10^7 - 8.6 \times 10^6 \approx 42 \times 10^6 - 9 \times 10^6 = 33 \times 10^6 = 3.3 \times 10^7$

Actual Computation: $4.23 \times 10^7 - 8.6 \times 10^6$

$= 4.23 \times 10^1 \times 10^6 - 8.6 \times 10^6$
$= 42.3 \times 10^6 - 8.6 \times 10^6$
$= (42.3 - 8.6) \times 10^6$
$= 33.7 \times 10^6$
$= 3.37 \times 10^7$

(c) **Mentally:** $2.8 \times 10^5 \approx 3 \times 10^5$ and $7.6 \times 10^{-2} \approx 8 \times 10^{-2}$

$\therefore \quad 2.8 \times 10^5 \times 7.6 \times 10^{-2} \approx 3 \times 10^5 \times 8 \times 10^{-2} = 24 \times 10^{5-2} = 24 \times 10^3 = 2.4 \times 10^4$

Actual Computation: $(2.8 \times 10^5) \times (7.6 \times 10^{-2})$

$= 2.8 \times 7.6 \times 10^5 \times 10^{-2}$
$= 21.28 \times 10^{5-2}$
$= 21.28 \times 10^3$
$= 2.128 \times 10^4$

(d) **Mentally:** $8.58 \times 10^7 \approx 9 \times 10^7$ and $3.25 \times 10^4 \approx 3 \times 10^4$

$\therefore \quad (8.58 \times 10^7) \div (3.25 \times 10^4) \approx (9 \div 3) \times (10^7 \div 10^4) = 3 \times 10^3$

Actual Computation: $(8.58 \times 10^7) \div (3.25 \times 10^4)$

$$= \frac{8.58 \times 10^7}{3.25 \times 10^4}$$
$$= \frac{8.58}{3.25} \times 10^7 \div 10^4$$
$= 2.64 \times 10^{7-4}$
$= 2.64 \times 10^3$

Example 12

The sides of a rectangle are of lengths 5.6×10^{-2} m and 2.7×10^{-2} m. Calculate (a) its perimeter, (b) its area, expressing your answers in the standard form.

Solution

(a) Perimeter of the rectangle $= 2(5.6 \times 10^{-2} + 2.7 \times 10^{-2})$ m
$= 2(5.6 + 2.7) \times 10^{-2}$ m
$= 2(8.3 \times 10^{-2})$ m
$= 1.66 \times 10^{-1}$ m

(b) Area of the rectangle $= (5.6 \times 10^{-2})$ m $\times (2.7 \times 10^{-2})$ m
$= 15.12 \times 10^{-4}$ m^2
$= 1.512 \times 10^{-3}$ m^2

Exercise 1d

1. Express the following in the form 10^n, where n is an integer.

(a) $10^{12} \times 10^3$ **(b)** $10^{19} \times 10^{-7}$
(c) $10^{-4} \times 10^{-5}$ **(d)** $10^{21} \div 10^3$
(e) $10^5 \div 10^{13}$ **(f)** $10^{-6} \div 10^7$
(g) $10^{14} \div 10^{14}$ **(h)** $10^{-20} \div 10^{-5}$
(i) $10^{16} \div 10^{-19}$ **(j)** $\dfrac{10^{12}}{10^{-6} \div 10^{-6}}$
(k) $\dfrac{10^9 \times 10^{-7}}{10^{-5}}$ **(l)** $\dfrac{10^{-4}}{10^{-7} \times 10^{-3}}$
(m) $\dfrac{10^{-6} \times 10^{-7}}{10^{-14} \times 10^2}$ **(n)** $\dfrac{10^{-3} \times 10^{15}}{10^{-7} \div 10^{-28}}$
(o) $\dfrac{10^{12} \div 10^{-9}}{10^{-7} \div 10^{-16}}$

2. Express the following numbers in the standard form.

(a) 237 **(b)** 5 600
(c) 912 400 **(d)** 612 006
(e) 28 000 000 **(f)** 0.043 5
(g) 0.000 77 **(h)** 0.008 306
(i) 0.296 **(j)** 74.8
(k) 159.4 **(l)** 70 600

3. Express the following in ordinary notation.

(a) 6.37×10^3 **(b)** 4.213×10^{-3}
(c) 8.1×10^{-5} **(d)** 1.729×10^4
(e) 3.82×10^{-1} **(f)** 9.8×10^6
(g) 5.09×10^{-3} **(h)** 2.47×10^2
(i) $3(4.7 \times 10^{-2})$ **(j)** $0.7(1.2 \times 10^3)$
(k) $\dfrac{3.6 \times 10^4}{10^3}$ **(l)** $\dfrac{6.55 \times 10^{-2}}{10^{-3}}$

4. Evaluate and then express $2(11 \times 10^3)^2$ in the form $A \times 10^n$ where $1 \leqslant A < 10$ and n is an integer.

5. Evaluate and then express $0.6(8.5 \times 10^4)$ in the standard form.

6. Evaluate and then express $21(3.0 \times 10^2) \div (7.0 \times 10^3)$ in the standard form.

7. Multiply 3.2×10^6 by 4×10^{-3}, giving the answer in the standard form.

8. Evaluate the following and give your answer in the standard form.

(a) $2.8 \times 10^4 + 3.2 \times 10^5$
(b) $6.3 \times 10^3 + 5.37 \times 10^4$
(c) $9.7 \times 10^2 + 0.3 \times 10^3$
(d) $6.527 \times 10^6 - 4.05 \times 10^6$
(e) $8.1 \times 10^3 - 2.4 \times 10^2$
(f) $2.3 \times 10^4 \times 1.2 \times 10^2$
(g) $3.7 \times 10^5 \times 1.5 \times 10^{-3}$
(h) $(8.4 \times 10^5) \div (2.1 \times 10^4)$
(i) $(6.4 \times 10^6) \div (1.6 \times 10^3)$
(j) $(7.2 \times 10^5) \div (2.4 \times 10^2)$
(k) $2.5 \times 10^6 \times 4.2 \times 10^{-4}$
(l) $3.4 \times 10^{-4} \times 2.2 \times 10^5$

9. Evaluate and then write $(4 \times 10^{-4}) + (8 \times 10^{-3})$ as a single number expressed in the standard form.

10. Given that $x = 2 \times 10^{-3}$ and $y = 7 \times 10^{-4}$, evaluate $x + 8y$, and express your answer in the standard form.

11. Evaluate $10^{-6} - 2.5 \times 10^{-7}$ and express your answer in the standard form.

12. Given that $x = 3.2 \times 10^6$ and $y = 5.0 \times 10^7$, evaluate and then express in the same standard form

    (a) xy, **(b)** $\frac{x}{y}$.

13. In the formula $R = \frac{M}{EI}$, substitute $M = 6 \times 10^4$, $E = 4.5 \times 10^8$, $I = 4 \times 10^2$ and evaluate R, giving your answer in the standard form.

14. The radius of a circular micro–organism is 2.8×10^{-7} cm. Calculate the circumference and area of the micro–organism, giving your answer in the standard form. $\left(\text{Take } \pi = \frac{22}{7}.\right)$

15. Evaluate where necessary and express the following in the standard form.

    (a) 785
    (b) 0.004 5
    (c) $(5.4 \times 10^3) \times (3.0 \times 10^2)$
    (d) $(44.1 \times 10^4) \div (7.0 \times 10^3)$

Use of Calculator

The examples below show how indices and numbers in the standard form are expressed and evaluated using a scientific calculator.

1. To find the value of a number raised to a power, we use the [x^y] key.

   For example,

   (a) to find 2^7, press 2 [x^y] 7 [=] to get the answer 128.

   (b) to find 5.4^3, press 5.4 [x^y] 3 [=] to get the answer 157.464.

   (c) to find 4^{-2}, press 4 [x^y] [(–)] 2 to get the answer 0.062 5.

2. To express the standard form of a number on a calculator, we use the [EXP] key.

   For example,

   (a) to express 2.56×10^{34}, press 2.56 [EXP] 34 [=] to get the result 2.56 E 34.

   (b) to express 35×10^{14}, press 35 [EXP] 14 [=] to get the result 3.5 E 15.

   (c) to express 46×10^{-5}, press 46 [EXP] [(–1)] 5 [=] to get the result 4.6 E –04.

3. **(a)** To evaluate $(5.74 \times 10^{15}) \div (3.8 \times 10^3)$, press 5.74 [EXP] 15 ÷ 3.8 [EXP] 3 [=] to get 1.510 526 316 E 12, i.e., $1.510\,5 \times 10^{12}$ (correct to 5 sig. figs.).

   (b) To evaluate $87 \div (43 \times 10^{12})$, press 87 ÷ 43 [EXP] 12 [=] to get 2.023 255 814 E –12 i.e., 2.023×10^{-12} (correct to 4 sig. figs.).

   (c) To evaluate $5.6 \times 10^{11} \div 7.2^5$, press 5.6 [EXP] 11 ÷ 7.2 [x^y] 5 [=] to get 28 941 800.45, i.e., 2.89×10^7 (correct to 3 sig. figs.).

Exercise 1e

Use your calculator to evaluate each of the following, giving your answer in the standard form correct to 4 significant figures.

1. $87.5^3 + 6.85^4$
2. $14.5^4 - 8.94^5$
3. $6.8^{12} \times 3.8^{-4}$
4. $15.76^4 \div 5.79^2$
5. $59.6^4 \div 14.3^5$
6. $3.2^{-5} + 1.16^{-1}$
7. $7.6^3 \times 2.7^{-2}$
8. $16.5^7 - 8.4^{10}$
9. $5\pi \times 7.9^2$
10. $\dfrac{4.9^3 + 7.5^2}{5.4^4 - 4.5^3}$
11. $\dfrac{7.89^5 + 9.4^7}{6.99^7 - 16.7^4}$
12. $\dfrac{3.6^7 \times 6.4^9}{1.5^{25} \div 6.7^{14}}$
13. $4.9 \times 10^9 + 5.6 \times 10^8$
14. $56.7 \times 10^6 - 3.7 \times 10^7$
15. $(7.8 \times 10^{-4}) \div (6.7 \times 10^{-6})$
16. $6.3 \times 10^4 \times 3.7 \times 10^7$
17. $(48.5 \times 10^2) \div (1.67 \times 10^3)$
18. $5.9 \times 10^{-4} \times 7.86 \times 10^7$
19. $8.76 \times 10^9 + 65 \times 10^5$
20. $(8.6 \times 10^4) \div (7 \times 10^{-4})$
21. $(4.9 \times 10^{15}) \div (2.3 \times 10^{21})$
22. $7.5 \times 10^9 \div 5.3^4$
23. $7.99 \times 10^4 \times 0.78^9$
24. $5.34 \times 10^{10} \div 7.5^{-4}$
25. $5.73 \times 10^{-5} \times 3.89^6$
26. $3.4 \times 10^7 + 12.3^4$
27. $10^7 - 5.76^7$

Problem Solving Strategies

The following examples deal with strategies for solving arithmetic problems.

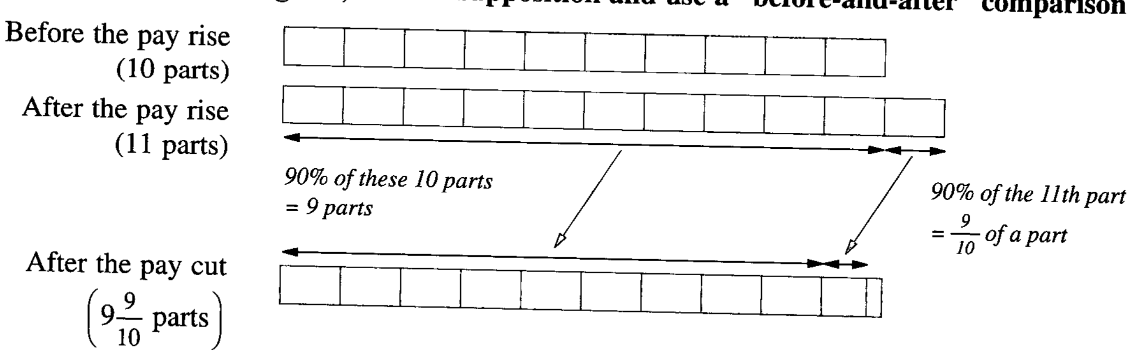
Example 13

In 1998, Mr Tan received a 10% pay rise. The following year his pay was cut by the same percentage when his company turned in a poor performance. After the pay cut, did Mr Tan take home more or less than before the pay increase?

(The answer can be derived using various strategies as shown below.)

Solution

Strategy 1: **Use a diagram, make a supposition and use a "before-and-after" comparison**



Suppose that Mr Tan's original pay was made up of 10 parts. After the 10% increase, his pay became 11 parts. With the 10% pay cut, 10 of these 11 parts were reduced to 9 parts and the 11th part decreased to $\frac{9}{10}$ of a part. After the pay cut, Mr Tan's pay was $9\frac{9}{10}$ of his original pay before the increase.

Thus, Mr Tan's final take-home pay was less than before the pay rise.

Strategy 2: Make a supposition and use a "before-and-after" comparison

Suppose Mr Tan's pay was $100 before the pay rise.

In 1998, his new pay = 110% of $100 = $110.

The following year, his pay = 90% of $110 = $99.

Thus, we can conclude that he received $1 less for every $100 he earned before 1998 and hence his final pay was less than before the pay rise.

Strategy 3: Use an equation

Let Mr Tan's pay before the pay rise in 1998 be $*x*.

In 1998, his pay = $\$\frac{110}{100} \times x = \$\,1.1\,x.$

In 1999, his pay = $\$\frac{90}{100} \times 1.1\,x = \$0.99\,x.$

Thus, he took home less in 1999 than before the pay rise in 1998.

Strategy 4: Use ratio

The ratio of Mr Tan's pay *before* and *after* pay rise = 100 : 110.

The ratio of his pay in 1998 and 1999 = 100 : 90 = 110 : 99.

∴ his pay *before* the pay rise : his pay *after* the pay cut = 100 : 99.

He received less pay in 1999 than before the pay rise in 1998.

Example 14

Mr Chen went to a computer shop to buy a personal computer. He was offered a 20% discount. A 3% GST should be levied on the discounted price. However, the new shop assistant made a mistake by levying the 3% GST on the price before discount. As a result of the shop assistant's mistake, find out

(a) if Mr Chen paid more than he should,
(b) by how much more or less the shop allocated for GST.

Solution

Strategy 1: Make a supposition

Suppose the marked price of the computer is $1 000.

Payment I (wrong option):

The actual price of the computer (inclusive of the 3% GST) = 103% of $1 000

$$= \frac{103}{100} \times \$1\,000$$
$$= \$1\,030$$

The computer costs Mr Chen $\frac{80}{100} \times \$1\,030 = \$824$ for every $1 000 of the marked price.

The 3% GST allocated = $30 for every $1 000 of the marked price.

Payment II (right option):

Discounted price = $\frac{80}{100} \times \$1\ 000 = \$800$

The 3% GST = 3% of \$800 = $\frac{3}{100} \times \$800 = \$24$

Mr Chen pays \$800 + \$24 = \$824 for the computer for every \$1 000 of the marked price.

The 3% GST = \$24 for every \$1 000 of the marked price.

(a) Thus, Mr Chen pays the same amount (\$824) under either mode of calculation.
(b) The shop allocated \$30 – \$24 = \$6 more for GST for every \$1 000 of the marked price.

Strategy 2: Use an equation

Let the marked price of the computer be $\$x$.

Payment I (wrong option):

The 3% GST = 3% of $\$x = \$0.03x$

The price of the computer (inclusive of GST) = $\$x + \$0.03x = \$1.03x$

The computer costs Mr Chen 80% of $\$1.03x = 0.80 \times \$1.03x = \$0.824x$

Payment II (right option):

The 3% GST = 3% of (80% of $\$x$) = $0.03 \times (\$0.80x) = \$0.024x$

The computer costs Mr Chen $\$0.80x + \$0.024x = 0.824x$

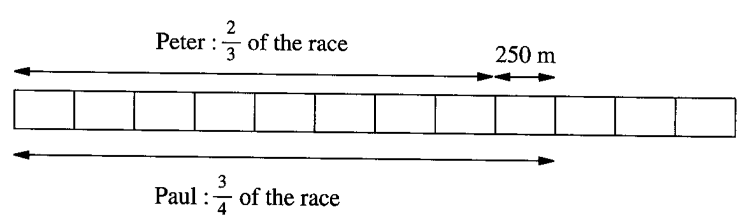
(a) Thus, Mr Chen pays the same amount ($\$0.824x$) under either mode of calculation.
(b) The shop allocated $\$0.03x - \$0.024x = \$0.006x$ more for GST.

Example 15

Peter and Paul enter a cross country race. When Peter completes $\frac{2}{3}$ of the race, Paul completes $\frac{3}{4}$ of the race and they are 250 m apart. Given that Peter is racing at a speed of 200 m per minute, what is the racing speed of Paul in m/min?

Solution

Strategy 1: Use a diagram and make a supposition



Suppose the race is made up of 12 equal parts.

From the diagram, when Paul completes $\frac{3}{4}$ of the race, he is one part or 250 m ahead of Peter.

Thus, the time taken by Peter to complete $\frac{2}{3}$ of the race = $\frac{8 \times 250}{200}$ = 10 min.

Paul covers 9×250 m in 10 min.

Thus, his speed is $\frac{9 \times 250}{10}$ = 225 m/min.

Strategy 2: **Use an equation**

Let Paul's speed be x m/min.

Since Peter and Paul take the same time to run their respective distances, their speeds and the individual distances covered are proportional. Thus, we have

$$x : 200 = \frac{3}{4} : \frac{2}{3}$$

i.e., $x = \frac{3}{4} \times \frac{3}{2} \times 200 = 225$

$\therefore$ Paul's speed in the race is 225 m/min.

Example 16

Susan paid $4.32 for a number of stickers. If the price of a sticker were cheaper by 3 cents, she could get 12 more stickers for the same amount of money. How much did she pay for a sticker and how many stickers did she buy?

Solution

Strategy: **Make a systematic list**

Prime factorising 432, we have $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$. Thus, the factors of 432 are 1, 2, 3, 4, 6, …, 18 ….

List them as shown in the table below.

Price of a sticker (in cents)	1	2	3	4	6	8	9	12	16	18
Possible no. of stickers bought	432	216	144	108	72	54	48	36	27	24

From the table, 9 and 12 from the top row differ by 3 (cents) and the respective numbers of stickers 48 and 36 from the second row differ by 12.

Thus, Susan paid 12 cents for each sticker and bought 36 stickers altogether.

Exercise 1f

1. Mr Lee bought two identical television sets. He sold one at a profit of 25% and the other one at a loss of 25%. Upon the sale of both television sets, did he gain, make a loss or break even?
2. John's pay was cut by 15% in a certain year. The following year he received a pay increase of 15%. Was his final pay more or less than before the pay cut?
3. The length of a rectangle is increased by 20% while its breadth is decreased by 20%. Is its final area greater than or lesser than its original area?
4. There are two piles of books with equal number of books in each pile. If 28 books are removed from one pile and 76 books are added to the other, the larger pile will have three times more books than the smaller pile. How many books are there in each pile initially?
5. Mr Sim pays $337.50 for a number of files. He can buy 10 more files with the same amount of money if each file costs 20 cents less. How much does he pay for a file and how many files does he buy?
6. Suppose Peter has $24 000 to invest. He places part of it in Bank A at an interest rate of 6% per year and the remaining part in Bank B at 5.5% per year. If he earns $1 412.50 in one year, how much did he deposit in each bank?
7. Jasmine has accepted a new job with an interesting pay scale. She will earn $10 for the first month, $20 the second month, $40 the third month, and so on. Each month her salary will be twice that of the previous month. Develop a formula that will give Jasmine's total income through n months. How much will she have received after 2 years, giving your answer in standard form correct to two significant figures?

Summary

A number written in the standard form is represented by $A \times 10^n$ where $1 \leqslant A \leqslant 10$ and n is an integer.

Review Questions 1

*1. Mr Huang sells a plot of land at a profit of 16%. If he had sold it at a profit of 20%, he would have received $18 000 more. Find the actual selling price of the land.

*2. A village lost 14% of its goats in a flood and 6% of the remainder died from diseases. Given that the number of goats left is 8 084, find the original number of goats in the village.

*3. An insurance company charges a basic premium of $20 and an additional 30 cents for every $1 000 of cover. If a man pays $95 premium, how much cover does he have?

*4. A man transferred S$420 000 to the United States for investment at 8% per annum simple interest, at a time when US$1 = S$1.84. After $2\frac{1}{2}$ years, he withdraws the interest portion back to Singapore. How much, in S$(correct to the nearest cent), will he receive if the exchange rate is US$1 = S$1.75?

*5. The value of a new car of a particular make, A, decreases by $22\frac{1}{2}$% during the first year. In the second year, its value decreases by $12\frac{1}{2}$% of its value at the beginning of the year. The value of a new car of another make, B, decreases by 25% during the first year. In the second year, its value decreases by 15% of its value at the beginning of the year. A new car of make A costs \$48 000 and a new car of make B costs \$42 000.

(a) By how much is a new car of make B cheaper than a new car of make A? Express your answer as a percentage of the price of a new car of make B.

(b) What is the difference between the values of the two cars at the end of the first year? Express this difference as a percentage of the value of the car of make A at the end of the first year.

(c) Calculate the difference between the values of the two cars at the end of the second year.

✶6. Three men, X, Y and Z, invest \$40 000, \$70 000 and \$30 000 respectively in a business. At the end of each year, $22\frac{1}{2}\%$ of the profit made is placed on reserve and the remainder is divided among the partners in proportion to their investments.

(a) Given that in a certain year, the profit is \$12 880, calculate the amount placed on reserve and the amount received by each of the partners.

(b) In another year, Y receives \$9 331. Calculate how much X and Z received and the amount placed on reserve.

7. Evaluate the following, giving your answers in the standard form.

(a) $(7.54 \times 10^3) \times (3.2 \times 10^5)$
(b) $(16.4 \times 10^{-3}) \div (0.82 \times 10^3)$
(c) $(3.8 \times 10^{-3}) \times (4.0 \times 10^5)$
(d) $(16.4 \times 10^2) \div (0.4 \times 10^{-2})$
(e) $(5.3 \times 10^{-2}) \times (3.1 \times 10^3)$
(f) $(5.4 \times 10^3) \div (0.9 \times 10^{-4})$
(g) $(3.7 \times 10^2) \times (4.0 \times 10^{-5})$
(h) $(37.1 \times 10^8) \div (7.0 \times 10^4)$
(i) $(3.3 \times 10^{-4}) \times (2.5 \times 10^7)$
(j) $(2.4 \times 10^{-5}) \div (0.8 \times 10^{-7})$
(k) $(3.4 \times 10^{-5}) \times (4.0 \times 10^3)$
(l) $(9.9 \times 10^3) \div (2.2 \times 10^{-1})$
(m) $(9.3 \times 10^{-2}) \times (1.1 \times 10^{-2})$
(n) $(6.8 \times 10^{-4}) \div (2.5 \times 10^{-2})$

8. Evaluate the following, giving your answers in the standard form.

(a) $1.3 \times 10^4 + 7.8 \times 10^3$
(b) $3.8 \times 10^8 - 7.9 \times 10^7$
(c) $10^6 + 4.1 \times 10^7$
(d) $10^{-6} - 7 \times 10^{-7}$
(e) $5.73 \times 10^7 + 6 \times 10^5$
(f) $8.65 \times 10^{-3} - 4 \times 10^{-4}$
(g) $3.56 \times 10^2 + 3.1 \times 10^3$
(h) $2.7 \times 10^{-2} - 9 \times 10^{-3}$
(i) $6.7 \times 10^{-4} + 8 \times 10^{-5}$
(j) $3.8 \times 10^{-4} - 6.8 \times 10^{-5}$
(k) $5.76 \times 10^{-2} + 6.7 \times 10^{-3}$
(l) $7.77 \times 10^{-5} - 9.9 \times 10^{-6}$
(m) $10^{-4} + 5 \times 10^{-5}$
(n) $10^7 - 9.8 \times 10^5$

9. Given that $a = 8 \times 10^5$ and $b = 4 \times 10^4$, find each of the following, expressing your answers in the standard form.

(a) $a + b$ **(b)** $a - b$ **(c)** ab
(d) $\frac{a}{b}$ **(e)** $\frac{b}{a}$ **(f)** $3a - 7b$
(g) $\frac{5b}{2a}$ **(h)** $\frac{7a}{2b}$

✶10. Three companies, Alpha, Better and Compact, offer cars for hire. Their charges, based on the number of days for which a car is hired and the number of kilometres for which the car is driven, are shown in the following table:

Company	*Cost (per day)*	*Cost (per kilometre)*
Alpha	\$66	Nil
Better	Nil	45¢
Compact	\$30	25¢

(a) Jean wishes to hire a car for 2 days to drive 400 km.

(i) Show that the cost of hiring a car from Compact is \$160.

(ii) Find the difference between the largest and smallest charges she might have to pay.

(b) Linda hires a car for 3 days. She finds that Alpha and Better make equal charges. How far does she intend to drive?

(c) Maureen wishes to hire a car for 6 days and finds that Better and Compact make equal charges. How far does she intend to drive?

(d) Norman hired a car for 4 days from Alpha in 1993. He calculates that the cost now is 20% more than it was in 1993. What was the cost of hiring a car for 4 days from Alpha in 1993? **(C)**

11. The Port Authority of Singapore (PSA) Corporation handled 15.9 million containers in 1999, making it the busiest port in the world.

(a) If each container has a length of 6 metres, what would be the total length, in metres, of all the containers when they are placed end-to-end?

(b) Given that the 15.9 million containers handled in 1999 is an improvement of 5.3% over the 1998 figure, how many containers did the PSA handle in 1998?

(c) If PSA expects the container traffic to grow at an annual rate of 6.5%, calculate the expected number of containers it will handle in the year 2000 and 2001.

(Give your answer to each of the above parts in standard form correct to 4 significant figures.)

12. Singaporeans consume about 1.02×10^5 tonnes of vegetables in 1999. The bulk of the vegetables are imported from neighbouring countries. The government is encouraging local farms in Singapore to increase their production from the present level of approximately 8 000 tonnes a year to 22 800 tonnes a year in the year 2004.

Express **(a)** 8 000 tonnes as a percentage of 1.02×10^5 tonnes,

(b) 22 800 tonnes as a percentage of 1.02×10^5 tonnes.

Problem Solving

1. With a $100 purchase, a housewife is offered 4 successive discounts of 25%, 20%, 10% and 5% in any order that she wishes. Which order of discounts will benefit her the most?

2. A man makes annual deposits of $1 000 in an account which pays an interest of 3% per annum. How much money will be in the account immediately after the 5th deposit?

3. John starts a business with $168 000 as capital. Six months later, Andrew joins John with a capital of $144 000. One month after that, Edward joins them with a capital of $324 000. At the end of the first year, the profit is $40 000. How much do John, Andrew and Edward get if their shares are in proportion to their investments?

4. Five brothers bought a car for $42 000. The eldest brother paid one third of the sum of the amounts paid by the other brothers. The second eldest brother paid one quarter of the sum of the amounts paid by the other brothers. The second youngest brother paid one fifth of the sum of the amounts paid by the other brothers. The youngest brother paid one sixth of the sum of the amounts paid by the other brothers. How much did the fifth brother pay?

CHAPTER 2

Congruence and Similarity

In this chapter, you will learn how to

- △ identify congruent and similar figures;
- △ apply some of the properties of congruent and similar figures.

Preliminary Problem

Do you notice similar and congruent triangles in the row of houses shown in the picture?

Congruent Figures

Look at the sets of congruent figures shown in Fig. 2.1 carefully. Do you notice that the different orientations of the figures will not change the fact that two figures are **congruent**?

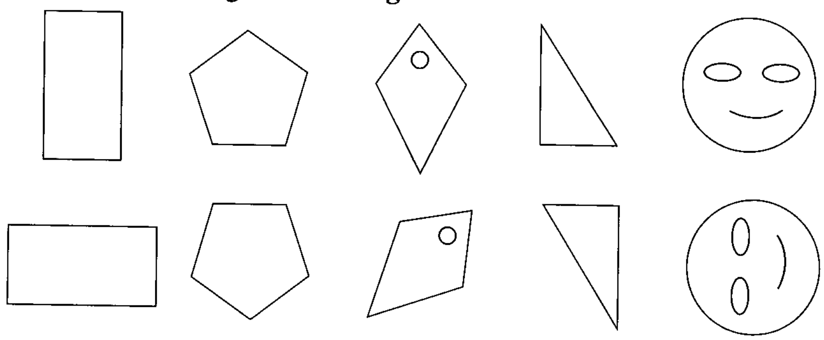


Fig. 2.1

The two quadrilaterals shown in Fig. 2.2 have equal areas but not the same shape. They are thus not congruent.

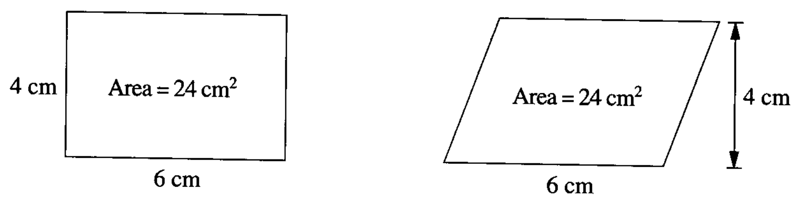


Fig. 2.2

The two regular hexagons shown in Fig. 2.3 have the same shape. However, they do not have the same size. They are thus not congruent. However, the two hexagons are **similar**.

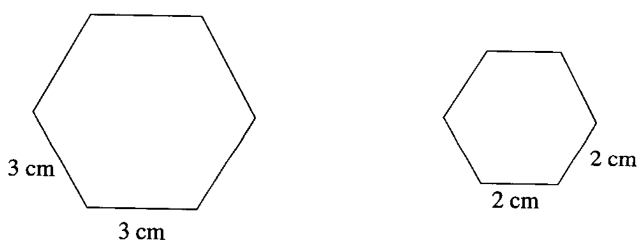


Fig. 2.3

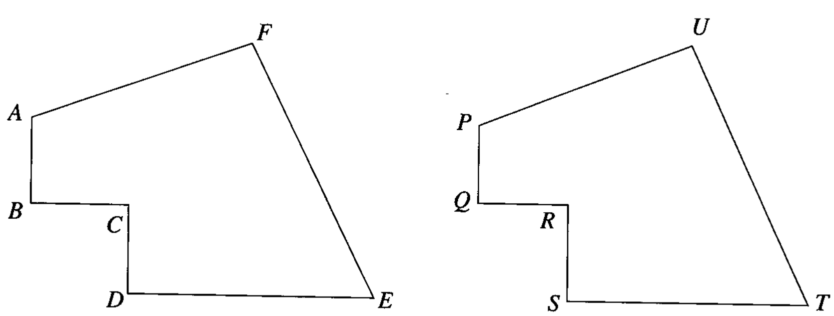
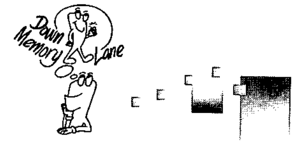


Fig. 2.4



Thales (about 625–545 BC) was the first person to formulate the theoretical study of geometry in order to lay the scientific foundation for the study of astronomy.

Can you draw two straight lines through a square to divide it into four congruent quadrilaterals which are not parallelograms of any kind?

The two congruent figures in Fig. 2.4 are obtained by placing two sheets of paper one on top of the other and cutting the shapes out. The sides and angles of these two congruent figures can be matched in the following way:

$$AF = PU, \quad CD = RS, \quad \hat{A} = \hat{P}, \quad \hat{D} = \hat{S},$$
$$AB = PQ, \quad DE = ST, \quad \hat{B} = \hat{Q}, \quad \hat{E} = \hat{T},$$
$$BC = QR, \quad EF = TU, \quad \hat{C} = \hat{R}, \quad \hat{F} = \hat{U}$$

In two congruent figures, the sides that match are called corresponding sides and the angles that match are corresponding angles. In Fig. 2.4, *AF*, *PU*; *AB*, *PQ*; *BC*, *QR*; *CD*, *RS*; *DE*, *ST* and *EF*, *TU* are pairs of **corresponding sides** while $\hat{A}$, $\hat{P}$; $\hat{B}$, $\hat{Q}$; $\hat{C}$, $\hat{R}$; $\hat{D}$, $\hat{S}$; $\hat{E}$, $\hat{T}$ and $\hat{F}$, $\hat{U}$ are pairs of **corresponding angles**.

The CD, Through the Ages with Congruency & Similarity, from the Dynamic Mathematics Series (DMS) has interesting tutorials and activities on congruent figures and triangles. This CD can be used as an interesting introduction or remedial lessons. Go through them before you start the lessons and use them for revision.

Notation for Congruence

The symbol ≡ means "is congruent to". Thus, in Fig. 2.4, we have $ABCDEF \equiv PQRSTU$. Notice that the letters representing the vertices of the figures are written in matching pairs (see Fig. 2.5).

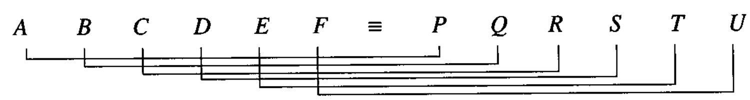
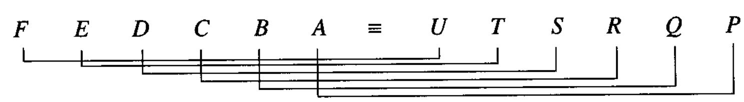
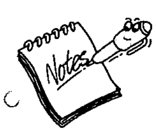


Fig. 2.5

In the statement



we still have the corresponding vertices matched. Hence, the statement $ABCDEF \equiv PQRSTU$ can be written as $FEDCBA \equiv UTSRQP$ or any other equivalent statement as long as the corresponding vertices match. Is the statement $BACDEF \equiv PQRSTU$ equivalent to $ABCDEF \equiv PQRSTU$?

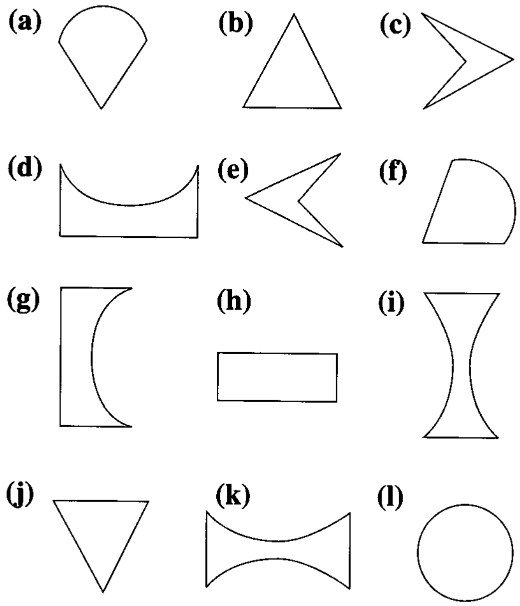


Can you name other situations where the notion of congruence is being used?

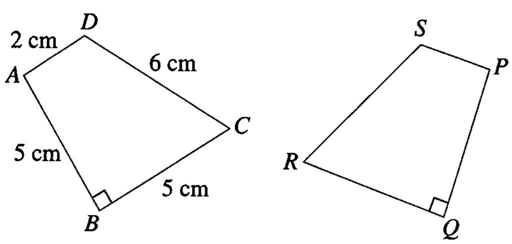
The concept of congruence is not restricted to the study of geometry. It plays an important part in everyday living too. We are able to replace a worn-out part of a car with a new one of "the same part number". The blocks used in the construction of building toys are standard in size. The medium-sized diapers of a certain brand are all "alike". In each of these examples, the idea of "sameness of size and shape" comes in and mathematically, we call this notion **congruence**.

Exercise 2a

1. Study the figures carefully and name the pairs that are congruent.

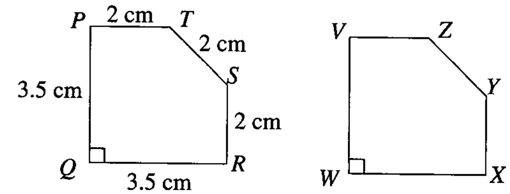


2. Given that $ABCD \equiv PQRS$, copy and complete the following:



(a) $PQ = AB =$ _______ cm
(b) $SR =$ _______ $= 6$ cm
(c) $PS =$ _______ = _______ cm
(d) $QR =$ _______ = _______ cm
(e) $P\hat{Q}R = \hat{}$ _______ = _______ °

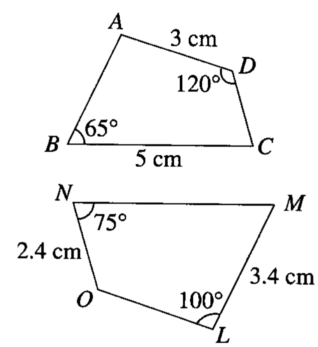
3. In the diagram below, $PQRST \equiv VWXYZ$.



Copy and complete the following:

(a) $PQ = VW =$ _______ cm
(b) $PT =$ _______ $= 2$ cm
(c) $QR =$ _______ = _______ cm
(d) $TS =$ _______ = _______ cm
(e) $SR =$ _______ = _______ cm
(f) $P\hat{Q}R = \hat{}$ _______ = _______ °

4. In the diagram below, $ABCD \equiv LMNO$. Write down the missing measurements.



Similar Figures

Fig. 2.6 shows some mugs which look alike but are of different sizes.

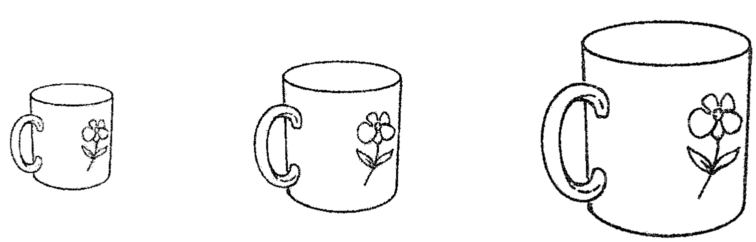


Fig. 2.6

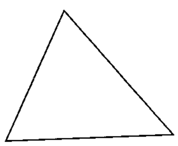
Can you name some other everyday situations where the concept of similarity is used?

When we use a magnifying glass to look at an object, we see a magnified image of the object. The magnified image of the object will have the same shape as the object. Two figures that have the same shape but not necessarily the same size are said to be **similar**. When we think of the concept of similarity, many everyday situations come to our mind. *For example*, a road map, the plan of a house, a toy plane that is a scale model of a full-sized plane and a figure that is a model of a full-sized figure are all illustrations of the concept of similarity.

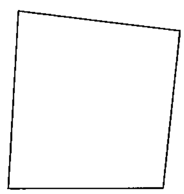
Polygons

Polygons have different names, depending on the number of corners or sides they have.

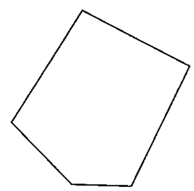
In Book 1 we learnt that a plane figure with three or more straight edges as its sides is called a **polygon** and each polygon is named after the number of sides it contains. The following are some common polygons for recapitulation.



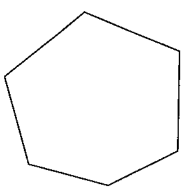
Triangle (3-sided)



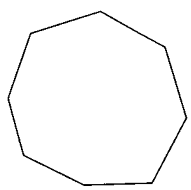
Quadrilateral (4-sided)



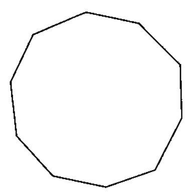
Pentagon (5-sided)



Hexagon (6-sided)



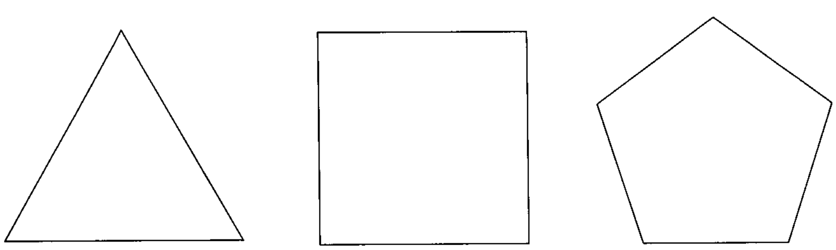
Octagon (8-sided)



Decagon (10-sided)

In addition, we also learnt that a polygon with n sides is called an n-gon. Thus a polygon with 15 sides is called a 15-gon and a polygon with 38 sides is called a 38-gon. A **regular polygon** is one in which all its sides and all its angles are equal. The following figures are some examples of regular polygons.

Do you know the special name given to a regular triangle and regular quadrilateral?



In-Class Activity

Consider a polygon which is magnified (see Fig 2.7).

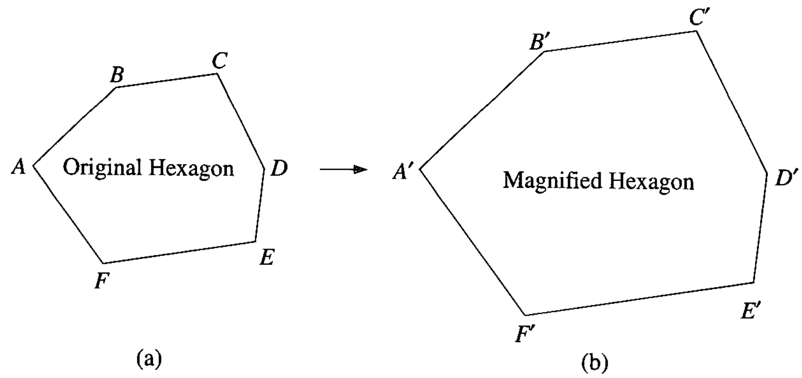


Fig. 2.7

The CD, Through the Ages with Congruency & Similarity, from the DMS provides interesting tutorials and activities on similar figures and triangles. Go through them before you start the lessons and use them for revision.

The hexagon in Fig. 2.7(a) increases in size without a change in shape.

1. Measure the following angles:

(a) $\hat{A}, \hat{A}'$ **(b)** $\hat{B}, \hat{B}'$ **(c)** $\hat{C}, \hat{C}'$

(d) $\hat{D}, \hat{D}'$ **(e)** $\hat{E}, \hat{E}'$ **(f)** $\hat{F}, \hat{F}'$

What do you notice about their sizes?

2. Measure the following sides:

(a) $AB, A'B'$ **(b)** $BC, B'C'$ **(c)** $CD, C'D'$

(d) $DE, D'E'$ **(e)** $EF, E'F'$

3. Find the values of the following ratios:

(a) $\frac{A'B'}{AB}$ **(b)** $\frac{C'D'}{CD}$ **(c)** $\frac{E'F'}{EF}$

What do you notice about their values?

From the above activities, we can conclude that two figures are similar when the corresponding angles remain unchanged and the length of each side is increased by the same factor, i.e.,

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'D'}{CD} = \frac{D'E'}{DE} = \frac{E'F'}{EF} = k, \text{ where } k \text{ is a constant.}$$

Hence, two polygons are similar if

1. all the corresponding angles are equal, and
2. the ratios of the corresponding sides are equal.

Notice that the angles of one polygon being equal to the angles of another polygon does not necessarily prove that the polygons are similar. *For example*, the square and the rectangle in Fig. 2.8 have their corresponding angles equal but are obviously not similar.

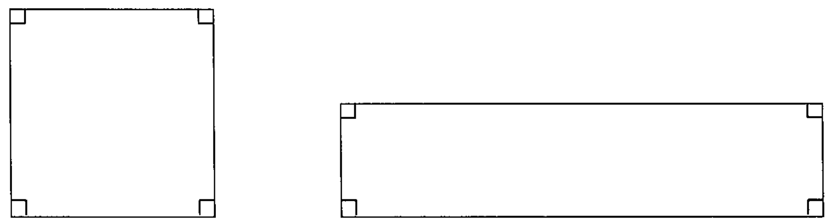


Fig. 2.8

Similarly, polygons in which the ratios of the corresponding sides are equal are not necessarily similar. *For example*, a rhombus and a square may have sides which are proportional but they are not similar (see Fig. 2.9).

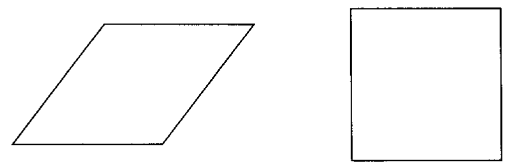


Fig. 2.9

Fig. 2.10 shows two similar right-angled triangles. Calculate the values of the ratios $\frac{AB}{PQ}$, $\frac{BC}{QR}$ and $\frac{AC}{PR}$. Notice that the value of each ratio is 1, i.e., the ratios of the corresponding sides are equal.

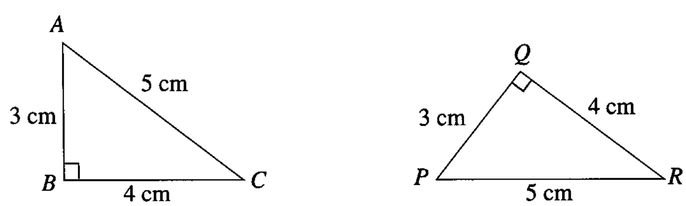


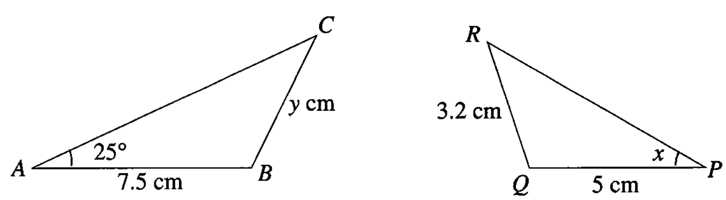
Fig. 2.10

Are all congruent figures similar? Are all similar figures congruent?

When two similar figures are of the same size, they are also congruent.

Example

Given that $\triangle ABC$ is similar to $\triangle PQR$, find the unknowns marked in the figures. (Figures shown are not drawn to scale.)



Solution

We have $\frac{AB}{PQ} = \frac{BC}{QR}$ (ratios of corresponding sides are equal)

i.e., $\frac{7.5}{5} = \frac{y}{3.2}$

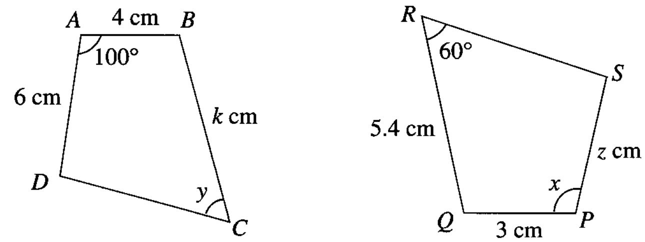
$\therefore \quad y = \frac{7.5 \times 3.2}{5} = 4.8$

$B\hat{A}C = Q\hat{P}R = 25°$ (corresponding angles are equal)

$\therefore \; x = 25°$

Example 2

Quadrilateral ABCD is similar to quadrilateral PQRS. Find the values of the unknowns marked. (Figures shown are not drawn to scale.)



Solution

$x = 100°$ and $y = 60°$ (corresponding angles are equal)

$$\frac{AB}{PQ} = \frac{AD}{PS} = \frac{BC}{QR}$$ (ratios of corresponding sides are equal)

i.e., $\frac{4}{3} = \frac{6}{z}$ and $\frac{4}{3} = \frac{k}{5.4}$

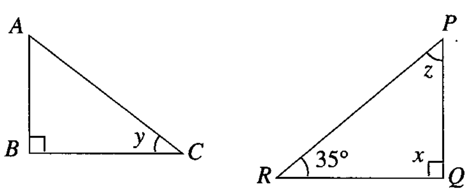
$\therefore\ z = \frac{6 \times 3}{4} = 4.5$ and $k = \frac{4 \times 5.4}{3} = 7.2$

Exercise 2b

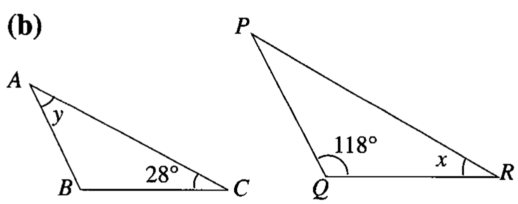
Figures shown in this exercise are not drawn to scale.

1. In each of the following triangles, $\triangle ABC$ is similar to $\triangle PQR$. Calculate the values of the unknowns marked.

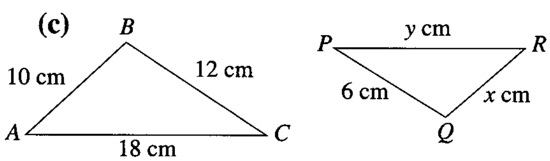
(a)



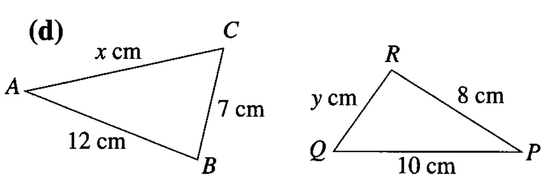
(b)



(c)

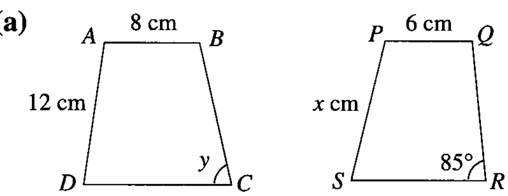


(d)

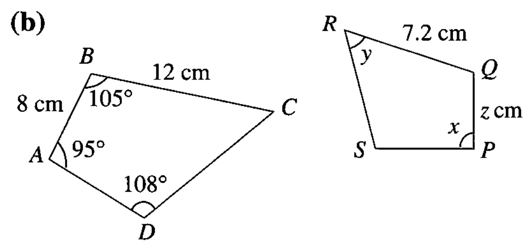


2. In each of the following cases, quadrilateral $ABCD$ is similar to quadrilateral $PQRS$. Find the values of the unknowns marked.

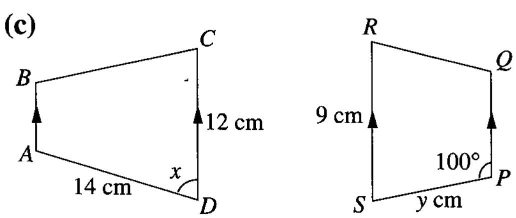
(a)



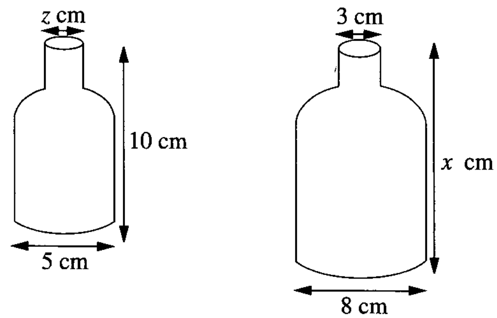
(b)



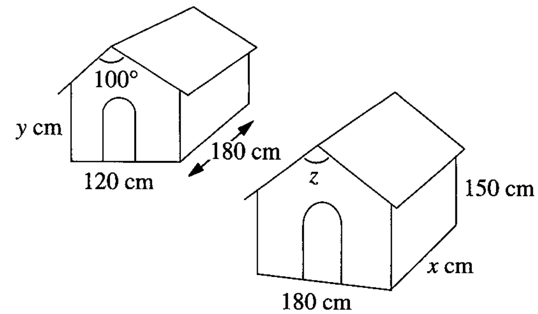
(c)



3. Calculate the unknown lengths of the two similar water-bottles shown below.



4. Calculate the unknowns marked in the two similar toy houses shown below.



Summary

1. Congruent figures are figures of the same shape and size.
2. Similar figures are figures in which
   (a) all the corresponding angles are equal, and
   (b) the ratios of the corresponding sides are equal.

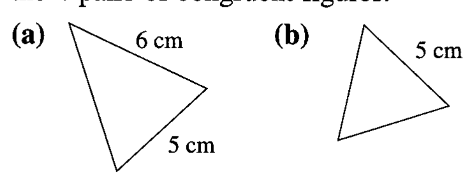
Review Questions 2

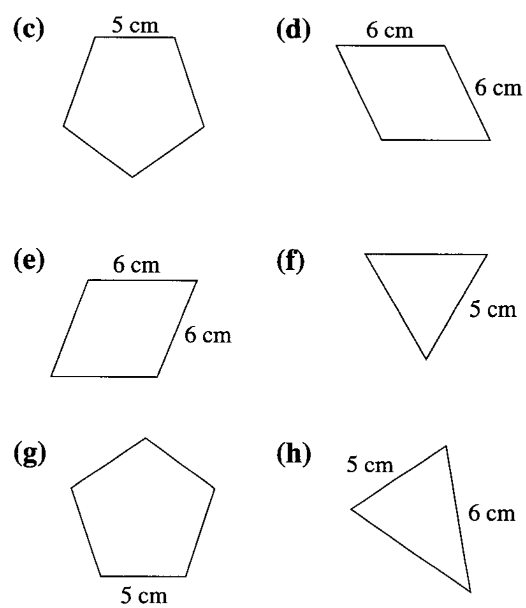
1. Given that $\triangle ABC$ is congruent to $\triangle PQR$, $\hat{A} = 70°$, $\hat{B} = 60°$ and $AB = 8$ cm, find
   (a) $\hat{P}$, (b) $\hat{Q}$, (c) $\hat{R}$, (d) PQ.

2. Given that $ABCD$ is congruent to $PQRS$, $\hat{A} = 100°$, $\hat{B} = 70°$, $\hat{C} = 95°$ and $PQ = 6$ cm, find
   (a) $\hat{P}$, (b) $\hat{Q}$, (c) $\hat{R}$, (d) $\hat{S}$,
   (e) AB.

3. Given that $\triangle ABC$ is similar to $\triangle PQR$, $\hat{A} = 50°$ and $\hat{B} = 68°$, find
   (a) $\hat{P}$, (b) $\hat{Q}$, (c) $\hat{R}$.

4. Study the figures below carefully and identify the 4 pairs of congruent figures.

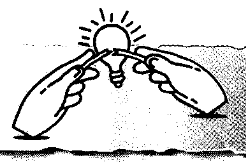




5. Given that $\triangle ABC$ is similar to $\triangle PQR$, $AB = 6$ cm, $AC = 8$ cm, $\hat{A} = 60°$ and $PR = 10$ cm, find **(a)** $\hat{P}$ and **(b)** PQ.

6. A television image of a teddy bear is 6 cm tall and that of its owner, a little girl, is 15 cm tall. If the actual height of the little girl is 120 cm, what is the height of her teddy bear?

7. A photograph is taken of a man 180 cm tall standing in front of his terrace house. The image of the man in the photograph is 9 cm tall and that of his house is 22.5 cm tall. Calculate the actual height of the house.

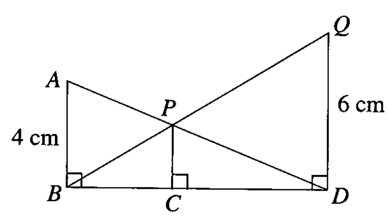


Problem Solving

1. A rectangular piece of land shown on a photograph measures 4.5 cm by 6.4 cm. If 1 cm on the photograph corresponds to 20 m on the ground, calculate the actual perimeter and area of the land.

2. Two teapots are similar in every respect. The height of the smaller teapot is 15 cm with a total surface area of 2 000 cm^2 and a volume of 6 000 cm^3. If the height of the larger teapot is 25 cm, calculate its total surface area and volume.

3. In the figure, AB, PC and QD are perpendicular to BCD. It is given that $AB = 4$ cm and $QD = 6$ cm.
   (a) Name three pairs of similar triangles.
   (b) Find the ratio of the length of BC to the length of CD.



CHAPTER 3

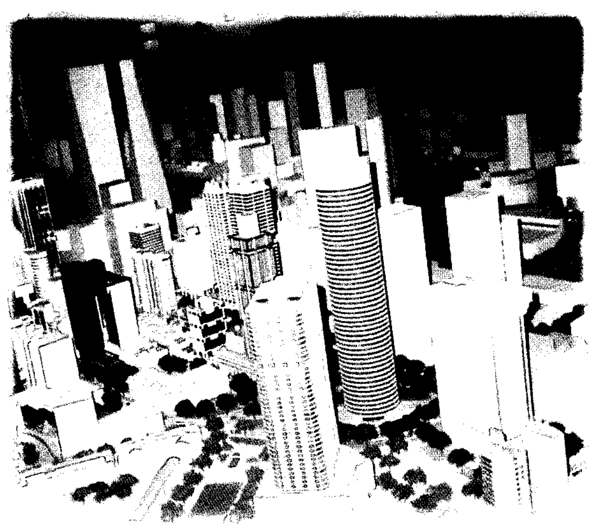
Scales and Maps

In this chapter, you will learn how to

- △ read and make scale drawings.

Preliminary Problem

Before a new model of aircraft is built, engineers will build a small scale model of the aircraft and test it in a wind tunnel. Similarly, scale models of buildings like the ones shown in the picture are made to show their final settings before actual construction is undertaken.



Can you think of other situations where scale models are used?

The complex effect of scale is a highly technical subject although the basic concepts and applications are quite simple.

Scale Drawings

In our daily activities, we sometimes need to have enlarged pictures or drawings or to have reduced drawings of actual objects. *For example*, if we wish to draw a plan of a badminton court in order to explain the rules of the game, we need to make our drawing very much smaller on paper or on a whiteboard. If we wish to show a diagram of the apparatus used for the preparation of oxygen, we can enlarge the diagram with the use of an overhead projector.

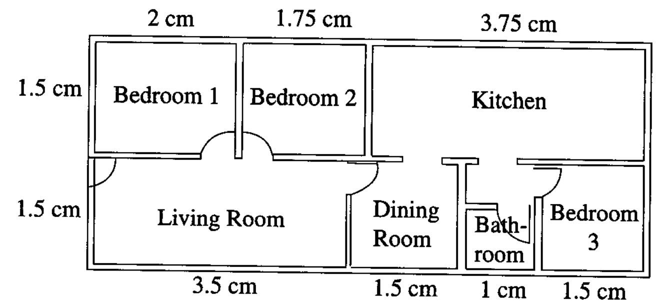


Fig. 3.1

Fig. 3.1 shows the floor plan of a terrace house. The dimensions of the plan are proportional to the corresponding actual dimensions of the house. Fig. 3.1 has been drawn to a scale of 1 cm to 2 m, i.e., 1 cm on the scale represents 2 m on actual ground. From Fig. 3.1, we find the following.

(a) The length of the living room is (3.5×2) m, i.e., 7 m and its width is (1.5×2) m, i.e., 3 m.
(b) The area of bedroom 1 is 1.5 cm × 2 cm on the plan.
Hence, actual area = $(1.5 \times 2)\text{ m} \times (2 \times 2)\text{ m} = 12\text{ m}^2$.
(c) The area of the living room = $3\text{ m} \times 7\text{ m} = 21\text{ m}^2$.
(d) The total area of the house = $(3 \times 2)\text{ m} \times (7.5 \times 2)\text{ m} = 90\text{ m}^2$.

Example 1

The scale of a building plan is 1 cm to 50 cm. Find (a) the actual length of one of the bedrooms if it is represented by a length of 9.2 cm and (b) the length on the plan that represents an actual length of 28 m.

Solution

(a) Since 1 cm represents 50 cm, i.e., 0.5 m on actual ground,
∴ 9.2 cm represents 9.2×0.5 m, i.e., 4.6 m on actual ground.

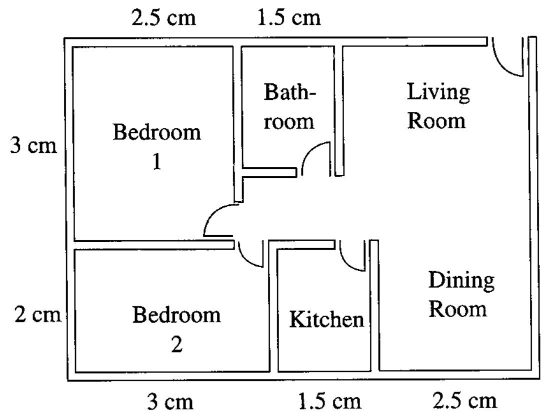
(b) Since 0.5 m is represented by 1 cm, i.e., 1 m is represented by 2 cm,
∴ 28 m will be represented by 28×2 cm, i.e., 56 cm on the plan.
Alternatively, we can use the **ratio method** to solve this problem.
1 cm represents 0.5 m.
Let x cm represent 28 m.

$$1 : 0.5 = x : 28$$
$$\frac{1}{0.5} = \frac{x}{28}$$
$$0.5x = 28$$
$$x = \frac{28}{0.5} = 56$$

∴ 28 m is represented by a length of 56 cm.

Exercise 3a

1. The diagram below shows the floor plan of a flat.



The scale is 1 cm to 1.5 m.

Find the actual

(a) dimensions and area of bedroom 1;
(b) dimensions and area of the kitchen;
(c) total area of the flat.

2. Make a scale drawing of a rectangular school hall which is 50 m long and 30 m wide using a scale of 1 cm to 5 m. Use your scale drawing to find the actual distance between the opposite corners of the hall.

3. A triangular field ABC is such that AB = 90 m, BC = 70 m and AC = 85 m. Make a scale drawing of it, using a scale of 1 cm to 10 m. From your scale drawing, find **(a)** the actual distance from A to the mid-point of BC and **(b)** the actual distance from B to the mid-point of AC.

4. A scale model of a house is made with the scale being 1 cm to 3 m.

(a) Given that the length of the model is 12 cm, calculate the actual length of the house.
(b) If the actual width of the house is 12 m, calculate the width of the model.

5. A model of an oil tanker is made to a scale of 1 cm to 15 m.

(a) If the height of the mast of the tanker is 85 m, find the height of the mast on the model.
(b) If the length of the oil tanker is found to be 210 m, find the length of the model.

6. A model of a school is made to a scale of 1 cm to 5 m.

(a) If the height of the school building is 28 m, find the height of the model.
(b) Find the actual length of the field if its length on the model is 22 cm.

7. On a scale drawing, the length of a ship is 45 cm. The actual length of the ship is 90 m. What is the scale used? If the width of the ship is 25 m, what is its width on the scale drawing?

8. The scale of a map is 1 cm to 100 km. What is the actual distance between two cities if they are

(a) 8 cm apart on the map;
(b) 2.4 cm apart on the map;
(c) 125 mm apart on the map?

9. The scale of a map is 1 cm to 5 km. What is the distance on the map between two towns if they are actually

(a) 25 km apart;
(b) 38 km apart;
(c) 12.5 km apart;
(d) 2 500 m apart?

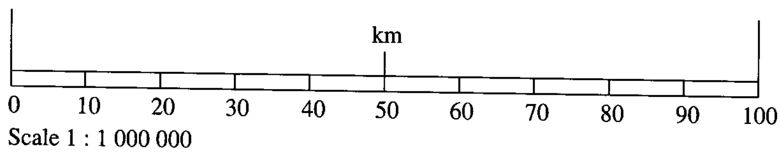
10. A model of a tower is made to a scale of 1 cm to 0.5 m. If the height of the tower on the model is 84 cm, find the height of the tower on another model made to a scale of 1 cm to 2 m.

11. A map is drawn to a scale of 5 cm to 2 km. A road on the map has a length of 12.5 cm. Find

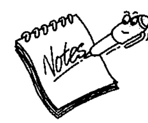
(a) the actual length of the road in km;
(b) the length of the road drawn on another map of scale 3 cm to 10 km.

Scales on Maps

All maps are scale drawings of the shapes of pieces of land. They are drawn many times smaller than the actual areas. The scale of a map is usually given at the top or a corner of the map. There are several ways of representing the scale of a map. *For example*, on a map of Malaysia, the following scale may be given:



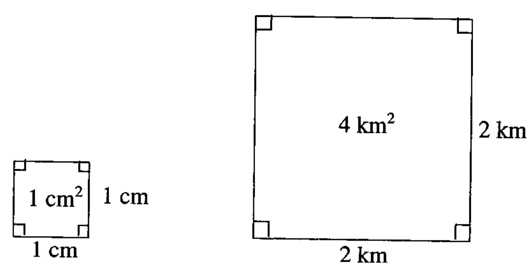
The scale bar tells us the actual distance represented by the respective length on the map. The scale bar is very useful if we wish to find the approximate distance between two places quickly. The scale represented in the form 1 : 1 000 000 means that 1 unit length (such as 1 cm) on the map represents 1 000 000 units on actual ground. The scale of 1 : 1 000 000 can also be expressed as a **representative fraction (R. F.)** of $\frac{1}{1\,000\,000}$. Thus, if the R. F. is $\frac{1}{200}$, the scale would be 1 : 200. It means that 1 mm represents 200 mm or 1 cm represents 200 cm or 1 m represents 200 m.



When we use R.F., the numerator must always be 1.

Area Scale

The **area scale** of a map is the square of its **linear scale**. If the scale of the map is 1 cm to 2 km, then 1 cm² on the map will represent $(2\text{ km})^2$, i.e., 4 km². Thus the area scale is 1 cm² : 4 km² while the linear scale is 1 cm : 2 km.



Example 2

A map has a scale of 1 cm to 3 km.

(a) What length on actual ground does a 3-cm length on the map represent?
(b) What length will represent 7.5 km?
(c) What is the R. F. of the map?

Solution

(a) 1 cm on the map represents 3 km on actual ground.
$\therefore$ 3 cm on the map represents (3×3) km on actual ground,
i.e., 9 km on actual ground.

(b) 3 km is represented by 1 cm on the map.

1 km will be represented by $\frac{1}{3}$ cm on the map.

$\therefore$ 7.5 km will be represented by $\left(7.5 \times \frac{1}{3}\right)$ cm on the map,

i.e., 2.5 cm on the map.

(c) 1 cm : 3 km = 1 cm : 300 000 cm

$\therefore$ the R. F. of the map is $\frac{1}{300\,000}$.

Example 3

A scale of 2 cm to 1 km is used for a map.

(a) The distance between two villages is 6 km. What is its length represented on the map?
(b) What is the area scale of the map?
(c) What is the R. F. of the map?

Solution

(a) 1 km is represented by 2 cm on the map.
6 km is represented by (6×2) cm on the map.
$\therefore$ the length between the two villages is 12 cm on the map.

(b) The scale of the map is 2 cm : 1 km
So the area scale is
$(2\text{ cm})^2 : (1\text{ km})^2$
$= 4\text{ cm}^2 : 1\text{ km}^2$
$= 1\text{ cm}^2 : \frac{1}{4}\text{ km}^2$

(c) 2 cm : 1 km
= 2 cm : 100 000 cm
= 1 cm : 50 000 cm
$\therefore$ the R. F. of the map is $\frac{1}{50\,000}$.

Example 4

On a map whose scale is 1 : 50 000, a lake is found to have an area of 16 cm².

(a) Find the actual area of the lake.
(b) If the lake is a square, find the length of one of its sides on actual ground.

Solution

(a) Since the scale is 1 : 50 000, 1 cm on the map represents 50 000 cm on actual ground, i.e., 1 cm represents 500 m or $\frac{1}{2}$ km.

1 cm^2 represents $\left(\frac{1}{2}\text{ km}\right)^2$, i.e., 1 cm^2 represents $\frac{1}{4}\text{ km}^2$.

$\therefore$ 16 cm^2 represents $\left(16 \times \frac{1}{4}\right)\text{ km}^2$, i.e., 4 km^2.

(b) Since the lake is a square, each side will be $\sqrt{4\text{ km}^2}$, i.e., 2 km.

Exercise 3b

1. A map is drawn to a scale of 1 : 50 000.
   (a) What is the actual distance represented by
   (i) 2 cm, (ii) 7.5 cm, (iii) 0.6 cm, (iv) 26 cm?
   (b) What length on the map represents the actual distance of
   (i) 4 km, (ii) 15 km, (iii) 250 m, (iv) 1 600 m?

2. A map is drawn to a scale of 1 cm : 2 km.
   (a) What is the actual distance represented by
   (i) 3 cm, (ii) 4.5 cm, (iii) 1 m, (iv) 0.25 cm?
   (b) What length represents the actual distance of
   (i) 5 km, (ii) 500 m, (iii) 9 km, (iv) 1.5 km?

3. The distance between two towns is 225 km. On a map with a scale of 1 cm : 50 km, how far apart would the towns be?

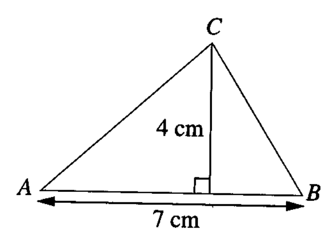
4. The scale of a map is 1 : 20 000. Find the distance in km of a road represented by $5\frac{1}{2}$ cm on the map.

5. Given that 1 cm on a map represents 2 km on the ground, calculate the area of a park on the map if the actual area of the park is 10 km^2.

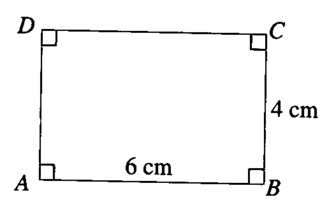
6. The scale of a map is 1 cm : 8 km. What area on the map would represent
   (a) 64 km^2, (b) 128 km^2, (c) 320 km^2, (d) 1 600 km^2?

7. On a scale of 1 cm : 2 km, the plan of the field measures 5 cm by 3.5 cm on the map. Find the actual area of the field.

8. In the given figure, *ABC* is a triangle of height 4 cm and base 7 cm, drawn to a scale of 1 cm : 3 km. Find the actual area of the triangle.



9. The figure on the right is a rectangle, drawn to a scale of 1 cm : 200 m. Given that AB = 6 cm and BC = 4 cm, find the actual
   (a) length and breadth of the rectangle;
   (b) area of the rectangle in km^2.



10. The scale of a map is 1 : 20 000. Find the area on the map which represents 124 km^2.

11. On a map drawn to a scale of 1 cm to 500 m, an airport has an area measuring 16 cm by 8.5 cm. Calculate, in hectares, the actual area of the airport. (1 ha = 10 000 m^2)

12. A map is drawn to a scale of 1 : 50 000.
    (a) Calculate the actual distance, in kilometres, represented by 4 cm on the map.
    (b) Two towns are 28 km apart. Calculate, in centimetres, their distance apart on the map.
    (c) On the map, a forest has an area of 12 cm^2. Calculate, in square kilometres, the actual area of the forest.

Summary

1. If the **linear scale** of a map is $1 : x$, it means that 1 cm on the map represents x cm on the actual piece of land.

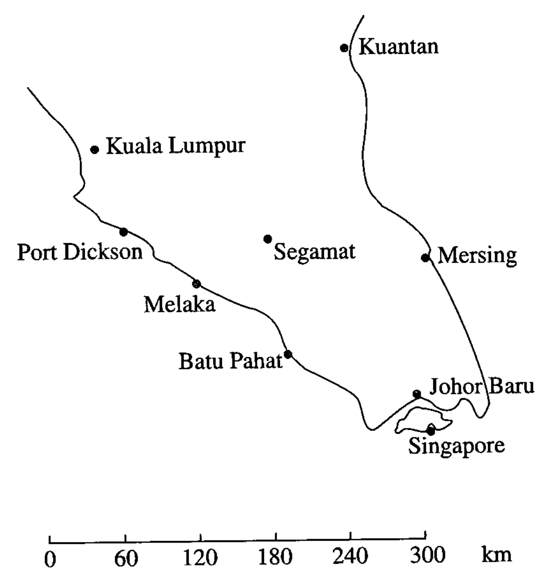
2. The **area scale** of a map is the square of its linear scale. For example, if the linear scale is $1 : x$, then the area scale is $1^2 : (x)^2$, i.e., $1 : x^2$.

Review Questions 3

1. Given that 2 cm on a map represents 3 km on the ground,
   (a) calculate the distance, in km, between two towns which are 7 cm apart on the map;
   (b) express the scale of the map in the form $1 : n$;
   (c) calculate, in cm^2, the area of the map which represents a lake of area 81 km^2 on the ground.

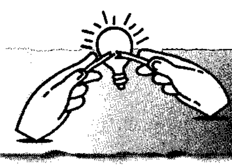
2. A map of a region is drawn to a scale of 1 : 25 000.
   (a) Calculate the actual distance, in kilometres, represented by 24 cm on the map.
   (b) Two HDB area offices are 3.5 km apart. Calculate, in centimetres, their distance apart on the map.
   (c) On the map, a reservoir has an area of 16 cm^2. Calculate, in square kilometres, the actual area of the reservoir.

3. Use the map of South Malaysia to answer the following questions. You may use a ruler to measure the approximate distances between any two places and then calculate the actual distances from the given scale.
   (a) What is the distance between Singapore and Kuantan?
   (b) How much would it cost to hire a taxi to go from Melaka to Kuala Lumpur if the taxi fare is 60 cents per km?
   (c) A car travels at an average speed of 60 km/h from Batu Pahat to Port Dickson. How long will the journey take?
   (d) A train takes 4 hours to travel from Johor Baru to Segamat. Find its average speed in km/h.
   (e) An express bus travels from Kuala Lumpur to Kuantan and it charges 8 cents per km per person. How much would it cost a man to travel from Kuala Lumpur to Kuantan?



4. Given that 5 cm on a map represents 6 km on the ground.
   (a) Calculate the actual distance, in km, between two towns which are 9.5 cm apart on the map.
   (b) A road has a length of 8.4 km, calculate its length on the map.
   ⁎**(c)** The area of a new township on the map is 15 cm^2, calculate its actual area in hectares.

5. The plan of a shopping complex is drawn to a scale of 1 : 400.
   (a) Find the length, in metres, of a corridor which is represented by a line 24.5 cm long on the plan.
   (b) The area of the floor of a fast food restaurant is 400 m^2, find its area on the plan.
   (c) A supermarket on the plan occupies an area of 0.25 m^2, calculate its actual area in hectares.



Problem Solving

1. On a map whose scale is 1 : 40 000, the area of a housing estate is 50 cm^2. Calculate the area, in cm^2, which represents the same housing estate on a second map whose scale is 1 : 20 000.

2. On a map whose scale is 1 : 50 000, a housing estate is represented by an area of 24 cm^2. Find, in cm^2, the area representing this housing estate on a map whose scale is 1 : 100 000.

3. A model of a ship is made to a scale of 1 : 200. The volume of a hall on the model is 250 cm^3. Find the volume of the hall on the actual ship in m^3.

Exploration

Look at the atlas which you use for your Geography lessons and find out the scales used in maps of Singapore, Southeast Asia, Asia and a world map. Are the scales used the same for all the different maps?

CHAPTER 4

Symmetry and Angle Properties of Polygons

In this chapter, you will learn how to

- △ identify line and rotational symmetry of plane figures;
- △ make use of the symmetrical properties of triangles, quadrilaterals and regular polygons;
- △ use symmetrical properties of simple solids.

Preliminary Problem

Look at the various stamps. Can you name the geometrical shapes of them?

Line Symmetry in Two Dimensions

In-Class Activity

1. Fold a sheet of paper into two equal parts and mark two points, *X* and *Y*, on the folded line (Fig. 4.1(a)).

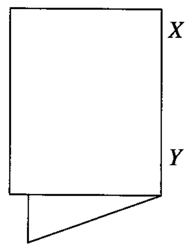
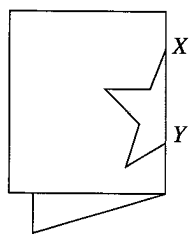


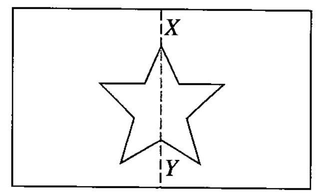
Fig. 4.1 (a)

2. Use a pair of scissors to cut out a shape, starting at *X* and ending at *Y* (Fig. 4.1(b)).



(b)

3. Unfold the paper. Do you get a shape similar to the one shown in Fig. 4.1(c) when you cut along the lines shown?



(c)

4. The "half-shapes" on both sides of the line *XY* in Fig. 4.1(c) are congruent. The shape cut out is said to be symmetrical about the line *XY*. The line *XY* is known as the **line of symmetry** or the **axis of symmetry**.

5. Place a small thin mirror along *XY* in Fig. 4.1(c), what do you notice about the reflected image and the actual figure? Sometimes, we also refer to the two congruent halves in Fig. 4.1(c) as **mirror images** of each other.

Are human beings symmetrical? Do you know of any animal that is not symmetrical?

Symmetry Around Us

Many things around us are symmetrical. Things in nature – animals and plants – often have symmetrical shapes too. (See Fig. 4.2.)

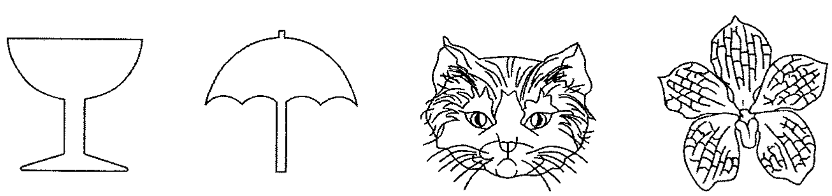


Fig. 4.2

Symmetry plays an important part in our environment. Look around you, at the school building, the school hall, your house, bridges, etc. Do you notice that symmetrical shapes are used commonly and appear almost everywhere?

The symmetry of a figure about an axis may be tested by folding it along that axis; if the figure is symmetrical, the part to the left of the axis will fit exactly onto the part to the right of the axis and vice versa (Fig. 4.1(c)).

Can you draw a triangle that has exactly two lines of symmetry?

A figure may have more than one line of symmetry. *For example*, the square shown in Fig. 4.3(a) has 4 lines of symmetry, the rectangle in Fig. 4.3(b) has 2 lines of symmetry and the star shape shown in Fig. 4.3(c) has 5 lines of symmetry.

Another interesting DMS CD is Space Trek through Symmetry. Go through the tutorial and activities on line symmetry. You will have a better understanding of line symmetry.

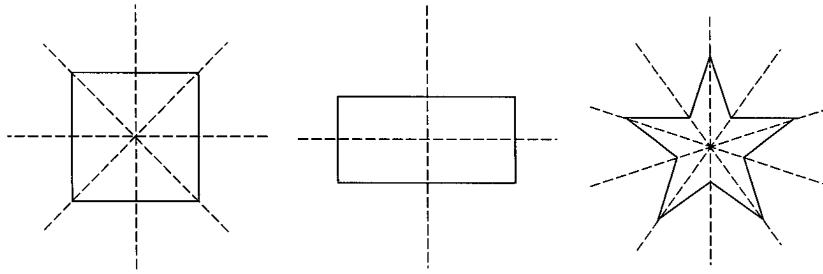


Fig. 4.3

In-Class Activity

1. Organise a class competition to make life-sized symmetrical masks. The materials used can be paper, cardboard, clay or even wood.
2. Collect specimens of insects and animals or photographs of these which have symmetrical shapes.

The Mirror Image of a Point

If P lies on XY, the mirror image of P with respect to XY is P itself.

Consider a line XY on a plane. To find the mirror image of a point P with respect to XY, we draw a line PM perpendicular to XY and produce PM to P' so that $PM = MP'$ as shown in Fig. 4.4. P' is the mirror image of P with respect to the line XY.

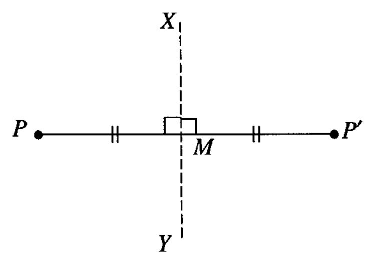


Fig. 4.4

The Mirror Image of a Line

To find the mirror image of PQ with respect to XY, we locate the images of P and Q. They are denoted by P' and Q' respectively as shown in Fig. 4.5. The line $P'Q'$ is the image of PQ with respect to XY.

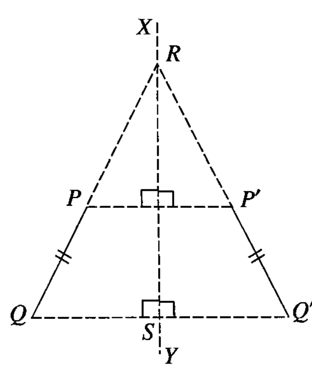


Fig. 4.5

$PQ = P'Q'$ and $Q\hat{R}S = Q'\hat{R}S$.

Exercise 4a

1. Study the eight capital letters below.

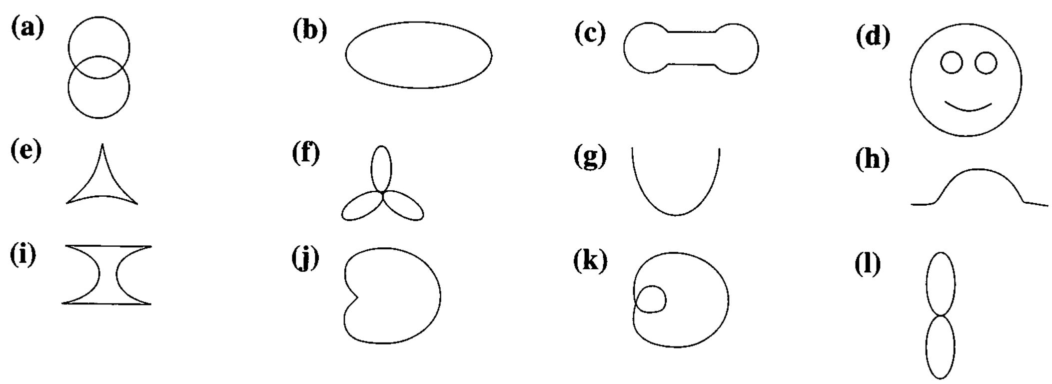
NEW MATHS

(a) Which letters have a vertical line of symmetry?
(b) Which letters have a horizontal line of symmetry?
(c) Which letters have two lines of symmetry?
(d) Which letters are not symmetrical?
(e) Copy those letters above which are symmetrical and draw lines of symmetry on each one.

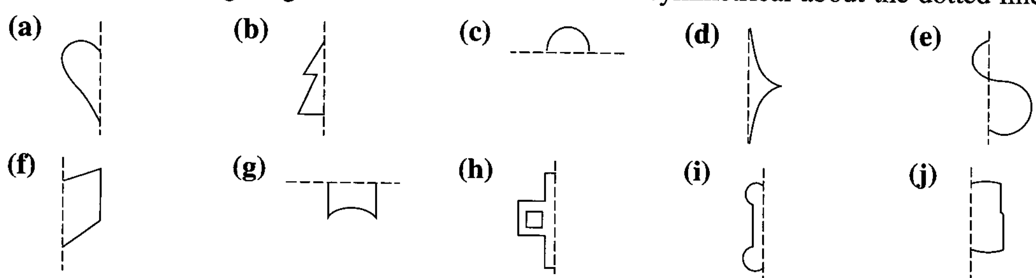
2. Show with dotted lines the line of symmetry of each of the following capital letters, where they exist:

B C D F G I J K L
O P Q R U V X Y Z

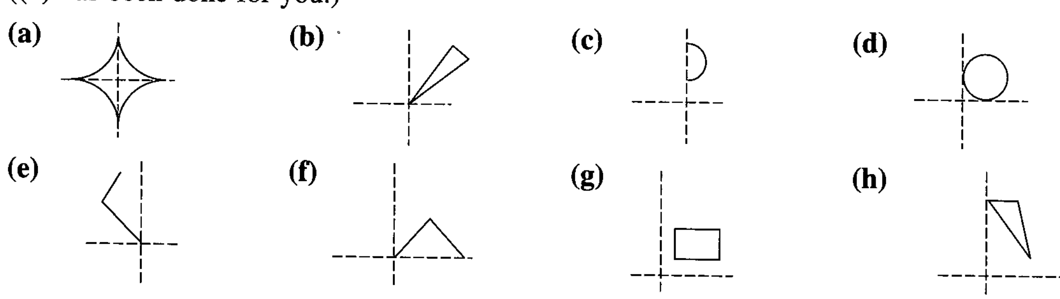
3. Copy the following diagrams and draw the axis of symmetry for each. How many lines of symmetry does each of them have?



4. Copy the following diagrams and make each of them symmetrical about the dotted line:

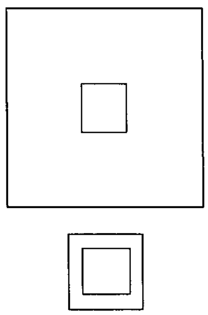


5. Copy the following diagrams and make each of them symmetrical about the two dotted lines. ((a) has been done for you.)



Rotational Symmetry

Seeing is believing.



Which inner square is larger?

In-Class Activity

1. Trace Fig. 4.6 on two separate pieces of tracing paper. Place one piece of the paper exactly on top of the other such that the two figures coincide, and then place a thumbtack at the centre of the figure.

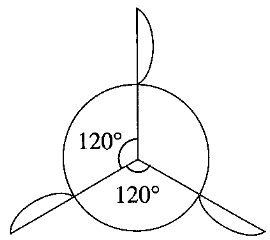


Fig. 4.6

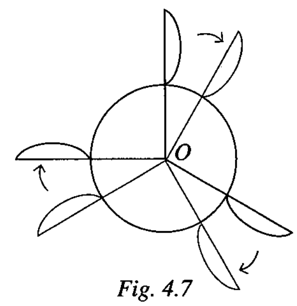


Fig. 4.7

2. Rotate the top tracing paper clockwise (see Fig. 4.7) until the two figures totally coincide again. What fraction of a complete turn has been made?

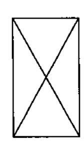
3. Rotate the top figure again until it covers the lower figure. What fraction of a complete turn has been made?

4. How many thirds of a turn have to be made before the top figure is to be back in its original position?

When a figure can be rotated to fit the outline of its original position, we say that the figure possesses **rotational symmetry**. The point O where we place the thumbtack is called the **centre of rotation.** Since the figure can be rotated 3 times to fit the original figure, the figure is said to have rotational symmetry of **order** 3, or the order of rotational symmetry is 3.

The following shows figures with different orders of rotational symmetry:

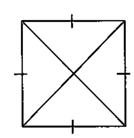
The CD, Space Trek through Symmetry, from the DMS has interesting and useful tutorials and activities that deal with basic concepts and identifications of rotational symmetry. Go through them in the computer lab or your school library.

(a) 

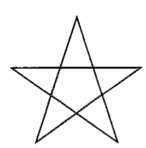
order 2

(b) 

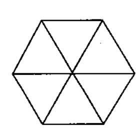
order 3

(c) 

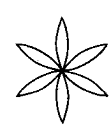
order 4

(d) 

order 5

(e) 

order 6

(f) 

order 6

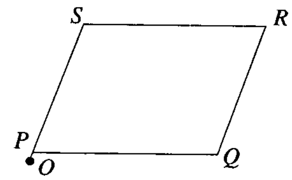
The order of rotational symmetry is the number of distinct ways in which a figure can map onto itself by rotation.

In the case of order 1, we say that there is no rotational symmetry.

If an identical figure can be obtained only by turning the original figure through 360°, we say that the order of rotational symmetry is 1. Hence, every figure has at least an order of rotational symmetry 1.

Example 7

The incomplete figure on the right has rotational symmetry of order 2, with O as the centre of rotation. Complete the figure by drawing the other half.



Solution

We start by locating the image of S with respect to O. Produce SO to S' such that $SO = S'O$ as shown in Fig. 4.8. S' is the image of S with respect to O. Mark the point P' such that $PO = P'O$. P' is the image of P with respect to O. We continue in the same way to obtain the images of R and Q which are denoted by R' and Q' respectively. Join $P'Q'$, $Q'R'$ and $R'S'$ to complete the figure which has rotational symmetry of order 2 as shown in Fig. 4.8.

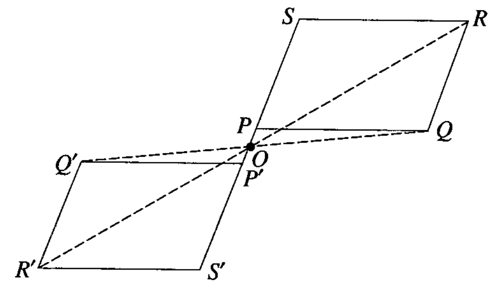
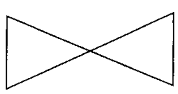
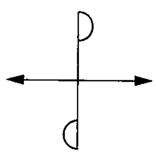
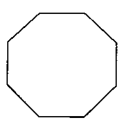
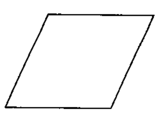
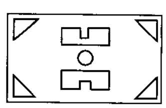
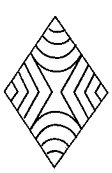
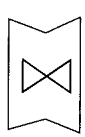
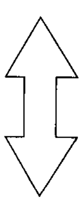
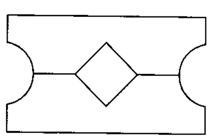
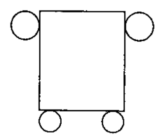
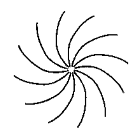


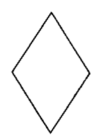
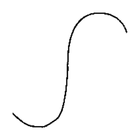
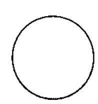
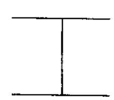
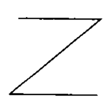
Fig. 4.8

Exercise 4b

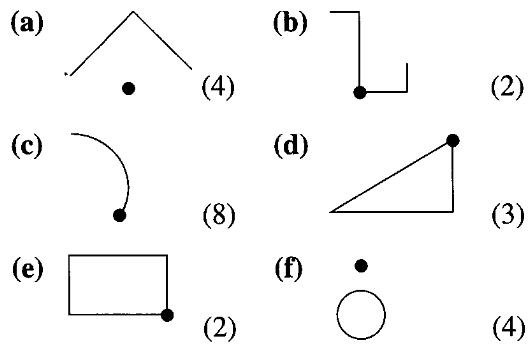
1. For each of the following figures, state **(i)** the number of lines of symmetry, **(ii)** the order of rotational symmetry:

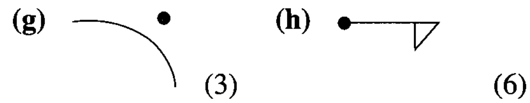
(a) 
(b) 
(c) 
(d) 
(e) 
(f) 
(g) 
(h) 
(i) 
(j) 
(k) 
(l) 

2. Copy the following diagrams and mark the centre of rotational symmetry for each:

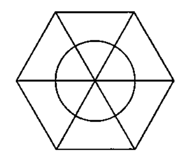
(a) 
(b) 
(c) 
(d) 
(e) 
(f) 
(g) 
(h) 

★3. Copy the following diagrams and complete each of them so that the point marked becomes the centre of symmetry with order of rotational symmetry as given in brackets:





4. Copy the figure and shade certain parts of the figure so that the resulting figure has rotational symmetry of
   (a) order 2,
   (b) order 3,
   (c) order 6.



The Symmetries of Regular Polygons

In-Class Activity

Fig. 4.9 shows some regular polygons.

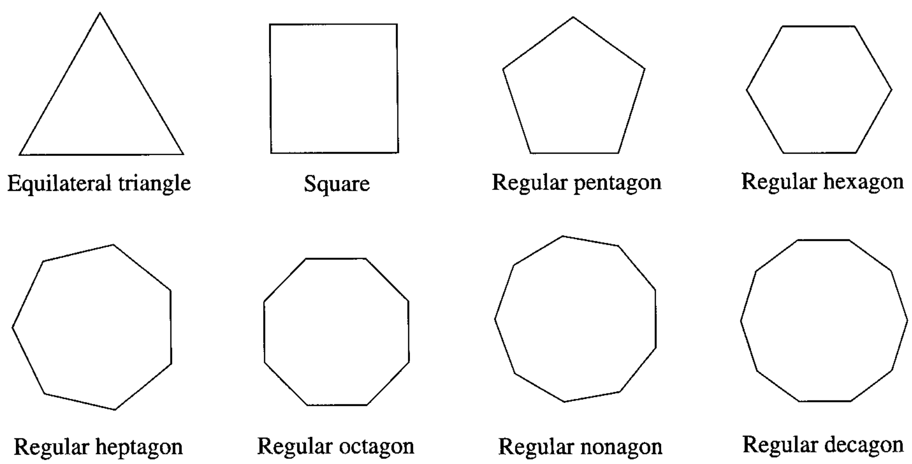
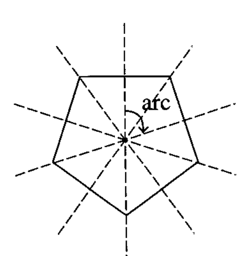


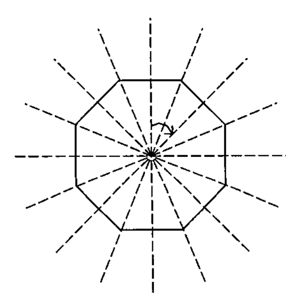
Fig. 4.9

1. Copy the polygons in Fig. 4.9 and use a colour pencil to mark their lines of symmetry.

2. Use a different colour pencil to mark the centres of rotational symmetry. Draw arcs showing the smallest angle of rotational symmetry (Fig. 4.10(a) and Fig. 4.10(b) are examples of what is to be done).



(a) Regular pentagon



(b) Regular octagon

Fig. 4.10

3. Copy and complete the table below.

Types of quadrilateral	*No. of lines of symmetry*	*Order of rotational symmetry*
Equilateral triangle		
Square		
Regular pentagon	5	$\frac{360°}{72°} = 5$
Regular hexagon		
Regular heptagon		
Regular octagon	8	$\frac{360°}{45°} = 8$
Regular nonagon		
Regular decagon		

4. Deduce
(a) the number of lines of symmetry, and
(b) the order of rotational symmetry of a regular polygon with n sides.

5. In the case of a regular pentagon, the smallest angle of rotation for which the pentagon can map onto itself is $72° = \frac{360°}{5}$. For a regular octagon, the smallest angle of rotational symmetry is $45° = \frac{360°}{8}$. What is the smallest angle of rotational symmetry for a general regular polygon with n sides?

Triangles with Symmetry

Isosceles Triangles

Fold a sheet of paper in half and cut off a right-angled triangle as shown in Fig. 4.11(a).

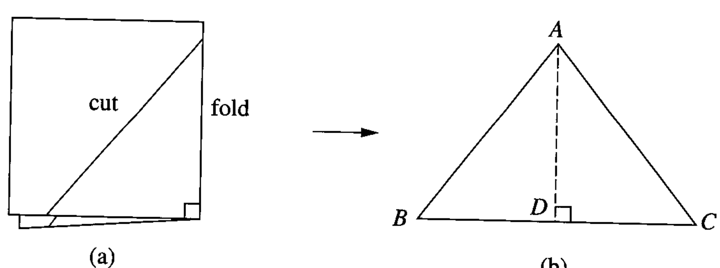


Fig. 4.11

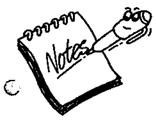
Open out the triangle and we obtain the shape as shown in Fig. 4.11(b).

How many lines of symmetry does the triangle have?

Can you say, without measuring, that the two sides AB and AC are equal and that the two angles B and C are equal? If so, why?

This triangle which has only one line of symmetry (line AD in Fig. 4.11(b)) is an **isosceles triangle**.

An isosceles triangle has
(a) two equal sides,
(b) two equal angles opposite the two equal sides.



An equilateral triangle has
(a) three equal sides,
(b) each angle equals to 60°.

Equilateral Triangles

If a triangle has three lines of symmetry, the triangle is an **equilateral triangle**.

Triangle ABC in Fig. 4.12 is an equilateral triangle. AP, BQ and CR are lines of symmetry. Since RC is a line of symmetry, $\hat{A} = \hat{B}$ and $CA = BC$.

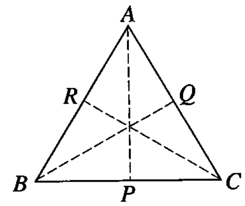


Fig. 4.12

Similarly, $\hat{A} = \hat{C}$ and $AB = BC$ (BQ is a line of symmetry)
$\hat{B} = \hat{C}$ and $AB = CA$ (AP is a line of symmetry)

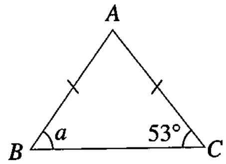
$$AB = BC = CA \text{ and } \hat{A} = \hat{B} = \hat{C} = \frac{180°}{3} = 60°$$

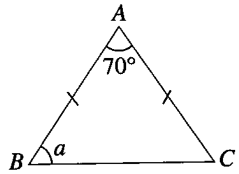
Exercise 4c

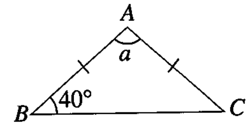
1. State whether each of the following statements is true or false:

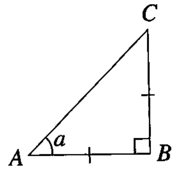
(a) An equilateral triangle is an isosceles triangle.
(b) An isosceles triangle is an equilateral triangle.
(c) An equilateral triangle has rotational symmetry of order 3.
(d) If a triangle has two equal sides, then it is an isosceles triangle.
(e) If a triangle has two equal sides, then it has two equal angles.
(f) If a triangle has two equal angles, then it has two equal sides.

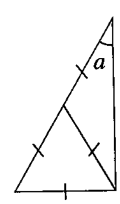
2. What is the size of the angle marked a in each of these diagrams?

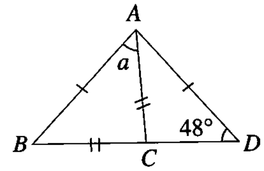
(a)


(b)


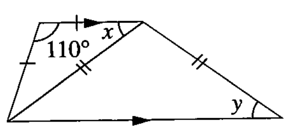
(c)


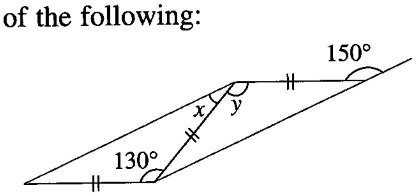
(d)


(e)


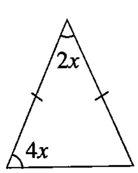
(f)


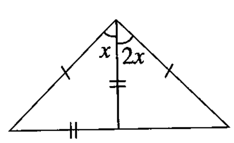
3. Find the value of x and of y in each of the following:

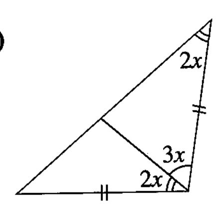
(a)


(b)


4. Find the value of x in each case.

(a)


(b)


(c)


Quadrilaterals with Symmetry

Kite

An example of a quadrilateral with one diagonal as the line of symmetry is the **kite** or the arrowhead.

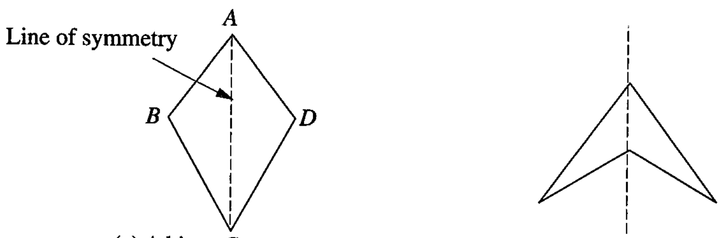


Fig. 4.13

Just for fun

How many blocks are there in the diagram below? Six or seven?

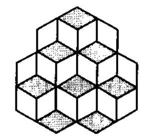


Fig. 4.13(a) shows a kite. The diagonal AC is the line of symmetry. AD and DC are mirror images of AB and BC with respect to AC respectively. Is $AB = AD$? Is $CB = CD$?

In fact, $\triangle ABC$ and $\triangle ADC$ are congruent. Thus, $AB = AD$, $CB = CD$, $A\hat{B}C = A\hat{D}C$, $B\hat{A}C = D\hat{A}C$ and $A\hat{C}B = A\hat{C}D$.

In addition, D is the mirror image of B with respect to AC. Hence, BD is perpendicular to AC.

From the above discussion, we can say that a kite is a quadrilateral which has

(i) two pairs of equal sides,
(ii) one pair of equal angles,
(iii) two diagonals perpendicular to each other,
(iv) a diagonal which bisects a pair of opposite angles.

Isosceles Trapezium

Can you think of a quadrilateral with one line of symmetry which is not a diagonal?

Fig. 4.14 shows such a quadrilateral. It is an **isosceles trapezium**.

Use the CD, Space Trek through Symmetry, from the DMS to gain a better insight on symmetry of polygons. Go through the section on symmetry in triangles and quadrilaterals.

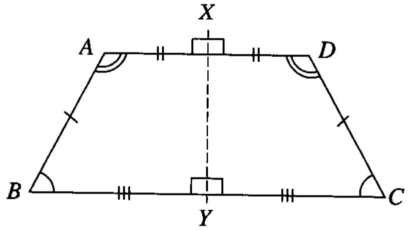


Fig. 4.14

We should have no difficulty establishing that an isosceles trapezium is a quadrilateral which has

(i) one pair of parallel sides (AD and BC in Fig. 4.14).
(ii) one pair of equal sides (AB and DC in Fig. 4.14).
(iii) two pairs of equal angles ($B\hat{A}D = C\hat{D}A$ and $A\hat{B}C = D\hat{C}B$).

Note: A general trapezium is a quadrilateral which has a pair of parallel sides.

Parallelogram

A quadrilateral which has rotational symmetry of order 2 about the centre is a **parallelogram**. Fig. 4.15 shows a parallelogram.

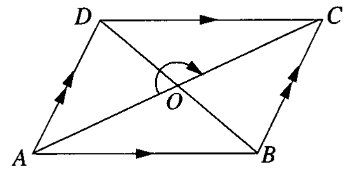


Fig. 4.15

To study the properties of a parallelogram, we can make a cardboard parallelogram $ABCD$. Pin the cardboard to a piece of paper with a thumbtack at the centre O. Trace around the parallelogram and rotate it 180° clockwise (see Fig. 4.16).

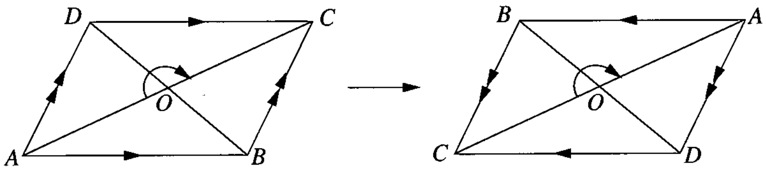
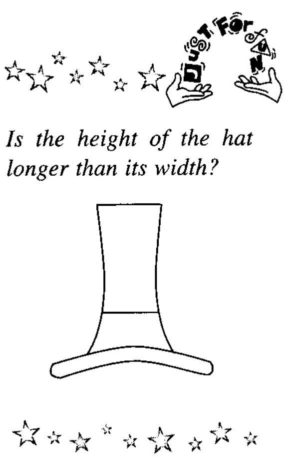


Fig. 4.16

Trace around it again. The two tracings should coincide. Thus the parallelogram $ABCD$ maps onto itself. AB coincides with CD and therefore $AB = CD$. Similarly, AD maps onto CB, AO maps onto CO and BO maps onto DO. Hence $AD = CB$, $AO = CO$ and $BO = DO$. Also, $D\hat{A}B$ coincides with $B\hat{C}D$ and $A\hat{B}C$ coincides with $C\hat{D}A$. Hence, $D\hat{A}B = B\hat{C}D$ and $A\hat{B}C = C\hat{D}A$.

From the above discussion, we can say that a parallelogram is a quadrilateral which has

(i) opposite sides equal and parallel,
(ii) opposite angles equal,
(iii) the diagonals bisecting each other.



Rhombus

A **rhombus** is a quadrilateral with both diagonals as lines of symmetry. It also has rotational symmetry of order 2.

Fig. 4.17 shows a rhombus. Diagonals AC and BD are lines of symmetry.

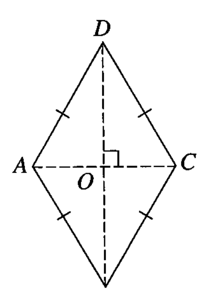


Fig. 4.17

Since a rhombus has rotational symmetry of order 2, it is a parallelogram and has all the properties of a parallelogram. Also, with one diagonal as line of symmetry, a rhombus is a kite. Hence, it has all the properties of a kite. An extra line of symmetry adds two more properties: all sides of a rhombus are equal and the two diagonals bisect the opposite angles.

Rectangle

A **rectangle** is also a quadrilateral which possesses rotational symmetry of order 2. Hence, it has all the properties of a parallelogram. But a rectangle has two lines of symmetry which are not diagonals (see Fig. 4.18).

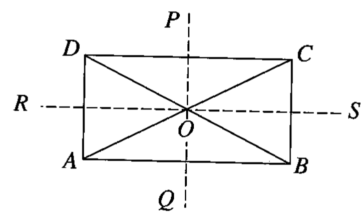


Fig. 4.18

(a) ☐

How many straight lines can you draw to divide the square into two congruent parts?

(b) ☐

How many lines can you draw to divide the rectangle into two congruent parts?

Since PQ and RS are lines of symmetry, we have

$$A\hat{B}C = B\hat{C}D = A\hat{D}C = B\hat{A}D$$

and $\quad OA = OB = OC = OD.$

But $\quad A\hat{B}C + B\hat{C}D + A\hat{D}C + B\hat{A}D = 360°$

$$A\hat{B}C = B\hat{C}D = A\hat{D}C = B\hat{A}D = \frac{360°}{4} = 90°$$

$$AC = OA + OC = OB + OD = BD$$

Hence, a rectangle has all the properties of a parallelogram and also all the angles of a rectangle are right angles and the two diagonals are equal.

Square

A **square** is a quadrilateral with four lines of symmetry as shown in Fig. 4.19.

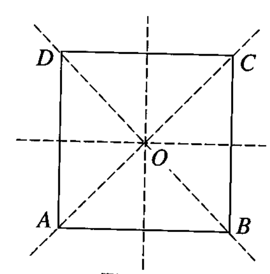


Fig. 4.19

A square is a quadrilateral which has all the properties of a rectangle and a rhombus. Can you explain?

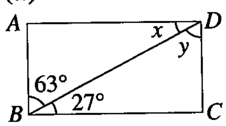
Exercise 4d

1. Sketch each of the following:
   - **(a)** A parallelogram that has all its sides equal but not all its angles equal.
   - **(b)** A rectangle that is not a square.
   - **(c)** A rhombus that is not a square.
   - **(d)** A parallelogram that has all its angles equal but not all its sides equal.
   - **(e)** A parallelogram that has all its sides and angles equal.
   - **(f)** A trapezium with three equal sides.
   - **(g)** An isosceles trapezium.

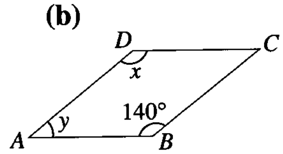
2. State whether each of the following is true or false. If it is false, explain why.
   - **(a)** A rectangle is a parallelogram.
   - **(b)** A trapezium is a parallelogram.
   - **(c)** A parallelogram is a trapezium.
   - **(d)** A rhombus is a square.

(e) A square is a rhombus.
(f) A parallelogram is a rhombus.
(g) A rhombus is a parallelogram.
(h) A kite is a rhombus.
(i) A rhombus is a kite.
(j) Opposite sides of a rectangle are equal.
(k) A parallelogram with one right angle is a rectangle.
(l) A rhombus with one right angle is a square.
(m) A rectangle with two adjacent sides equal is a square.

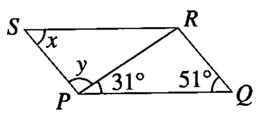
3. Find the value of x and of y in each of the following diagrams:

(a)


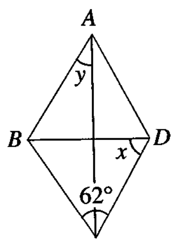
ABCD is a rectangle.

(b)


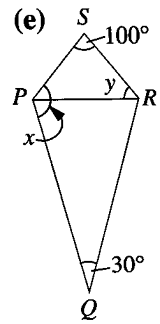
ABCD is a parallelogam.

(c)


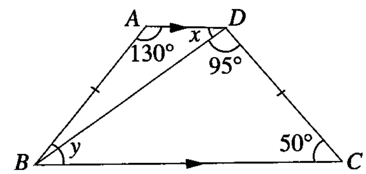
PQRS is a parallelogram.

(d)


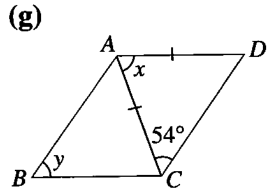
ABCD is a rhombus.

(e)


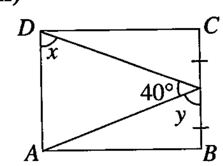
PQRS is a kite.

(f)


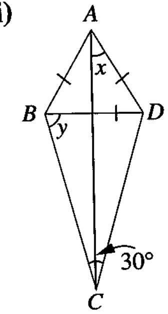
ABCD is a trapezium.

(g)


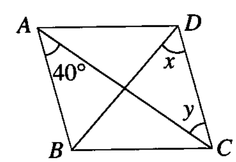
ABCD is a parallelogram.

(h)


ABCD is a rectangle.

(i)


ABCD is a kite.
AB = *AD* = *BD*

(j)


ABCD is a rhombus.

Plane Symmetry

The diagrams below show a cuboid being cut by a plane symmetrically in two different ways.

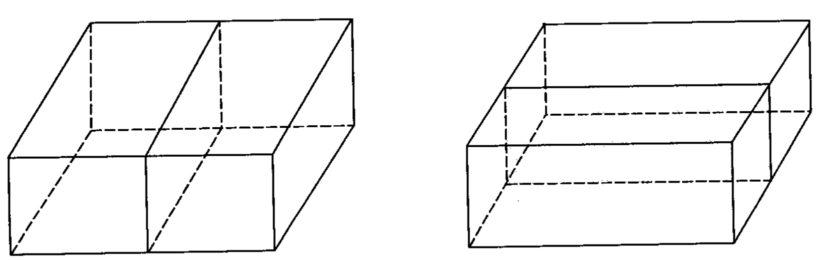

(a) (b)

Fig. 4.20

Can you find a few more planes of symmetry for the cuboid other than those shown above?

The cuboid is said to be **symmetrical to a plane** since that part of the cuboid on one side of the plane is a mirror image of the part on the other side of the plane. We call the plane a **plane of symmetry**.

Can you find a few more axes of rotational symmetry for the cuboid other than XY?

You can find very interesting animated sections on plane symmetry and rotational symmetry on solids in the CD, Space Trek through Symmetry, from the DMS. Have fun going through these sections!

Fig. 4.21 shows a cuboid where $AB \neq BC$ rotating about a line XY which passes through the centre of two parallel faces.

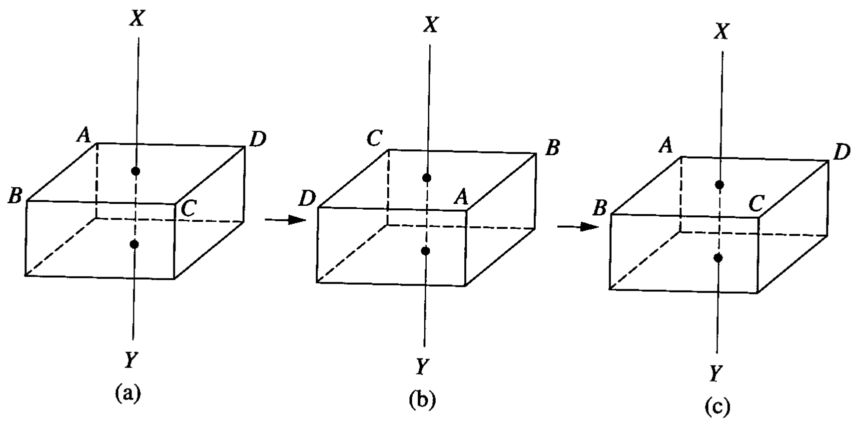


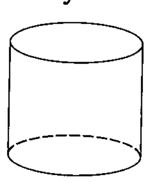
Fig. 4.21

XY is called the **axis of rotational symmetry**. The cuboid is rotated 180° clockwise about XY. The cuboid looks the same although we notice that the position of the vertex A of the upper face of the cuboid has changed (see Fig. 4.21(b)). If the cuboid is rotated another 180° clockwise about XY, the vertex A returns to the original position (see Fig. 4.21(c)). As in the case of rotational symmetry in two dimensions, we say that the cuboid has a rotational symmetry of order 2 about the axis XY.

Exercise 4e

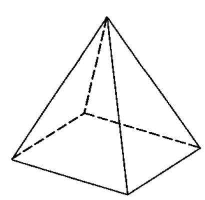
1. Copy the following figures and, for each, draw two planes of symmetry and one axis of rotational symmetry:

(a)



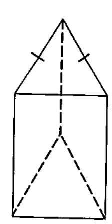
A right cylinder

(b)



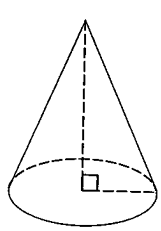
A right pyramid with a rectangular base

(c)



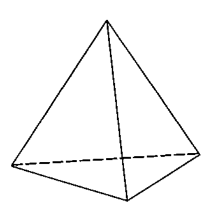
A right prism with an equilateral triangle as the base

(d)



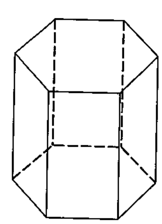
A right circular cone

(e)



A regular tetrahedron

(f)



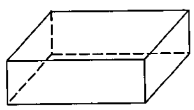
A regular right hexagonal prism

∗2. How many planes of symmetry are there in a cube? Does a cube have more planes of symmetry than a cuboid?

∗3. How many planes of symmetry does a right pyramid with a square base have altogether?

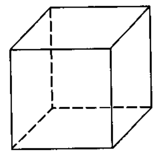
∗4. State **(i)** the number of planes of symmetry, **(ii)** the number of axes of rotational symmetry for the following solids:

(a)



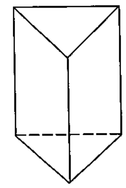
A cuboid

(b)



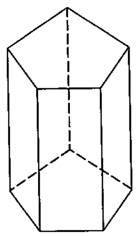
A cube

(c)



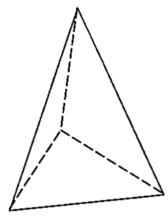
A right prism with an equilateral triangle as the base

(d)



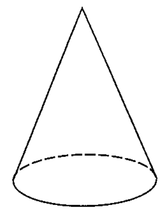
A right pentagonal prism

(e)



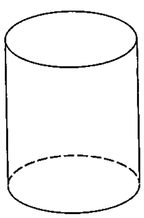
A regular tetrahedron

(f)



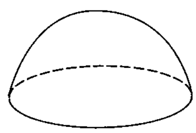
A right circular cone

(g)



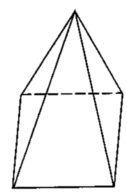
A right circular cylinder

(h)



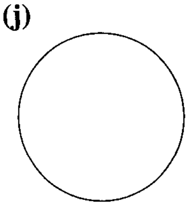
A hemisphere

(i)



A square pyramid

(j)

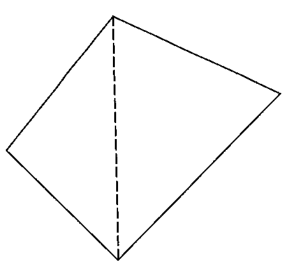


A sphere

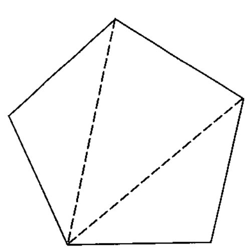
Sum of the Interior Angles of a Polygon

In-Class Activity

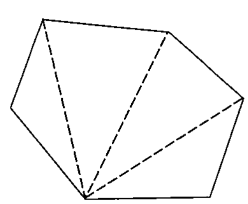
1. Draw the following polygons where one vertex has been joined to the other vertices.



Quadrilateral



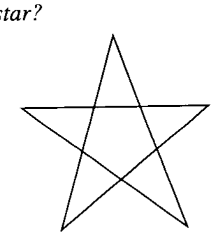
Pentagon



Hexagon

Problems

What types of polygons and how many of each kind can you find in this star?



2. How many triangles does each of the above have? What is the sum of the interior angles of the above polygons?

3. Repeat the above process for polygons with 7, 8, 9 and 10 sides and find the number of triangles each polygon can form.

4. Copy and complete the following table:

Name of Polygon	*No. of sides*	*No. of triangles formed*	*Sum of interior angles (in terms of right angles)*
Quadrilateral	4	2	$2 \times 2 = (4 - 2) \times 2$
Pentagon	5	3	$3 \times 2 = (5 - 2) \times 2$
Hexagon	6	4	$4 \times 2 = (6 - 2) \times 2$
Heptagon	7		
Octagon	8		
Nonagon	9		
Decagon	10		
n-gon	n		

5. From the above table, what can you say about the number of triangles formed by a polygon in relation to its number of sides? In general, we have the following conclusion:

the sum of the interior angles of an n-sided polygon is $(n - 2) \times 180°$ or $(n - 2) \times 2$ rt $\angle$s or $(2n - 4)$ rt $\angle$s.

Sum of the Exterior Angles of a Polygon

Let us consider a 6-sided polygon in which each side is produced (Fig. 4.22). The interior angles of the polygon are named a, b, c, d, e and f while the exterior angles are named p, q, r, s, t and u.

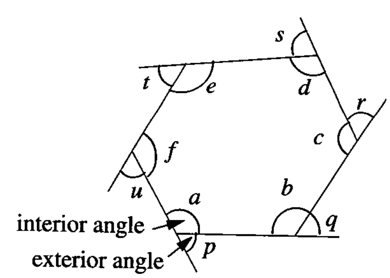


Fig. 4.22

Now, $a + p = 180°$, $b + q = 180°$
$c + r = 180°$, $d + s = 180°$
$e + t = 180°$, $f + u = 180°$

i.e., $a + p + b + q + c + r + d + s + e + t + f + u = 1\,080°$
$(a + b + c + d + e + f) + (p + q + r + s + t + u) = 1\,080°$

But we know that the sum of the interior angles of a 6-sided polygon is 8 right angles, i.e., 720°.

Thus, $720° + (p + q + r + s + t + u) = 1\,080°$
$p + q + r + s + t + u = 360°$

By using this method, the sum of the exterior angles of a polygon with 5 sides, 7 sides or any other number of sides can be shown to be equal to 360°.

In general, we have the following:

the sum of the exterior angles of any polygon is 360°.

Example 2

Find the sum of the interior angles of a polygon with 20 sides.

Solution

The sum of the interior angles $= (20 - 2) \times 180°$
$= 3\,240°$

Example 3

Calculate the size of an interior angle of a regular octagon.

Solution

Method 1:
A regular octagon has 8 equal exterior angles. Since the sum of the exterior angles is 360°, each exterior angle $= \frac{360°}{8} = 45°$.

Now, the sum of the interior and exterior angles at any vertex is 180° as they are adjacent angles on a straight line.
∴ an interior angle $= 180° - 45° = 135°$

Method 2:
Sum of the interior angles of an octagon $= (2 \times 8 - 4)$ rt ∠s
$= 12 \times 90°$

∴ each interior angle of the regular octagon $= \frac{12 \times 90°}{8} = 135°$

We can use the open tool, Geometers' Sketch Pad (GSP), to explore and investigate the property that the sum of the exterior angles of any polygon is equal to 360°. Besides GSP, there are a few internet sites that have Java applets illustrating this property. Log on to one of these sites and verify it. (Check with your teacher for the site addresses.)

Example 4

Find the number of sides of a regular polygon whose interior angles are 156° each.

Solution

Method 1:
Each interior angle = 156°
Each exterior angle $= 180° - 156° = 24°$
Now, the sum of the exterior angles is 360°.
The number of sides, n, is given by $n \times 24° = 360°$

$$n = \frac{360°}{24°} = 15$$

∴ the polygon has 15 sides.

Method 2:

Let the number of sides of the polygon be n.

Sum of the interior angles = $(2n - 4) \times 90°$

$$\text{One interior angle} = \frac{(2n-4) \times 90°}{n}$$

$$156° = \frac{(2n-4) \times 90°}{n}$$

$$156n = 180n - 360$$

$$24n = 360$$

$$n = \frac{360}{24} = 15$$

$\therefore$ the polygon has 15 sides.

Exercise 4f

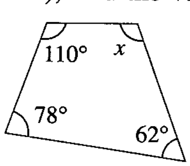
1. Calculate the sum of the interior angles of the following polygons:
   (a) a 10-gon, (b) a 12-gon,
   (c) a 15-gon, (d) an 18-gon.

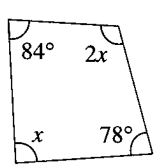
2. Find the size of each interior angle of a regular polygon with
   (a) 5 sides, (b) 20 sides,
   (c) 24 sides, (d) 36 sides.

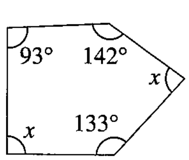
3. Find the number of sides of a regular polygon with an interior angle of
   (a) 140°, (b) 160°,
   (c) 162°, (d) 170°.

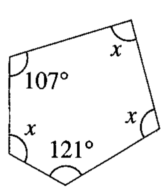
4. Find the number of sides of a regular polygon with an exterior angle of
   (a) 45°, (b) 36°,
   (c) 30°, (d) 12°.

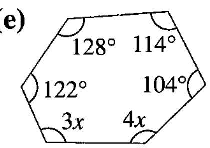
*5. For each of the following figures (not drawn to scale), find the value of x:

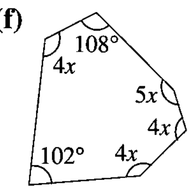
(a)


(b)


(c)


(d)


(e)


(f)


6. The angles of a quadrilateral are $3x$, $4x$, $5x$ and $6x$. Find x. What are the angles in degrees?

7. The exterior angles of a triangle are $3y°$, $4y°$ and $5y°$. Find y. What are the interior angles of the triangle?

8. In a quadrilateral $PQRS$, the angles P, Q, R and S are $3x$, $4x$, $5x$ and $8x$ respectively. Find **(a)** $\hat{P}$ and **(b)** $\hat{S}$.

9. A polygon has n sides. Three of its exterior angles are 70°, 25° and 15° while the remaining $(n - 3)$ exterior angles are each equal to 50°. Find n.

*10. Given that $ABCDE$ is a regular pentagon, calculate
   (a) $A\hat{B}C$, (b) $B\hat{C}A$, (c) $B\hat{D}E$.

*11. Given that $ABCDEFGH$ is a regular octagon, calculate
   (a) $A\hat{B}C$, (b) $A\hat{C}D$, (c) $A\hat{F}G$.

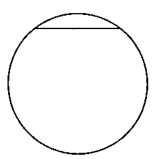
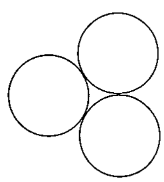
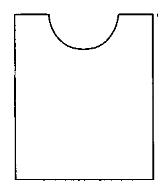
*12. The points A, B, C and D are consecutive vertices of a regular polygon with 20 sides. Calculate
   (a) $A\hat{B}C$, (b) $A\hat{C}D$, (c) $A\hat{B}D$.

Summary

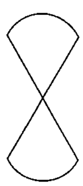
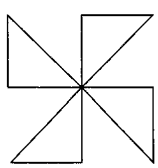
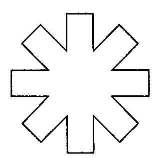
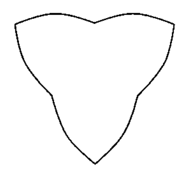
1. A line is a **line of symmetry** of a figure if the "half-shapes" on either side of the line are congruent. This figure is symmetrical. A symmetrical figure may have more than 1 line of symmetry.
2. A point is a **centre of rotational symmetry** of a figure if the figure maps onto itself under rotation about the point.
3. The **order of rotational symmetry** is the number of different ways in which a figure can map onto itself by rotation. In the case of order 1, we say that there is no rotational symmetry.
4. A plane is a **plane of symmetry** of a solid if the "half-shapes" on either side of the plane are congruent.
5. A line is an **axis of rotational symmetry** of a solid if the solid is invariant under rotation about that line.
6. The **sum of the interior angles of a polygon** is given by $(2n - 4)$ right angles where n is the number of sides of the polygon.
7. The **sum of the exterior angles of a polygon** is equal to 360°.

Review Questions 4

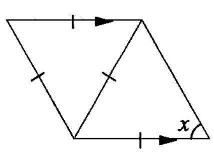
1. Copy each of the following diagrams and draw the axes of symmetry for each of them:

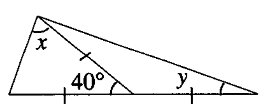
(a) 
(b) 
(c) 
(d) 

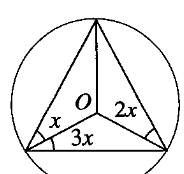
2. For each of the following figures, state **(i)** the number of lines of symmetry and **(ii)** the order of rotational symmetry:

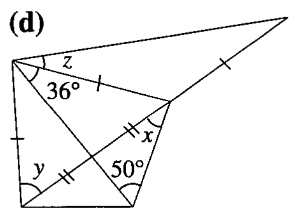
(a) 
(b) 
(c) 
(d) 

3. Find the unknown angles marked x and y in the following figures:

(a) 

(b) 

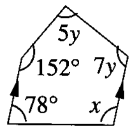
(c) 

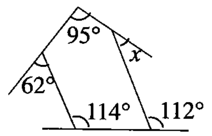
(d) 

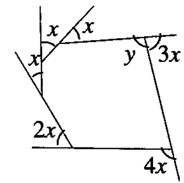
4. State the order of rotational symmetry of each of the following letters:

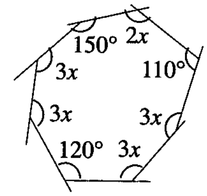
(a) A	(b) I	(c) M	(d) N
(e) X	(f) W	(g) Z	(h) H

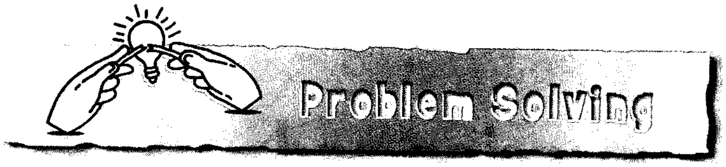
5. Find the values of the unknown marked in the following figures:

(a) 

(b) 

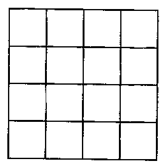
(c) 

(d) 



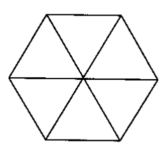
Problem Solving

1. The diagram has 16 small squares. On separate diagrams shade exactly two small squares so that the resulting figure will have



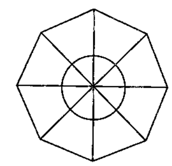
   (a) two lines of symmetry,
   (b) one line of symmetry,
   (c) rotational symmetry of order 2,
   (d) no rotational symmetry,
   (e) no line of symmetry.

2. The diagram shows a regular hexagon with six equilateral triangles. On separate diagrams, shade exactly two of the triangles so that the resulting figure will have



   (a) two lines of symmetry,
   (b) one line of symmetry,
   (c) rotational symmetry of order 2,
   (d) no rotational symmetry.

3. The diagram shows a regular octagon with an inner circle. On separate figures, shade certain parts of the figure so that the resulting figure will have

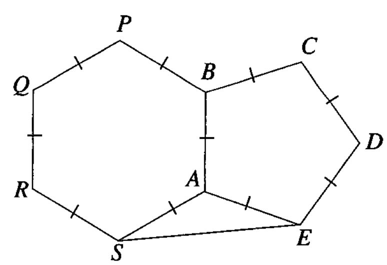


   (a) one line of symmetry,
   (b) two lines of symmetry,
   (c) four lines of symmetry,
   (d) eight lines of symmetry,
   (e) no line of symmetry,
   (f) no rotational symmetry.

4. If the sum of the interior angles of a polygon is four times the sum of its exterior angles, find the number of sides of the polygon.

5. The ratio of an interior angle to an exterior angle of a regular polygon is 11 : 2. Find the number of sides of the polygon.

6. In the figure, $ABCDE$ is a regular pentagon and $ABPQRS$ is a regular hexagon. Calculate the value of $A\hat{S}E$.



CHAPTER 5

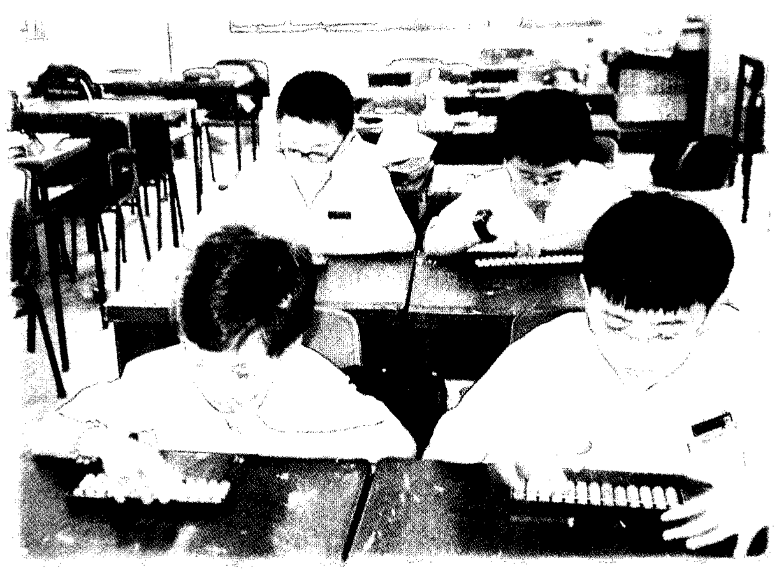
Expansion and Factorisation of Algebraic Expressions

In this chapter, you will learn how to

- △ expand products of simple algebraic expressions;
- △ factorise algebraic expressions.

Preliminary Problem

The children are learning how to use the abacus to help them calculate faster. Many algebraic properties can be used to help us calculate the answers for expressions faster. An example is the use of factorisation to calculate $2001^2 - 1999^2$. Do you know the answer?



Expansion of Algebraic Expressions

We shall use the distributive property to expand algebraic expressions.

Example 1

Expand (a) $(a + b)(a + b)$, (b) $(a - b)(a - b)$,
(c) $(a + b)(a - b)$.

Solution

(a) Let $(a + b) = k$

Now $(a + b)(a + b) = k(a + b)$
$= ka + kb$ (Distributive property)
$= (a + b)a + (a + b)b$ [Replace k by $(a + b)$.]
$= a^2 + ba + ab + b^2$ (Distributive property)
$= a^2 + ab + ab + b^2$ (Commutative property)
$= a^2 + 2ab + b^2$

$\therefore \quad (a + b)^2 = a^2 + 2ab + b^2$

(b) Let $(a - b) = k$

Now $(a - b)(a - b) = k(a - b)$
$= ka - kb$ (Distributive property)
$= (a - b)a - (a - b)b$ [Replace k by $(a - b)$.]
$= a^2 - ba - ab + b^2$ (Distributive property)
$= a^2 - ab - ab + b^2$ (Commutative property)
$= a^2 - 2ab + b^2$

$\therefore \quad (a - b)^2 = a^2 - 2ab + b^2$

(c) Let $(a + b) = k$

Now $(a + b)(a - b) = k(a - b)$
$= ka - kb$ (Distributive property)
$= (a + b)a - (a + b)b$ [Replace k by $(a + b)$.]
$= a^2 + ba - ab - b^2$ (Distributive property)
$= a^2 + ab - ab - b^2$ (Commutative property)
$= a^2 - b^2$

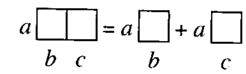
$\therefore \quad (a + b)(a - b) = a^2 - b^2$

The results of *EXAMPLE 1* are very useful in expanding expressions of a similar nature. These results are summarised below:

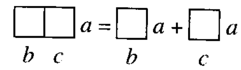
1. $\boldsymbol{(a + b)^2 = a^2 + 2ab + b^2}$
2. $\boldsymbol{(a - b)^2 = a^2 - 2ab + b^2}$
3. $\boldsymbol{(a + b)(a - b) = a^2 - b^2}$

For Your Information

Distributive property:
$a \times (b + c)$
$= (a \times b) + (a \times c)$

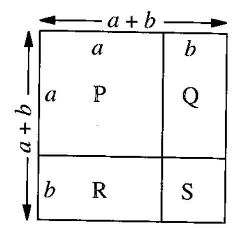


$(b + c) \times a$
$= (b \times a) + (c \times a)$



Commutative property:
$a \times b = b \times a$

Consider the area of a square whose sides are $(a + b)$.



What is the sum of the areas of P, Q, R and S? Do you obtain $a^2 + 2ab + b^2$?

Investigate

Can you use geometrical diagrams to illustrate the results of the other 2 identities, $(a - b)^2$ and $(a + b)(a - b)$?

Example 2

Expand the following expressions:

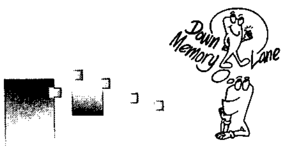
(a) $(2a + 3b)^2$, *(b)* $(5x - 3y)^2$, *(c)* $(3x + 2y)(3x - 2y)$.

Solution

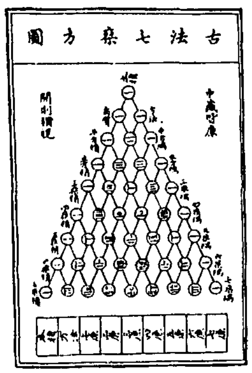
(a) $(2a + 3b)^2 = (2a)^2 + 2(2a)(3b) + (3b)^2$ (Use result 1.)
$= 4a^2 + 12ab + 9b^2$

(b) $(5x - 3y)^2 = (5x)^2 - 2(5x)(3y) + (3y)^2$ (Use result 2.)
$= 25x^2 - 30xy + 9y^2$

(c) $(3x + 2y)(3x - 2y) = (3x)^2 - (2y)^2$ (Use result 3.)
$= 9x^2 - 4y^2$



The Chinese were familiar with binomial expansions of $(a + b)^3$ and other higher orders. It appears that the famous "Pascal Triangle" was known in China long before Blaise Pascal was even born. The diagram below appeared in the 13th century text "Detailed Analysis of the Mathematics Rules in the Nine Chapters".



Example 3

Use the algebraic results shown above to evaluate the following:

(a) 201×199, *(b)* 797^2, *(c)* $39^2 + 78 + 1$.

Solution

(a) $201 \times 199 = (200 + 1)(200 - 1)$
$= 200^2 - 1^2$ (result 3)
$= 40\ 000 - 1$
$= 39\ 999$

(b) $797^2 = (800 - 3)^2$
$= 800^2 - 2(800)(3) + 3^2$ (result 2)
$= 640\ 000 - 4\ 800 + 9$
$= 635\ 209$

(c) $39^2 + 78 + 1 = 39^2 + 2(39)(1) + 1^2$ (result 1)
$= (39 + 1)^2$
$= 40^2$
$= 1\ 600$

Exercise 5a

1. Expand the following expressions:

(a) $(a + 4)^2$ **(b)** $(m + 7)^2$ **(c)** $(p - 2q)^2$ **(d)** $(4e - d)^2$ **(e)** $(2y - 1)^2$

(f) $(1 - 5c)^2$ **(g)** $\left(m - \frac{1}{4}\right)^2$ **(h)** $\left(\frac{3}{4}a - \frac{1}{4}b\right)^2$ **(i)** $(3x + 2y)^2$ **(j)** $\left(5x - \frac{1}{4}y\right)^2$

(k) $(2x - 5y)(2x + 5y)$ **(l)** $(3x - 4y)(3x + 4y)$

(m) $(t^3 - 4t)(t^3 + 4t)$ **(n)** $(x^4 - 3y)(x^4 + 3y)$

(o) $(x^3 - y)^2$ **(p)** $\left(\frac{3}{4}a - c^2\right)^2$ **(q)** $(-2t + 4)^2$ **(r)** $(-3x - 2y)(-3x + 2y)$

(s) $(5x^2 + 3y)(5x^2 - 3y)$ **(t)** $(5xy^2 + 2ab)^2$ **(u)** $(2a^3b - 3)^2$ **(v)** $(n^2 - t^3)^2$

(w) $\left(\frac{5}{8}x^2 - 2xy\right)^2$ **(x)** $\left(\frac{3}{4}a + \frac{2}{3}b\right)^2$

2. Use the algebraic results obtained to evaluate each of the following:

(a) 502×498	**(b)** 305×295	**(c)** 98×102	**(d)** $1\,203^2$
(e) $9\,001^2$	**(f)** 899^2	**(g)** 892^2	**(h)** $3\,205^2$
(i) $69^2 + 138 + 1$	**(j)** $78^2 + 312 + 4$	**(k)** $501^2 - 1\,002 + 1$	**(l)** $301^2 - 602 + 1$

Further Expansions

We shall now learn more about expanding algebraic products.

Find a method which you can use to solve the following problem without using a calculator.

$$\frac{88\,888\,888}{[(88\,888\,889)^2 - (88\,888\,888 \times 88\,888\,890)]}$$

Example 4

Expand the following:

(a) $(a + b)(c + d)$, *(b)* $(a + b)(c + d + e)$.

Solution

Method 1

(a) Let $c + d = k$

Now $(a + b)(c + d) = (a + b)k$
$= ak + bk$ (Distributive property)
$= a(c + d) + b(c + d)$ [Replace k by $(c + d)$.]
$= ac + ad + bc + bd$ (Distributive property)

(b) Let $(c + d + e) = k$

Now $(a + b)(c + d + e) = (a + b)k$
$= ak + bk$ (Distributive property)
$= a(c + d + e) + b(c + d + e)$
[Replace k by $(c + d + e)$.]
$= ac + ad + ae + bc + bd + be$

The CD, The Undersea World of Algebra, from the DMS has very interesting animations on expansion of algebraic expressions. Use it to learn about algebraic expansion and then proceed to do the activities.

Method 2

Notice from the above that in order to multiply two algebraic expressions, we multiply each term of one expression by each term of the other and simplify the result.

The above can be worked out more easily by using the following method.

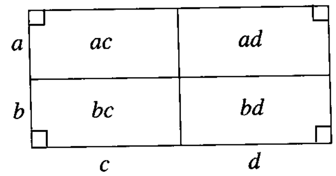
(a) $(a + b)(c + d) = ac + ad + bc + bd$ (Multiply across each term.)

(b) $(a + b)(c + d + e) = ac + ad + ae + bc + bd + be$

Method 3

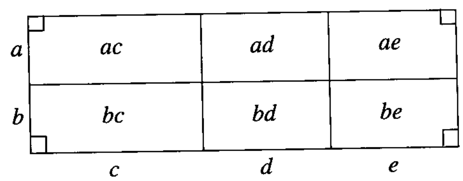
The above result may also be illustrated by the following figures:

(a)



$(a + b)(c + d) = ac + ad + bc + bd$ shows that the area of the rectangle whose sides are $(a + b)$ and $(c + d)$ is equal to the sum of the areas of the four smaller rectangles.

(b)



$$(a + b)(c + d + e) = ac + ad + ae + bc + bd + be$$

You can find out the age of your friend without being told by asking your friend to carry out the following instructions in the given order:

1. *Write down your age.*
2. *Add 5 to it.*
3. *Double the result.*
4. *Add 10 to it.*
5. *Multiply the result by 5.*
6. *Subtract 100 from it.*

Ask your friend to give you the final result and cross out the last digit to obtain your friend's age. Try this mathematical game with different people. Do you always get their age right? If so, can you explain why?

Example 5

Expand the following:

(a) $(a + 3)(a + 4)$
(b) $(2d + 3e)(d + 2e)$
(c) $(x - 2y)(x + 5y)$
(d) $(2x - y)(3x + 2y + 1)$

Solution

(a) $(a + 3)(a + 4) = a^2 + 4a + 3a + 12$
$= a^2 + 7a + 12$

(b) $(2d + 3e)(d + 2e) = 2d^2 + 4de + 3de + 6e^2$
$= 2d^2 + 7de + 6e^2$

(c) $(x - 2y)(x + 5y) = x^2 + 5xy - 2xy - 10y^2$
$= x^2 + 3xy - 10y^2$

(d) $(2x - y)(3x + 2y + 1) = 6x^2 + 4xy + 2x - 3xy - 2y^2 - y$
$= 6x^2 + xy + 2x - 2y^2 - y$

Example 6

Simplify the following:

(a) $(x + 5)(x - 4) - (x + 2)(x - 3)$, *(b)* $(x - 2y)(2x + y) - (3x + y)(5x - 4y)$.

Solution

(a) $(x + 5)(x - 4) - (x + 2)(x - 3)$
$= x^2 - 4x + 5x - 20 - (x^2 - 3x + 2x - 6)$ (Don't forget the brackets.)
$= x^2 + x - 20 - x^2 + x + 6$
$= 2x - 14$

(b) $(x - 2y)(2x + y) - (3x + y)(5x - 4y)$
$= 2x^2 + xy - 4xy - 2y^2 - (15x^2 - 12xy + 5xy - 4y^2)$ (Don't forget the brackets.)
$= 2x^2 - 3xy - 2y^2 - 15x^2 + 7xy + 4y^2$
$= -13x^2 + 4xy + 2y^2$

Note: Care must be taken with regard to signs.

Example 7

Find the following products:

(a) $(x + 2)(x + 5)$, *(b)* $(2x^2 + 3x + 2)(5x - 4)$,
(c) $(2x^3 - 7x + 6)(3x - 4)$.

Solution

(a) The product of $(x + 2)(x + 5)$ can be obtained by the distributive rule or by multiplying across each term. The following shows another method of obtaining the product of two polynomials.

$$\begin{array}{rcl} x + 2 & & \\ \times \quad x + 5 & & \\ \hline 5x + 10 & \longleftrightarrow & 5(x + 2) \\ x^2 + 2x \quad & \longleftrightarrow & x(x + 2) \\ \hline x^2 + 7x + 10 & & \end{array}$$

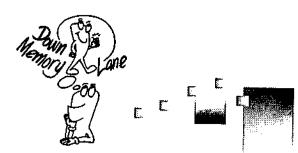
i.e., $(x + 2)(x + 5) = x^2 + 7x + 10$

(b)
$$\begin{array}{rcl} 2x^2 + 3x + 2 & & \\ \times \quad 5x - 4 & & \\ \hline -8x^2 - 12x - 8 & \longleftrightarrow & -4(2x^2 + 3x + 2) \\ 10x^3 + 15x^2 + 10x \quad & \longleftrightarrow & 5x(2x^2 + 3x + 2) \\ \hline 10x^3 + 7x^2 - 2x - 8 & & \end{array}$$

i.e., $(2x^2 + 3x + 2)(5x - 4) = 10x^3 + 7x^2 - 2x - 8$

(c)
$$\begin{array}{rcl} 2x^3 + 0x^2 - 7x + 6 & & \\ \times \quad 3x - 4 & & \\ \hline -8x^3 + 0x^2 + 28x - 24 & \longleftrightarrow & -4(2x^3 - 7x + 6) \\ 6x^4 + 0x^3 - 21x^2 + 18x \quad & \longleftrightarrow & 3x(2x^3 - 7x + 6) \\ \hline 6x^4 - 8x^3 - 21x^2 + 46x - 24 & & \end{array}$$

i.e., $(2x^3 - 7x + 6)(3x - 4) = 6x^4 - 8x^3 - 21x^2 + 46x - 24$



The Chinese regard odd numbers as yang (male) and even numbers as yin (female). In the days of Pythagoras, the number 1 was considered to be the original source of all numbers. The odd numbers were considered to be "masculine" numbers and the even numbers as "feminine" numbers.

Note: As there is no x^2 term in the first line, $0x^2$ is added. This is repeated in the third and fourth lines. After some practice, $0x^2$ may be omitted and a space is left where a term does not exist.

Exercise 5b

1. Expand the following:

(a) $(a + 3)(a + 7)$ **(b)** $(a - 5)(a - 6)$ **(c)** $(c - 2)(c + 7)$
(d) $(x + 1)(x - 9)$ **(e)** $(n + 9)(n + 9)$ **(f)** $(n - 9)(n - 9)$
(g) $(x + 3y)(x - y)$ **(h)** $(b - 4c)(b + 4c)$ **(i)** $(m + 2p)(m - 3p)$
(j) $(e + f)(e + 5f)$ **(k)** $(1 + 7a)(1 - 7a)$ **(l)** $(b - c)(-b + c)$
(m) $(1 - 2a)(1 + 5a)$ **(n)** $(4m + n)(m - 3n)$ **(o)** $(2d + e)(d + 3e)$
(p) $(y + 9)(2y + 1)$ **(q)** $(ab - 1)(ab - 1)$ **(r)** $(e - 10d)(e - 2d)$
(s) $(x - y)(x - 3y)$ **(t)** $(a - 5b)(a + 7b)$

2. Simplify the following:

(a) $(x - 7)(x + 8) + (x + 2)(x - 3)$ **(b)** $(x + 2y)(x - 6y) - (x - 3y)(x - y)$
(c) $(2x - 5y)(3x + y) + 5y(4x + y)$ **(d)** $(a - 3b)^2 - (3a - b)^2$
(e) $4(a + 3)^2 + 3(a - 4)(a + 7)$ **(f)** $2(2a + 1)(3a - 2) - 3(4a - 1)^2$
(g) $5p^2 - 4(p + q)(2p - 3q)$ **(h)** $2p(5p - 4q) - (3p + 2q)^2$

✶3. Find the following products:

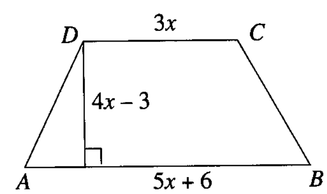
(a) $(x^2 + x + 1)(x + 2)$ **(b)** $(x^2 - x - 1)(x + 1)$ **(c)** $(x^2 + 2x - 1)(x - 1)$
(d) $(x^2 - 2x + 3)(x - 2)$ **(e)** $(a^2 + 3a - 2)(a + 3)$ **(f)** $(a^2 - 3a + 4)(a - 3)$
(g) $(a^2 + 4a + 1)(2a - 1)$ **(h)** $(a^2 - 4a + 2)(3a + 2)$ **(i)** $(2x^2 - 3)(x + 5)$

✶4. If $x^2 + y^2 = 14$ and $xy = 5$, find the value of $(x + y)^2$.

✶5. Given that $a + b = 10$ and $a^2 - b^2 = 40$, find the value of $a - b$.

✶6. If $x^2 + y^2 = 86$ and $xy = -16$, find the value of $(x - y)^2$.

✶7. The figure on the right shows $ABCD$ as a trapezium in which $AB = (5x + 6)$ cm, $DC = 3x$ cm and the height between the parallel lines is $(4x - 3)$ cm. Show that the area of the trapezium is $16x^2 - 9$.



Factorisation

In Book 1, we saw that in $15 = 3 \times 5$, 3 and 5 are called factors of 15. Similarly, in $ax + ay = a(x + y)$, a and $(x + y)$ are called the factors of $ax + ay$ and in $ax + ay + az = a(x + y + z)$, a and $(x + y + z)$ are the factors of $ax + ay + az$. The process of writing an algebraic expression as a product of two or more other algebraic expressions, which are called factors of the first algebraic expression, is called **factorisation**

If you want to confirm that your answers are correct, you can multiply the factors together to check whether you get back the given expression.

Example 8

Factorise the following:

(a) $3x + 12y$ *(b)* $x^2 + xy$ *(c)* $5d^3 - 20d^2e$
(d) $ad + dc + df$ *(e)* $2pq + 6p^2q - 4p^3q$

Solution

(a) $3x + 12y = 3(x + 4y)$ (b) $x^2 + xy = x(x + y)$

(c) $5d^3 - 20d^2e = 5d^2(d - 4e)$ (d) $ad + dc + df = d(a + c + f)$

(e) $2pq + 6p^2q - 4p^3q = 2pq(1 + 3p - 2p^2)$

Exercise 5c

Factorise the following expressions, where possible.

1. $c^2 + cb$
2. $4x + 8$
3. $am + ap$
4. $b^2 - 3bc$
5. $n^4 - n^3$
6. $2d^2 + 6de^2$
7. $3x^2 - 9xy$
8. $4d^2e - 16de$
9. $p^4 + p^6$
10. $y^2 + xy^2$
11. $5d - 25d^2$
12. $64y - 4x^2$
13. $2xp - rq$
14. $7d^3 - 28d^2k$
15. $2a^2b^3 - a^2b^2$
16. $d(x - y) - d^2$
17. $4a^2b^2 + 2ab$
18. $5m^2 - 10mn$
19. $c^2d^2 - 3de$
20. $5a^2 - 25ab$
21. $9a^5 - 6a^3$
22. $16x^3 - 8x^2$
23. $15abx - 9bx^2$
24. $2a^3 - 6a^2b$
25. $7ab^3 - 7a^2b$
26. $2a^3 - 4a^2y$
27. $x + 6xy - 9xz$
28. $14acd + 21c^2d - 7cd^2$
29. $2ar^2 + 2arh$
30. $8m - 8mn - 8p$
31. $2a^2b - 3ab^2c + 4acd$
32. $11p^2q + 22p^3q^2 - 33pq^2$
33. $x(2a + b) + 3y(2a + b)$
34. $2x^2(a + b) - 3y^3(a + b)$
35. $2y(x + 9y) + 3z(x + 9y)$
36. $3a(x - 3) - 6b(x - 3)$
37. $5y(m + 3n) + 2z(m + 3n)$
38. $7a^2 - 7a^3 + 7$
39. $2x^9 - 4y^2 + 8z^3$
40. $n^2a^2 - na^2 + na$
41. $p^2a^2 - p^2a^3 - p^3a^3$
42. $3x^2 + 9x^3y + 12x^2y^2$
43. $p^2z^3 + p^3z^4 + p^4z^5$
44. $abc + a^2b^2c^2 - a^3b^3c^3$

Perfect Squares and Difference of Two Squares

At the beginning of the chapter, we found out that

(1) $a^2 + 2ab + b^2 = (a + b)^2$,
(2) $a^2 - 2ab + b^2 = (a - b)^2$, and
(3) $a^2 - b^2 = (a + b)(a - b)$.

$(a + b)^2$ and $(a - b)^2$ are called **perfect squares** and $(a^2 - b^2)$ is called the **difference of two squares** These identities are useful in helping us to factorise certain expressions.

Example 9

Factorise the following:

(a) $x^2 + 6x + 9$ *(b)* $t^2 - 12t + 36$

(c) $k^2 - 81$ *(d)* $9a^2 - (b + c)^2$

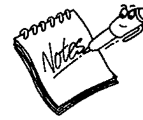
Solution

(a) $x^2 + 6x + 9 = x^2 + 2(3)(x) + 3^2$
$= (x + 3)^2$

(b) $t^2 - 12t + 36 = t^2 - 2(6)(t) + 6^2$
$= (t - 6)^2$

(c) $k^2 - 81 = k^2 - 9^2$
$= (k + 9)(k - 9)$

(d) $9a^2 - (b + c)^2 = (3a)^2 - (b + c)^2$
$= [3a + (b + c)][3a - (b + c)]$
$= (3a + b + c)(3a - b - c)$



Check your answers by expanding the factors and see whether you get back the original expressions.

Use the CD, The Undersea World of Algebra, from the DMS to explore factorisation of algebraic expressions. There are many interesting activities to help you understand the topic better.

Using factorisation can sometimes help us to do mental calculations in arithmetic.

Example 10

Evaluate the following by factorisation:

(a) $79 \times 83 - 69 \times 83$, *(b)* $103^2 - 9$.

Solution

(a) $79 \times 83 - 69 \times 83 = 83(79 - 69)$
$= 83(10)$
$= 830$

(b) $103^2 - 9 = 103^2 - 3^2$
$= (103 + 3)(103 - 3)$
$= 106 \times 100$
$= 10\ 600$

Exercise 5d

Factorise the following:

1. $a^2 - 25$
2. $x^2 - 64$
3. $4c^2 - b^2$
4. $36d^2 - 1$
5. $n^2 - 100m^2$
6. $p^2 - 81q^4$
7. $a^2b^2 - 1$
8. $121 - y^2$
9. $36x^2 - y^8$
10. $49c^2 - d^2e^2$
11. $c^2 - (d + 2)^2$
12. $4x^2 + 4xy + y^2$
13. $e^2 - 6ef + 9f^2$
14. $16n^2 + 8ne + e^2$
15. $9f^2 + 24fg + 16g^2$
16. $4c^2 - 8cd + 4d^2$
17. $25p^2 - 10pq + q^2$
18. $49y^2 + 42yz + 9z^2$
19. $9m^2 + 12mn + 4n^2$
20. $n^2 - n + \frac{1}{4}$
21. $\frac{1}{4}m^2 + nm + n^2$

★22. $\frac{9}{4}p^2 + \frac{3}{2}pq + \frac{1}{4}q^2$

★23. $16x^2 + 4xy + \frac{1}{4}y^2$

24. $9p^2 - 12pq + 4q^2$
25. $x^4 + 2x^2y + y^2$
26. $9(x - 3y)^2 - 16(2x + y)^2$
27. $4(6a - 5b)^2 - (3a - b)^2$

*28. $4m^2n^2 + 20mn + 25$

*29. $9x^2y^2 + 42xyz + 49z^2$

*30. $36a^2 + 6a^3 + 54a$

*31. $-x^2 + 2xy - y^2$

*32. $-m^6 + 2m^3n^3 - n^6$

*33. $8x^2y^4 - 56x^3y^3 + 98x^4y^2$

34. $1 - 25x^2y^2$

35. $x^2y^2 - 36z^2$

36. $x^2y^2 - 36y^2$

*37. $49x^4y^4 - 25z^4$

*38. $225x^3 - 169xy^2$

*39. $a^4 - b^4$

40. $\frac{a^2}{25} - \frac{4b^2}{9}$

*41. $50a^3 - 8ab^2$

*42. $\frac{x^2}{9} - \frac{y^2z^2}{16}$

43. $4x^2 - \frac{9}{25}$

44. $27a^2 - 3$

45. $32a^2b - 2b^3$

*46. $a^2b^2 - \frac{c^4}{64}$

Evaluate the following by factorisation:

47. $7.7^2 - 2.3^2$

48. $26.7^2 - 23.3^2$

49. $533^2 - 467^2$

50. $59^2 - 41^2$

51. $903^2 - 97^2$

52. $78^2 - 22^2$

53. $892^2 - 8^2$

54. $3\,205^2 - 205^2$

55. $43 \times 56 + 43 \times 44$

56. $37 \times 63 + 37^2$

57. $36 \times 49 + 36 \times 51$

58. $84^2 - 84 \times 74$

59. $245 \times 92 - 235 \times 92$

60. $27 \times 365 - 27 \times 265$

61. $5.16 \times 5.6 + 5.16 \times 4.4$

62. $3.26 \times 37 + 3.26 \times 63$

63. $587 \times 23 - 23 \times 487$

64. $395 \times 47 - 47 \times 385$

Factorisation by Grouping

Sometimes it is possible to find a common factor by grouping terms of an expression.

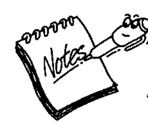
For example, $x^2 - xy + bx - by = x(x - y) + b(x - y)$
$= (x - y)(x + b)$

At other times it may be necessary to re-group the terms of an expression in order to find the common factor.

For example, $xd + yc + xc + yd = (xd + xc) + (yc + yd)$
$= x(d + c) + y(c + d)$
$= (d + c)(x + y)$

Factorisation may be simplified by changing the signs of the factors.

For example, $4(a - b) + x(b - a) = 4(a - b) - x(a - b)$
$= (a - b)(4 - x)$



$(b - a) = -(a - b)$
$(q - p) = -(p - q)$

Example 11

Factorise the following:

(a) $3x(p - 2q) - 2y(p - 2q)$

(b) $x^2 - 3x - xy + 3y$

(c) $x^3 - x^2 - 1 + x$

(d) $(a + 2b)^2 - (a + 2b)(3a - 7b)$

Solution

(a) $3x(p - 2q) - 2y(p - 2q)$
$= (p - 2q)(3x - 2y)$

(b) $x^2 - 3x - xy + 3y = x(x - 3) - y(x - 3)$
$= (x - 3)(x - y)$

(c) $x^3 - x^2 - 1 + x$
$= x^2(x - 1) - (1 - x)$
$= x^2(x - 1) + (x - 1)$
$= (x - 1)(x^2 + 1)$

(d) $(a + 2b)^2 - (a + 2b)(3a - 7b)$
$= (a + 2b)[(a + 2b) - (3a - 7b)]$
$= (a + 2b)[a + 2b - 3a + 7b]$
$= (a + 2b)(9b - 2a)$

Exercise 5e

Factorise the following where possible. Where there are no factors, state so.

1. $xa + xy + 3a + 3y$
2. $m^2 + mb + mc + bc$
3. $ac + ad - bd - bc$
4. $p^2 - pq - 2p + 2q$
5. $y + z - cy - cz$
6. $ax + ay - x^2 - xy$
7. $ac + ad - bd + bc$
8. $a^2 - 5a - ac + 5c$
9. $4q + 4p - q^2 - pq$
10. $7ab + 7ac + b + c$
11. $y^3 + y^2 + y + 1$
12. $a^2b + a^2c + 4c + 4b$
13. $m^2 - 3a + 3n - mn$
14. $2 - 4c - 2c^4 + c^3$
15. $10x + 10y - dy - dx$
16. $2c - 2d - d^2 + dc$
17. $py - qy - pq + q^2$
18. $e^2 - e - p^2 + p^2e$
19. $3bc - ac - 6ab + 2a^2$
20. $a^2 + a - 4 - 4a$
21. $4a - 4b - ma - mb$
22. $a - q + ap - pq$
23. $ad + bc - ac - bd$
24. $ax - by - ay + bx$
25. $px - py + qx - qy$
26. $ax + bx - ay - by$
27. $ap - bp - aq + bq$
28. $2am + an + 2bm + bn$
29. $2px + 3qx + 4py + 6qy$
30. $4ax - 2ay - 6bx + 3by$
31. $x^2 + yz + xz + xy$
32. $5p^2 + 15pq - 2pr - 6qr$
33. $a^3 - a^2 + a - 1$
34. $ap + bp - 2a - 2b$
35. $ax^2 + a^2x - 3xy - 3ay$
36. $24mn - ab - 3an + 8bm$
37. $8pq - 3qr + 12pr - 2q^2$
38. $a + b - ac - bc$
39. $6ab - 3a - 2b + 1$
40. $1 + p^2 + p^3q + pq$
41. $p^2q - pqr - 2pr + 2r^2$
42. $49x^2 - 7x + 7ax - a$
43. $4ax + 6by + 4bx + 6ay$
44. $3p^2 + 6pq - 4pr - 8qr$
45. $4a^4 + 6a^3 + 2a^2 + 3a$
46. $x^2y^2 - 5x^2y - 5xy^2 + xy^3$
47. $5a^2 - a(2b - 30)$
48. $4x^2 - 2x(a + b)$
49. $m^3 + m^2(2m - 1)$
50. $p(4m - n) - 2p(m + 2n)$
51. $(2a - b)(x + y) + (2a - b)(2x - 5y)$
52. $(3p + q)(2x - 3y) - (3p + q)(x - 4y)$
53. $(a - 3b)(4x + 5y) - (a - 3b)(5y + z)$
54. $6p(3x + 2y) - (3p + 2q)(3x + 2y)$
55. $(p + 4q)^2 + 3p + 12q$
56. $(p - 3q)^2 + (p - 3q)(p + 2q)$
57. $(2x + 5y)^2 - (2x + 5y)(3x - 2y)$
58. $(3x + 2y)^2 - 6x - 4y$
59. $3a - 6b + (a - 2b)^2$
60. $2x + 4y - 3(x + 2y)^2$
61. $(a + 3b)(2a - b) + (a + 3b)^2$

Summary

1. Important results:
   (a) $(a + b)^2 = a^2 + 2ab + b^2$
   (b) $(a - b)^2 = a^2 - 2ab + b^2$
   (c) $(a + b)(a - b) = a^2 - b^2$

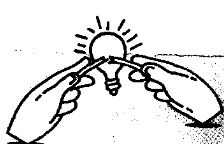
2. Factorisation of algebraic expressions can be done by
   (a) extracting common factors from all terms of the given expressions,
   (b) grouping terms in such a manner that the new terms obtained have a common factor.

Review Questions 5

1. Expand the following expressions:
   (a) $\left(\frac{2}{3}xy - 3\right)^2$ (b) $\left(3a + \frac{4}{5}b\right)^2$
   (c) $\left(-\frac{1}{4}a - \frac{1}{6}b\right)^2$
   (d) $\left(\frac{1}{4}abc - \frac{3}{2}x^2yz\right)^2$
   (e) $\left(\frac{3}{4}xy + \frac{1}{3}ab\right)\left(\frac{3}{4}xy - \frac{1}{3}ab\right)$
   (f) $\left(\frac{x}{2} + \frac{y}{4}\right)\left(\frac{x}{2} - \frac{y}{4}\right)$
   (g) $(2a + 3b)(3a + 4b)$
   (h) $(3a - 5b)(4a - b)$
   (i) $(5x + 2y)(x - 3y)$
   (j) $(7x - 4y)(x + 3y)$

2. Factorise the following:
   (a) $4x - 8(y - 2z)$
   (b) $5a^2 + 10a(b + c)$
   (c) $(x + y)(a + b) - (y + z)(a + b)$
   (d) $(2x + y)^2 - 3(2x + y)$
   (e) $(2a - 3b)(p + q) + (a - b)(p + q)$
   (f) $5(m - 2n) - (m - 2n)^2$
   (g) $ax + by - ay - bx$
   (h) $x^2 - 2xy + xz - 2yz$
   (i) $3a^3 - 2a^2 + 3a - 2$
   (j) $x^3 + x^2 - 4x - 4$
   (k) $9mn + 12m + 6n + 8$
   (l) $5mn - 10m - 3n + 6$

3. Factorise the following completely:
   (a) $6p^4 - 24q^2$
   (b) $2p^4 - 18p^2q^2$
   (c) $x^4y^2 - 4x^2y^4$
   (d) $32xy^4 - 2x^5$
   (e) $64a^4 - 4b^4$
   (f) $m^8 - 81$
   *(g) $p^2 - 9q^2 + 6qr - r^2$
   *(h) $4a^2 - b^2 - 2bc - c^2$



Problem Solving

1. Simplify $(a - b + c)^2 - (b - c + a)^2$.

2. Factorise the following:
   (a) $3x^{2a} + 6x^a$
   (b) $9y^{2a} + 15y^a$
   (c) $a^{4x} + 2a^{2x}$
   (d) $a^{2x} + a^{x+1}$
   (e) $4a^{2n} + 8a^n$
   (f) $a^b + a^{b+1} + a^{b+2}$
   (g) $a^b - a^{b-1} - a^{b-2}$
   (h) $2a^b + 4a^{b+1} + 4a^{b+2}$
   (i) $3x^a + 6x^a + 9x^{a+6}$

3. Factorise the following:
   (a) $x^2y^2 + 1 - x^2 - y^2$
   (b) $9a^{n+2} + 4a^{n-2}b^2 + 12a^nb$
   (c) $x^4 - 2x^2y^2 + y^4$
   (d) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Revision Exercise I No. 1

1. A man borrows $2 000 and agrees to repay $2 560 at the end of 4 years. What is the rate of simple interest?

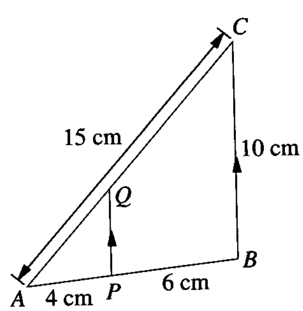
2. What is the largest number of books at $3.50 each that you can buy with $30? How much will you have left over?

3. An iron bar can be cut into 16 pieces, each 15 cm long. If 12-cm pieces were required instead, how many pieces could be cut from the bar?

4. A map is drawn using a scale of 1 cm to 8 km.
   (a) Find the R. F. of the map.
   (b) Find in cm, the distance on the map between two places which are 72 km apart.
   (c) Find in cm^2, the area on the map of a forest which has an area of 496 km^2.

5. State the number of axes of symmetry of
   (a) a regular octagon,
   (b) a parallelogram,
   (c) the word OXO.

6. **(a)** In the figure, $\triangle APQ$ is similar to $\triangle ABC$. Given that $AP = 4$ cm, $PB = 6$ cm, $BC = 10$ cm and $AC = 15$ cm, find the lengths of AQ and PQ.



   (b) Find the number of sides of a regular polygon whose exterior angle is 18°.

7. Simplify
   (a) $\sqrt{9a^4}$, **(b)** $\sqrt{p^4q^6}$,
   (c) $\sqrt[3]{27x^3}$, **(d)** $\sqrt[3]{8a^3b^9}$.

8. Multiply each of the following:
   (a) $a^2b^2c + 6b^2c^3 - 5a^2bc$ **by** $2ab^2c$
   (b) $-xy^3 - 2xy^2 + 4x^2y^2$ **by** $-3xz^3$
   (c) $3x^3 - 2x^2y^2 + 8xy^3$ **by** $3xy$
   (d) $5x + 6y$ **by** $-3x + 7y$

9. In 1997, the total trade in Singapore was $382 218 million. In 1998 the total trade dropped to $353 627 million due to the Asian economic crisis.
   (a) Express 382 218 million in standard form giving your answer correct to 3 significant figures.
   (b) Calculate the percentage drop in trade in 1998 as compared with 1997 figures, giving your answer correct to 2 decimal places.
   (c) Given that the total gross domestic product in 1997 was $141 262 million, how many times larger was the total trade figure as compared with the gross domestic product? Give your answer correct to 4 significant figures.

10. If $5x - 8y = 3(x - y)$, find the numerical value of $\frac{x}{3y}$.

Revision Exercise I No. 2

1. Find the quantity of which
   (a) 12.15 km is $67\frac{1}{2}\%$,
   (b) 5.4 kg is 45%.

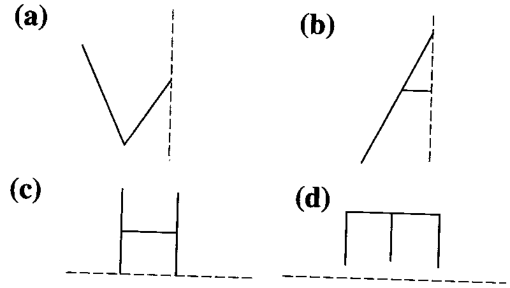
2. 20 tonnes of rice can feed 300 soldiers for 72 days. For how many days can 32 tonnes of rice feed 540 soldiers?

3. A sum of $360 is shared among 3 people in the ratio 2 : 3 : 7. Calculate the largest and smallest shares.

4. Simplify
   (a) $3x^2 - 5x - \{7x - [-2x^2 + x + 4(x - 2)]\}$,
   (b) $8x - [-(5x - 4) + 2(x - 1)]$.

5. Out of 240 pupils, 78 prefer Literature, 94 prefer Geography and the rest prefer History. Draw a pie chart to illustrate this information and state the angles in each sector.

6. Copy the following figures and complete them so that they are symmetrical about the dotted line:



7. Simplify the following expressions:
   (a) $(2a + 5b)^2 - (a + 3b)(a - 6b)$
   (b) $(4a + b)(4a - b) + (a - b)^2$

8. A map is drawn to a scale of 1 : 40 000.
   (a) Two towns are 18 km apart. Calculate, in cm, their distance apart on the map.
   (b) On the map, a park has an area of 18 cm². Calculate, in km², the actual area of the park.

9. The interior angle of a regular polygon is 35 times its exterior angle. How many sides has the polygon?

10. Factorise
   (a) $5x^2y - 15xy^2 - 25xy$
   (b) $2ax + 3by - 2ay - 3bx$

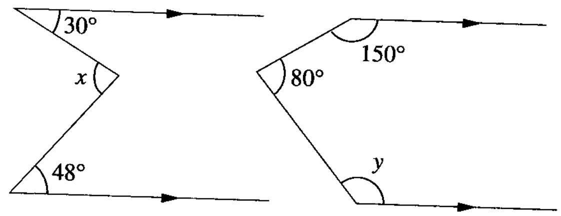
Revision Exercise I No. 3

1. Find the result of
   (a) decreasing \$900 by 25%,
   (b) increasing \$70 by 40%.

2. Find the simple interest on \$120 in 5 years at $4\frac{1}{2}\%$ per annum.

3. Two towns X and Y are 460 km apart. A car leaves X for Y at 65 km/h and a slower car leaves Y for X at the same time at 50 km/h. How long will it take the cars to meet each other?

4. Find the angles marked x and y in the following figures.



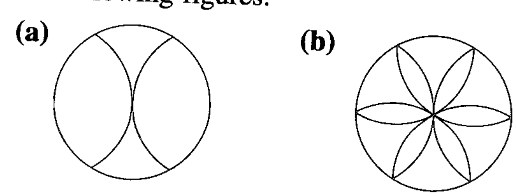
5. Evaluate each of the following, giving your answer in the standard form.
   (a) $(6 \times 10^3) \times (7.5 \times 10^4)$
   (b) $(2.8 \times 10^7) \div (7 \times 10^{-4})$
   (c) $7.2 \times 10^5 + 8.7 \times 10^4$
   (d) $6.7 \times 10^8 - 9.0 \times 10^7$

6. Write down the expansion of
   (a) $(2a - 5b)^2$;
   (b) $(2a + 5b)^2$;
   (c) $(2a - 5b)(2a + 5b)$.

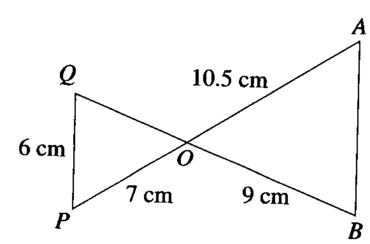
7. (a) $ABCDEF$ is a regular hexagon. Calculate $B\hat{A}C$ and $A\hat{C}D$.
   (b) Construct a rhombus of sides 5 cm each with one of its diagonals 7.6 cm long. Measure the length of the other diagonal.

8. A map is drawn using a scale of 5 cm to 8 km. Find the actual area of a forest measuring 75 cm² on the map.

9. State the number of axes of symmetry of the following figures.



10. In the figure, $\triangle POQ$ is similar to $\triangle AOB$. Given that OP = 7 cm, OA = 10.5 cm, PQ = 6 cm and OB = 9 cm, find the lengths of OQ and AB.



Revision Exercise I No. 4

1. A man works for 6 days and is paid \$51. How much would he be paid for working 9 days at the same rate?

2. Find the sum of money which amounts to \$2 800 in 5 years at 8% simple interest per annum.

3. Solve the following equations:

(a) $\frac{3}{4}(x + 1) + \frac{1}{2}(2x + 1) = 3\frac{1}{4}$

(b) $2x - 3 = \frac{1}{3}(x - 7)$

4. A cyclist took $3\frac{1}{3}$ hours to cover 46 km. For the first 30 km, he cycled at 15 km/h. Find his speed for the last part of the journey.

5. (a) The interior angle of a regular polygon is twice its exterior angle. Find the number of sides of the polygon.

(b) Four interior angles of a hexagon are 80°, 90°, 100° and 120° while the remaining angles are each equal to $x°$. Find x.

6. A model of a house is made using a scale of 1 : 100. If the height of the actual gate is 2 m and the area of the hall of the model is 16 cm², calculate

(a) the height of the gate of the model;

(b) the area of the actual hall.

7. The floor of a rectangular room measures 14 m by 10 m. Find, by scale drawing and measurement, the length of one of its diagonals.

8. Consider the following sequence of subtraction of fractions:

$$\frac{1}{1} - \frac{1}{2} = \frac{1}{2}$$
$$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$
$$\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$
$$\vdots$$
$$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{132}$$

(a) Write down the 5th line of the sequence.

(b) Find the value of x.

9. Simplify the following without using a calculator and express your answers in the standard form.

(a) $(7.2 \times 10^{6}) \times (4 \times 10^{-10})$

(b) $(4.72 \times 10^{5}) - (8.8 \times 10^{4})$

(c) $(2.54 \times 10^{3}) + (3.92 \times 10^{4})$

(d) $(2.89 \times 10^{2}) \div (3.4 \times 10^{-4})$

10. Solve the following equations:

(a) $(3x - 7) - (7x - 3) = 6$

(b) $3x + 4(x + 1) = 2(3x - 1)$

Revision Exercise I No. 5

1. Express the following as fractions:

(a) 0.375 **(b)** 1.64

(c) 76% **(d)** 3.2%

2. In 1999, there were 1 080 Singaporeans infected with the Aids virus, 150 more than in 1998. Of these, 11 were babies who caught the virus from their mothers, 126 were women and 88 of these women were married. Calculate

(a) the percentage increase in the number of Aids cases from 1998 to 1999,

(b) the percentage of male Aids carriers in the 1 080 Singaporeans,

(c) the percentage of women Aids carriers who were married.

3. Coffee powder of grade A at \$10 per kg and coffee powder of grade B at \$8 per kg are blended in the ratio 3 : 2. Find the cost of 1 kg of the blended coffee.

4. How long will it take \$1 250 to earn a simple interest of \$250 at 4% per annum?

5. Construct $\triangle ABC$ in which AB = 8 cm. $B\hat{A}C = 46°$ and $A\hat{B}C = 68°$. Measure the length of AC.

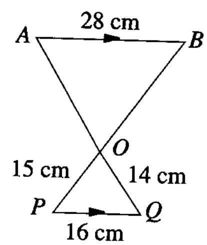
6. Simplify

(a) $2a^2b(a^2 + 3ab - 5b^2)$,

(b) $-3xy^2(2x^2 - 4xy + 7y^2)$.

7. Six of the interior angles of a plane 7-sided polygon are each equal to $x°$ while the remaining angle is $(x + 18)°$. Calculate x.

8. In the figure, $\triangle OBA$ is similar to $\triangle OPQ$. If AB = 28 cm, PQ = 16 cm, OP = 15 cm and OQ = 14 cm, calculate the length of AO and of BO.



9. Expand

(a) $(2xy + 1)(4x^2y^2 - 2xy + 1)$,

(b) $(3 - 2a)(3 + 2a)(9 + 4a^2)$.

10. The R. F. of a map is 1 : 400 000.

(a) Find the distance between two towns on the map which are actually 47 km apart.

(b) The distance between two towns on the map is 12.8 cm apart. Find the distance on actual ground.

(c) The area of a housing estate on the map is 3.2 cm^2, find its actual area, giving your answer in km^2.

CHAPTER 6

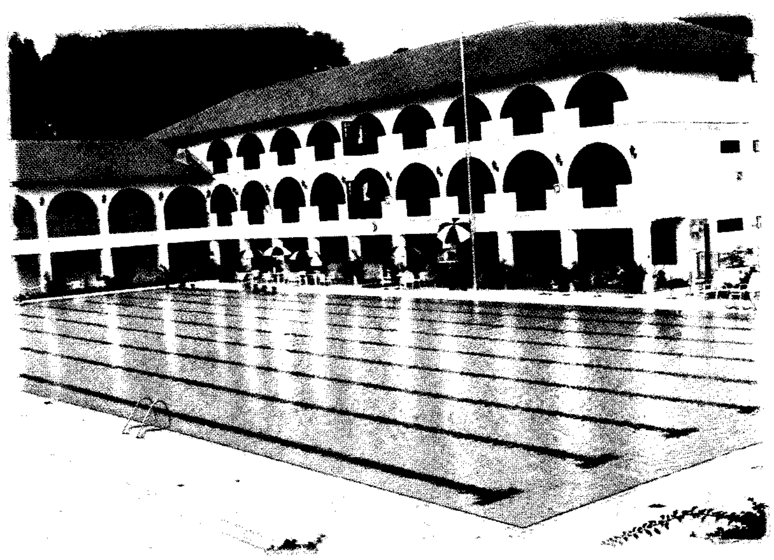
Solving Quadratic Equations by Factorisation

In this chapter, you will learn how to

- △ solve quadratic equations by factorisation;
- △ solve problems involving quadratic equations.

Preliminary Problem

We can solve several common types of mathematical problems by using quadratic equations.



For example, given that a swimming pool has an area of 1 200 m^2 and its width is 26 m shorter than its length, can you form a quadratic equation and use it to find the dimensions of the swimming pool?

Factorisation of Quadratic Polynomials

The general form of a quadratic polynomial is $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$. We shall learn how to factorise quadratic polynomials.

Example 1

Factorise $x^2 - 5x + 6$.

Solution

The first term is x^2, which is the product of x and x. Therefore, the first term in each bracket must be x, i.e.,

$$x^2 - 5x + 6 = (x \quad\quad)(x \quad\quad).$$

The last term is 6. The possible factors of 6 are ±1 and ±6 or ±3 and ±2. Thus we have the following choices:

$$(x + 1)(x + 6)$$
$$(x - 1)(x - 6)$$
$$(x + 3)(x + 2)$$
$$(x - 3)(x - 2)$$

The only pair of factors which gives $-5x$ as the middle term is $(x - 3)(x - 2)$.

$\therefore \quad x^2 - 5x + 6 = (x - 3)(x - 2)$

The method below will help you to obtain the factors more easily.

x	-3	$-3x$
x	-2	$-2x$
x^2	6	$-5x$

This is done by writing down the factors of the first term and the last term in two columns. Then cross-multiply the factors and write the products in the third column. Add the products of the third column. If the result is the same as the second term of the original expression, then the factors will be those shown in the first two rows.

Example 2

Factorise $3x^2 - 17x + 20$.

Solution

The factors of $3x^2$ are $3x$ and x, while those of 20 are ±1 and ±20 or ±4 and ±5 or ±2 and ±10.

Notes

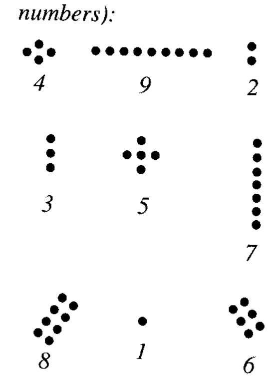
A polynomial is an algebraic expression that contains more than two terms, especially the sum of terms containing different integral powers of the same variable(s).

Just For Fun

Simplify this expression:
(88 – 8) (88 – 18)
(88 – 28) ... (88 – 188)

Down Memory Lane

The Chinese were familiar with the magic squares more than 3 000 years ago. According to popular myth, an emperor was walking along the banks of the Yellow River when he saw a tortoise which had black and white circles on its back, forming a magic square as follows (coloured circles represent even numbers and white circles represent odd numbers):



By trial and error,

First trial

$3x$	20		$20x$
x	1		$3x$
$3x^2$	20		$23x$

Second trial

$3x$	-4		$-4x$
x	-5		$-15x$
$3x^2$	20		$-19x$

Third trial

$3x$	-5		$-5x$
x	-4		$-12x$
$3x^2$	20		$-17x$

$\therefore \quad 3x^2 - 17x + 20 = (3x - 5)(x - 4)$

With sufficient practice, the possible combinations and trials can be done mentally and many steps in the working may be omitted.

Note: Some of the quadratic expressions cannot be factorised in this way. We shall look at such problems in Book 3.

The CD, Jungle Survival with Quadratic Equation, from the DMS deals with recognising quadratic equations, solving quadratic equations by factorisation, and solving word problems that leads to solving a quadratic equation. Have fun using the CD!

Exercise 6a

Factorise the following expressions wherever possible:

1. $a^2 + 6a + 8$	**2.** $c^2 + 3c + 2$	**3.** $m^2 + 9m + 8$
4. $b^2 + 11b + 28$	**5.** $e^2 - 4e + 4$	**6.** $x^2 - 11x + 24$
7. $m^2 + 9m + 20$	**8.** $p^2 + 8p + 21$	**9.** $a^2 - 9a + 14$
10. $m^2 - 13m + 36$	**11.** $x^2 - 9x + 18$	**12.** $p^2 + 7p + 12$
13. $a^2 - 14a + 48$	**14.** $c^2 - 6c + 15$	**15.** $x^2 - 13x + 40$
16. $m^2 - 8m + 15$	**17.** $n^2 - 18n + 17$	**18.** $ce^2 + 5ec - 6c$
19. $q^2 - 7q - 18$	**20.** $pn^2 + 3np - 28p$	**21.** $y^2 + 5y - 24$
22. $a^2 + 11a - 16$	**23.** $d^2 - d - 20$	**24.** $x^2y - 2xy - 3y$
25. $ab^2 + 5ba - 36a$	**26.** $e^2 - 7e - 14$	**27.** $1 - 2b - 24b^2$
28. $1 + 14x + 49x^2$	**29.** $1 - k - 20k^2$	**30.** $p^2 - p + \frac{1}{4}$
31. $a^2 + 8ab + 16b^2$	**32.** $x^2 - 10xy + 25y^2$	**33.** $4p^2 + 2p + \frac{1}{4}$
34. $2x^2 + 7xy - 15y^2$	**35.** $14m^2 - 29m - 15$	**36.** $14 - 31x + 15x^2$
37. $3a^2 - 10a + 7$	**38.** $4x^2 - 22x + 24$	**39.** $2x^2 + 11x + 12$
40. $12x^2 + 10x - 12$	**41.** $6a^2 + 19a - 20$	**42.** $5p^2 - 13p + 6$
43. $5p^2 + 7p - 6$	**44.** $6p^2 - 7p - 20$	**45.** $4a^2 + 7a + 3$
46. $4a^2 - 7a + 3$	**47.** $4m^2 + 8m + 3$	**48.** $6p^2 + 7p - 20$
***49.** $5p^2q^2 - 7pq - 6$	***50.** $4m^2 - 8mn + 3n^2$	***51.** $6p^2 - 7pq - 20q^2$
***52.** $12p^2 + 14pq - 40q^2$	***53.** $5p^2 + 13pq + 6q^2$	***54.** $6a^2b^2 - 19ab - 20$
***55.** $6x^2y^2 + 5xy - 6$	***56.** $6x^2 - 5xy - 6y^2$	***57.** $6x^2 + 33xy + 36y^2$
***58.** $8x^2y^2 - 44xy + 48$	***59.** $3a^2 + 10ab + 7b^2$	***60.** $14 - 31xy + 15x^2y^2$
***61.** $14x^2 - 11xy - 15y^2$	***62.** $2x^2y^2 - 7xy - 15$	***63.** $6x^2 - 9xy - 81y^2$
***64.** $10a^2 + 5ab - 15b^2$	***65.** $4m^2n^2 + 20mn + 25$	***66.** $a^2b^2 + 14ab + 49$

Solving Quadratic Equations by Factorisation

A quadratic equation is a polynomial equation in which the highest power of the unknown variable is two.

The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.

In the algebra of real numbers, if two numbers P and Q are such that $P \times Q = 0$, then either $P = 0$ or $Q = 0$ or both P and Q are equal to 0. We shall use this principle to solve quadratic equations.

Example 3

Solve the following equations:

(a) $(x-3)(x-4) = 0$ *(b)* $(2x-3)(4x-5) = 0$
(c) $x^2 - 3x - 28 = 0$ *(d)* $2x^2 - 7x + 6 = 0$

Solution

(a) $(x-3)(x-4) = 0$

Either $(x-3) = 0$ or $(x-4) = 0$

$\therefore x = 3$ or $x = 4$

(b) $(2x-3)(4x-5) = 0$

Either $(2x-3) = 0$ or $(4x-5) = 0$

$\therefore 2x = 3$ or $4x = 5$

$x = \frac{3}{2}$ or $x = \frac{5}{4}$

(c) $x^2 - 3x - 28 = 0$

$(x-7)(x+4) = 0$

$x - 7 = 0$ or $x + 4 = 0$

$\therefore x = 7$ or $x = -4$

x	-7	$-7x$
x	4	$4x$
x^2	-28	$-3x$

(d) $2x^2 - 7x + 6 = 0$

$(2x-3)(x-2) = 0$

$(2x-3) = 0$ or $(x-2) = 0$

$\therefore x = \frac{3}{2}$ or $x = 2$

$2x$	-3	$-3x$
x	-2	$-4x$
$2x^2$	6	$-7x$

Investigate

Assume $x = y$, multiply both sides by x : $x^2 = xy$
Subtract y^2 from both sides:

$x^2 - y^2 = xy - y^2$

Factorise both sides:

$(x+y)(x-y) = y(x-y)$

Divide both sides by $x - y$:

$x + y = y$

But $x = y$,

$2y = y.$

Divide both sides by y:

$\therefore 2 = 1$

Is it true that $2 = 1$?

Can you identify the mistake in the steps above?

Exercise 6b

1. Solve the following equations:

(a) $b^2 - 16 = 0$ **(b)** $4n^2 - 25 = 0$
(c) $64 - a^2 = 0$ **(d)** $3x^2 - 3 = 0$
(e) $2e^2 - 50 = 0$ **(f)** $4p^2 - 100 = 0$
(g) $m^2 - \frac{1}{4} = 0$ **(h)** $d^2 - \frac{16}{25} = 0$
(i) $\frac{4}{9} - \frac{x^2}{25} = 0$

2. Solve the following equations:

(a) $x^2 + 8x = 0$ **(b)** $3x^2 - 4x = 0$
(c) $3d - 81d^2 = 0$ **(d)** $4a^2 - 16a = 0$
(e) $7x^3 + 21x^2 = 0$ **(f)** $5x^2 + 15x = 0$

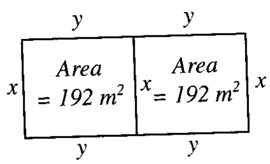
3. Solve the following equations:

(a) $e^2 - 16e + 64 = 0$
(b) $d^2 + 6d - 27 = 0$

(c) $a^2 + 12a + 36 = 0$
(d) $q^2 + 7q = 60$
(e) $b^2 - 7b - 120 = 0$
(f) $1 + 3a = 10a^2$
(g) $k^2 - 2k = 63$
(h) $3p^2 - 10p + 8 = 0$
(i) $2m^2 + 5m - 3 = 0$
(j) $4b^2 + 16b + 15 = 0$
(k) $4 - 3t - t^2 = 0$
(l) $15 + 2s - s^2 = 0$
(m) $12 - 7a + a^2 = 0$
(n) $3a^2 - 12a - 15 = 0$
(o) $2p^2 - 5p = -2$

★4. If $x = 3$ is a solution of the equation $x^2 + kx + 15 = 0$, find the value of k. Hence find the other solution of the equation.

Brian intends to build two semi-detached houses each with an area of 192 m² as shown below.



However, he is not sure what the measurements of x and y should be.

Can you help him select the cheapest design to build (this will be the one with the smallest wall length, i.e., 3x + 4y)?

Copy and complete the table to get the answer.

x	y	$3x + 4y$
192	1	580
96	2	
48		
24		
16		
12		
6		
3		
2		
1		

Which design is the cheapest to build? Explain why.

Problem Solving Involving Quadratic Equations

Many mathematical and real-life problems can be solved with the help of quadratic equations.

Example 4

Find two consecutive positive odd numbers such that the sum of their squares is equal to 130.

Solution

Strategy 1: Make a systematic list

The odd numbers are: 1, 3, 5, 7, 9, 11, 13, ….

Their squares are: 1, 9, 25, 49, 81, 121, 169, ….

From the above list we have 49 + 81 = 130

∴ the two consecutive positive odd numbers are 7 and 9.

Strategy 2: Use an equation

Let one number be x. The next consecutive odd number will be $(x + 2)$.

Hence
$$x^2 + (x+2)^2 = 130$$
$$x^2 + x^2 + 4x + 4 = 130$$
$$2x^2 + 4x - 126 = 0$$
$$x^2 + 2x - 63 = 0$$
$$(x - 7)(x + 9) = 0$$
$$\therefore \quad x = 7 \quad \text{or} \quad x = -9$$

When $x = 7$, $(x + 2) = 9$

$x = -9$ is not a solution because it is a negative number.

∴ the two consecutive positive odd numbers are 7 and 9.

Check: $7^2 + 9^2 = 49 + 81 = 130$.

Example 5

A man is now 5 times as old as his son. Four years ago, the product of their ages was 52. Find their present ages.

Solution

Let the boy be x years old now.

Therefore his father is $5x$ years old.

4 years ago, their ages were $(x - 4)$ and $(5x - 4)$ respectively.

Hence
$$(x-4)(5x-4) = 52$$
$$5x^2 - 24x + 16 = 52$$
$$5x^2 - 24x - 36 = 0$$
$$(5x+6)(x-6) = 0$$
$$\therefore \quad x = -\frac{6}{5} \quad \text{or} \quad x = 6$$

Since the boy cannot be $-\frac{6}{5}$ years old, the boy must now be 6 years old and his father 30 years old.

Check: 4 years ago, the boy was 2 years old and his father 26 years old. $(2 \times 26 = 52)$.

Just for fun

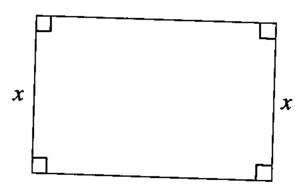
Paul and Julie had a date one Saturday. They agreed to meet at Marina Square at 8 p.m. Julie thought that her watch was faster by 5 minutes but in actual fact, it was slower by 5 minutes. Paul thought that his watch was slower by 5 minutes but in actual fact, it was faster by 5 minutes. Julie deliberately turned up 10 minutes late while Paul turned up 10 minutes earlier. Who turned up first and how long did she/he have to wait for the other person?

Example 6

The perimeter of a rectangle is 20 cm and its area is 24 cm². Calculate the length and breadth of the rectangle.

Solution

Let the breadth of the rectangle be x cm.



$$\therefore \quad \text{the length of the rectangle} = \frac{20 - 2x}{2} = (10 - x) \text{ cm}$$

$$\text{Area of the rectangle} = x(10 - x) = 24$$
$$10x - x^2 = 24$$
$$x^2 - 10x + 24 = 0$$
$$(x-4)(x-6) = 0$$
$$\therefore \quad x = 4 \quad \text{or} \quad x = 6$$

When $x = 4$, length $= 10 - 4 = 6$ cm

When $x = 6$, length $= 10 - 6 = 4$ cm

Since we normally assign the longer side to length, the length = 6 cm and the breadth = 4 cm.

Check: When the length = 6 cm and the breadth = 4 cm,
its perimeter $= 2(6 + 4)$ cm $= 20$ cm and
its area $= (6 \times 4)$ cm² $= 24$ cm².

Example 7

A shop proprietor bought x CD-Roms for $108.

(a) Write down an expression in terms of x, for the cost price, in dollars, of one CD-Rom.

(b) The proprietor priced each CD-Rom to sell at a profit of $1.50. Find an expression, in terms of x, for the selling price of each CD-Rom.

(c) Given that he was able to sell only 10 CD-Roms at this price and the remaining CD-Roms were sold at a price of $7.50 each, find an expression, in terms of x, for the total amount of money he received for the CD-Roms.

(d) If the proprietor received $120 altogether, form an equation in x and show that it reduces to $x^2 - 24x + 144 = 0$. *Hence find the value of x.*

Solution

(a) The price of each CD-Rom = $\$\frac{108}{x}$.

(b) The selling price of each CD-Rom = $\$\left(\frac{108}{x} + \frac{3}{2}\right)$

$= \$\frac{216 + 3x}{2x}$

(c) The total amount he received for the CD-Roms = $10\left(\frac{216 + 3x}{2x}\right) + (x - 10)7.50$

$= \frac{1\,080 + 15x + 7.5x^2 - 75x}{x}$

(d) We have $\frac{1\,080 + 15x + 7.5x^2 - 75x}{x} = 120$

i.e. $1\,080 + 7.5x^2 - 60x = 120x$

$7.5x^2 - 180x + 1\,080 = 0$

$x^2 - 24x + 144 = 0$

$(x - 12)(x - 12) = 0$

$\therefore \quad x = 12.$

Example 8

Mr Tan makes monthly visits to his parents in Malacca, a distance of 240 km from Singapore. He finds that if he increases the average speed by 10 km/h, he could save a total of 20 minutes for the journey. Find the speed at which Mr Tan originally travelled.

Solution

Let the speed at which Mr Tan normally travels be x km/h.

Time taken by Mr Tan = $\frac{240}{x}$ hours.

When the speed = $(x + 10)$ km/h, the time taken = $\frac{240}{x + 10}$ hours.

We have $\frac{240}{x} - \frac{240}{x+10} = \frac{20}{60}$

i.e. $\frac{240(x+10) - 240x}{x(x+10)} = \frac{1}{3}$

$$x(x + 10) = 3(2\,400)$$

$$x^2 + 10x - 7\,200 = 0$$

$$(x - 80)(x + 90) = 0$$

$\therefore \quad x = 80 \quad \text{or} \quad x = -90$ (not applicable)

Thus the speed at which Mr Tan originally travelled is 80 km/h.

Exercise 6c

1. Find the whole number such that four times the number subtracted from three times the square of the number makes 15.

2. Find the whole number such that twice its square added to itself makes 10.

3. Find two consecutive positive numbers such that the sum of their squares is equal to 113.

4. Find two consecutive positive odd numbers such that the sum of their squares is 74.

5. Find two consecutive positive even numbers such that the sum of their squares is 164.

6. The difference between two numbers is 9 and the product of the numbers is 162. Find the two numbers.

7. A rectangular field, 70 m long and 50 m wide, has a path of uniform width around it. If the area of the path is 1 024 m^2, find the width of the path.

8. The base and height of a triangle are $(x + 3)$ cm and $(2x - 5)$ cm respectively. If the area of the triangle is 20 cm^2, find x.

9. The difference between two numbers is 3. If the square of the smaller number is equal to 4 times the larger number, find the numbers.

10. The length of a rectangle is 5 cm longer than its width and its area is 66 cm^2. Find the perimeter of the rectangle.

11. Two positive numbers differ by 7 and the sum of their squares is 169. Find the numbers.

12. Two positive numbers differ by 5 and the square of their sum is 169. Find the numbers.

13. Mr Ong makes regular business trips to Kuala Lumpur, a distance of 420 km from Singapore.
    (a) On his way up to Kuala Lumpur he travels along the trunk road at an average speed of x km/h. Write down an expression, in terms of x, for the time taken, in hours, to travel from Singapore to Kuala Lumpur.
    (b) On his return journey to Singapore, he travels along the North South Highway and increases the average speed by 15 km/h. Write down an expression, in terms of x, for the time taken, in hours, to travel from Kuala Lumpur to Singapore.
    (c) If the time difference between the two journeys is 40 minutes, form an equation in x and show that it reduces to $x^2 + 15x - 9\,450 = 0$.
    (d) Solve the equation and find the time taken for the trip from Singapore to Kuala Lumpur.

14. Mr Kumar lives in the eastern part of Singapore. He visits his aged parents, who lives 36 km away, every weekend. He finds that if he increases the average speed of his vehicle by 12 km/h, he could save 9 minutes of his travelling time. Find the speed at which he travelled before the increase in speed.

15. The exchange rate for Australian dollar in January 2000 was AU\$100 = S\$$x$. In June 2000 the exchange rate had become AU\$100 = S\$$(x - 5)$. Mr Chong found that he could get an extra AU\$32 for every S\$672 that he exchanged in June as compared in January. Form an equation in x and solve it.

16. The price of petrol in Singapore was x cents per litre in December 1999. In April 2000 the price had increased by 12 cents per litre.
 (a) How many litres of petrol could be bought with \$58 in December 1999?
 (b) How many litres of petrol could be bought with \$58 in April 2000?
 (c) If the difference in the number of litres of petrol bought in December 1999 and April 2000 is $4\frac{11}{16}$, form an equation in x and show that it reduces to $x^2 + 12x - 14\,848 = 0$.
 (d) Solve the equation in (c) and use it to find the number of litres of petrol that could be bought with \$34 in December 1999, giving your answer correct to 1 decimal place.

Division of Polynomials (Optional)

We have learnt how to multiply a polynomial by another and to factorise a polynomial into its factors. We shall now look at how one polynomial may be divided by another.

Let us look at an example of division in arithmetic.

To divide 132 by 11, the steps taken can be shown as follows:

$$\begin{array}{r} 12 \\ 11\,\overline{)\,132} \\ \underline{11} \\ 22 \\ \underline{22} \\ 0 \end{array}$$

The steps taken above can also be shown in a different way when we write $132 = 10^2 + 3 \times 10 + 2 = 100 + 30 + 2$ and 11 as $10 + 1$. The pattern of division will be as follows:

$$\begin{array}{rcl} 10 + 2 & & \\ 10 + 1\,\overline{)\,100 + 30 + 2} & & \\ \underline{100 + 10} & \longleftrightarrow & 10(10 + 1) \\ 20 + 2 & & \\ \underline{20 + 2} & \longleftrightarrow & 2(10 + 1) \\ 0 & & \end{array}$$

If we substitute x for 10 above, we have

$$\begin{array}{rcl} x + 2 & \longleftarrow & \textit{quotient} \\ \text{divisor} \longrightarrow x + 1\,\overline{)\,x^2 + 3x + 2} & \longleftarrow & \textit{dividend} \\ \underline{x^2 + x} & \longleftrightarrow & x(x + 1) \\ 2x + 2 & & \\ \underline{2x + 2} & \longleftrightarrow & 2(x + 1) \\ 0 & & \end{array}$$

We call $(x^2 + 3x + 2)$ the **dividend**, $(x + 1)$ the **divisor** and $(x + 2)$ the **quotient**.

Thus, when the division is exact, we have

dividend = divisor × quotient.

Example 9

(a) Divide $(2x^2 + 11x + 14)$ by $(x + 2)$. *(b) Divide $(6x^2 - 7x - 9)$ by $(2x + 3)$.*

Solution

(a) $(2x^2 + 11x + 14) \div (x + 2) = 2x + 7$

$$\begin{array}{r|l} & 2x + 7 \\ \hline x + 2 & 2x^2 + 11x + 14 \\ & 2x^2 + 4x \\ \hline & 7x + 14 \\ & 7x + 14 \\ \hline & 0 \end{array}$$

(b) $(6x^2 - 7x - 9) \div (2x + 3)$

$$\begin{array}{r|l} & 3x - 8 \\ \hline 2x + 3 & 6x^2 - 7x - 9 \\ & 6x^2 + 9x \\ \hline & -16x - 9 \\ & -16x - 24 \\ \hline & 15 \end{array}$$

In (b), we cannot get an exact quotient when $(6x^2 - 7x - 9)$ is divided by $(2x + 3)$. The number 15 is called a **remainder**. Thus we have

$$(6x^2 - 7x - 9) = (2x + 3)(3x - 8) + 15$$

or

$$(6x^2 - 7x - 9) \div (2x + 3) = (3x - 8) + \frac{15}{2x + 3}$$

Thus, when the division is not exact, we have

dividend = divisor × quotient + remainder.

Example 10

Divide $x^3 + 2x - 7$ by $(x - 2)$.

Solution

$$\begin{array}{r|l} & x^2 + 2x + 6 \\ \hline x - 2 & x^3 \qquad + 2x - 7 \\ & x^3 - 2x^2 \\ \hline & 2x^2 + 2x \\ & 2x^2 - 4x \\ \hline & 6x - 7 \\ & 6x - 12 \\ \hline & 5 \end{array}$$

$\therefore \quad (x^3 + 2x - 7) \div (x - 2) = x^2 + 2x + 6 + \frac{5}{x - 2}$

Exercise 6d (Optional)

Do the following divisions:

1. $(x^3 + 2x^2 - 7x) \div x$
2. $(2x^4 + 3x^2 - 4x) \div 2x$
3. $(5x^3 + 2x^2 - 4x) \div 3x$
4. $(18x^3 + 4x^2 - 12x) \div 3x$
5. $(x^2 - 7x + 12) \div (x - 4)$
6. $(2x^2 - 13x + 21) \div (x - 3)$
7. $(6x^2 - 5x + 1) \div (2x - 1)$
8. $(5x^2 + 7x - 8) \div (x - 1)$
9. $(7x^2 + x - 71) \div (x + 3)$
10. $(14x^2 + 2x - 368) \div (x - 5)$
11. $(4x^2 - 7x - 160) \div (x - 7)$
12. $(3x^3 + 4x - 5) \div (x + 1)$
13. $(x^3 + 7x^2 - 4x) \div (x + 3)$
14. $(2x^3 + 4x + 459) \div (x + 6)$
15. $(5x^3 + 11x^2 - 5x + 1) \div (x + 2)$
16. $(8x^3 + 42x^2 + 18x + 17) \div (x + 5)$
17. $(9x^3 + 6x^2 + 4x + 5) \div (3x + 1)$
18. $(12x^3 + 12x^2 + 7x + 1) \div (2x + 1)$
19. $(3x^3 - 7) \div (x - 3)$
20. $(2x^4 - x^3 - 2x^2 + 8x) \div (2x - 1)$

Summary

If two factors P and Q are such that $P \times Q = 0$, then either $P = 0$ or $Q = 0$ or both P and Q are equal to 0. This principle is used to solve quadratic equations.

Review Questions 6

1. Factorise the following

(a) $a^2 + 15a - 16$ (b) $a^2 - 20a + 19$
(c) $x^2 + 13x + 36$ (d) $4p^2 - 12pq + 9q^2$
(e) $25p^2q^2 + 10pq + 1$
(f) $9 + 6y + y^2$
(g) $1 + 12xy + 36x^2y^2$
(h) $49 - 4a^2$ (i) $90x^2y^2 - 10$

2. Solve the following equations:

(a) $12x - 20 = x^2$ (b) $11x^2 = 26x + 21$
(c) $12x + 9 = 5x^2$ (d) $2x^2 - 11x + 5 = 0$
(e) $2x^2 - 5x - 3 = 0$
(f) $8x^2 + 41x + 36 = 0$
(g) $15a^2 - 7a - 88 = 0$
(h) $28a^2 - 17a = 65$

∗3. x is a number such that when $(x + 1)^2$ is divided by $(x - 2)$, the quotient is 16 and the remainder is $(x - 3)$. What are the values of x?

∗4. A man walks for x hours at a speed of $(x + 1)$ km/h and cycles for $(x - 1)$ hours at a speed of $(2x + 5)$ km/h. If the total distance travelled is 90 km, find x.

∗5. The length and breadth of a rectangle are $(5x + 3)$ cm and $(3x - 2)$ cm. Write down in terms of x,

(a) the perimeter, (b) the area.

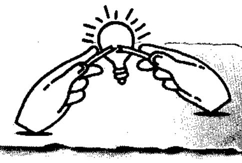
If the area of the rectangle is 230 cm², find the value of x and hence write down the perimeter of the rectangle.

6. Solve the following equations:

(a) $9n^2 - n - \frac{2}{3} = 0$ ∗(b) $10x - \frac{1}{x} = 3$
∗(c) $\frac{x}{2} + \frac{6}{x} = 4$ ∗(d) $x + \frac{6}{x} = 5$
∗(e) $3x - 8 = \frac{16}{x}$ ∗(f) $3x - 9 = \frac{27}{1 - 5x}$

∗7. A man travels by a car from Town A to Town B, a total distance of 100 km, at an average speed of x km/h. He finds that the time for the journey would be shorter by 25 minutes if he increased his average speed by 12 km/h. Find x.

∗8. If each pupil in a class sends a New Year greeting card to every classmate, the total number of cards sent out will be 870. Find the number of pupils in the class.



Problem Solving

1. If $x^2 - x + 1 = 2$, find the value of $1 + x - x^2$.

2. Given that $(a + b)^2 = 24$ and $ab = 3$, find the value of $(2a - 2b)^2$.

3. If n is a natural number, show that for all values of n, $(n^3 + 11n)$ is divisible by 6.

4. Solve the following equations:
   (a) $(2x - 3)(y + 1) = 0$
   (b) $(y - 1)^2 = 9$
   (c) $x(y + 1) + (y + 1) = 0$
   (d) $3xy + 2y - 12x - 8 = 0$
   (e) $x(x + 6) = 5(x + 4)$
   (f) $x(x - 4) - 10 = 2(x - 5)$
   (g) $8x - 2 + 4ax - a = 0$
   (h) $2ap - a + 4 - 8p = 0$
   (i) $xy - 2x - 6 + 3y = 0$
   (j) $3 - 3x + cx - c = 0$

5. Given that $4x^2 - 12xy + 9y^2 = 0$, find the value of the ratio $\frac{2x}{y}$.

6. Given that $16x^2 + 24xy + 9y^2 = 0$, find the value of the ratio $\frac{x}{y}$.

CHAPTER 7

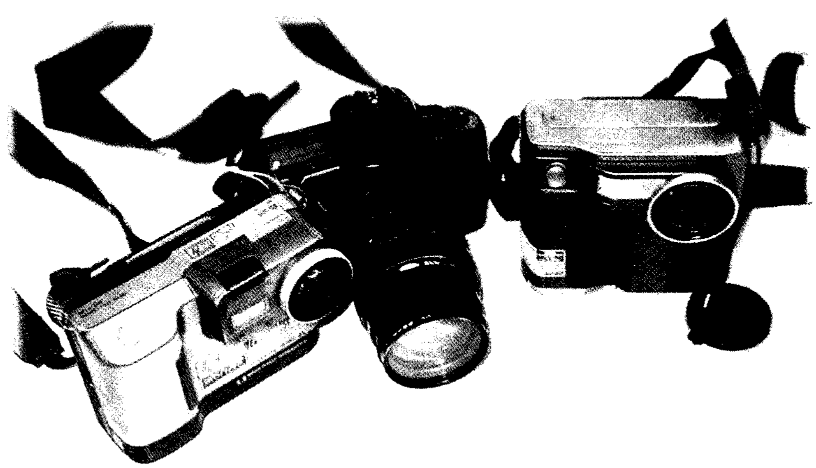
Algebraic Manipulation and Formulae

In this chapter, you will learn how to

- manipulate algebraic fractions;
- solve equations involving algebraic fractions;
- transform simple formulae.

Preliminary Problem

To develop the cameras shown in the picture, engineers have to rely on the important lens formula: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$. With the knowledge of this formula, they can then produce the cameras they want.



Simple Algebraic Fractions

The rules which are associated with numerical fractions are also applicable to algebraic fractions. One important rule to remember is:

Notes: $\frac{a}{b} \neq \frac{a+c}{b+c}$

The value of a fraction remains unchanged if both its numerator and denominator are multiplied or divided by the same non-zero number or expression.

i.e., $\frac{a}{b} = \frac{a \times c}{b \times c}$ **and** $\frac{a}{b} = \frac{a \div c}{b \div c}$**, where** $c \neq 0$**.**

Example 1

Simplify (a) $\frac{42}{63}$; *(b)* $\frac{24a^2b^3}{42a^5b}$.

Solution

(a) $\frac{42}{63} = \frac{2}{3}$ (b) $\frac{24a^2b^3}{42a^5b} = \frac{4b^2}{7a^3}$

Note: We obtain the results shown above by cancelling in the usual way, i.e. by dividing the numerator and the denominator by their common factors.

Exercise 7a

Simplify the following expressions:

1. $\frac{15x^3}{20x^4}$
2. $\frac{14x^5}{7x^2y}$
3. $\frac{16a^2b^2}{48a^3b}$
4. $\frac{23a^5b^7}{69a^3b^2c}$
5. $\frac{3m^2np^4}{12mn^3p^2}$
6. $\frac{15ac^7}{75a^3bc^2}$
7. $\frac{(4abc^2)^2}{24a^3bc^7}$
8. $\frac{(-2ab^2)^2}{8a^3b^2}$
9. $\frac{16a^2bc}{48ab^2c^4}$
10. $\frac{5(x^2y^3)^3}{5xy^4}$
11. $\frac{(4a)^2b^3}{(-2ab^2)^3}$
12. $\frac{(-2x)^3y^2z}{(4xy)^3}$
13. $\frac{(3a^3b^2)^2}{(2a^2b)^3}$
14. $\frac{27a^2bx^3}{(-9ax)^2}$
15. $\frac{-8a^2b^4x}{(-2abx)^3}$
16. $\frac{4x^2(x+y)}{8x^3(x+y)}$
17. $\frac{8a^2(a+b)^2}{16b(b+a)}$
18. $\frac{7ab^2(x+y)}{21b^3(x-y)}$
19. $\frac{9ab(x-3)}{18a^2b(x-9)}$
20. $\frac{x(a-b)^2}{(a-b)^3}$

*21. $\frac{9a^n}{18a^{n+1}}$

*22. $\frac{7a^{n+2}b}{21a^nb^2}$

*23. $\frac{5a^2b^3}{15a^{n+2}b^{m+4}}$

*24. $\frac{9x^{n+5}y^{n+2}}{15x^ny^{n+1}}$

*25. $\frac{15(a+b)^n}{18(a+b)^{n+1}}$

*26. $\frac{8a^n(b+c)^{n+1}}{32a^{n-1}(b+c)^{n+2}}$

Example 2

Simplify the following expressions:

(a) $\frac{3a^2b + 6ab^2}{a^2 + 2ab}$ *(b)* $\frac{x^2 - 3xy - 10y^2}{3x - 15y}$ *(c)* $\frac{2m^2 - 4m}{m^2 - 4}$

Factorise both the numerator and the denominator.

The denominator of a fraction cannot be 0. This also applies to the denominator of an algebraic fraction.

Thus in $\frac{x^2 - 3xy - 10y^2}{3x - 15y}$, $3x - 15y \neq 0$ *i.e.,* $x \neq 5y$.

Solution

(a) $\frac{3a^2b + 6ab^2}{a^2 + 2ab}$

$= \frac{3ab(a + 2b)}{a(a + 2b)}$

$= 3b$

(b) $\frac{x^2 - 3xy - 10y^2}{3x - 15y}$

$= \frac{(x - 5y)(x + 2y)}{3(x - 5y)}$

$= \frac{x + 2y}{3}$

(c) $\frac{2m^2 - 4m}{m^2 - 4}$

$= \frac{2m(m - 2)}{(m + 2)(m - 2)}$

$= \frac{2m}{m + 2}$

Note: Cancellation can usually be done after both the numerator and denominator have been completely factorised. NEVER cancel individual terms of the numerator or the denominator.

Are the following processes correct? Explain.

$\frac{{}^{1}\cancel{4} + \cancel{10}^{2}}{{}_{2}\cancel{8} - \cancel{5}_{1}} = \frac{3}{1}$, $\frac{3\cancel{a^2}\cancel{b} + {}^{3}\cancel{6}\cancel{a}b^2}{\cancel{a^2} + 2\cancel{ab}} = 3 + 3b^2$, $\frac{2\cancel{m}^2 - \cancel{4}m}{\cancel{m}^2 - \cancel{4}} = 2 - m$

Exercise 7b

Simplify the following fractions. If there is no simpler form, state so.

1. $\frac{a^2 + 2ac}{3a}$
2. $\frac{xy + 3y}{2x + 6}$
3. $\frac{p + n}{p - n}$
4. $\frac{5m - 4n}{m - n}$
5. $\frac{a^2 - b^2}{3a + 3b}$
6. $\frac{de}{d^2 - de}$
7. $\frac{y^2 - xy}{xy + x^2}$
8. $\frac{c - cd}{cd}$
9. $\frac{3m^2 - mn}{4m - 4n}$
10. $\frac{2xy + 4y}{x^2 + 2xy}$
11. $\frac{ab^2 + a^2b}{3a + 3b}$
12. $\frac{6e - 3f}{4e - 2f}$
13. $\frac{5e + 5d}{7e + 7d}$
14. $\frac{m^2}{m^2 - mp}$
15. $\frac{3a - b}{3a + b}$
16. $\frac{x^2 - y^2}{(x - y)^2}$
17. $\frac{c^2 + c}{cd + d}$
18. $\frac{a + b}{a^2 - b^2}$
19. $\frac{p^2 - p - 6}{p - 3}$
20. $\frac{(2c - 2d)^2}{(3c - 3d)^2}$
21. $\frac{m^2 + 4m - 32}{mq + 8q}$
22. $\frac{3x^2 + 5xy - 2y^2}{4x^2 + 7xy - 2y^2}$
23. $\frac{(2 - a)(2 - b)}{3a - 6}$
24. $\frac{p^2 + 6p - 7}{p^2 - p}$
25. $\frac{m^2 - 9}{m^2 - 7m + 12}$
26. $\frac{c^2 + 7c + 10}{c^2 + 5c}$
27. $\frac{q^2 - 2q - 15}{q^2 - 3q - 10}$
28. $\frac{x^3y - 2x^2y^2}{2xy^3 - x^2y^2}$
29. $\frac{(a - c)^2}{ac - a^2}$
30. $\frac{e^2 + ed - 6d^2}{e^2 - 3ed + 2d^2}$
31. $\frac{9p^2 - q^2}{q^2 - 2pq - 3p^2}$
32. $\frac{8 - 2m - m^2}{2m^2 - 3m - 2}$
33. $\frac{(e + f)^2 - d^2}{(d + f)^2 - e^2}$
34. $\frac{(x + y)^2 - 4y^2}{x^2 - 9y^2}$
35. $\frac{a^2 - ab - ac + bc}{a^2 + ab - ac - bc}$
36. $\frac{a^2 + am - an - mn}{a^2 + am + an + mn}$
37. $\frac{ax - ay - x + y}{3x - 3y}$
38. $\frac{b^2 - a^2}{2a^2 + ab - 3b^2}$
39. $\frac{3d - 6}{d^2 + d - 6}$
40. $\frac{2y^2 - 17y + 21}{y^2 - 6y - 7}$

Multiplication and Division of Algebraic Fractions

Just for fun

Find the positive integers x, y and z such that

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1.$

We shall now learn how to multiply and divide algebraic fractions.

Example 3

Simplify (a) $\frac{4}{7} \times \frac{21}{32}$; *(b)* $\frac{ab}{c^2d} \times \frac{4cd}{6a^2b}$.

Solution

(a) $\frac{4}{7} \times \frac{21}{32} = \frac{3}{8}$

(b) $\frac{ab}{c^2d} \times \frac{4cd}{6a^2b} = \frac{2}{3ac}$

Note: We obtain the results above by cancelling in the usual way, i.e., by dividing the numerator and the denominator by their common factors.

Example 4

Simplify (a) $\frac{3}{10} \times \frac{35}{54} \div \frac{14}{15}$; *(b)* $\frac{x}{y} \times \frac{x^2z}{y^2} \div \frac{xz^2}{2y}$.

Solution

(a) $\frac{3}{10} \times \frac{35}{54} \div \frac{14}{15}$

$= \frac{3}{10} \times \frac{35}{54} \times \frac{15}{14}$

$= \frac{5}{24}$

(b) $\frac{x}{y} \times \frac{x^2z}{y^2} \div \frac{xz^2}{2y}$

$= \frac{x}{y} \times \frac{x^2z}{y^2} \times \frac{2y}{xz^2}$

$= \frac{2x^2}{y^2z}$

Note: To divide by a fraction, multiply by its reciprocal.

The reciprocal of $\frac{14}{15}$ is $\frac{15}{14}$. The reciprocal of $\frac{xz^2}{2y}$ is $\frac{2y}{xz^2}$.

The reciprocal of 0 is not defined. $\therefore$ $\frac{2x}{0}$ is not defined.

Exercise 7c

Simplify the following, giving your answers in the lowest terms.

1. $\frac{15a^2}{8ab^3c} \times \frac{4c}{5ab}$
2. $\frac{12ap^2}{8a^2b^3} \times \frac{16a^3b}{6bp}$
3. $\frac{3x^2}{9x^3y} \div \frac{8xy^2}{24(xy)^3}$
4. $\frac{(abc)^2}{(2c)^3} \div \frac{a^2b^3}{6b^2}$
5. $\frac{3m^2p^4}{14m^3p^3} \times \frac{7m^4p^2}{12mp^3}$
6. $\frac{9m^2n^5}{15n^2p^4} \div \frac{21m^3}{10np^3}$
7. $\frac{12x^3}{5a^2b^3} \div \frac{7x}{15ab}$
8. $\frac{a^2b}{4c^2d} \times \frac{2c^4d^5}{3ab^2}$

9. $\frac{12ab^2c^3}{4d^3} \times \frac{3d^5}{15a^2b^3}$

10. $\frac{14x^2y^3}{15z^4} \times \frac{20z^2}{28x^4y^4}$

11. $\frac{18a^4b^4}{7x^2y^3} \div \frac{6a^2b^3}{14xy^2}$

12. $\frac{3xy^2}{5x^3yz^2} \times \frac{15x^5y^2}{9xy^5} \div \frac{2x^5}{3z^2}$

13. $\frac{a^3b^2c}{3ad^4} \times \frac{2bd^2}{5ac} \div \frac{3b^3}{15ac^3}$

14. $\frac{3x^2y^3}{8z^2} \times \frac{6y^2z^3}{5x^4} \div \frac{9y^2}{10a^2z}$

15. $\frac{12a^3b}{3ab^2} \div \frac{4abc}{3ad} \times \frac{14d^2}{7bc}$

16. $\frac{5a}{9b} \times \frac{ac^2}{2b} \div \frac{c^3}{8b^4}$

17. $\frac{5h^3}{(5h)^2} \times \frac{15hk}{(8h)^2} \div \frac{75k}{16u^2}$

18. $\frac{6uv}{9v^3} \div \frac{8u^3v^2}{27u} \times \frac{3v}{16u^2}$

19. $2y \div \frac{4y}{5xy} \times \frac{64xy}{100x^3y^4}$

20. $\frac{ab^n}{bc} \times \frac{c^nd}{cd} \div \frac{b^{n+2}}{c^{n+3}}$

21. $\frac{2a^{n+2}}{b^n} \div \frac{5b^{n+2}}{4a^nc} \times \frac{6a^3}{15c^2}$

22. $\frac{(a+b)^n}{bc^2} \div \frac{(a+b)^{n+3}}{abc}$

A man buys a brand new watch. He finds that it is 30 seconds faster per hour than his old grandfather clock. However, the grandfather clock is slower by 30 seconds per hour than the official standard time. Is the new watch accurate?

The man decides to adjust the time on the new watch and the grandfather clock so that it coincides with the official time at 6 a.m. one morning. Give the time shown by the watch and the grandfather clock at 14 00 that day.

Highest Common Factor of Algebraic Expressions

The **Highest Common Factor** (HCF) of two or more algebraic expressions can be found in a way similar to that for numbers.

Example 5

Find the HCF of $12p^3q^2$, $8p^2qr^3$ and $4p^2q^3r$.

Solution

$12p^3q^2 = 2 \times 2 \times 3 \times p \times p \times p \times q \times q = 2^2 \times 3 \times p^3 \times q^2$

$8p^2qr^3 = 2 \times 2 \times 2 \times p \times p \times q \times r \times r \times r = 2^3 \times p^2 \times q \times r^3$

$4p^2q^3r = 2 \times 2 \times p \times p \times q \times q \times q \times r = 2^2 \times p^2 \times q^3 \times r$

The largest number of factors common to all three expressions is $2^2 \times p^2 \times q$.

$\therefore$ the HCF is $4p^2q$.

Alternative method:

$4p^2q$	$12p^3q^2$	$8p^2qr^3$	$4p^2q^3r$
	$3pq$	$2r^3$	q^2r

Since there is no more common factor in $3pq$, $2r^3$ and q^2r, the HCF is $4p^2q$.

Example 6

Find the HCF of $18ab^2c^3$, $6a^2bc^3$ and $24ab^2c^2$.

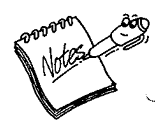
Solution

$18ab^2c^3 = 2 \times 3^2ab^2c^3 = 2 \times 3^2 \times a \times b^2 \times c^3$

$6a^2bc^3 = 2 \times 3a^2bc^3 = 2 \times 3 \times a^2 \times b \times c^3$

$24ab^2c^2 = 2^3 \times 3ab^2c^2 = 2^3 \times 3 \times a \times b^2 \times c^2$

$\therefore$ the HCF is $2 \times 3 \times a \times b \times c^2 = 6abc^2$.



With sufficient practice, one can get the HCF by inspection, without having to write out each expression in its expanded form.

Least Common Multiple of Algebraic Expressions

The method to find the **Least Common Multiple** (LCM) of algebraic expressions is also similar to that for numbers.

Example 7

Find the LCM of $12p^3q^2$, $8p^2qr^3$ and $4p^2q^3r$.

Solution

$12p^3q^2 = 2 \times 2 \times 3 \times p \times p \times p \times q \times q = 2^2 \times 3 \times p^3 \times q^2$

$8p^2qr^3 = 2 \times 2 \times 2 \times p \times p \times q \times r \times r \times r = 2^3 \times p^2 \times q \times r^3$

$4p^2q^3r = 2 \times 2 \times p \times p \times q \times q \times q \times r = 2^2 \times p^2 \times q^3 \times r$

The smallest group of factors which contains all the factors of the three expressions is $2^3 \times 3 \times p^3 \times q^3 \times r^3$, i.e., $24p^3q^3r^3$.

$\therefore$ the LCM is $24p^3q^3r^3$.

Alternative method:

$4p^2q$	$12p^3q^2$	$8p^2qr^3$	$4p^2q^3r$
q	$3pq$	$2r^3$	q^2r
r	$3p$	$2r^3$	qr
	$3p$	$2r^2$	q

$\therefore$ LCM $= 4p^2q \times q \times r \times 3p \times 2r^2 \times q$
$= 24p^3q^3r^3$

Example 8

Find the LCM of $18ab^2c^3$, $6a^2bc^3$ and $24ab^2c^2$.

Solution

$18ab^2c^3 = 2 \times 3^2ab^2c^3$

$6a^2bc^3 = 2 \times 3a^2bc^3$

$24ab^2c^2 = 2^3 \times 3ab^2c^2$

$\therefore$ the LCM is $2^3 \times 3^2a^2b^2c^3 = 72a^2b^2c^3$.

With sufficient practice, one can get the LCM by inspection.

Exercise 7d

1. Find the HCF of the following:

(a) $abxy$ and a^2bc
(b) $6pqr$ and $15qrs$
(c) $8xy^2z^3$ and $12x^2y^2z^2$
(d) $14a^2bc$ and $21ab^2$
(e) abm, acm and bcm
(f) $4x^3y$, $6x^2y^2$ and $8xy^3$
(g) $3x^5y^2$, $12x^2y^4$ and $15x^3y^2$
(h) $4abc^3$, $5a^3bc$ and $6ab^3c$

2. Find the LCM of the following:

(a) $abxy$ and a^2bc
(b) $8xy^2z^3$ and $12x^2y^2z^2$
(c) $16x^4y^3$ and $24xy^3$
(d) $6x^3y^4$ and $9x^3y$
(e) abm, acm and bcm
(f) $4x^3y$, $6x^2y^2$ and $8xy^3$
(g) $3x^5y^2$, $12x^2y^4$ and $15x^3y^2$
(h) $4abc^3$, $5a^3bc$ and $6ab^3c$

3. Find the HCF and the LCM of the following:

(a) $3a^2b$ and $4ab^2$
(b) $6ab^2c^2$ and $10a^2b^3$
(c) $4xyz^2$ and $12x^2yz$
(d) $6a^2b$, $9b^2c$ and $3c^2a$
(e) $4x^2y^2z^2$, $6x^3yz$ and $12xy^2z$
(f) $2mnp^2$, $3n^2pq$ and $6m^2pq^2$
(g) $a^4bx^2y^3$, $3a^2xy^4$ and $2a^3b^2y^3$
(h) $4pqr^3$, $8p^2qs^2$ and $10q^3rs$

Addition and Subtraction of Algebraic Fractions

Now, let us learn how to do addition and subtraction involving algebraic fractions.

Investigate

Within four consecutive years, Mrs Li gave birth to four lovely children. Today, x years later, Mr and Mrs Li find out that the product of their four children's ages is 3 024.

How old is each child now, assuming all of them are of different ages?

Example 9

Simplify $\frac{a+2b}{6} - \frac{a+2b}{4}$.

Solution

$$\frac{a+2b}{6} - \frac{a+2b}{4} = \frac{2(a+2b) - 3(a+2b)}{12} \quad \text{(The LCM of 6 and 4 is 12.)}$$

$$= \frac{2a+4b-3a-6b}{12}$$

$$= \frac{-a-2b}{12}$$

Example 10

Simplify (a) $\frac{2}{c} - \frac{3}{2c}$; (b) $\frac{2a + 3c}{3b} + \frac{a - c}{b}$.

Solution

(a) $\frac{2}{c} - \frac{3}{2c} = \frac{4 - 3}{2c} = \frac{1}{2c}$ (The LCM of c and $2c$ is $2c$.)

(b) $\frac{2a + 3c}{3b} + \frac{a - c}{b} = \frac{2a + 3c + 3(a - c)}{3b}$ (The LCM of $3b$ and b is $3b$.)

$$= \frac{2a + 3c + 3a - 3c}{3b}$$

$$= \frac{5a}{3b}$$

Just For Fun

During a party, the host distributed balloons to his guests in this way: He gave the first guest 2 balloons plus $\frac{1}{9}$ of the remainder; the second guest, 4 balloons plus $\frac{1}{9}$ of the remainder; the third guest, 6 balloons plus $\frac{1}{9}$ of the remainder; the fourth guest, 8 balloons plus $\frac{1}{9}$ of the remainder; and so on, until the last guest got the final remainder. If all the guests got an equal number of balloons, how many guests were there and how many balloons did the host give out altogether?

Example 11

Simplify $\frac{4}{2x - 4y} + \frac{3}{3x - 6y}$.

Solution

$$\frac{4}{2x - 4y} + \frac{3}{3x - 6y} = \frac{4}{2(x - 2y)} + \frac{3}{3(x - 2y)}$$

$$= \frac{2}{x - 2y} + \frac{1}{x - 2y}$$

$$= \frac{3}{x - 2y}$$

Example 12

Simplify $\frac{1}{\frac{1}{p} - \frac{1}{q}}$.

Solution

$$\frac{1}{\frac{1}{p} - \frac{1}{q}} = \frac{pq}{\left(\frac{1}{p} - \frac{1}{q}\right)pq}$$

$$= \frac{pq}{q - p}$$

Alternatively, $\frac{1}{\frac{1}{p} - \frac{1}{q}} = 1 \div \left(\frac{q - p}{pq}\right)$

$$= 1 \times \left(\frac{pq}{q - p}\right)$$

$$= \frac{pq}{q - p}$$

Notes

Multiply both the numerator and denominator by pq.

Exercise 7e

Express the following as fractions with a single denominator:

1. $\frac{x}{2} + \frac{x-1}{4}$
2. $\frac{3y}{4} - \frac{y-1}{2}$
3. $\frac{z}{2} - \frac{x+2}{3}$
4. $\frac{1}{3x} - \frac{1}{3y}$
5. $\frac{4}{ac} + \frac{2}{ab}$
6. $3 - \frac{n-p}{m}$
7. $\frac{4x^3y^2}{8xy^2} - \frac{x^2}{4}$
8. $a + \frac{b}{ca} - \frac{c}{ab}$
9. $\frac{3ab}{5x} - \frac{ab}{2x} - \frac{ab}{10x}$
10. $\frac{2y+1}{5} - \frac{3y-2}{10} + \frac{y}{2}$
11. $\frac{b+4}{6} - \frac{b}{3} + \frac{b-3}{12}$
12. $\frac{a}{2} + \frac{a}{3} - \frac{3a}{8}$
13. $\frac{c-1}{5} - \frac{2c+3}{3}$
14. $\frac{2y-3}{3} + \frac{y-2}{4}$
15. $\frac{a+1}{5} - \frac{a+1}{10} - \frac{a}{15}$
16. $\frac{e-4}{5} + 1$
17. $\frac{2}{3}(x-y) - \frac{3}{5}(x+y)$
18. $\frac{1}{3x} + \frac{1}{5x}$
19. $\frac{2(c+d)}{5} - \frac{2(c-d)}{10}$
20. $\frac{3}{c-d} - \frac{2}{d-c}$
21. $\frac{3-y}{x} + \frac{1-x}{y}$
22. $\frac{3m}{n} - \frac{m+n}{n}$
23. $\frac{3(b+6)}{b} - \frac{2(b+1)}{3b}$
24. $\frac{x+1}{4} - \frac{x+2}{3} + \frac{x+3}{6}$

*25. $\frac{a+3x}{2a} + \frac{a-x}{6a} - \frac{2x+a}{3a}$

*26. $\frac{b-c}{bc} + \frac{c-a}{ca} + \frac{a-b}{ab}$

*27. $\frac{6}{x-2y} + \frac{4}{2y-x}$

*28. $\frac{5}{2(e-f)} + \frac{4}{3(f-e)}$

29. $\frac{a}{2a-b} + 4$

30. $\frac{6q}{2q+3r} - 3$

*31. $\frac{4c}{10c-5d} + \frac{2d}{6c-3d}$

*32. $\frac{1}{4e-2f} - \frac{1}{f-2e}$

*33. $\frac{2m-3}{9m-6} - \frac{m-2}{6m-4}$

*34. $\frac{a+1}{2a-8} - \frac{a+2}{12-3a}$

35. $\frac{4}{e-3} + \frac{3}{e+2}$

*36. $\frac{x-3}{5-2x} + \frac{x+2}{2x-5}$

*37. $\frac{1}{a+\frac{1}{2}}$

*38. $\frac{\frac{1}{4}c}{c+\frac{1}{3}}$

*39. $\frac{x+\frac{1}{z}}{x-\frac{1}{2z}}$

*40. $\frac{p+\frac{1}{r}}{r+\frac{1}{p}}$

*41. $\frac{1}{\frac{1}{m}+\frac{1}{r}}$

*42. $\frac{\frac{1}{2}a}{b-\frac{1}{3}}$

*43. $\frac{\frac{1}{3}d-\frac{1}{2}}{\frac{1}{4}d+\frac{1}{3}}$

*44. $\frac{\frac{1}{x}+\frac{1}{y}}{\frac{2}{x}}$

Equations Involving Algebraic Fractions

We shall now learn how to solve equation involving algebraic fractions.

Example 13

Solve the equation $\frac{b-2}{5} + \frac{b-1}{3} = 1.$

Solution

$$\frac{b-2}{5} + \frac{b-1}{3} = 1.$$

Multiply by the LCM of 5 and 3 throughout, i.e., multiply by 15:

$$3(b-2)+5(b-1)=15$$
$$3b-6+5b-5=15$$
$$8b=15+5+6$$
$$8b=26$$
$$b=\frac{26}{8}=3\frac{1}{4}$$

Example 14

Solve $\frac{6}{2b-5}-\frac{4}{b-3}=0.$

Solution

$$\frac{6}{2b-5}-\frac{4}{b-3}=0$$

Multiply by $(2b-5)(b-3)$ throughout:

$$6(b-3)-4(2b-5)=0$$
$$6b-18-8b+20=0$$
$$-2b+2=0$$
$$b=1$$

Check:
We need $(2b-5)\neq 0$ and $(b-3)\neq 0$ which mean $b\neq 2\frac{1}{2}$ and $b\neq 3$.

Exercise 7f

1. Solve the following equations:

(a) $\frac{2x-3}{5}=\frac{x-2}{2}$ **(b)** $\frac{m}{m+2}=\frac{3}{5}$ **(c)** $\frac{1}{v-2}=\frac{2}{v-1}$

(d) $\frac{2}{x-3}=4$ **(e)** $\frac{x}{2}+\frac{x-2}{3}=4$ **(f)** $\frac{3x-4}{7}+\frac{2-x}{3}=0$

(g) $\frac{7p}{8}+3=\frac{9p}{10}$ **(h)** $1+\frac{5}{e}=\frac{7}{3}$ **(i)** $\frac{4}{u+3}-\frac{3}{u+2}=0$

(j) $e+\frac{e}{2}+\frac{e}{3}=11$ **(k)** $\frac{5}{a+4}-\frac{2}{a-2}=0$ **(l)** $\frac{x}{2}+5=\frac{2x}{3}$

(m) $\frac{x+3}{4}=\frac{2x-3}{5}$ **(n)** $6+\frac{2x-1}{4}=x$ **(o)** $\frac{2}{x}=\frac{5}{6}$

(p) $\frac{7}{x}=2$ **(q)** $\frac{10}{3x}+2=\frac{6}{x}$ **(r)** $\frac{2x}{3}-\frac{2x}{5}=4$

2. Solve the following equations:

(a) $\frac{3m-1}{5}-\frac{2m-3}{3}=1$ **(b)** $\frac{x}{4}-\frac{5x+8}{6}=\frac{2x-9}{3}$

(c) $\frac{m}{2}+\frac{m+1}{7}=m-2$ **(d)** $\frac{2a-3}{2}-\frac{a+2}{3}=\frac{a+1}{4}$

(e) $\frac{x+2}{6}+\frac{x}{3}=\frac{2-x}{2}$ **(f)** $\frac{4x+1}{5}-\frac{x}{2}=\frac{x}{3}-\frac{x}{4}$

(g) $\frac{2a+3}{2} + \frac{a}{4} = \frac{a}{6} - \frac{2a}{3}$

(h) $\frac{2n-1}{5} - \frac{n+3}{2} = \frac{3n-5}{5}$

(i) $\frac{d+3}{3} - \frac{2d-3}{2} = d - \frac{5}{6}$

(j) $\frac{7x-4}{15} + \frac{x-1}{3} = \frac{3x-1}{5} - \frac{7+x}{10}$

✲**(k)** $\frac{3}{x+1} - \frac{1}{2x+2} = 5$

✲**(l)** $\frac{5}{2x-1} - \frac{4}{4x-2} - \frac{3}{6x-3} = 1$

✲**(m)** $\frac{5}{6x} + \frac{6}{7x} - \frac{9}{14x} = 4$

✲**(n)** $\frac{3}{2-x} + \frac{5}{4-2x} - \frac{1}{x-2} = 4$

Problem Solving Involving Algebraic Fractions

Problems involving algebraic fractions may be solved using the various problem solving heuristics we learnt earlier.

Example 15

Mrs Li bought some oranges. She gave $\frac{1}{2}$ of them to her sister, $\frac{1}{4}$ of the remainder to her neighbour, $\frac{3}{5}$ of those left to her children and had 6 left in the end. How many oranges did Mrs Li buy?

Solution

Strategy 1: **Work backwards**

(1) The last 6 oranges represent $\frac{2}{5}$ of those left since Mrs Li gave $\frac{3}{5}$ of those left to her children.

$\therefore$ she had $6 \times \frac{5}{2} = 15$ oranges left after giving some to her neighbour.

(2) Her neighbour received $\frac{1}{4}$ of the remaining oranges after Mrs Li had given $\frac{1}{2}$ of the oranges she had bought to her sister. Thus the 15 oranges represent the other $\frac{3}{4}$.

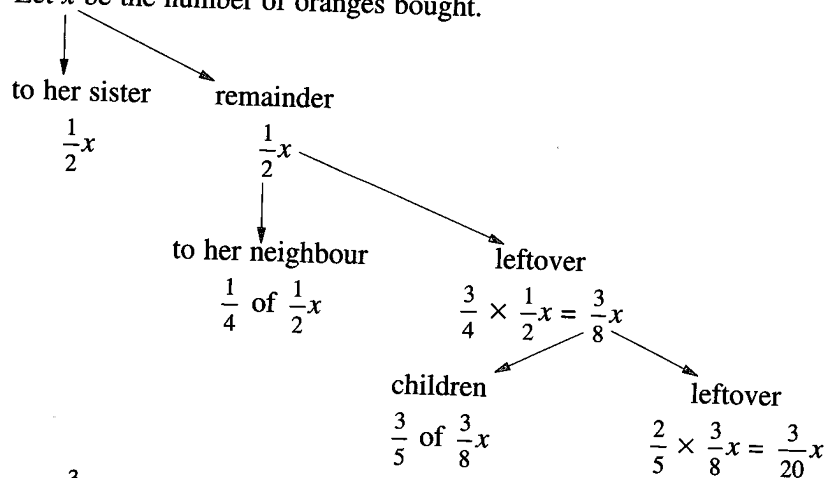
$\therefore$ Mrs Li had $15 \times \frac{4}{3} = 20$ oranges left after giving half to her sister.

(3) Since her sister received $\frac{1}{2}$ of the oranges bought, this means that the 20 oranges represent the other half.

$\therefore$ Mrs Li bought $20 \times 2 = 40$ oranges.

Strategy 2: Use an equation

Let x be the number of oranges bought.



$$\therefore \quad \frac{3}{20}x = 6$$
$$x = 6 \times \frac{20}{3}$$
$$= 40$$

Example 16

When a number is subtracted from 52 and the result is divided by 6, the answer obtained is twice the original number. What is the number?

Solution

Let the number be a.

$$\frac{52 - a}{6} = 2a$$
$$52 - a = 12a$$
$$52 = 13a$$
$$\frac{52}{13} = a$$
$$a = 4$$

Check:

$52 - 4 = 48$ and $48 \div 6 = 8$;
8 is twice 4.

Example 17

A motorist took $3\frac{1}{3}$ hours to drive 160 km. He drove part of the way at an average speed of 50 km/h and the rest of the way at 45 km/h. What is the distance he travelled at 50 km/h?

Solution

Let x km be the distance he travelled at 50 km/h.

Hence, time taken to travel x km = $\frac{x}{50}$ h.

Teri says to David, "I can tell you when your birthday is if you tell me the answer to the following:

1. *Multiply the number of your birth month by 5.*
   (If you were born in September, then multiply 9 by 5, etc.)
2. *Add 7.*
3. *Multiply the result by 4.*
4. *Add 13.*
5. *Multiply that total by 5.*
6. *Add the number of the day on which your birthday falls.*
   (If you were born on 3rd April, add 3, etc.)
7. *Subtract 205 from the total.*

Now, your answer will be either a three- or a four-digit figure. If it is a three-digit number, the first digit will be the month and the next two digits will be the day of your birthday. If it is a four-digit figure, the first two digits will be the month and the last two digits will be the day of your birthday. Am I right?"

How does this trick work?

He travelled $(160 - x)$ km at 45 km/h.

Time taken to travel $(160 - x)$ km $= \frac{160 - x}{45}$ h.

The total time taken was $3\frac{1}{3}$ hours.

$$\frac{x}{50} + \frac{160 - x}{45} = 3\frac{1}{3}$$

$$\left(\frac{x}{50} \times 450\right) + \left(\frac{160 - x}{45} \times 450\right) = \left(\frac{10}{3} \times 450\right)$$

$$9x + 10(160 - x) = 1\ 500$$

$$9x + 1\ 600 - 10x = 1\ 500$$

$$-x = 1\ 500 - 1\ 600$$

$$x = 100$$

Check:

100 km at 50 km/h requires 2 hours.

Thus $(160 - 100) = 60$ km at 45 km/h requires $1\frac{1}{3}$ hours.

$2h + 1\frac{1}{3}h = 3\frac{1}{3}h$

Exercise 7g

1. When 25 is added to a number and the result is halved, the answer is 3 times the original number. What is the number?

2. When 18 is added to $\frac{1}{4}$ of a number, the result is $2\frac{1}{2}$ times the original number. What is the number?

3. A number, when added to 5, gives the same result as when $\frac{2}{3}$ of it is subtracted from 6. What is the number?

4. 5 is subtracted from 4 times a number and the result is then doubled. If the answer is 6, what is the original number?

5. When a number is added to 4, the result is equal to subtracting 10 from three times of it. What is the number?

6. How can the number 39 be divided into two parts in order that the sum of $\frac{2}{3}$ of one part and $\frac{3}{4}$ of the other part is 28?

7. Meiling and Meimei divide \$69 into 2 shares. $\frac{3}{4}$ of Meiling's share is equal to $\frac{2}{5}$ of Meimei's share. How much does each get?

8. A mother is 21 years older than her new-born daughter. How old will the daughter be when her age is $\frac{1}{4}$ that of her mother's?

9. Mary's age is $\frac{2}{3}$ that of Peter's. Two years ago Mary's age was $\frac{1}{2}$ of what Peter's age will be in 5 years' time. How old is Peter now?

10. A half of what John's age was 4 years ago is equal to one third of what it will be in 5 years' time. How old is John now?

11. If a piece of wood is 5 cm longer than a second piece, and $\frac{3}{4}$ of the second piece is equal to $\frac{3}{5}$ of the first, what is the length of the second piece?

12. A man walks for some distance at 8 km/h, and for an equal distance at 5 km/h. The total time he takes is $3\frac{1}{4}$ hours. Find the distance he has walked altogether.

13. A man cycles for some time at 16 km/h and returns at 15 km/h. The total time taken is $7\frac{3}{4}$ hours. Find the distance he has cycled altogether.

14. A cyclist goes from one village to another at 28 km/h. He returns at 24 km/h. If the return journey takes two hours longer than the outward journey, what is the distance between the villages?

15. A bus travels at 36 km/h and arrives at its destination half an hour late. If it travels at 42 km/h, it arrives at the same destination half an hour earlier. Find the journey's distance.

16. A man travels regularly between two cities. He takes $4\frac{2}{3}$ hours if he travels at his usual speed. He finds that if he increases his speed by 3 km/h he can reduce the time taken by $\frac{1}{3}$ hour. What is his usual speed?

17. A motorist does the first part of his journey at an average speed of 54 km/h. He then increases his speed to 60 km/h for the rest of the journey. If he travels 225 km in 4 hours, what is the distance travelled for the first part of his journey?

18. Mrs Lim buys 30 kg of cheese at a certain price. She finds that if she had bought some cheaper cheese costing 95 cents less per kg, she could have had $32\frac{1}{2}$ kg for the same amount of money. What is the price per kg of the cheese which is more expensive?

19. A boy has a certain number of sweets. If he eats 16 a day, they will last him 2 days longer than if he eats 18 a day. How many sweets has he?

20. A certain number of matches are needed to fill 24 boxes, with each box containing the same number of matches. When 4 less matches are put into each box, there are enough for 28 boxes. Find the total number of matches.

Just For Fun

1. If

$$\begin{array}{r} ABCDE \\ \times \quad 4 \\ \hline EDCBA, \end{array}$$

find A, B, C, D and E where none of them is zero.

2. If $\frac{PORK}{CHOP} = C$ *and* $C > 2$, *find the value of each of these letters.*

For Your Information

Algebra is a branch of mathematics which deals with relations and properties of numbers by means of letters and other general symbols.

Changing the Subject of a Formula

Consider the following two sentences:

(1) Michael is the younger brother of Simon.
(2) Simon is the elder brother of Michael.

These two sentences actually have the same meaning.

The only difference is that 'Michael' is the subject of the first sentence and is put at the beginning of the sentence while 'Simon' is the subject of the second sentence and is placed at the beginning of the second sentence.

Similarly, an algebraic formula may be expressed differently to suit a particular purpose. For example, the formula for the area of a rectangle is $A = lb$ where A is the area of the rectangle, l its length and b its breadth. We say that A is the subject of the formula.

We can rearrange the formula to make either l or b the subject of the formula. This is done as shown below:

$$A = lb$$

$\therefore \quad l = \frac{A}{b}$ Divide both sides by b.

and $\quad b = \frac{A}{l}$ Divide both sides by l.

The three formulae $A = lb$, $l = \frac{A}{b}$ and $b = \frac{A}{l}$ are equivalent. They may be used for different purposes.

The following examples illustrate the technique of changing the subject of a formula.

Example 18

Make P the subject of the formula $I = \frac{PRT}{100}$.

Solution

$$I = \frac{PRT}{100}$$

$PRT = 100I$ — Multiply both sides by 100.

$P = \frac{100I}{RT}$ — Divide both sides by RT.

Example 19

Make p the subject of the formula $3b = 2p - 7$.

Solution

$$3b = 2p - 7$$

$2p = 3b + 7$ — Add 7 to both sides.

$p = \frac{3b + 7}{2}$ — Divide both sides by 2.

Example 20

Make x the subject of the formula $y = \frac{2 - x}{3 + 2x}$.

Solution

$$y = \frac{2 - x}{3 + 2x}$$

$y(3 + 2x) = 2 - x$ — Multiply both sides by $(3 + 2x)$.

$$3y + 2xy = 2 - x$$

$$2xy + x = 2 - 3y$$

$x(2y + 1) = 2 - 3y$ — Factorise the left-hand side.

$x = \frac{2 - 3y}{2y + 1}$ — Divide both sides by $(2y + 1)$.

Just For Fun

Each of the letters below represents a certain value. Try figuring out the value of each letter and then work out the sums.

(a)
```
   S E N D
+  M O R E
 ---------
 M O N E Y
```

(b)
```
   W I R E
+  M O R E
 ---------
 M O N E Y
```

There is only one answer for (a) but more than one possible answer for (b).

Exercise 7h

1. Make a the subject of the given formulae:

(a) $ax = y$ (b) $a(p - 4) = q$

(c) $ax + by = c$ (d) $p(a + b) = c$

(e) $2a - 3m = 4a - 7$ (f) $5b - 2a = 3c$

(g) $\frac{a}{m} + b = c$ (h) $x = \frac{2a}{3} + 5z$

(i) $\frac{p + a}{5} = 3p$ (j) $R = m(a + g)$

(k) $2b = ax + a$ (l) $2m = 65 - 4a$

2. Make the letter in the brackets the subject of the given formulae:

(a) $A = \frac{1}{2}bh$ (b) (b) $T = \frac{kx^2}{2l}$ (k)

(c) $\frac{F}{m} = \frac{v - u}{t}$ (t) (d) $3k = \frac{12 + 2l}{5}$ (l)

(e) $\frac{a}{b} - \frac{a}{c} = 1$ (a) (f) $\frac{a}{3} + \frac{b}{4} = \frac{c}{5}$ (a)

(g) $A = \frac{h}{2}(a + b)$ (a) (h) $S = \frac{n}{2}(a + l)$ (l)

(i) $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ (q) (j) $s = ut + \frac{1}{2}gt^2$ (u)

(k) $p + \frac{p}{q} = q$ (p)

(l) $l^2 = \frac{k}{m} - \frac{a}{2m^2}$ (a)

∗3. Make the letter in the brackets the subject of the formulae:

(a) $p = 2(l + b)$ (l)

(b) $A = 2\pi r(r + h)$ (h)

(c) $a(c - 3) = bc$ (c)

(d) $a + b = \frac{ax}{c}$ (a)

(e) $\frac{1}{a} + \frac{1}{b} = 1$ (b)

(f) $a = \frac{5}{4}(b - 18)$ (b)

(g) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ (b) (h) $\frac{a}{2c} + \frac{b}{4} = 1$ (a)

(i) $\frac{1}{a} + \frac{1}{b} = \frac{b}{a}$ (a) (j) $x = \frac{y(z - y)}{z}$ (z)

(k) $\frac{b - x}{a} = \frac{x}{c}$ (x) (l) $x = \frac{1}{y + 1} + 2$ (y)

(m) $w = \frac{ar}{R - r}$ (r) (n) $y = \frac{3 - x}{1 + 2x}$ (x)

(o) $b = \frac{abc}{4 - c}$ (c) (p) $\frac{m}{p - q} = r$ (q)

(q) $p - q = \frac{2p}{3}$ (p) (r) $\frac{b}{2} = \frac{5 - x}{2 + 3x}$ (x)

Summary

1. The value of a fraction remains unchanged if both its numerator and its denominator are multiplied or divided by the same non-zero number or expression.

i.e., $\frac{a}{b} = \frac{a \times c}{b \times c}$ and $\frac{a}{b} = \frac{a \div c}{b \div c}$.

2. To divide by a fraction, multiply by its reciprocal.

Review Questions 7

1. Express each of the following as a single fraction in its simplest form.

(a) $\frac{1}{x} + \frac{1}{3x}$ (b) $\frac{x - 2}{3} - \frac{2x - 3}{9}$

(c) $\frac{x + 3}{5} - \frac{1 - x}{6}$ (d) $\frac{2}{x} + \frac{3}{4x}$

(e) $\frac{10}{3x} - \frac{4}{x}$ (f) $\frac{x}{2x - 1} - \frac{4}{5}$

(g) $3x + \frac{2}{x}$ (h) $\frac{9x + 8}{5} + 2$

(i) $\frac{2}{2x - 1} - \frac{3}{5x + 1}$ (j) $\frac{3}{x + 1} - \frac{2}{3x + 4}$

(k) $\frac{1}{x-2} + \frac{5}{3x-7}$ **(l)** $\frac{2}{x+y} - \frac{3}{x-y}$

(m) $\frac{5}{2x-1} + \frac{3}{2x+1}$ **(n)** $\frac{2}{x-y} - \frac{3}{x}$

(o) $\frac{5}{x-2y} - \frac{3}{4x}$

2. Solve the following equations:

(a) $\frac{7}{2x} = \frac{5}{4}$ **(b)** $\frac{5}{x-3} = \frac{3}{7}$

(c) $\frac{x+5}{6} = \frac{3x-5}{8}$ **(d)** $\frac{x}{3} + 2x = 5$

(e) $\frac{12}{2x+1} = \frac{1}{x}$

(f) $\frac{x-1}{4} - \frac{2x-1}{6} = 3$

(g) $3 + \frac{2x+1}{3} = x$ **(h)** $\frac{4+x}{3} = \frac{2x-5}{5}$

(i) $\frac{1-2x}{4} + \frac{2-x}{2} = 4$

(j) $\frac{x+1}{4} - \frac{x}{5} = \frac{x}{6}$

(k) $\frac{x-1}{2} - \frac{x+1}{3} = x$

(l) $\frac{5}{4x} - \frac{3}{2x} + \frac{2}{x} = 5$

3. Make the letter in the brackets the subject of the formulae:

(a) $x = \frac{y}{y+1}$ (y) **(b)** $a = \frac{1-t}{1+t}$ (t)

(c) $c = \frac{a-b}{a}$ (a) **(d)** $x = \frac{3a+b}{a}$ (a)

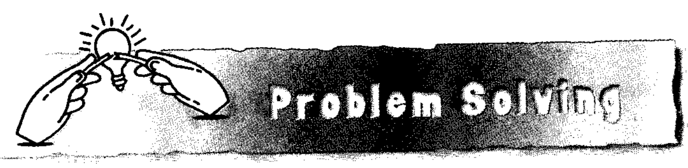
(e) $a = p + \frac{prt}{100}$ (p) **(f)** $k = h + \frac{2hk}{5}$ (h)

4. What is the number which when multiplied by 2 and added to 8 gives the same result as when it is divided by 2 and has 32 added to it?

5. When a number is divided by 4 and has 28 added to it, the result is equal to twice the number. Find the number.

6. The difference between the reciprocal of two consecutive positive even numbers is $\frac{1}{12}$. Find the two numbers.

7. One number is 3 times as large as the other and the difference between their reciprocals is $\frac{1}{6}$. Find the two numbers.



1. If $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{d}$, express b in terms of a, c and d.

2. $M is just sufficient to pay for the wages of one qualified teacher for x days or the wages of one relief teacher for y days. For how many days will $M be just sufficient to pay for the wages of one qualified teacher and one relief teacher? Give your answer in terms of x and y.

3. A man had a bag of sweets. He gave his son one sweet and $\frac{1}{7}$ of the remaining sweets. From what was left, he gave his daughter two sweets and $\frac{1}{7}$ of the remainder. The two children found that they had the same number of sweets. How many sweets were there initially in the bag?

4. The square of x is equal to the square root of y. Express y in terms of x.

5. Simplify

(a) $\frac{a - \frac{1}{b}}{b - \frac{1}{a}}$, **(b)** $\frac{a + \frac{1}{b}}{b + \frac{1}{a}}$.

CHAPTER 8

Simultaneous Linear Equations

In this chapter, you will learn how to

- △ solve a pair of simultaneous linear equations by
  (a) elimination, **(b)** substitution;
- △ apply the technique to solve some practical problems like the one mentioned below.

Preliminary Problem

The 7 apples and 4 pears cost the boy $4.10 while the 5 apples and 7 pears cost the girl $5.00. How much is each apple and each pear?

In Book 1, we learnt how to solve linear equations with one variable such as $2x - 5 = 7$, $4y - 3 = 2y + 14$, $5t - 7 = 2t + 5$, etc. Each of the equations has one unique solution.

Let us now consider expressions with two unknowns such as $3x + 2y$.

If $x = 2$ and $y = 3$, then $3x + 2y = 3(2) + 2(3) = 12$.

If $x = 1$ and $y = -2$, then $3x + 2y = 3(1) + 2(-2) = -1$.

We see that when x and y are assigned different values, the expression $3x + 2y$ takes on different values. Suppose the value of an expression is given, as 7, what then are the values of x and y?

Let us consider a table of values for $3x + 2y$ using different values of x and y.

x	0	0	1	1	2	2	3	5	7	–1	–2	–3
y	2	3	1	2	0	–3	–1	–4	–7	3	6	8
$3x + 2y$	4	6	5	7	6	0	7	7	7	3	6	7

Notice from the table above that there are several pairs of values of x and y which satisfy the equation $3x + 2y = 7$. In fact, if we continue substituting other pairs of values of x and y, we will find that there are infinitely many pairs of x and y that will satisfy $3x + 2y = 7$. We say that there are more than one pair of values of x and y which will satisfy the equation $3x + 2y = 7$.

Let us consider another linear equation with two unknowns, $2x + y = 4$.

Examine the table of values for $2x + y$ below by using different values of x and y.

x	0	0	1	1	2	2	3	3	4	–1	–2	–3
y	3	4	1	2	0	–1	–1	–2	–4	–3	8	1
$2x + y$	3	4	3	4	4	3	5	4	4	–5	4	–5

Again, we see that there are several pairs of values of x and y which satisfy the equation $2x + y = 4$.

However, if we take the two equations $3x + 2y = 7$ and $2x + y = 4$ together and check whether there is any pair of values of x and y that satisfies both equations, we will find that one particular pair, $x = 1$ and $y = 2$, satisfies the two equations simultaneously.

We say that $x = 1$ and $y = 2$ is the solution to the simultaneous linear equations $3x + 2y = 7$ and $2x + y = 4$.

Thus, to solve a linear equation with two unknowns, two different equations are needed. Similarly, to solve a linear equation with three unknowns, three different equations are usually needed, and so on.

There are three common methods of solving a pair of simultaneous equations. We shall introduce only two methods here: the **elimination method** and the **substitution method**. The third method, the **graphical solution method**, will be dealt with in Chapter 9.

Solve the following pair of simultaneous equations:

$3x + 4y = 8$,
$6x + 8y = 16$.

Explain the results obtained.

Solving Simultaneous Linear Equations Using Elimination Method

The following examples illustrate the process of solving a pair of simultaneous equations by the **elimination method**.

Example 1

Solve the simultaneous equations

$$3x - y = 12,\ 2x + y = 13.$$

Solution

$3x - y = 12$ ——— (1)

$2x + y = 13$ ——— (2)

Add equation (1) to equation (2), i.e., $(3x - y) + (2x + y) = 12 + 13$.

When this is done, the terms in y cancel out and we are left with one unknown x.

i.e., $(3x - y) + (2x + y) = 12 + 13$
$5x = 25$
$x = 5$

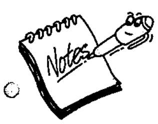
Substitute $x = 5$ into (1): $3(5) - y = 12$
$y = 3$

$\therefore$ $x = 5$ and $y = 3$ is the solution of the simultaneous equations.

Check: Substitute $x = 5$, $y = 3$ into (1) and (2):

In (1), LHS = $3(5) - 3 = 12$ = RHS

In (2), LHS = $2(5) + 3 = 10 + 3 = 13$ = RHS



It is a good practice to check the answers as errors may occur. However, you are not expected to have these steps in your solution.

Example 2

Solve the simultaneous equations

$$3x + 7y = 17,\ 3x - 6y = 4.$$

Solution

$3x + 7y = 17$ ——— (1)

$3x - 6y = 4$ ——— (2)

If we subtract equation (2) from equation (1), the terms in x cancel out.

i.e., $(3x + 7y) - (3x - 6y) = 17 - 4$
$13y = 13$
$y = 1$

Use the CD, Building a City with Linear Equation, from the DMS to explore solving a pair of simultaneous equations by using the method of elimination. Go through the activities provided.

Substitute $y = 1$ into (1): $3x + 7(1) = 17$

$$3x = 10$$
$$x = \frac{10}{3}$$
$$= 3\frac{1}{3}$$

$\therefore$ the solution set is $x = 3\frac{1}{3}$, $y = 1$.

Sometimes it is necessary to change the coefficients of one of the unknowns in the equations before we can eliminate it by addition or subtraction.

Example 3

Solve the simultaneous equations

$$13x - 6y = 20,$$
$$7x + 4y = 18.$$

Solution

$13x - 6y = 20$ —— (1)

$7x + 4y = 18$ —— (2)

The coefficients of y in both equations will be numerically equal if we multiply (1) by 2 and (2) by 3.

Note: The LCM of 6 and 4 is 12.

(1) × 2: $26x - 12y = 40$ —— (3)

(2) × 3: $21x + 12y = 54$ —— (4)

(3) + (4): $47x = 94$

$x = 2$

Substitute $x = 2$ into (1): $13(2) - 6y = 20$

$$6 = 6y$$
$$y = 1$$

$\therefore$ the solution set is $x = 2$, $y = 1$.

Do you think it will be easier if we eliminate x first? Why?

In the process of solving simultaneous linear equations, either one of the unknowns may be eliminated first.

Just For Fun

Under the law of a certain country, anyone caught poaching in the King's private forest is punished by being hanged or beheaded. However, before the culprits are hanged or beheaded, they have to make a statement. If the statement is true, they will be beheaded; if it is false, they will be hanged.

One day, a man was caught poaching in the King's private forest and the statement he made was:

"I shall be hanged."

Now, this is where the problem lies. If he was hanged, then his statement would be true and therefore he should be beheaded.

But if he was beheaded, then what he had said was true. But he did not say that he would be beheaded. Should he be hanged or be beheaded? This type of problem is called a logical paradox.

A paradox is a statement which contradicts itself.

You may wish to read more about logical paradoxes in your library or from the internet.

Example 4

Solve the simultaneous equations

$$\frac{2x}{3} - \frac{y}{9} = 6, \; x - \frac{y}{3} = 6.$$

Solution

$$\frac{2x}{3} - \frac{y}{9} = 6 \quad \text{—— (1)}$$

$$x - \frac{y}{3} = 6 \quad \text{—— (2)}$$

$(1) \times \frac{3}{2}$: $$x - \frac{y}{6} = 9 \quad \text{—— (3)}$$

$(3) - (2)$: $$\left(x - \frac{y}{6}\right) - \left(x - \frac{y}{3}\right) = 9 - 6$$

$$-\frac{y}{6} + \frac{y}{3} = 3$$

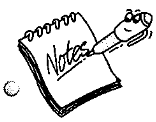
$$\frac{y}{6} = 3$$

$$y = 18$$

Substitute $y = 18$ into (2): $x - \frac{18}{3} = 6$

$$x = 12$$

$\therefore$ the solution set is $x = 12$, $y = 18$.



Alternatively, we can eliminate the fractional part by multiplying (1) by 9 and (2) by 3.

(1) × 9: $6x - y = 54$ —(3)
(2) × 3: $3x - y = 18$ —(4)
(3) − (4): $3x = 36$
$x = 12$

Substitute $x = 12$ into (4):

$3(12) - y = 18$
$y = 18$

$\therefore$ the solution set is $x = 12$, $y = 18$.

Exercise 8a

1. Solve the following simultaneous equations:

(a) $x + y = 16$, $x - y = 0$
(b) $x - y = 5$, $x + y = 19$
(c) $x - y = 6$, $x + y = 12$
(d) $2x + y = 23$, $4x - y = 19$
(e) $3x + 2y = 13$, $3x - 2y = 5$
(f) $5x - 2y = 12$, $3x + 2y = 12$
(g) $4x - y = 7$, $4x + 3y = 11$
(h) $3x - 2y = 9$, $2x - 2y = 7$
(i) $5x - 6y = 14$, $5x - 5y = 15$

2. Solve the following simultaneous equations:

(a) $3x - 2y = 5$, $2y - 5x = 9$
(b) $2x + 3y = 10$, $3y - x = 7$
(c) $3x - 5y = 13$, $5y + x = 7$
(d) $6x - y = 23$, $3y + 6x = 11$
(e) $7x + 2y = 33$, $3y - 7x = 17$
(f) $3x - y + 14 = 0$, $2x + y + 1 = 0$
(g) $3y - 2x + 15 = 0$, $2x - 2y + 19 = 0$
(h) $5x + 7y - 17 = 0$, $7y + 3x - 27 = 0$

3. Solve the following simultaneous equations:

(a) $2x + y = 3$, $3x - 2y = 1$
(b) $x + 3y = 38$, $3x - y = 24$
(c) $x = 3y - 2$, $9y = 4x - 7$
(d) $3x - 2y = 6$, $6y = 5x + 30$
(e) $3x = 7 + y$, $5x - 9y = 41$
(f) $7x = 5y + 45$, $2x - 5y = 4$
(g) $7y + 24x = 9$, $11y - 6x = 36$
(h) $12y + 15x = 21$, $5y - 3x = 85$

✶4. Solve the following simultaneous equations:

(a) $13 + 2y = 9x$, $3y = 7x$
(b) $2x - 3y = 5$, $3x - \frac{2y - 3}{5} = 4$
(c) $3x - y = 23$, $\frac{x}{3} + \frac{y}{4} = 4$
(d) $2x - 3y = 24$, $\frac{5x}{3} - \frac{y}{2} = 12$

(e) $\frac{5x}{8} + \frac{7y}{18} = 6$
$3x - y = 3$

(f) $\frac{x-3}{5} = \frac{y-7}{2}$
$11x = 13y$

(g) $\frac{1}{5}(x - 2) = \frac{1}{4}(1 - y)$
$26x + 3y + 4 = 0$

(h) $\frac{1}{3}(x + y) = \frac{1}{5}(x - y)$
$3x + 11y = 4$

(i) $\frac{1}{5}(x + y) = \frac{1}{7}(x - y)$
$3x + 17y = 2$

(j) $4(2x - y + 3) = 0$
$2(x + y) - 3(x - y) = 6$

(k) $0 = 3x - 7y - 5 = y - x + 3$

(l) $3x + 5 = 8y + 4 = 7x - 7y + 1$

(m) $\frac{x}{3} + \frac{y}{4} = 3x - 7y - 37 = 0$

(n) $\frac{1}{9}(5x + 2y) = x + y = 2x + 3y + 1$

(o) $0.3x + 0.1y = 1.7$
$0.4x + 0.3y = 1.1$

(p) $1.2x - 0.8y = 0.4$
$y + 0.1x = 0.3$

Solving Simultaneous Linear Equations Using Substitution Method

The following examples illustrate the process of solving a pair of simultaneous linear equations by the **substitution method**.

Example 5

Solve the simultaneous equations

$$7x - 2y = 21,\ 4x + y = 57.$$

Solution

$7x - 2y = 21$ —— (1)

$4x + y = 57$ —— (2)

From (2): $y = 57 - 4x$ —— (3)

Substitute (3) into (1):
$$\begin{aligned} 7x - 2(57 - 4x) &= 21 \\ 7x - 114 + 8x &= 21 \\ 15x &= 114 + 21 \\ 15x &= 135 \\ x &= 9 \end{aligned}$$

Substitute $x = 9$ into (3):
$$\begin{aligned} y &= 57 - 4(9) \\ &= 57 - 36 \\ &= 21 \end{aligned}$$

$\therefore$ the solution set is $x = 9$, $y = 21$.

In this example, it is easier to substitute y in terms of x. Can you give a reason for this?

Investigate

Consider the following simultaneous equations:

$2x + y = 6$ — (1)

$x = 1 - \frac{1}{2}y$ — (2)

Substitute (2) into (1);

$2\left(1 - \frac{1}{2}y\right) + y = 6$

$2 - y + y = 6$

$\therefore\ 2 = 6!$

Do you know where the problem lies?

Example 6

Solve the simultaneous equations

$$3x + 2y = 7,\ 9x + 8y = 22.$$

Solution

$3x + 2y = 7$ —— (1)

$9x + 8y = 22$ —— (2)

From (1), $2y = 7 - 3x$

$y = \dfrac{7-3x}{2}$ —— (3)

Substitute (3) into (2): $9x + 8\left(\dfrac{7-3x}{2}\right) = 22$

$9x + 28 - 12x = 22$

$-3x = -6$

$x = 2$

Substitute $x = 2$ into (3): $y = \dfrac{7-3(2)}{2} = \dfrac{1}{2}$

$\therefore$ the solution set is $x = 2$, $y = \dfrac{1}{2}$.

Do you think it will be easier to solve this question by the elimination method?

The CD, Building a City with Linear Equation, from the DMS includes a section on solving pairs of simultaneous equations by using the method of substitution. Go through the tutorials before proceeding to try the activities. You may then proceed to explore the applications given at the later part of the CD.

Exercise 8b

1. Use the substitution method to solve the following simultaneous equations:

(a) $x + y = 7$, $x - y = 5$
(b) $2x - y = 4$, $x + 2y = 7$
(c) $3x - y = 0$, $2x + y = 5$
(d) $6x - 5y = 10$, $5x + y = -2$
(e) $2x - 7y = 5$, $3x + y = -4$
(f) $4x + y = -12$, $2x + 5y = -6$
(g) $5x - y = 5$, $3x + 2y = 29$
(h) $4x - 3y = 8$, $3x + 5y = 6$
(i) $5x + 3y = 11$, $4x - y = 2$
(j) $2x - y = 11$, $x + 4y = 1$
(k) $3x - 5y = 6$, $2x - 3y = 5$
(l) $2x + 5y = 12$, $4x + 3y = -4$

2. Use either the substitution or the elimination method to solve the following simultaneous equations:

(a) $5x - 2y = 4$, $3x + y = 9$
(b) $2x - 3y = 4$, $x - 5y = 9$
(c) $x + y = 13$, $3x - 5y = 7$
∗(d) $\dfrac{x}{3} + \dfrac{y}{5} = 2$, $2x - y = 1$
∗(e) $\dfrac{x}{5} + y = -2$, $\dfrac{x}{3} - y = 10$
∗(f) $\dfrac{2x}{5} + \dfrac{y}{4} = -2$, $\dfrac{x}{5} + 1 = y$
∗(g) $\dfrac{x+y}{3} = 3$, $\dfrac{3x+y}{5} = 1$
∗(h) $\dfrac{x}{2} - \dfrac{2y}{5} = 4$, $\dfrac{x}{4} + y = -4$
∗(i) $\dfrac{x}{3} + \dfrac{y}{2} = 4$, $\dfrac{2x}{3} - \dfrac{y}{6} = 1$
∗(j) $2x + \dfrac{y}{4} = 6$, $4x - y = 6$
(k) $3x + 7y = 2$, $6x - 5y = 4$
(l) $5x + 5y = 3$, $10x - 15y = -4$
(m) $8x - 2y = 1$, $2x - y = 0$
(n) $5x + 2y = 3$, $x - 4y = -6$
(o) $4y = x + 1$, $2y = \dfrac{2x+3}{2}$

Problem Solving Involving Simultaneous Equations

Many mathematical and real-life problems can be solved by using the technique of solving simultaneous equations.

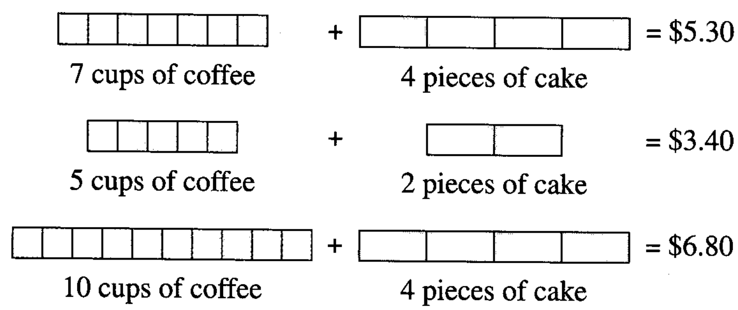
Example 7

7 cups of coffee and 4 pieces of cake cost \$5.30 while 5 cups of coffee and 2 pieces of cake cost \$3.40. Find the cost of one cup of coffee and one piece of cake.

Solution

Strategy 1: Draw a diagram

We can use diagrams to solve the above problem.



∴ the extra 3 cups of coffee cost \$6.80 – \$5.30 = \$1.50

∴ 1 cup of coffee costs \$0.50.

and 5(\$0.50) + 2 pieces of cake = \$3.40

2 pieces of cake = \$3.40 – \$2.50 = \$0.90

∴ 1 piece of cake costs \$0.45.

Strategy 2: Use an equation

We can also solve this problem by using simultaneous equations.

Let $\$x$ be the cost of one cup of coffee and $\$y$ be the cost of one piece of cake.

$$7x + 4y = 5.3 \quad \text{—— (1)}$$
$$5x + 2y = 3.4 \quad \text{—— (2)}$$

(2) × 2: $10x + 4y = 6.8$ —— (3)

(3) – (1): $3x = 1.5$

$\therefore \quad x = 0.5$

Substitute $x = 0.5$ into (2): $5(0.5) + 2y = 3.4$

$$2y = 3.4 - 2.5$$
$$= 0.9$$
$$y = 0.45$$

Hence one cup of coffee costs \$0.50 and one piece of cake costs \$0.45.

Example 8

Find two numbers whose sum is 67 and whose difference is 3.

Solution

Let x be the greater number and y be the smaller number.

$$x + y = 67 \quad \text{——— (1)}$$
$$x - y = 3 \quad \text{——— (2)}$$

(1) + (2): $2x = 70$
$x = 35$

Substitute $x = 35$ into (1): $35 + y = 67$
$y = 67 - 35$
$= 32$

$\therefore$ the two numbers are 32 and 35.

Just For Fun

Spiders, dragonflies and houseflies are kept in three separate containers. It is found that there are 20 heads, 136 legs and 19 pairs of wings altogether in the three containers. Find the number of spiders, dragonflies and houseflies given that a spider has 8 legs, a dragonfly has 6 legs and 2 pairs of wings and a housefly has 6 legs and 1 pair of wings.

Example 9

If 1 is added to the numerator and 2 to the denominator of a fraction, its value will be $\frac{2}{3}$. If 2 is subtracted from the numerator and 1 from the denominator, its value will be $\frac{1}{3}$. What is the fraction?

Solution

Let x be the numerator and y be the denominator of the fraction, i.e., let the fraction be $\frac{x}{y}$.

$$\frac{x+1}{y+2} = \frac{2}{3} \quad \text{——— (1)}$$
$$\frac{x-2}{y-1} = \frac{1}{3} \quad \text{——— (2)}$$

By clearing the fractions and simplifying, we get

$$3x - 2y = 1 \quad \text{——— (3)}$$
$$3x - y = 5 \quad \text{——— (4)}$$

(3) – (4): $-y = -4$
$y = 4$

Substitute $y = 4$ into (3): $3x - 2(4) = 1$
$3x - 8 = 1$
$3x = 9$
$x = 3$

$\therefore$ the fraction is $\frac{3}{4}$.

Example 10

The sum of the digits of a two-digit number is 8. When the number with the same digits reversed is subtracted from the number, the difference is 18. What is the number?

Solution

Let x be the *tens* digit and y be the *units* digit.

The number is $10x + y$.

The reversed number is $10y + x$.

$$x + y = 8 \quad \text{——— (1)}$$
$$10x + y - (10y + x) = 18$$
$$10x + y - 10y - x = 18$$
$$9x - 9y = 18 \quad \text{——— (2)}$$

(2) ÷ 9: $x - y = 2$ ——— (3)

(1) + (3): $x + y + x - y = 8 + 2$

$$2x = 10$$
$$x = 5$$

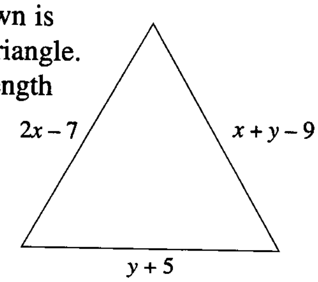
Substitute $x = 5$ into (1): $5 + y = 8$

$$y = 8 - 5$$
$$= 3$$

∴ the required number is 53.

Exercise 8c

1. Find two numbers whose sum is 138 and whose difference is 88.
2. One third of the sum of two angles is 60° and one quarter of their difference is 28°. Find the two angles.
3. The sum of two numbers is 36 and their difference is 9. Find the two numbers.
4. One fifth of the sum of two angles is 24° and half their difference is 14°. Calculate the two angles.
5. 8 kg of potatoes and 5 kg of carrots cost \$28 whereas 2 kg of potatoes and 3 kg of carrots cost \$11.20. What is the cost of 1 kg of each item?
6. 6 stools and 4 chairs cost \$58 but 5 stools and 2 chairs cost \$35. Find the cost of each stool and each chair.
7. Adding unity to the numerator as well as the denominator of a fraction makes it equal to $\frac{4}{5}$. Subtracting 5 from each makes it equal to $\frac{1}{2}$. What is the fraction?
8. A belt and a wallet cost \$42 while 7 belts and 4 wallets cost \$213. Calculate the cost of each item.
9. The figure shown is an equilateral triangle. Calculate the length of each side and give your answer in cm.

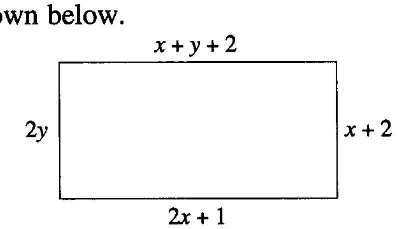


10. A fraction equals $\frac{1}{2}$ if 1 is subtracted from both the numerator and denominator. It is equal to $\frac{2}{3}$ if 1 is added to both the numerator and denominator. Find the fraction.

11. Find two numbers given that their sum is 48 and the smaller number is equal to one-fifth of the larger number.

12. \$80 is divided between two men such that one-quarter of one person's share is equal to $\frac{1}{6}$ of the other. How much will each man receive?

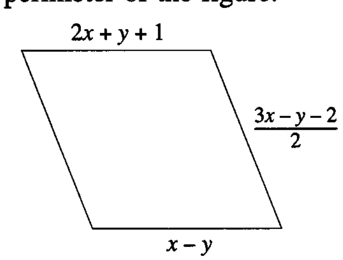
13. Find the perimeter, in cm, of the rectangle shown below.



14. I think of a pair of numbers. If I add 7 to the first, I obtain a number which is twice the second. If I add 20 to the second, I obtain a number which is four times the first. What are the numbers?

15. The figure shown is a rhombus. Its measurements are in cm. Calculate

(a) the length of a side,
(b) the perimeter of the figure.



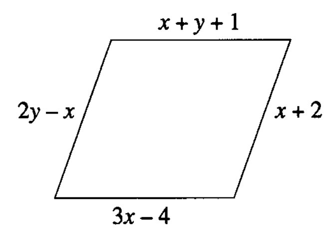
16. I have fifteen coins. Some are valued at 5 cents and others at 20 cents. Their total value is \$1.65. How many 5-cent coins and 20-cent coins do I have?

17. In five years' time, a father will be three times as old as his son. Four years ago, the father was six times as old as his son. Find their present ages.

18. A two-digit number is such that the sum of the digits is 11. When the number with the same digits reversed is subtracted from this number, the difference is 9. What is the number?

19. The difference between two numbers is 10 and their sum is equal to four times the smaller number. What are the numbers?

20. The sides of the parallelogram shown in the figure are given in centimetres. Find x and y. Hence, find the perimeter of the parallelogram.



21. The sum of two numbers is 40. If 2 is added to the larger number, the result is equal to twice the smaller number. What are the two numbers?

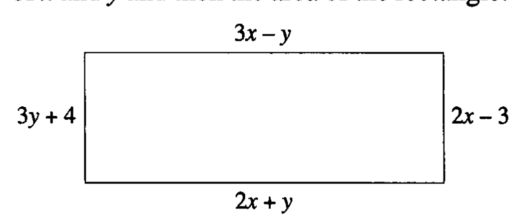
22. A man buys 36 stamps, all of which are either 50¢ or 10¢ pieces and the total value is \$11.60. How many stamps of each kind does he buy?

23. The length of a rectangle is greater than its breadth by 2 cm. If the length is increased by 4 cm and the breadth decreased by 3 cm, the area remains the same. Find the length and breadth of the rectangle.

24. \$2 500 is to be deposited in Banks A and B. The interest rate per annum for Bank A is 6% while that for Bank B is $6\frac{1}{2}$%. After one year, their interests are equal. How much money was deposited in each bank?

25. A train takes 7 hours to travel from station A to station B. If its speed is increased by 10 km/h, it will take 1 hour less to complete the journey. Find the distance between the two stations.

26. The figure below shows the lengths of the sides of a rectangle in cm. Find the values of x and y and then the area of the rectangle.



More Problem Solving Strategies

We learnt that to solve a pair of simultaneous equations involving two unknowns, we need to have two equations, and similarly to solve simultaneous equations involving three unknowns, normally we will need three equations. We shall now look at an example in which there are three unknowns but only two equations.

Example 11

A rooster costs $5 and a hen costs $3. Chicks are sold at 3 for a dollar. A farmer bought 100 birds of these 3 types for $100. How many of each type of bird did he buy?

Solution

Strategy: Solve part of the problem

Let the farmer buy x roosters, y hens and z chicks.

Thus we have $x + y + z = 100$ —— (1)

$$5x + 3y + \frac{1}{3}z = 100 \text{ —— (2)}$$

This is only a part of the problem, i.e., two equations with three unknowns. Solving part of a problem is a strategy we can sometimes use to obtain a full solution. We now proceed to solve the simultaneous equations (1) and (2).

(2) × 3 $\quad 15x + 9y + z = 300$ —— (3)

(3) – (1) $\quad 14x + 8y = 200$

$$\therefore \quad y = \frac{200 - 14x}{8} = \frac{100 - 7x}{4}$$

Since the number of birds of each type bought must be positive and a whole number, $y > 0$ and x can only take values that are multiples of 4. The values that will give a positive value of y as well as a whole number are 4, 8 and 12.

When $x = 4$, $y = \frac{100 - 7(4)}{4} = 18$ and $z = 100 - 4 - 18 = 78$

When $x = 8$, $y = \frac{100 - 7(8)}{4} = 11$ and $z = 100 - 8 - 11 = 81$

When $x = 12$, $y = \frac{100 - 7(12)}{4} = 4$ and $z = 100 - 12 - 4 = 84$

Thus, there are three possible sets of answers to this problem.

i.e., the farmer could have bought
4 roosters, 18 hens and 78 chicks, or
8 roosters, 11 hens and 81 chicks, or
12 roosters, 4 hens and 84 chicks for $100.

Exercise 8d

∗1. A rabbit costs \$3.50, ducklings are sold at 3 for \$1 and chicks cost 50 cents each. A farmer paid \$100 exactly for 100 of these animals. How many of each did he buy? (There are 5 possible sets of answers.)

∗2. A cow costs $3\frac{1}{2}$ gold coins. Goats are sold at 3 for 4 gold coins and piglets are sold at 2 for a gold coin. A man uses exactly 100 gold coins to buy 100 animals. How many of each did he buy? (There are 3 possible sets of answers.)

Summary

1. A pair of simultaneous linear equations in two variables can be solved by
   - **(a)** elimination method or
   - **(b)** substitution method.
2. Problems involving simultaneous equations may be solved by
   - **(a)** assigning variables to unknowns,
   - **(b)** forming the equations,
   - **(c)** solving the equations,
   - **(d)** giving the solution to the problem.

Review Questions 8

1. Solve the following simultaneous equations:
   - **(a)** $7x + 2y = 10$, $5x + 2y = 6$
   - **(b)** $9x + 4y = 28$, $4y - 11x = -12$
   - **(c)** $2x - 5y = 22$, $2x - 3y = 14$
   - **(d)** $6x - y = 16$, $3x + 2y = -12$
   - **(e)** $x + y = 0.5$, $x - y = 1$
   - **(f)** $4x + 3y = 0$, $5y + 53 = 11x$
   - **(g)** $8x + 3y = 14$, $2x + y = 4$
   - **(h)** $2x + 0.4y = 8$, $5x - 1.2y = 9$
   - **(i)** $5x - 4y = 4$, $2x - y = 2.5$
2. 5 apples and 4 oranges cost \$3.40 while 7 apples and 6 oranges cost \$4.90. Find the cost of an apple and an orange.
3. I think of a pair of numbers. If I add 11 to the first, I obtain a number which is twice the second. If I add 20 to the second, I obtain a number which is twice the first. What are the numbers?
4. If A gives B \$3, B will have twice as much as A. If B gives A \$5, A will have twice as much as B. How much does each have?
5. If the larger of two numbers is divided by the smaller, the quotient and the remainder are 2 each. If 5 times the smaller number is divided by the larger, the quotient and the remainder are still 2 each. Find the two numbers.
6. If the selling price of 5 pears and 4 mangoes is \$1.75 while that of 8 pears and 5 mangoes is \$2.45, what is the price of each pear and each mango?
7. At a basketball game, adult tickets were sold at \$1.00 each and student tickets at 75¢ each. If 150 tickets were sold and \$140 was collected, how many tickets of each kind were sold?
8. The sum of the numerator and denominator of a fraction is 17. If 3 is added to the numerator, the value of the fraction will be 1. What is the fraction?
9. The denominator of a certain fraction exceeds its numerator by 4. If $\frac{2}{3}$ is added to the reciprocal of the fraction, the sum becomes 3. Find the fraction.

10. In four years' time a father will be three times as old as his son. Six years ago he was seven times as old as his son. How old are they now?

11. A shopkeeper mixed coffee powder worth \$2.50 per kg with coffee powder worth \$3.50 per kg, and sold 20 kg of the mixture at \$2.80 per kg. Find the weights of the 2 grades of coffee powder that he mixed together.

12. Two cars leave a town at the same time and travel in opposite directions. The speed of one car is 12 km/h more than the other. They are 444 km apart after 3 hours. Find the speed of each car.

13. A motorist drove for 2 hours at one speed and then for 3 hours at another speed. He covered a distance of 252 km. If he had travelled 4 hours at the first speed and one hour at the second speed, he would have covered 244 km. Find the two speeds.

14. The sum of the digits of a two-digit number is 12, and the *units* digit is twice the *tens* digit. Find the number.

Problem Solving

1. Solve the following simultaneous equations:

(a) $\frac{5}{x} - \frac{6}{y} = 1, \quad \frac{17}{x} + \frac{30}{y} = 16$

(b) $\frac{2}{x+y} = \frac{1}{2x+y}, \quad 3x + 4y = 9$

(c) $125^{x+y} = 25, \quad 3^{x-y} = 81$

(d) $2^x \times 32^y = 2^{13}, \quad 8^x \times 2^{2y} = 1$

(e) $\frac{4}{x} + \frac{15}{y} = 15, \quad \frac{7}{5x} - \frac{6}{y} = 3$

(f) $x + 2y = 7, \quad x^2 + y^2 = 1 + 4x$

2. A farmer keeps hens and rabbits on his farm. One day, he counted a total of 70 heads and 196 legs. How many more hens does he have than rabbits?

3. In a block of flats there are 24 units of three types: the Luxury Unit, the Superior Unit and the Deluxe Unit. The Luxury Unit can accommodate 8 people, the Superior Unit 7 people and the Deluxe Unit 5 people. Given that the total number of people living in this block is 160, how many of each type of flats are there?

Revision Exercise II No. 1

1. The difference between the selling prices of an article at a gain of 5% and at a loss of 5% is $12.50. What is the cost price of the article?

2. Solve the following equations:
   (a) $5(x^2 - 9) = 3(x - 3)^2$
   (b) $(2 - x)(4 + x) + x^2 = 14$

3. What is the Representative Fraction (R.F.) of a map which has a scale of 1 cm to 2.5 km?

∗4. Simplify each of the following:
   (a) $\frac{4p^2q^2 - 8p^3q - 14pq^3}{2pq}$
   (b) $\frac{6a^3b^4 + 12a^4b^2 - 9a^2b^3}{-3a^2b^2}$

5. Factorise the following:
   (a) $5x^2 - 25x + 20$
   ∗(b) $10a^2 - 21ab + 9b^2$
   ∗(c) $28a^2 + 11ab - 24b^2$
   (d) $a^2b^2 - 25$

6. If $5a = \frac{3x - 4}{2y - 3x}$, make x the subject of the formula and use your result to obtain the value of x when $a = 1$ and $y = 5$.

7. Solve the following simultaneous equations:
$$5x - 2y = 16,$$
$$x + 3y + 7 = 0.$$

8. Make a the subject of the formula
$$3x = \frac{a + b}{2a - b}.$$

9. Express each of the following as a fraction with a single denominator.
   (a) $\frac{5(a + b)}{3} + \frac{a - b}{5}$
   (b) $\frac{1}{2a} - \frac{2}{7a} + \frac{1}{3}$

10. If the numerator of a fraction is increased by 1, the value of the fraction becomes 1. If the numerator is increased by 2 and the denominator is decreased by 3, the value of the fraction becomes 2. What is the fraction?

Revision Exercise II No. 2

1. How long will it take a car to cross a bridge 900 m long if it is travelling at 54 km/h?

2. Write down the expansion of
   (a) $(a + b)(a^2 - ab + b^2)$,
   (b) $(x - y)(x^2 + xy + y^2)$.

3. A profit of 12% can be obtained if an article is sold for $95.20. Find the percentage profit or loss if the article is sold for $76.50.

4. Solve the following equations:
   (a) $\frac{x}{2} + \frac{2x}{5} = \frac{x}{15} + 3$
   (b) $\frac{2x - 5}{6} - \frac{x - 2}{4} = \frac{3}{8}$

5. Simplify
   (a) $\frac{10a^6 - 12a^4 + 34a^2}{2a^2}$
   (b) $\frac{33a^2b + 12ab^2 - 3ab}{3ab}$

6. Make x the subject of the formula
$$\frac{1}{a} = \frac{2}{b} - \frac{3}{x}.$$

7. Solve the simultaneous equations
$$2(x - 1) + 3(y + 1) = 4,$$
$$5(x - 2) + 2(y + 2) = 7.$$

8. Factorise the following:
   (a) $4x^2 + 24x + 35$
   (b) $5x^2 - 2x - 3$
   (c) $6x^2 - 23x + 21$

9. Solve the following equations:
   (a) $x(2 - 3x) + 1 = 0$
   (b) $5x(x + 1) - 18 = 2x(2x - 1)$

10. A man paid $101 for 2 pairs of trousers and 3 shirts. He found out that 7 pairs of trousers and 9 shirts would cost $331. Find the cost of
   (a) a pair of trousers,
   (b) a shirt.

Revision Exercise II No. 3

1. Given that $S = \frac{a}{1-r}$, express r in terms of S and a. Find r when $S = 11\frac{1}{9}$ and $a = 10$.

2. A car travels 75 km in $1\frac{1}{4}$ hours and then travels for $\frac{1}{2}$ hour at an average speed of 80 km/h. Find the average speed for the whole journey.

3. Two felt pens and three rulers cost \$3.30 while five felt pens and seven rulers cost \$8.10. Calculate the cost of three felt pens and four rulers.

4. Solve the following equations:
   (a) $\frac{x}{3} - \frac{x}{4} = \frac{3}{8}$ **(b)** $\frac{2}{3a} + \frac{1}{4a} = \frac{5}{6}$

5. A boy paid \$1.60 for 4 exercise books and 2 ball-point pens. If a ball-point pen costs 5¢ more than an exercise book, find the cost of each ball-point pen and each exercise book.

6. Given that $\frac{1}{a} = \frac{1}{2b} + \frac{1}{3c}$, express c in terms of a and b. Hence or otherwise evaluate c when $a = 2$ and $b = 5$.

7. One side of a rectangle is 2 cm longer than the other. If its area is 195 cm², find its perimeter.

8. Factorise the following expressions:
   ✶**(a)** $x^2 - 38x + 345$
   (b) $x(x + 2) + x(x - 5) - 5$
   (c) $9(x^2 + y^2) - 25y^2$

9. Solve the following equations:
   (a) $2(x^2 - 9) = (x + 3)(x - 6)$
   ✶**(b)** $x(x^2 - x - 12) = (x + 2)(x + 3)^2$

10. Simplify the following expressions:
   (a) $(x + 2)(x - 5) - 3x(x - 4) + 7x(x + 3)$
   (b) $\frac{1}{x-1} + \frac{1}{x+1}$

Revision Exercise II No. 4

1. Solve the following equations:
   (a) $\frac{4x}{9} - \frac{x}{6} = \frac{1}{5}$
   (b) $\frac{7x-3}{5x} + \frac{1}{x} = 3$

2. A bicycle wheel has a diameter of 29 cm. Find the number of revolutions it makes when it travels half a kilometre. Give your answer correct to the nearest whole number.

3. The energy, E, of an object of mass m kg travelling at a height of h metres with velocity v m/s is $E = \left(\frac{mv^2}{2} + mgh\right)$ joules. Make m the subject of the formula.

4. In $\triangle ABC$, $\hat{C} = 90°$ and $\hat{A} = 60°$. The bisectors of $\hat{B}$ and $\hat{C}$ meet at D. Show that $B\hat{D}C = 120°$.

5. Factorise
   (a) $(x + 2)^2 + x^2 + 6x + 8$,
   ✶**(b)** $(x + y)^2 + 5(x + y) + 6$,
   ✶**(c)** $a^4 - b^4$.

✶6. Solve the simultaneous equations

$$\frac{1}{7}(x + 2) + y = 4,$$
$$\frac{1}{4}(x - 1) + 2y = 7.$$

7. Make x the subject of the formulae:
   (a) $\frac{x}{a} - b = \frac{x}{c}$,
   (b) $ax + b = cx - d$.

8. Factorise
   (a) $3p^3q - 12p^2q^2 + 9pq^3$,
   (b) $2x^3y + 8x^2y^2 + 8xy^3$,
   (c) $4a^2 - 22ab + 30b^2$.

9. If 1 is added to both the numerator and the denominator of a fraction, its value becomes $\frac{3}{4}$. If 3 is subtracted from both the numerator and the denominator, its value becomes $\frac{1}{2}$. Find the fraction.

10. The table below shows the indices to measure the quality of the teaching of science in five Asean countries for the years 1996 to 1999, 1 being the worst and 10 being the best quality.

	1996	1997	1998	1999
Singapore	8.08	8.15	8.16	8.73
Malaysia	5.89	7.00	6.85	7.02
Thailand	3.47	4.15	3.72	4.72
Indonesia	4.90	4.53	4.40	4.57
Philippines	5.18	4.34	4.77	4.91

(a) Construct line graphs for the indices of Singapore, Malaysia and Indonesia for the period 1996 to 1999 in a single diagram.

(b) Calculate the percentage increase in the index for Malaysia from 1996 to 1997.

(c) Calculate the percentage change in the index for Singapore from 1997 to 1998 and from 1998 to 1999. Can you suggest a reason for the small percentage increase from 1997 to 1998 for Singapore?

(d) Suggest a reason for the drop in the indices for Malaysia, Thailand and Indonesia from 1997 to 1998.

Revision Exercise II No. 5

1. A boy used 3.214 for π instead of 3.142 in calculating the area of a circle of radius 7.6 cm. Find the percentage error giving your answer correct to 3 significant figures.

2. The ratio of the weights of two boys is 7 : 8. If the heavier boy weighs 48 kg, what is the weight of the lighter boy?

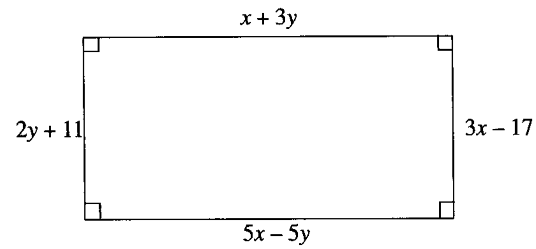
3. Given that $A = k(R^2 - r^2)$, express k in terms of A, R and r. Find the value of k if $A = 50$, $R = 5$ and $r = 3$.

4. A group of children shared \$$x$ among themselves. Each of them got \$7 and there were \$4 left. If there were \$5 more, each would get \$8. Find x and the number of children.

∗5. Solve the simultaneous equations

$$3x + 2y = -2,$$
$$\frac{3x+2}{2} + \frac{y-2}{3} = 0.$$

6. In the given figure, the lengths are in cm. Find the area of the rectangle.



7. (a) If $v = u + at$, find t in terms of u, v and a. Find the value of t when $v = 12$, $u = 2$ and $a = 4$.

(b) The product of two consecutive positive integers is 240. Find the two numbers.

8. Simplify the following expressions:

(a) $\frac{2}{x+2} - \frac{3}{x-3}$ **(b)** $\frac{x-5}{x+6} - \frac{x+5}{x-6}$

(c) $\frac{1}{2x} - \frac{3}{x+6}$

9. Make x the subject of the formulae:

(a) $a = bx + x$ **(b)** $a = \frac{ax+b}{cx-b}$

10. (a) Factorise

(i) $x^4 + x^2 - 2$,

(ii) $x^4 - 13x^2 + 36$.

(b) Evaluate the following without using a calculator:

(i) $243 \times 244 - 243^2$

(ii) $196^2 - 195^2$

Mid-Year Examination Specimen Paper 1

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** allowed to be used.*

1. Find, using factorisation, the value of

(a) $73.7^2 + 26.3 \times 73.7$ [2]

(b) $9.85^2 - 0.15^2$ [2]

2. A, B and C undertook a piece of work for which \$675 was paid. A worked 4 hours a day for 8 days, B worked 6 hours a day for 6 days and C worked 8 hours a day for 5 days. If all three were paid the same rate, calculate how much A was paid. [4]

3. The scale of a map is 4 cm to x km. Find, in terms of x, the actual area of a lake which has an area of 26 cm^2 on the map. [3]

4. If $T = \frac{k}{l}(x - l)$, find l in terms of T, k and x. Given that $T = 3$, $k = 15$ and $x = 6$, find the value of l. [4]

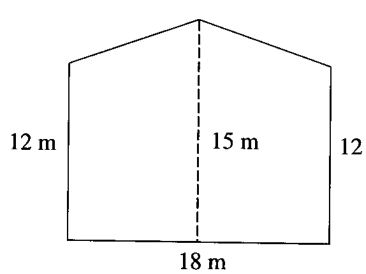
5. Which of the letters Z, N, O, M and H have

(a) just one axis of symmetry, [2]

(b) rotational symmetry of order 2? [2]

6. A rectangular box, 21 cm by 19 cm by 11 cm internally, is made of wood 0.5 cm thick. If the box has a lid, find the volume of the wood used in cm^3. [3]

7. The cross-section of a barn is shown below.



Find the air space in the barn which is 30 m in length. [4]

8. Solve the simultaneous equations

$4x + y = 7, \quad 2x + 3y = 11.$ [3]

9. If x is divided by a certain number, the result is 11 and the remainder is 6. If $x + 8$ is divided by the same number, the result is 13. Find x and the number. [4]

10. Factorise the following expressions:

(a) $5(3x - 4)^2 - 45$ [2]

(b) $2(x^2 + 3) + 7(x^2 - 1)$ [2]

11. Simplify the following expressions:

(a) $(3x - 4)(2x - 3) - x(2x - 5)$ [2]

(b) $(x + y)^2 - (x - y)^2$ [2]

***12.** Solve the following equations:

(a) $3x(x + 4) + 28(x + 2) + 21 = 0$ [2]

(b) $2(4x^2 + 23x) - 105 = 0$ [2]

13. **(a)** If $(a - b)^2 = 19$ and $(a + b)^2 = 37$, find the values of

(i) $8ab$ **(ii)** $3a^2 + 3b^2$. [3]

(b) Given that $2x - 3y = 5(x - 2y)$, find the numerical value of $\frac{3x}{4y}$. [2]

Part II (50 marks) **Time: 1 h 15 min**

*Answer **all** the questions. Calculators may be used.*

Section A (22 marks)

1. Simplify each of the following:

(a) $(2x + 1)(3x - 1) - (x + 2)(3x - 4)$ [2]

(b) $(4x + 2y)(x - 3y) - (4x - y)(2x - 3y)$ [2]

2. Solve the equations

(a) $5x^2 + 7x = 0$, [2]

(b) $2\frac{1}{4}x + 3\frac{1}{2} = \frac{1}{6}x$. [2]

3. A rectangle is measured as 6.3 cm by 7.8 cm. If its actual area is 51.2 cm^2, find the percentage error in the area. [3]

4. A man drove 60 km at 30 km/h, 60 km at 40 km/h and 60 km at 50 km/h subsequently. What was his average speed for the whole journey? [4]

5. (a) A sum of money is divided among three people in the ratio 7 : 6 : 5. If the largest share is $210, what is the difference between the smallest share and the largest share? [3]

(b) A wholesaler allows a retailer a trade discount of 25% on an article listed at $54. The retailer gives a cash discount of 5% on the list price. What profit does the retailer make? [4]

Section B (28 marks)

6. (a) A man sold his car for $18 415 at a loss of $27\frac{1}{2}$%. How much did the car cost him originally? [3]

(b) Find the simple interest on $1 500 invested for 8 months at 10% per annum. [3]

7. (a) Find the size of the exterior angle of a regular 16-sided polygon. [3]

(b) Three of the interior angles of a hexagon are 128°, 128° and 134° while the other three interior angles are $4x°$, $5x°$ and $6x°$. Find the value of x. [4]

8. (a) Given that $A = 2x^2 - 3x + 5$ and $B = 7x^2 - 4x + 9$, simplify $7A - 3B$. [3]

(b) The sum of the ages of Peter and Jane is 36 years. Six years ago, Peter was twice as old as Jane. Find their ages in 5 years' time. [4]

9. (a) An American tourist went on a tour of Singapore and Malaysia in 2000. He exchanged US$2 540 for Singapore dollars at a rate of US$1 = S$1.710 4. While in Singapore, he spent S$2 560 and exchanged the remaining Singapore dollars for Malaysian ringgit at a rate of S$1 = RM2.213 5. While in Malaysia, he exchanged a further US$860 for Malaysian ringgit at a rate of US$1 = RM3.786 4. He spent RM4 780 and exchanged the remaining ringgits for US dollars at a rate of US$1 = RM3.816 5. How many complete US dollars can he get? [4]

(b) A firm hires out vans.
A van can be hired for $40 per day plus an insurance coverage of $15 per van for period of less than a week. If the total distance travelled during the total period of hire is 150 km or less, no extra charge is made. However, every extra kilometre travelled over 150 km is charged at 10¢ per extra kilometre.

(i) A van was hired for 4 days and travelled 400 km in this time. Calculate the total hire charge. [2]

(ii) A man who hired a van for 2 days was charged $108. Calculate the total distance he travelled. [2]

Mid-Year Examination Specimen Paper 2

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** allowed to be used.*

1. Express
(a) 82.785 correct to 3 significant figures, [1]
(b) 1.298 4 correct to 2 decimal places, [1]
(c) 0.002 37 in standard form, [1]
(d) 0.08 as a fraction in its lowest terms. [1]

2. Make x the subject of the formula
$\frac{w}{k} = \frac{2x}{a - x}$. [4]

3. A man borrows $8 000 from a bank which charges simple interest of 5.5% per annum. Calculate the amount that the man has to pay at the end of 3 years. [4]

4. A man and a woman shared a lucky draw prize of $6 000 in the ratio 2 : 3. The woman divides her part of the share among her mother, her two daughters and herself in the ratio 3 : 1 : 1 : 5. How much will her mother get? [4]

5. Factorise the following expressions:
(a) $9x^2 - 12x + 4$ [2]
(b) $75x - 27xy^2$ [2]

6. Solve the following equations:
(a) $(x-1)^2 - 16 = 0$ [2]
(b) $\frac{x-3}{4} = \frac{2}{x+4}$ [2]

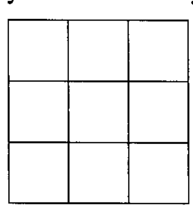
7. A regular polygon of $2n$ sides has interior angles 30° greater than the interior angles of a regular polygon with n sides. Find n. [4]

8. Evaluate each of the following, giving your answer in the standard form:
(a) $2.43 \times 10^4 + 7.64 \times 10^3$ [1]
(b) $8.35 \times 10^5 - 9.4 \times 10^4$ [1]
(c) $3.6 \times 10^{-4} \times 5.0 \times 10^{-5}$ [1]
(d) $(8.4 \times 10^3) \div (2.1 \times 10^{-6})$ [1]

9. (a) Express $\$\frac{4}{25}$ in cents. [1]
(b) Find 28% of 4.8 kg. [1]
(c) Divide 2.47 by 7.6 exactly. [1]
(d) How many cm are there in 12.5% of 3.2 m? [1]

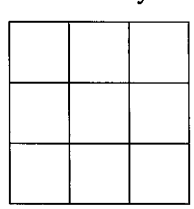
10. Simplify
(a) $\frac{3(a-b)}{5} - \frac{3a+4b}{10} + \frac{a-b}{2}$ [2]
(b) $\frac{x+y}{2} - \frac{x-y}{3} + \frac{4(3x-2y)}{6}$ [2]

11. (a) On the diagram, shade THREE small squares so that the final diagram will have exactly two lines of symmetry.



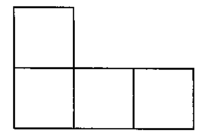
[1]

(b) On the diagram, shade FIVE small squares so that the final diagram will have just one line of symmetry.



[1]

(c) Add one more square to the diagram so that the resulting diagram will have rotational symmetry of order 2. Also mark on the diagram the centre of rotational symmetry.



[2]

12. The angles of a triangle are in the ratio 3 : 4 : 8. Find the largest angle. [3]

13. Solve the simultaneous equations
$\frac{x}{5} + \frac{y}{3} = 4, \quad x - \frac{y}{2} = 7.$ [3]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used.*

Section A (22 marks)

1. On a map with a scale of 1 : 4 000, what is the actual distance represented by 25 cm on the map? What will be the actual area of a lake on the map with an area of 8 cm^2? [4]

2. A cyclist travels 30 km at 20 km/h and a further 80 km at 25 km/h. Find
(a) the total time taken for the whole journey, [2]
(b) the average speed for the whole journey. [2]

3. (a) Solve the equation
$xy - 3x + 4y - 12 = 0$ [3]
(b) A man deposited $8 500 into a bank that pays 3.125% compound interest for 3 years. Calculate the amount he will receive at the end of the term, giving your answer correct to the nearest 50 cents. [4]

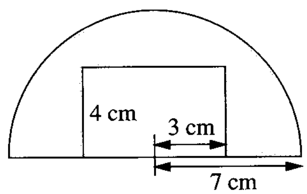
4. A cylindrical pipe is 7 m long, the external diameter 4.8 cm and its thickness 3 mm. Calculate the volume of metal used in making the pipe. [4]

If the density of the metal is 4.2 g/cm^3, find the mass of the metal used, giving your answer in kg. $\left(\text{Take } \pi \text{ to be } \frac{22}{7}\right)$ [3]

Section B (28 marks)

5. Construct a quadrilateral $ABCD$ such that AB = 6 cm, BC = 7 cm, AD = 4.8 cm, CD = 8.2 cm and $A\hat{B}C = 124°$. Measure $A\hat{D}C$, correct to the nearest degree. [6]

6. (a) Find the area and perimeter of the shaded portion of the figure.



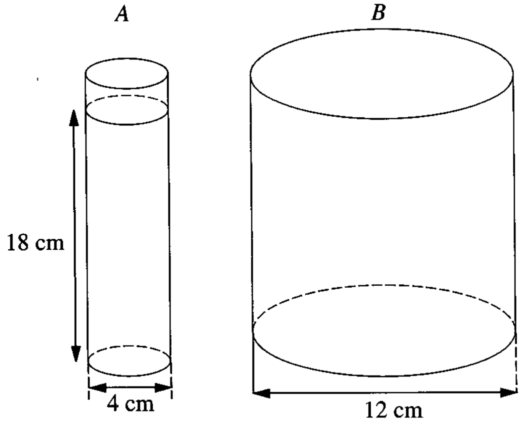
[4]

(b) Solve the following equations:

(i) $\frac{6-x}{3} - \frac{1-x}{4} = 1\frac{1}{3}$ [2]

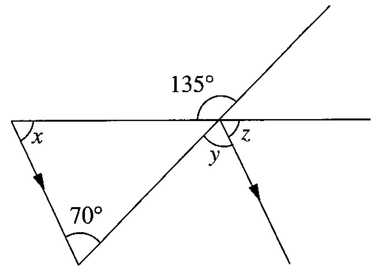
(ii) $3x - 3\frac{1}{3} = 7 + 1\frac{2}{3}x + 1\frac{2}{3}$ [2]

7. The diagram below shows two cylinders, A and B. Initially A contains water to a depth of 18 cm. The water in A is then poured into B. Calculate the depth of the water in B and the total surface area of B that is in contact with the water. (Take π = 3.14)



[7]

8. (a) Find the angles marked x, y and z in the following figure. [3]



(b) Eight men took 12 days to repair a road 4 km long. How long would it take 6 men to repair 1 km of the road? [2]

(c) A cyclist rides 35 km at 10 km/h and a further 30 km at 12 km/h. Find his average speed for the whole journey. [2]

Mid-Year Examination Specimen Paper 3

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** allowed to be used.*

1. Solve the simultaneous equations
$3x + y = 3$, $7x - y = 2$. [3]

2. In $\triangle XYZ$, $XY = XZ$ and W is a point on XZ such that $WY = YZ$. If $Y\hat{X}Z = 52°$, find the value of $X\hat{Y}W$. [4]

3. Factorise the following:

(a) $(2a - b)^2 - 4c^2$ [2]

(b) $18x^3 - 8xy^2$ [2]

4. Expand and simplify

(a) $(x + y)(2x - y) - y^2 + xy$, [2]

(b) $(2x - y)^2 - (x + 2y)^2$. [2]

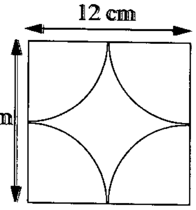
5. Simplify

(a) $\frac{x-2}{2} + \frac{x+3}{4} - \frac{x-5}{8}$, [2]

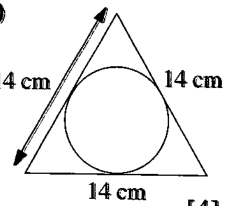
(b) $\frac{x}{x-2} + \frac{1}{x+2}$. [2]

6. Write down the number of axes of symmetry and the order of rotational symmetry for each of the following figures:

(a)



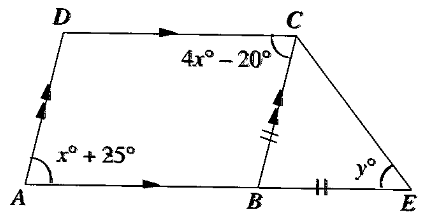
(b)



[4]

7. Given that $T = \frac{l(x-a)}{x-b}$, make x the subject of the formula. [3]

8. In the figure, $ABCD$ is a parallelogram, ABE is a straight line and $BC = BE$. Find the values of x and y. [4]



9. A rectangular tank 1.5 m long and 58 cm wide contains water to a depth of 65 cm. What is the volume of the water in the tank? The water in the tank is then poured into an empty tank 2 m long and 1 m wide. Find the depth of the water in this tank. [3]

10. The actual length of a square is 5 cm. A man measures it as 5.2 cm. Find the percentage error in the man's measurement of the area. [3]

11. Find the value of x if
   (a) $x : 4 = 56 : 32$, [2]
   (b) $9 : 11 = 81 : (100 - x)$. [2]

12. The interior angle of a regular polygon of n sides is 19 times the exterior angle. Find the value of n. [3]

13. (a) Rewrite 54.2×10^{-3} as a decimal number. [1]
   (b) Express 18.047 correct to
      (i) 3 significant figures, [1]
      (ii) 2 decimal places, [1]
      (iii) the nearest tenth. [1]

14. Solve the equation

$$\frac{1}{4}(3x-1) + \frac{1}{6}(7x-3) = \frac{1}{3}(5x+2).$$ [3]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used.*

Section A (22 marks)

1. Given that $a = 1.37 \times 10^{-10}$, $b = 2.53 \times 10^{-9}$ and $c = 9.43 \times 10^{-8}$, evaluate each of the following, giving your answer in the standard form.
   (a) $8a + 3b + 5c$ [2]
   (b) $4c - 3b + 17a$ [2]

2. A cylindrical pipe is 2 m long and has an internal diameter of 84 mm. How many litres of water can the pipe hold? [4]

3. (a) The coordinates of the point of intersection of the straight lines $y = mx + 6$ and $y = 2x + c$ are $(-5, 1)$. Find the values of m and c. [4]
   (b) A man drove for 2 hours at a speed of 36 km/h, 1 hour at 60 km/h and $1\frac{1}{2}$ hours at 50 km/h. Find his average speed for the whole journey. [3]

4. (a) Factorise $3x^2 - 12$ completely. [2]
   (b) A car travels at a constant speed for 450 km. If the average speed was increased by 15 km/h, the journey would have taken 90 minutes less. Find the speed of the car. [5]

Section B (28 marks)

5. A map of a town is drawn to a scale of 1 : 20 000.
   (a) A stretch of a highway on the map measures 12.4 cm. Calculate the actual length of this stretch of road in metres. [2]
   (b) A school has an area of 30 000 m^2. Find the area of this school on the map. [3]

6. (a) Given that $a = \frac{b-x}{c-x}$, express x in terms of a, b and c. Find the value of x if $a = -2$, $b = 8$ and $c = 2$. [4]
   (b) Solve the equation $2(x^2 + 1) - 5x = 0$. [3]

7. A pair of the opposite sides of a rectangle is $(3y + x)$ cm and $5x$ cm in length. Another pair is $(2x + y + 2)$ cm and $3y$ cm in length. Find x, y and the area of the rectangle. [6]

8. (a) Find the number of sides of a regular polygon if each interior angle is 160°. [2]

   (b) If an exterior angle of an octagon is 80° while the other seven exterior angles are each equal to $2x°$, calculate the value of x. [3]

9. Loading half a load of clothes into a washing machine instead of a full load every other day wastes 130 litres of water. Calculate the total amount of water wasted in a year of 366 days if a household washes their clothes with a half load. How much money will the household have to pay per year, if the cost of water is $1.34 per cubic metre? [5]

Mid-Year Examination Specimen Paper 4

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** allowed to be used.*

1. A cyclist travelling at 20 km/h takes 3 hours longer to travel a certain distance than a motorist travelling at 60 km/h. Find the distance travelled. [3]

2. Simplify each of the following:

   (a) $(4a - 3b) - 2(a - b)$ [2]

   (b) $(x + 1)(3x - 5) - (x - 4)(x + 1)$ [2]

3. Find two consecutive even numbers whose product is 48. [3]

4. Given that $a = 3$, $b = -2$ and $c = 4$, find the value of each of the following:

   (a) $c^2 - 4ab$ [1]

   (b) $ac \div b^2$ [1]

   (c) $3a - b^3 + c$ [1]

5. The length of a rectangle is increased by 20% while its width is decreased by 10%. Calculate the percentage change in the area of the rectangle. [3]

6. Construct a quadrilateral $PQRS$ where $PQ = PR = 12$ cm, $PS = 10$ cm, $Q\hat{R}P = 64°$ and $R\hat{P}S = 43°$. Measure QR and RS. [5]

7. Solve the following equations:

   (a) $x - 2 = \frac{x - 4}{3}$ [2]

   (b) $4\frac{4}{5}(2x - 1) = \frac{1}{2}(3x - 2)$ [2]

8. A father is now four times as old as his son. Five years ago, he was seven times as old as his son. How old are they now? [4]

9. If $A = P(1 + rt)$, find t in terms of A, P and r. Given that $P = 200$, $A = 235$ and $r = 0.05$, find the value of t. [4]

10. The scale of a map is 1 : 400 000. Find the area of a piece of land represented by an area of 5.6 cm^2 on the map. Give your answer in km^2. [3]

11. Evaluate each of the following, giving your answer in the standard form:

    (a) $(2.88 \times 10^4) \div (3.6 \times 10^{-4})$ [1]

    (b) $3.72 \times 10^{10} + 9.5 \times 10^9$ [1]

    (c) $2.5 \times 10^6 - 5.9 \times 10^5$ [1]

12. Solve the simultaneous equations $x - y = 4$, $2x = 13 - 3y$. [3]

13. By watering plants with a container instead of a running hose, Mrs Kumar finds that she can save 115 litres of water per watering session. If Mrs Kumar normally waters plants three times a week, how much water could she save in a year? How much will this water-saving habit translate into money saved if each litre of water is charged at 0.136 cents? [Give your answer correct to the nearest cent.] [5]

14. Solve the equation $\frac{x - 5}{3} - \frac{x - 4}{6} = 5$. [3]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used.*

Section A (22 marks)

1. A circular pond 1.2 m in diameter is filled with water to a depth of 24 cm. Water is drained through a 24-mm-diameter pipe. If the water in the pipe is moving at 150 m per minute, how long does it take to empty the pond? [4]

2. Factorise the following completely:
   (a) $3xy + 2y - 12x - 8$ [2]
   (b) $x^4 - 16$ [2]

3. The sum of the digits of a two-digit number is 7 and the difference between the number and that by reversing its digits is 9. Find the number. [6]

4. **(a)** A wire is in the form of a circle of radius 2.8 cm. If the wire is bent to form the shape of a square, find the perimeter and area of the square. Taking $\pi = 3.14$, give your answer correct to 3 significant figures. [5]
   (b) Solve the equation $6x^3 = 384$. [3]

Section B (28 marks)

∗5. **(a)** Solve the following simultaneous equations

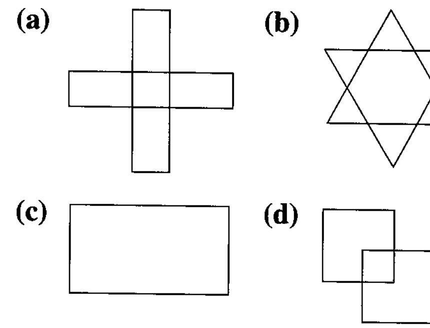
$$\frac{5x+y}{9} + \frac{x+y}{5} = 2,$$

$$\frac{7x-3}{2} - \frac{y-x}{3} = 1.$$ [4]

   (b) Solve the equation $(2x-3)^2 = (4x-1)(x-6)$. [3]

6. **(a)** An athlete runs round a circular track of diameter 196 m. How many complete rounds must he make in order to cover a distance of 2.4 km? [4]
   (b) A rectangle of length 36 cm has an area of 504 cm². Calculate its perimeter. [4]

7. State the number of axes of symmetry and the order of rotational symmetry for each of the following figures:



[8]

8. Simplify the following algebraic fractions:
   (a) $\frac{1}{x} + \frac{1}{y} - \frac{2}{z}$ [2]
   (b) $\frac{2}{x-3} - \frac{1}{2x-5}$ [3]

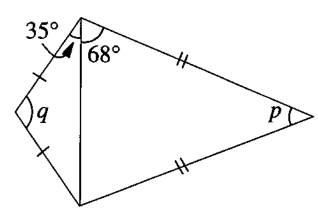
Mid-Year Examination Specimen Paper 5

Part I (50 marks) Time: 1 h

*Answer **all** the questions. Calculators are **not** allowed to be used.*

1. A rectangle measures 45 cm by 32 cm. If its width is decreased by 8 cm and its area remains unchanged, find its length. [3]

2. Find the angles marked p and q in the figure below.



[3]

3. If $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, make f the subject of the formula. Find the value of f when $u = 1.5$ and $v = 2.5$. [4]

4. Expand the following:
   (a) $(2x-1)(3-4x)$ [1]
   (b) $(x+2)(x^2-5)$ [2]
   (c) $(2x+3)(2-3x-5x^2)$ [2]

5. Factorise the following:

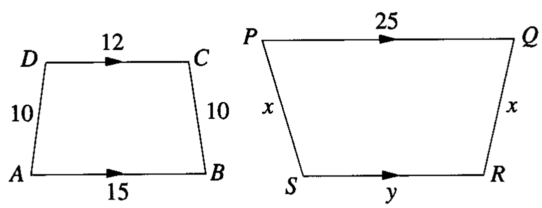
(a) $x^2 - 3x - 10$ [1]
(b) $x^2 + 2x - xy - 2y$ [2]
(c) $27x^2 - 12y^4$ [2]

6. Solve the simultaneous equations

$$\frac{x}{3} + \frac{y-4}{5} = -\frac{1}{5},$$
$$\frac{3x-2}{5} + \frac{2y-5}{6} = -\frac{1}{10}.$$ [4]

7. X can complete a piece of work in 9 days and Y can complete the same work in 18 days. How long will X and Y take to complete the work together? [4]

8. In the diagram, trapezium $ABCD$ is similar to trapezium $PQRS$. Calculate the values of x and y. [4]



9. Express the following as a fraction in its simplest form:

(a) $\frac{x}{3} + \frac{x}{4} + \frac{x}{5}$ [2]

(b) $\frac{1}{x+2} + \frac{2}{x+3}$ [2]

10. Make D the subject of the formula

$$A = \frac{BD}{C+D}.$$ [3]

11. Solve the equations:

(a) $(x - 3)^2 - 36 = 0,$ [2]
(b) $x^2 - 5x - 14 = 0.$ [2]

12. A man bought 12 shirts for $180 and sold them for $18 each. What was his percentage gain? [3]

13. A boy is 42 years younger than his father. In 8 years' time, he will be $\frac{1}{4}$ times as old as his father. Find their present ages. [4]

Part II (50 marks) Time: 1 h 15 min

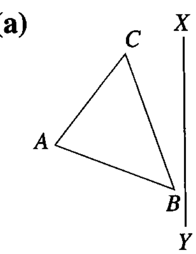
*Answer **all** the questions. Calculators may be used.*

Section A (22 marks)

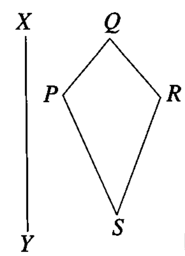
1. A cyclist travelling at 14 km/h takes 6 hours 45 minutes to cover a certain journey. How long would it take a car travelling at 52 km/h to cover the same journey? Give your answer correct to the nearest minute. [4]

2. Draw the mirror image of the following figures so as to form a symmetrical pattern about the line XY.

(a)



(b)



[4]

3. A man would gain 20% by selling a bag for $10.20 and 25% by selling a pen for $12. As there is a minor defect in the bag, he sold it for $7.20. For what price must he sell the pen if there is to be no loss or gain on the two sales? [6]

4. The Primary Production Department (PPD) in Singapore aims to increase the local fish production from the 1999 level of 3 000 to 40 000 tonnes per year in its 10-year plan. This represents an increase from 3% to 32% of total consumption. Calculate the total fish consumption in 1999 and 2009. [4]

5. When DBS Bank lowers its rate of interest from 5.5% to 5% per annum, a man saves $2 370 on simple interest in 3 years. Find the amount of money the man borrowed from the bank. [4]

Section B (28 marks)

6. Evaluate each of the following, giving your answer in the standard form:

(a) $2.64 \times 10^{-2} - 5.8 \times 10^{-3}$ [1]

(b) $6.84 \times 10^{5} + 3.6 \times 10^{6}$ [1]

(c) $\dfrac{25\pi + 15.4^{3}}{2.8^{5} - \sqrt{840}}$ [2]

7. A salesman sold 40 pairs of shoes a day and the total sales were $620. Given that some of the shoes were sold at $10 a pair and the rest $20, how many pairs of shoes worth $20 a pair did he sell? [6]

8. The product of two numbers is 108. If one number is greater than the other by 3, find the possible values of the two numbers. [6]

9. The scale of a map is 1 cm to 500 m.

(a) If the distance between two towns on the map is 8.4 cm, calculate its actual distance in km. [2]

(b) A railway track has a length of 14.8 km, calculate its length on the map. [2]

(c) A town has an area of 4.8 km², find its area on the map. [3]

10. Mr Ong invests $25 000 in fixed deposit that pays interest of 3% per annum. If Mr Ong reinvests the interest, how much money can he get if he withdraws his money after 3 years. Give your answer correct to the nearest 50 cents. [5]

CHAPTER 9

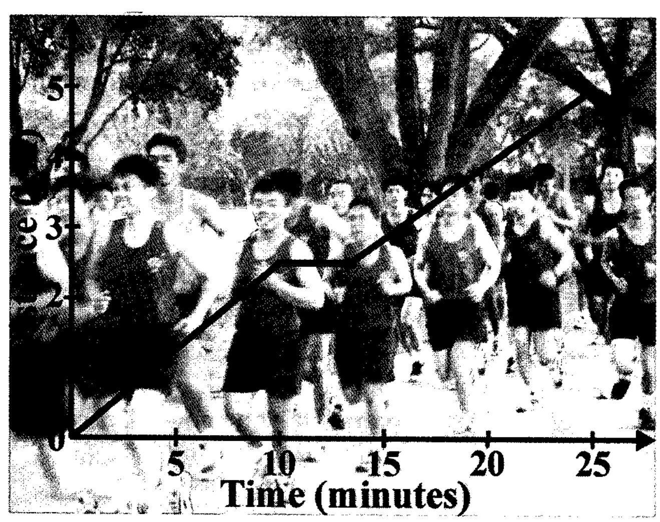
Linear Graphs and Their Applications

In this chapter, you will learn how to

- △ plot coordinate points on a graph;
- △ plot straight line graphs;
- △ solve simultaneous linear equations graphically;
- △ interpret and use graphs in practical situations;
- △ draw graphs using data from practical situations.

Preliminary Problem

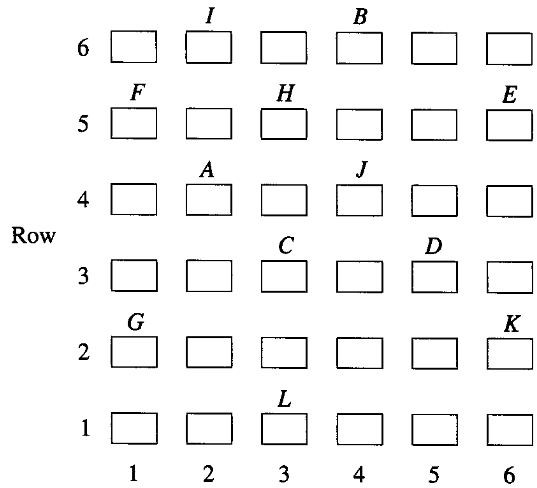
Linear graphs are used in a lot of daily situations. For example, we use a travel graph to show a journey undertaken by a moving object, like a moving vehicle. The picture displays a linear graph of a runner.



Rectangular Coordinates in Two Dimensions

Have you ever noticed how a seat in a cinema is identified? How about the position of a chess piece on the chessboard and the location of a place in a sectional map in the Singapore Street Directory? Can you use the same idea to identify a desk in a classroom where the desks are arranged neatly in rows and columns?

At the beginning of a new school year, some teachers will prepare a seating arrangement plan of a class (as shown on the right) in order to know the students quickly. Each box represents the position of a pupil.



FRONT OF THE CLASS
Fig. 9.1

To identify each student more easily, the teacher can connect each student with the column and the row in which he or she is sitting. Thus, student A sits in column 2, row 4, while student B sits in column 4, row 6 and so on.

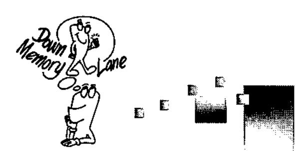
Use the CD, The Business of Graphs, from the DMS to get a visual presentation of rectangular coordinate system. There are also many interesting activities provided in the CD.

The teacher can write a pair of numbers against the name of a student in the class list as follows:

$A(2, 4)$	$B(4, 6)$	$C(3, 3)$
$D(5, 3)$	$E(6, 5)$ and so on.	

From the pair of numbers (2, 4), we know that student A is in column 2 and in row 4. The pair of numbers (4, 6) tells us that student B is in column 4 and in row 6.

Can you write down the pairs of numbers corresponding to students F, G, H, I, J, K and L?



Rene Descartes (1596–1650), the great French mathematician, developed the idea of combining algebra with geometry. This resulted in the formation of a new field of mathematics called analytical geometry or Cartesian geometry in honour of him.

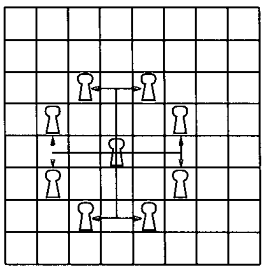
Is the order in which the two numbers are written important? That is, do (5, 3) and (3, 5) indicate the same position?

The pairs of numbers (2, 4), (4, 6), (3, 3), (5, 3), (6, 5) and so on are examples of **ordered pairs**. Do you know why they are called ordered pairs?

Figure 9.2 displays the same classroom plan on graph paper with horizontal lines and vertical lines drawn through the centres of the boxes showing the positions of the students. The horizontal lines and vertical lines are numbered as shown.

Just for Fun

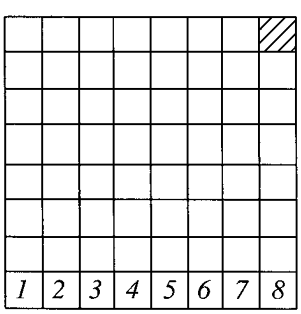
In chess, the knight is normally represented by a horse's head. A knight moves to its new position by moving two steps (two squares) horizontally (either left or right) and one step vertically (either upwards or downwards) or two steps vertically and one step horizontally as shown in the diagram.



(a) If a knight starts from position 1 in the diagram below, what is the minimum number of moves required for it to reach the shaded square?

(b) Find the minimum number of moves required in each case if the knight starts from 2, 3, 4, 5, 6, 7 or 8.

(c) If the knight starts from position 1, is it possible for it to visit all the 64 squares once only? (Use a pencil to cross out the squares it has visited.)



Thus, the first number in each ordered pair used to locate a student refers to the horizontal scale while the second number refers to the vertical scale. To simplify it even further, we can now use a point (indicated by crosses) to replace the boxes to show the students' positions. This gives us an idea of locating a point in a plan.

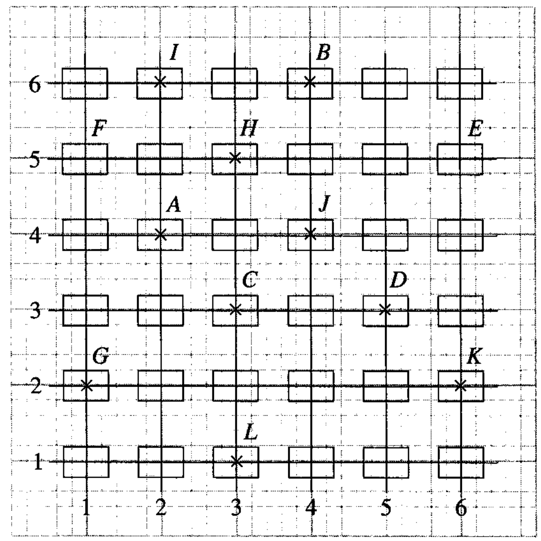


Fig. 9.2

The Rectangular or Cartesian Plane

In Book 1 we discussed how to locate a point representing a real number on a number line. We shall now discuss how to use an ordered pair of real numbers (a, b) to determine a point in a plane.

Fig. 9.3 below shows a **rectangular** or **Cartesian** plane consisting of two number lines Ox and Oy intersecting at right angles at the point O.

The point O is known as the **origin**. Ox and Oy are called the **coordinate axes** or the **horizontal axis** and **vertical axis** respectively, or more simply as the ***x*-axis** and ***y*-axis**.

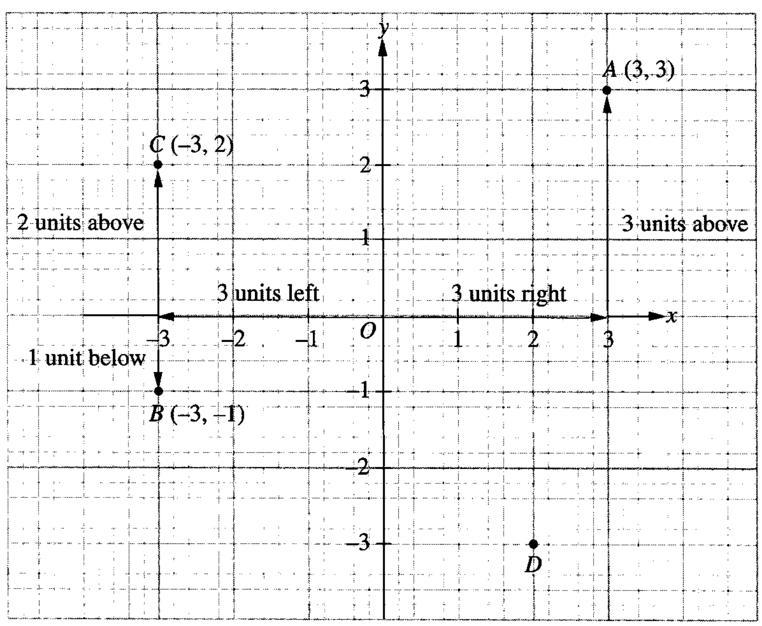


Fig. 9.3

Coordinates of a Point

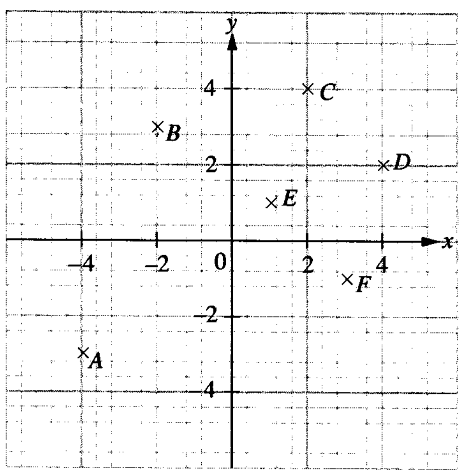
The position of any point in the plane can be determined by its distance from each of the axes. In Fig. 9.3, point A is 3 units to the **right** of the y-axis and 3 units **above** the x-axis; its position is described by the ordered pair (3, 3). The ordered pair (–3, –1) determines point B since it is 3 units to the **left** of the y-axis and 1 unit **below** the x-axis. Point C is 3 units to the **left** of the y-axis and 2 units **above** the x-axis and thus it is represented by the ordered pair (–3, 2).

Can you name the ordered pair that determines point D? What ordered pair represents the origin O?

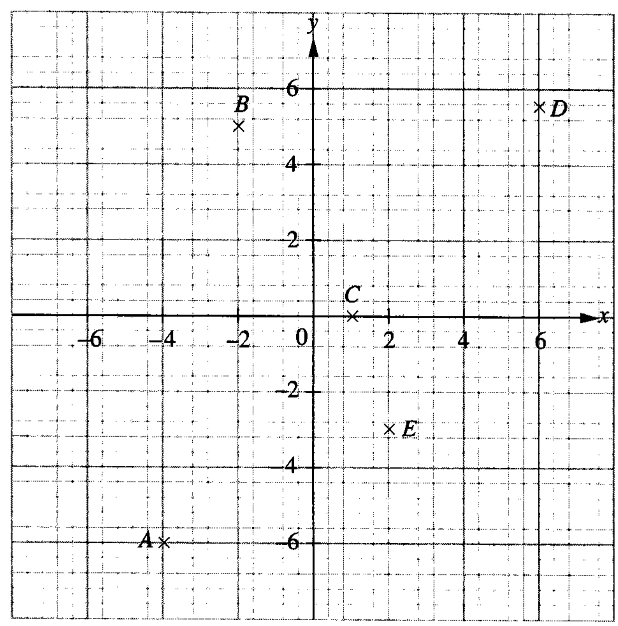
From the above discussion, each point P in the plane is located by an ordered pair (a, b). We call a the **x-coordinate** (or **abscissa**) of P, and b the **y-coordinate** (or **ordinate**). We say that P has **coordinates** (a, b) and refer to P as the point (a, b) or $P(a, b)$. Thus, A has coordinates (3, 3). The x-coordinate and y-coordinate of B are –3 and –1 respectively. C is the point (–3, 2).

Exercise 9a

1. Write down the coordinates of the points in the diagram below:



2. Write down the coordinates of the points shown in the diagram below:



3. Plot the following points on a piece of graph paper:
$A(2, 5)$, $B(1, 2)$, $C(-3, -1)$, $D(6, -2)$, $E(3, -2)$, $F(-1, 2)$.

4. Plot the following points on a piece of graph paper:
$W(-4, 0)$, $X(3, 6)$, $Y(0, 4)$, $Z(5, -5)$.

5. Plot each set of the given points on graph paper. Join the points in order with straight lines and identify the geometrical shapes obtained.

(a) (6, 4), (–6, 4), (–6, –4), (6, –4).

(b) (0, 5), (–6, 0), (0, –5), (6, 0).

(c) (0, 0), (0, 8), (5, 4).
(d) (1, 0), (0, 3), (−1, 4), (−5, −2).
(e) (4, 0), (0, 5), (−4, 0).
(f) (5, 2), (−1, 3), (−1, −3), (5, −2).
(g) (0, −3), (6, −3), (4, 2), (−1, 2).

6. The vertices of a right-angled triangle are $A(1, 0)$, $B(7, 0)$ and $C(1, 8)$. Write down the coordinates of the mid-points of all the three sides of the triangle.

7. Plot the following points on a piece of graph paper:
(3, −5), (2, −3), (1, −1), (0, 1), (−1, 3), (−2, 5), (−3, 7).
Do you notice that the points lie in a special pattern? Describe the pattern.

8. **(a)** Plot the following points on a piece of graph paper:
(−3, −1), (−2, 0), (−1, 1), (0, 2), (1, 3), (2, 4), (3, 5).
(b) Draw a line through all the points.
(c) What is the shape obtained?

9. **(a)** Plot the following points on a piece of graph paper:
(−3, 5), (−2, 0), (−1, −3), (0, −4), (1, −3), (2, 0), (3, 5).
(b) Join all the points with a curve.

Graphs of Linear Equations in Two Variables

In Book 1 we solved equations in one variable, such as

$$2x + 8 = 16, \quad 1\frac{4}{5}x - \frac{3}{5} = 1\frac{1}{5}x + 10\frac{1}{5} \quad \text{and} \quad 2(x - 3) = 4(3x + 4).$$

Each equation has exactly one solution which is a real number. *For example*, the solution or the value of the variable x that satisfies the equation $2x + 8 = 16$ is 4. It can be represented by a point on a number line as shown in Fig. 9.4.

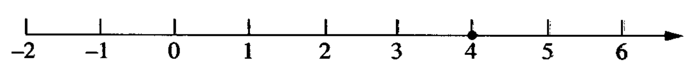


Fig. 9.4

In Chapter 8 of this book we learnt that in solving a linear equation in two variables such as $y = 2x + 1$, we should look at all pairs of numbers x and y that satisfy the equation. If we let $x = 1$, then $y = 2 \times 1 + 1 = 3$. Thus the pair of numbers $x = 1$, $y = 3$ satisfies the equation and hence is a solution to the equation. We know that we can obtain infinitely many solutions to the equation by simply selecting a value for x and then finding the corresponding value of y such that the pair of values satisfies the equation. Obviously, we cannot list all the solutions of the equation. Can we find a way to display geometrically all the solutions?

The table below shows five solutions of the equation $y = 2x + 1$.

x	−2	−1	0	1	2
$y = 2x + 1$	−3	−1	1	3	5

The five pairs of numbers (x, y) appearing in the table above are plotted as points in the Cartesian plane as shown in Fig. 9.5. Do you notice any pattern formed by the points? In Fig. 9.6, a straight line is drawn through the five points.

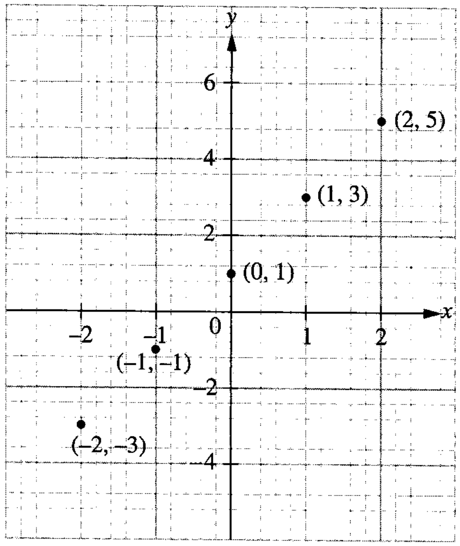


Fig. 9.5

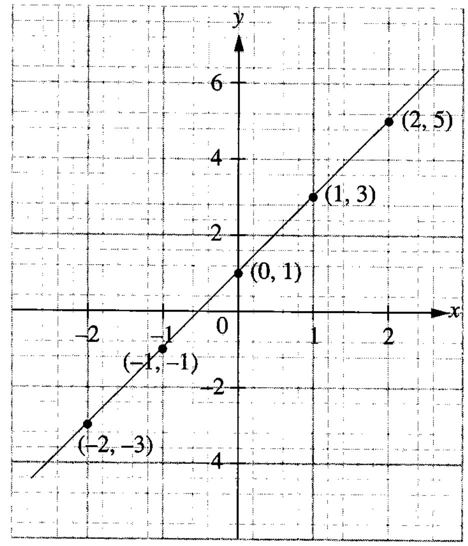


Fig. 9.6

The line is said to be a **graph** of the equation $y = 2x + 1$. Do you agree that every pair of numbers (x, y) satisfying $y = 2x + 1$ appears as a point somewhere on the line and every point on the line has coordinates that satisfy $y = 2x + 1$? The graph of an equation is the set of all points whose coordinates satisfy the equation.

The graph of every linear equation in two variables is a straight line. Since two points determine a line, we require two ordered pairs that satisfy the equation to draw the graph. A third ordered pair is obtained for checking.

Choice of Appropriate Scales for Graphs

Before we proceed to draw a graph, we have to choose a suitable scale. If the scale of the graph to be drawn is given, then follow it. If not, the following guidelines are useful:

1. Use a convenient scale for both the x- and y-axes. *For example,* 1 cm to represent 1 unit, 2 units, 4 units, 5 units or 10 units. Avoid using awkward scales like 1 cm to represent 2.3 units or 1 cm to represent 7 units.

2. The scale used for the x-axis need not be the same as that used for the y-axis. You can use, for example, 1 cm to represent 2 units on the x-axis and 1 cm to represent 5 units on the y-axis.

3. Choose a suitably large scale so that the graph will be more than half the size of the given graph paper. The bigger the graph, the more accurate will be the results obtained from it. Look at the largest and smallest values of x and do a rough calculation to decide the scale that would give the largest possible graph. Do the same for the y values.

Example 7

Draw the graph of $y = 3x - 1$. From your graph, find

(a) the values of y when $x = -0.8$ and 1.3,

(b) the values of x when $y = -2.8$, 0 and 0.8.

Solution

The table below gives the values of x from -1 to 1 and the corresponding values of y.

x	-1	0	1
$y = 3x - 1$	-4	-1	2

Fig. 9.7 shows the graph of $y = 3x - 1$.

(a) To find the value of y when $x = -0.8$, we draw a vertical line from the horizontal axis where $x = -0.8$ to meet the graph and then continue from the graph horizontally to meet the y-axis. Read off the value of y from the y-axis. From the graph, $y = -3.4$ when $x = -0.8$. Similarly, $y = 2.9$ when $x = 1.3$.

(b) To find the value of x when $y = -2.8$, we draw a horizontal line from the vertical axis where $y = -2.8$ to meet the graph and continue from the graph vertically to meet the x-axis. Read off the value of x from the x-axis. From the graph, $x = -0.6$ when $y = -2.8$. Similarly, $x \approx 0.3$ when $y = 0$ and $x = 0.6$ when $y = 0.8$.

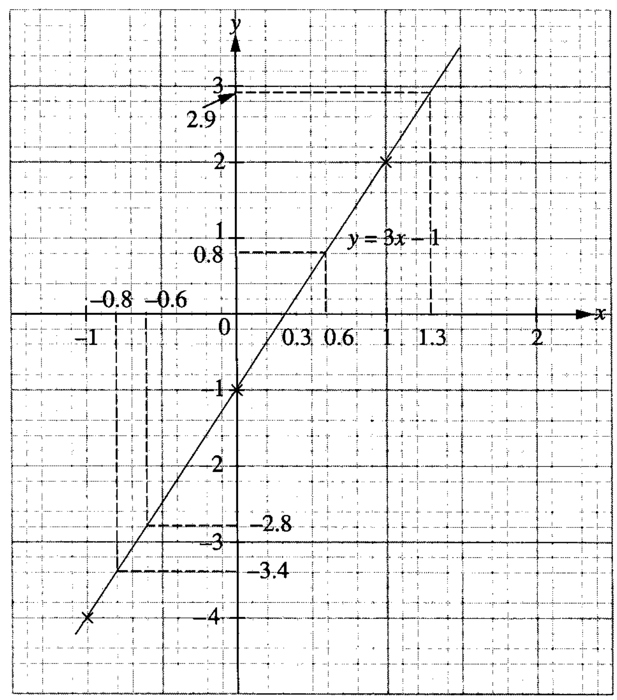


Fig. 9.7

Can you check the accuracy of the above result by using the equation $y = 3x - 1$ to find the values of y?

Exercise 9b

1. For each question, complete the table. Plot the coordinates and draw the graph of the equation.

(a) $y = x$

x	0	1	2	3	4
$y = x$					

(b) $y = x + 3$

x	-2	-1	0	1	2
$y = x + 3$					

(c) $y = -2x$

x	–1	0	1	2	3
$y = -2x$					

(d) $y = x - 2$

x	–3	–2	–1	0	1
$y = x - 2$					

2. Using the values of x from –1 to 3, construct a table showing some points whose coordinates satisfy each of the following equations. Plot the points and draw the graph of the equation.

(a) $y = x + 2$ **(b)** $y = -x$ **(c)** $y = x - 1$ **(d)** $y = 2x + 1$

3. **(a)** Given the equation $y = 3x + 5$, copy and complete the table below.

x	–1	0	1
$y = 3x + 5$			

(b) Plot the points and draw a straight line through the points.

(c) From the graph, find

(i) the values of y when $x = -2$, 0.6 and 1.5,

(ii) the values of x when $y = -1$, 0.8 and 2.9.

4. **(a)** Given the equation $y = 4x$, copy and complete the table below.

x	–1	0	1
$y = 4x$			

(b) Draw the graph of the equation $y = 4x$.

(c) From the graph, find

(i) the values of y when $x = -0.5$, 1.5 and 2.5,

(ii) the values of x when $y = -2$, 1.6 and 3.6.

The CD, The Business of Graphs, from the DMS has useful tutorials and activities on drawing and reading linear graphs. Go through them. Then proceed to plot linear graphs in the Equation Plotter in the CD. You can also use the open tool, Graphmatica, to explore the shapes of linear graphs that you encounter in this chapter.

Graphs of Equations of Lines Parallel to the Coordinate Axes

In-Class Activity

You may go through the activity on your own.

1. **(a)** Write down the coordinates of the points A, B, C, D, E and F in Fig. 9.8.

(b) What is the y-coordinate of each point?

(c) If a straight line can be drawn through these points, draw the line. Do you agree that the equation of the line is $y = 3$?

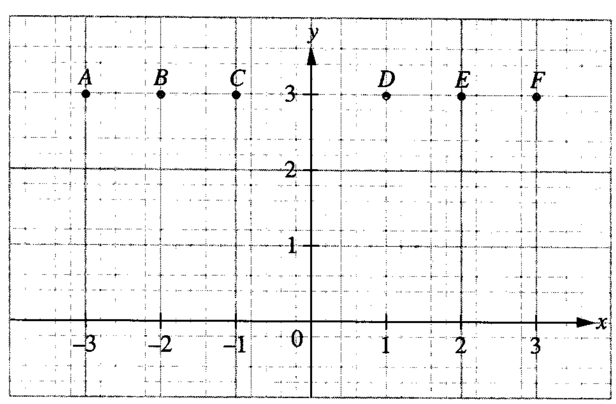


Fig. 9.8

2. **(a)** Write down the coordinates of the points P, Q, R, S, T and U in Fig. 9.9.
   (b) What is the x-coordinate of each point?
   (c) If a straight line can be drawn through these points, draw the line. What is the equation of the line?

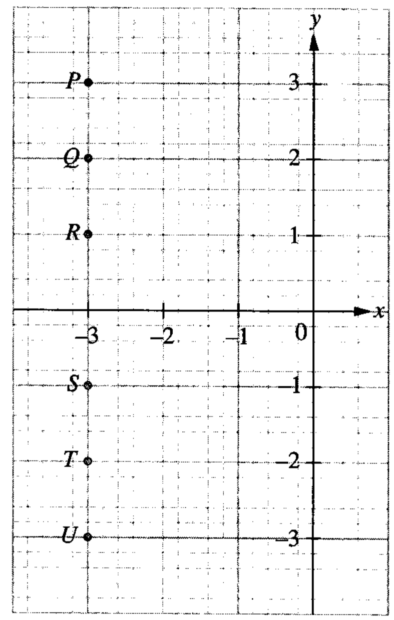


Fig. 9.9

Graphs of Equations of the Form $y = c$

From question 1 of the above In-Class Activity, you notice that $y = 3$ is a horizontal straight line passing through the point (0, 3) and parallel to the x-axis (See Fig. 9.10).

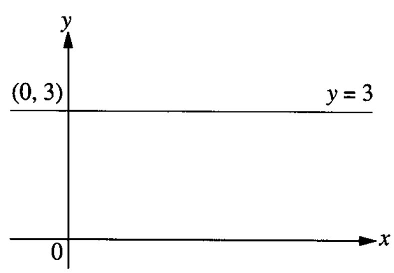
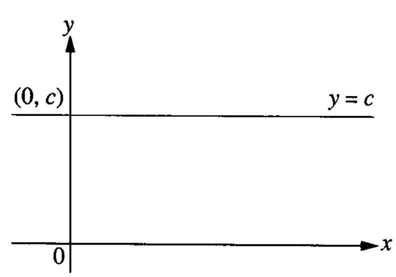


Fig. 9.10

In general, the graph of the equation $y = c$ is a line passing through the point $(0, c)$ and parallel to the x-axis.

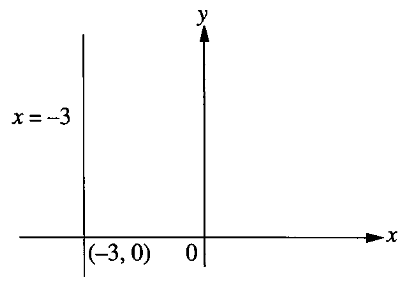


Note: $y = 0$ represents the x-axis.

Graphs of Equations of the Form $x = a$

From question 2 of the In-Class Activity, you notice that $x = -3$ is a vertical straight line passing through the point (–3, 0) and parallel to the y-axis. (See Fig. 9.11).

In general, the graph of the equation $x = a$ is a line passing through the point (a, 0) and parallel to the y-axis.



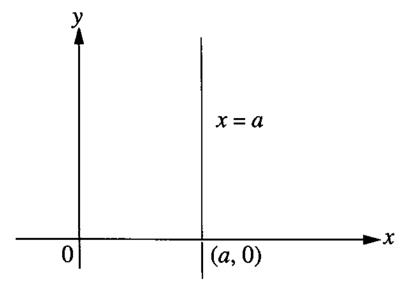


Fig. 9.11

Note: What equation represents the y-axis?

Exercise 9c

1. Each of the following sets of points lies on a line. Write down the equation of the line.

(a) (2, 2), (–1, 2), (–3, 2), (5, 2), (10, 2), (0, 2).

(b) (–6, 9), (–2, 9), (4, 9), (–1, 9), (7, 9), (9, 9).

(c) (0, –3), (–5, –3), (1, –3), (8, –3), (–8, –3), $\left(-1\frac{1}{2}, -3\right)$.

(d) $\left(-\frac{1}{3}, -\frac{1}{2}\right)$, $\left(4, -\frac{1}{2}\right)$, $\left(-4, -\frac{1}{2}\right)$, $\left(-7, -\frac{1}{2}\right)$, $\left(8, -\frac{1}{2}\right)$.

(e) (10, 0), $\left(4\frac{1}{2}, 0\right)$, (–6, 0), (–9, 0), (0, 0).

2. Write down the equations of the lines on which the following points lie.

(a) (12, –5), (12, 6), (12, –9), (12, –3), (12, 10), (12, 5).

(b) (5, 4), $\left(5, \frac{1}{2}\right)$, (5, –6), (5, –11), (5, 0), (5, 12).

(c) (–4, 4), (–4, 0), (–4, –10), (–4, 8), (–4, –4), (–4, 7).

(d) (0, –8), (0, 6), $\left(0, \frac{1}{2}\right)$, (0, 0), (0, 16), (0, –15).

(e) $\left(-\frac{1}{4}, 9\right)$, $\left(-\frac{1}{4}, \frac{1}{2}\right)$, $\left(-\frac{1}{4}, -4\right)$, $\left(-\frac{1}{4}, -10\right)$, $\left(-\frac{1}{4}, 0\right)$, $\left(-\frac{1}{4}, -\frac{1}{4}\right)$.

3. State the equations of the lines on which the following points lie.

(a) (6, –6), (–8, –6), (15, –6), (1, –6), (–9, –6).

(b) (3, 9), (3, 27), (3, –8), (3, –81), (3, 0).

(c) (–10, 2), (–10, –1), (–10, –10), (–10, 10), (–10, 5).

(d) (4, 8), (8, 8), (–7, 8), (–14, 8), (9, 8).

(e) $\left(-2, -\frac{1}{3}\right)$, $\left(4, -\frac{1}{3}\right)$, $\left(\frac{2}{3}, -\frac{1}{3}\right)$, $\left(-4\frac{1}{2}, -\frac{1}{3}\right)$, $\left(10, -\frac{1}{3}\right)$.

4. Write down the equation of each horizontal line in Fig. 9.12, including the x-axis.

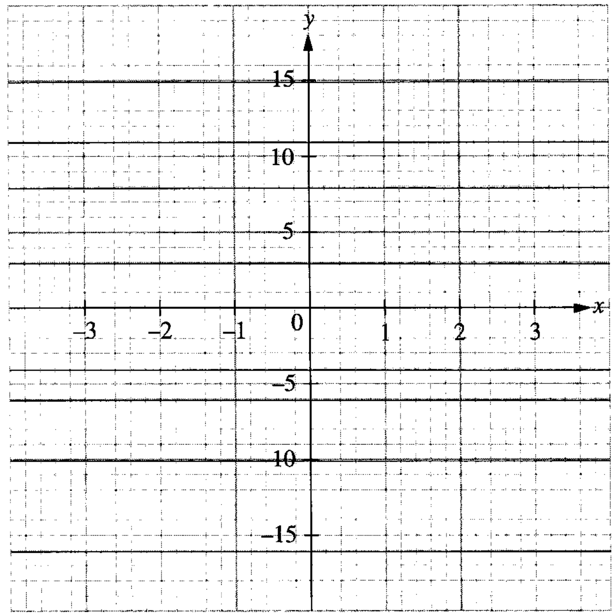


Fig. 9.12

5. Write down the equation of each vertical line in Fig. 9.13, including the y-axis.

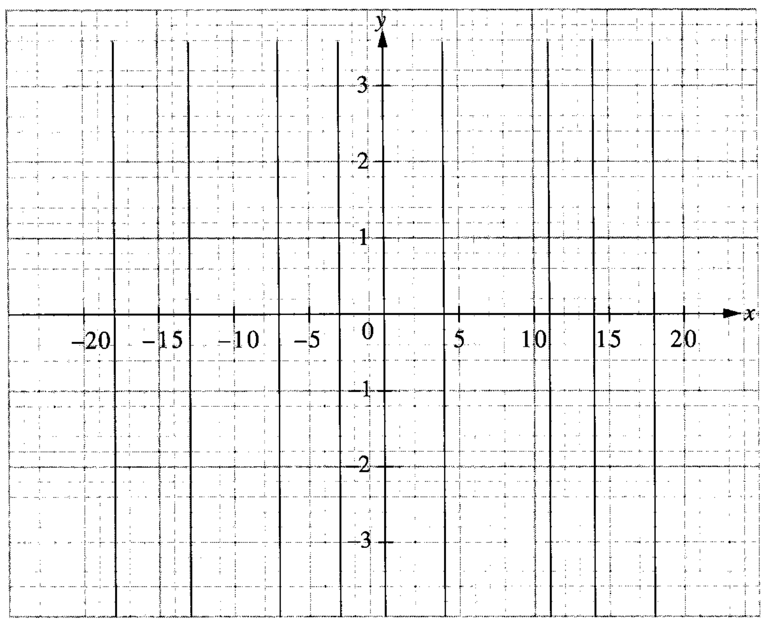


Fig. 9.13

6. Write down the coordinates of four points on each of the following lines:

(a) $y = 3\frac{1}{2}$	**(b)** $x = -6$	**(c)** $x = \frac{2}{5}$	**(d)** $y = -7$
(e) $y = 4.2$	**(f)** $x = -3.3$	**(g)** $x = 20$	**(h)** $y = -16$

7. Draw the graphs of the following equations:

(a) $x = 3$	**(b)** $y = 10$	**(c)** $y = -4$	**(d)** $y = -7$
(e) $x = 5$	**(f)** $x = -5$	**(g)** $y = 3.5$	**(h)** $x = -2.5$
(i) $x = 7.5$	**(j)** $x = 5\frac{1}{2}$	**(k)** $y = 8$	**(l)** $y = -\frac{4}{3}$

In-Class Activity

You may work in pairs for this activity.

1. On the same piece of graph paper and using the same axes and scales, draw the graphs of

   (a) $y = x$, **(b)** $y = 2x$,
   (c) $y = 3x$, **(d)** $y = -2x$,
   (e) $y = -4x$.

   Is there any common feature among the five lines?

2. **(a)** Draw the graph of each of the following equations on the same piece of graph paper:

   (i) $y = x$ **(ii)** $y = 2x$
   (iii) $y = -x$ **(iv)** $y = -2x$
   (v) $y = \frac{1}{2}x$ **(vi)** $y = -\frac{1}{4}x$
   (vii) $y = -3x$ **(viii)** $y = \frac{3}{4}x$
   (ix) $y = 6x$

   (b) Does each graph pass through the origin?
   (c) Which graph is the steepest?

Graphs of Equations of the Form $y = mx$

From the above activity, you notice that the graph of $y = mx$, where m is a constant, is a straight line passing through the origin.

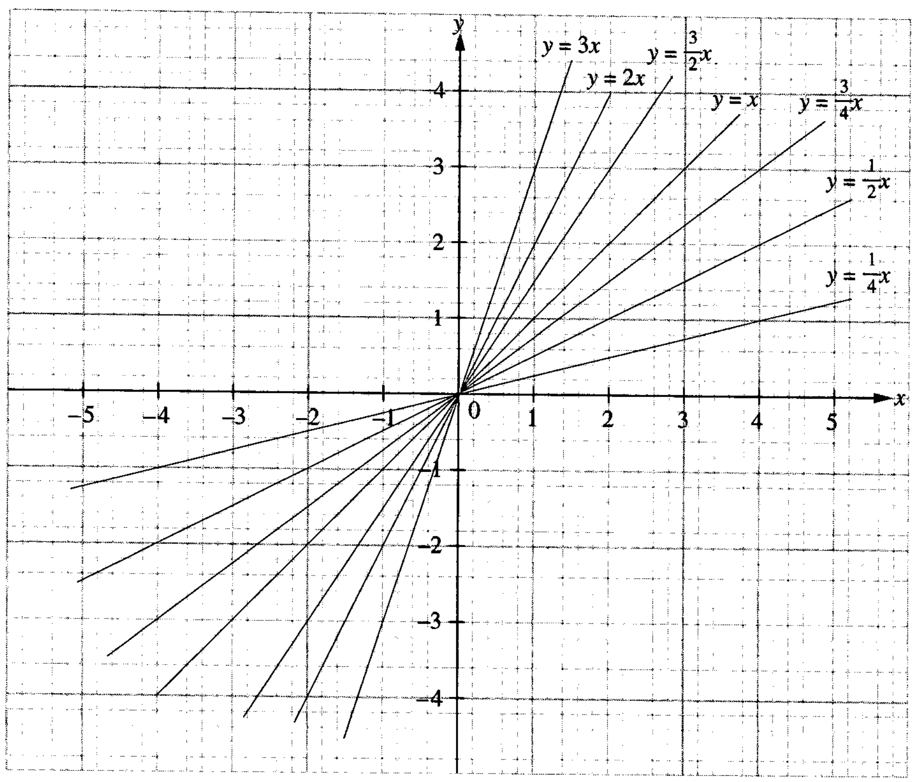


Fig. 9.14

Fig. 9.14 shows a series of graphs of the form $y = mx$. Do you notice that for each line, m has a positive value and the line rises from left to right? What is the relationship between the steepness of a line and the numerical value of m?

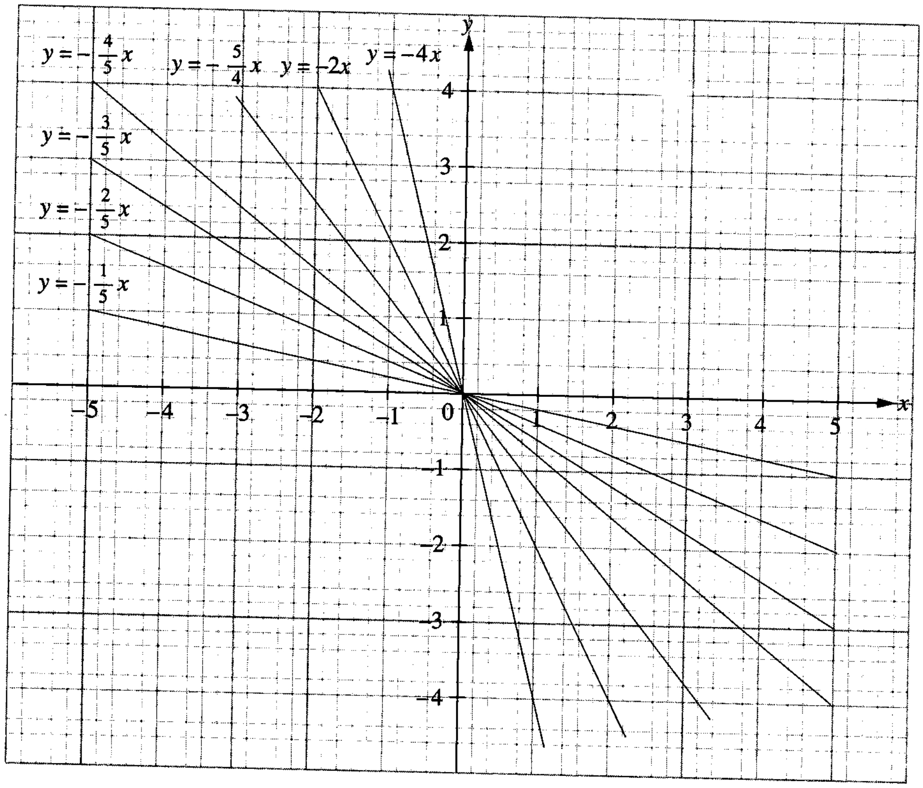


Fig. 9.15

Fig. 9.15 shows another series of graphs of the form $y = mx$ where m is negative. Do you notice that the lines **fall** from **left** to **right**? What is the relationship between the steepness of a line and the numerical value of m?

In-Class Activity

You may work in a group of three or four.

1. **(a)** Draw the graphs of the following equations on the same graph paper:
   (i) $y = 5 - 2x$ **(ii)** $y = 4x + 3$
   (iii) $y = -5 - 3x$ **(iv)** $y = -\frac{1}{2}x + 1$
   (v) $y = 3x - 2$ **(vi)** $y = -\frac{3}{2}x + \frac{5}{2}$
   (b) Does each graph pass through the origin?
   (c) At what point does each graph cut the y-axis?

2. On the same piece of graph paper and using the same scales and axes, draw the graphs of
   (a) $y = 2x$, **(b)** $y = 2x + 2$,
   (c) $y = 2x + 4$, **(d)** $y = 2x - 3$,
   (e) $y = 2x - 5$.

   What do you observe from these lines? Are there any similarities?

3. On the same piece of graph paper and using the same scales and axes, draw the graphs of
   (a) $y = -4x$, **(b)** $y = -4x + 5$,
   (c) $y = -4x + 1$, **(d)** $y = -4x - 2$,
   (e) $y = -4x - 5$.

   What similarities do the lines have?

4. On the same piece of graph paper and using the same axes and scales, draw the graphs of
   (a) $y = x + 3$, **(b)** $y = 2x + 3$,
   (c) $y = 3x + 3$, **(d)** $y = -x + 3$,
   (e) $y = -2x + 3$, **(f)** $y = -3x + 3$.

   Are there any similarities among the six lines?

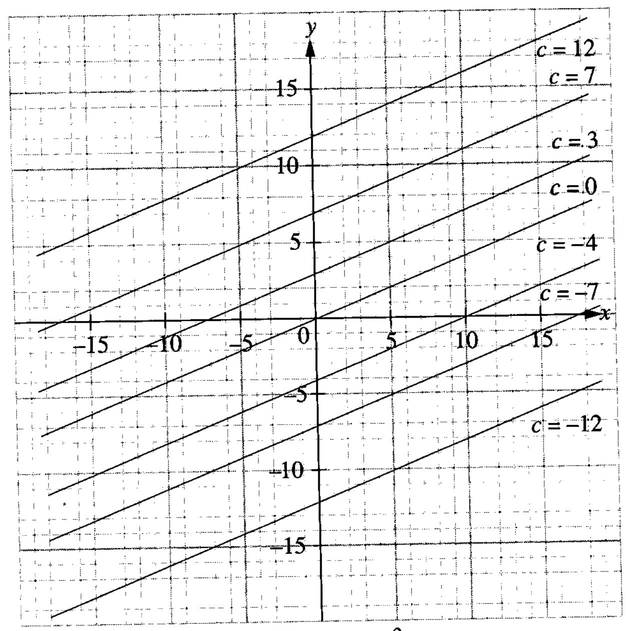
5. On the same piece of graph paper and using the same scales and axes, draw the graphs of
   (a) $y = x - 2$,
   (b) $y = 2x - 2$,
   (c) $y = 3x - 2$,
   (d) $y = -x - 2$,
   (e) $y = -2x - 2$.

   What similarities do the lines have?

Graphs of Equations of the Form $y = mx + c$

From questions 2 and 3 of the In-Class Activity, you notice that if m remains the same while c takes on different values, the graphs of equations of the form $y = mx + c$ are parallel lines cutting the y-axis at points with coordinates given by $(0, c)$.

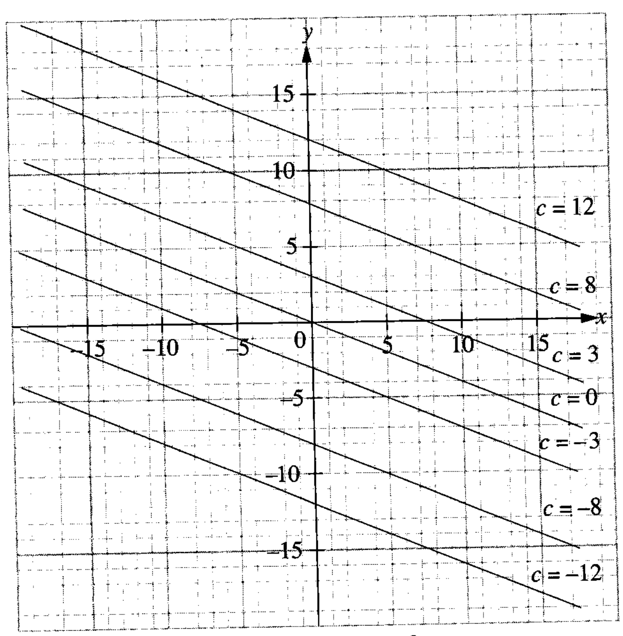
Fig. 9.16(a) shows a series of graphs of the form $y = mx + c$ where m takes a constant positive value $\frac{2}{5}$ while c takes on different values. The lines are parallel and rise from left to right.



$y = mx + c$ where $m = \frac{2}{5} (> 0)$

Fig. 9.16 (a)

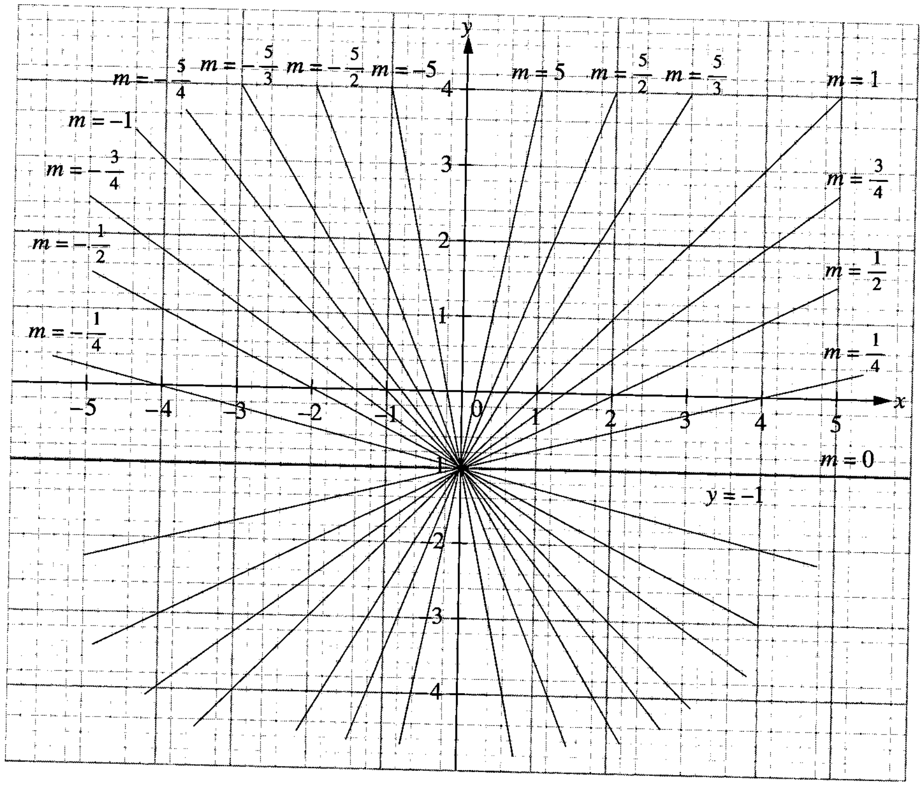
Fig. 9.16(b) shows another series of graphs of the form $y = mx + c$. Here m has a constant negative value of $-\frac{2}{5}$ while c takes on different values. The lines are parallel but fall from left to right.



$y = mx + c$ where $m = -\frac{2}{5} (< 0)$

Fig. 9.16 (b)

From questions 4 and 5 of the In-Class Activity, you notice that graphs of equations of the form $y = mx + c$ all pass through the point $(0, c)$ if c remains unchanged while m varies. Fig. 9.17 shows a series of graphs of $y = mx + c$ where c takes a constant value of -1 while m takes on different values. The diagram shows an interesting pattern of graphs passing through a common point $(0, -1)$.



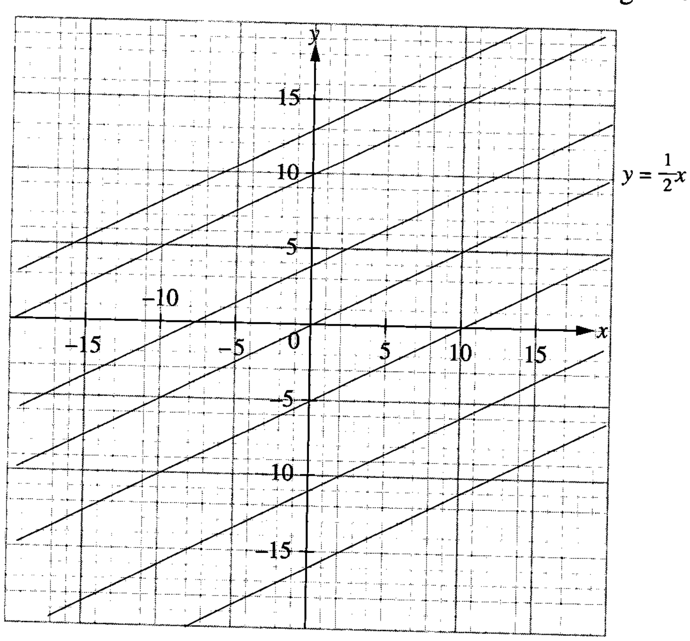
$y = mx + c$ where $c = -1$

Fig. 9.17

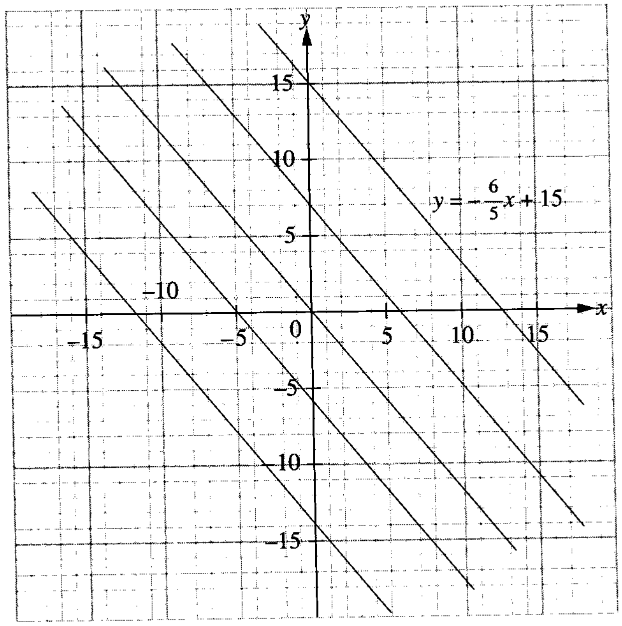
Exercise 9d

1. Write down the equations of the parallel lines in the diagrams below.

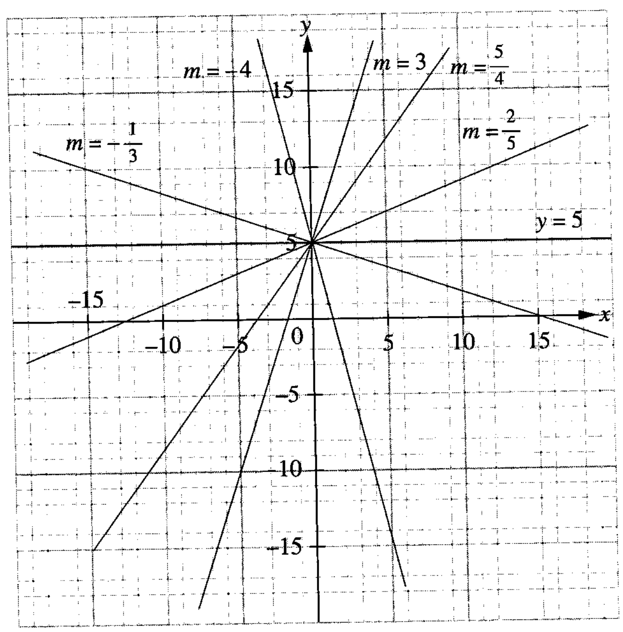
(a)



(b)



2. Write down the equations of the lines shown below.



3. On separate diagram, draw the graphs of the following equations:

(a) $2y = 3x$ **(b)** $3y = -5x + 2$ **(c)** $3y = 5x - 3$ **(d)** $2y + 3x = 7$

(e) $4y + x = 2$ **(f)** $2y = 4x + 3$ **(g)** $x + 2y = 0$ **(h)** $y = 4x - 8$

(i) $y = -\frac{1}{3}x$ **(j)** $y = 5x$ **(k)** $y = 4(x + 1)$ **(l)** $y = 10 - 2x$

***4.** Draw the graph of each of the following equations on the same graph paper:

(a) $y = 2$ **(b)** $y = 6$ **(c)** $y = 2x - 2$ **(d)** $y = 2x - 6$

Do you obtain a parallelogram from these four lines? Write down the coordinates of the vertices of the parallelogram.

∗5. Draw the graph of each of the following equations on the same graph paper:

(a) $y = -x + 6$ **(b)** $y = x - 2$ **(c)** $y = -x + 10$ **(d)** $y = x + 2$

What figure is formed by these four lines? Write down the coordinates of the vertices of this figure.

∗6. Draw the graph of each of the following equations on the same graph paper:

(a) $y = \frac{1}{3}x$ **(b)** $y = \frac{1}{3}x + 4$ **(c)** $y = -\frac{1}{3}x + 4$ **(d)** $y = -\frac{1}{3}x + 8$

What figure is formed by these four lines? Write down the coordinates of the vertices of this figure.

∗7. Draw the graph of each of the following equations on the same graph paper:

(a) $y = -\frac{3}{2}x + 9$ **(b)** $y = \frac{1}{2}x + 5$ **(c)** $y = -\frac{1}{2}x + 11$ **(d)** $y = \frac{3}{2}x - 9$

What figure is formed by these four lines? Write down the coordinates of the vertices of this figure.

∗8. Draw the graph of each of the following equations on the same graph paper:

(a) $y = -2x + 10$ **(b)** $y = x + 4$ **(c)** $y = \frac{1}{4}x + 7$ **(d)** $y = x - 2$

What figure is formed by these four lines? Write down the coordinates of the vertices of this figure.

Solving Simultaneous Linear Equations Using Graphical Method

In Chapter 8, we learnt the elimination and substitution methods of solving simultaneous linear equations. Here, we will learn the **graphical method** of solving simultaneous linear equations.

Example 2

Draw the graphs of the linear equations $2x + 3y = 5$ and $3x - y = 2$ on the same rectangular plane. Solve the simultaneous equations using the graphs drawn.

Solution

Obtain a table of values for each equation.

$2x + 3y = 5$

x	–2	1	4
y	3	1	–1

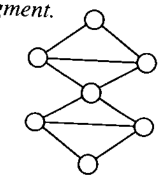
$3x - y = 2$

x	0	1	2
y	–2	1	4

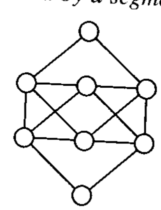
Fig. 9.18 on the next page shows the graphs of $2x + 3y = 5$ and $3x - y = 2$. The graphs intersect at the point (1, 1). Thus, the solution of the simultaneous linear equations is $x = 1$ and $y = 1$.

Just for Fun

1. Fill in the 7 circles with the digits 1, 2, 3, 4, 5, 6 and 7 so that no two consecutive integers are joined by a segment.



2. Fill in the circles below with the digits 1, 2, 3, 4, 5, 6, 7 and 8 so that no two consecutive integers are joined by a segment.



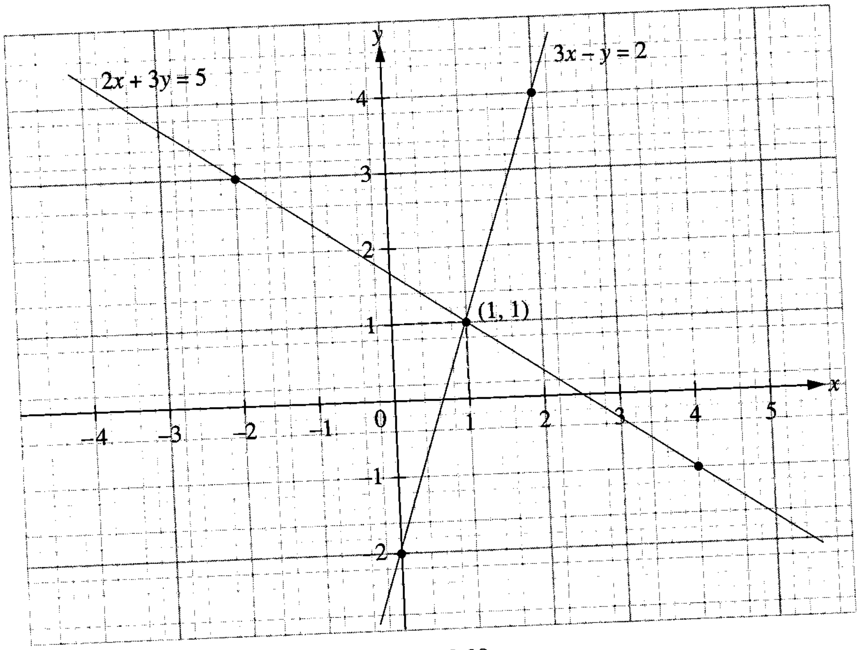


Fig. 9.18

Example 3

Solve the simultaneous equations $2x - 5y = 32$,
$2x + 3y = 0$ *graphically.*

Solution

Construct a table of values for each equation.

$2x - 5y = 32$

x	−4	1	6
y	−8	−6	−4

$2x + 3y = 0$

x	−3	0	3
y	2	0	−2

Fig. 9.19 shows that the graphs intersect at the point (6, −4). Thus the solution of the simultaneous linear equations is $x = 6$ and $y = -4$.

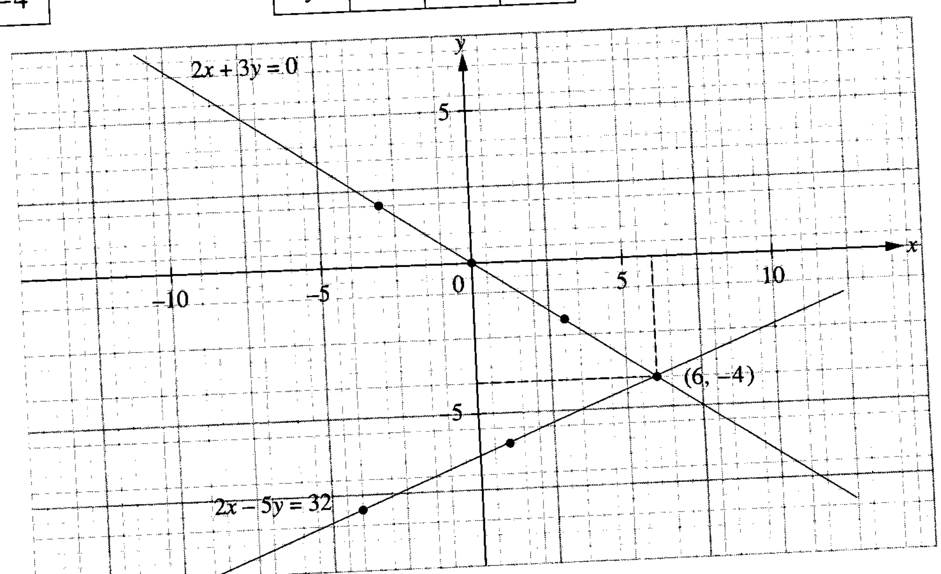


Fig. 9.19

Example 4

Solve the following simultaneous equations graphically:

(a) $x + y = 1$, $3x + 3y = 3$; *(b)* $x + y = 3$, $3x + 3y = 15$.

Solution

(a) Fig. 9.20 shows that the graphs of $x + y = 1$ and $3x + 3y = 3$ are identical; i.e., the two lines coincide. Thus, the graphs have an infinite number of common points. Therefore, the simultaneous equations $x + y = 1$ and $3x + 3y = 3$ have an infinite number of solutions.

Note: Simultaneous linear equations have an infinite number of solutions if their graphs drawn on the same rectangular plane are identical.

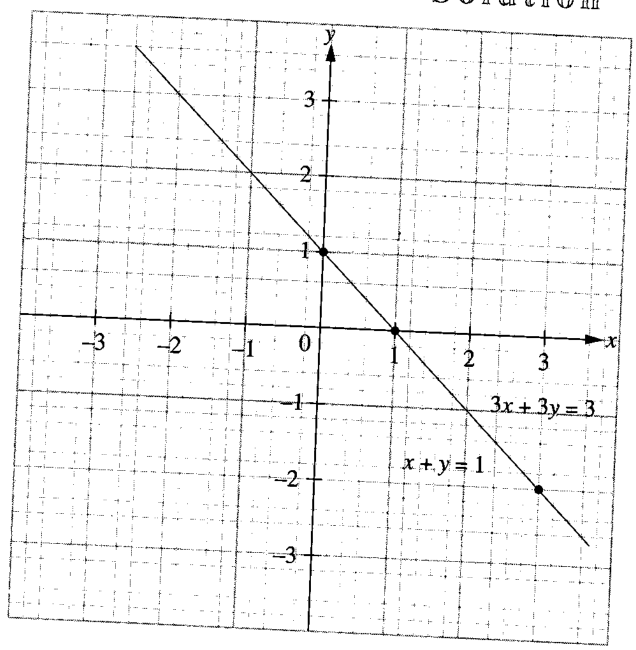


Fig. 9.20

(b) Fig. 9.21 shows that the graphs of $x + y = 3$ and $3x + 3y = 15$ are parallel lines. Thus they do not intersect and have no common points. Therefore, the simultaneous equations $x + y = 3$ and $3x + 3y = 15$ have no solution.

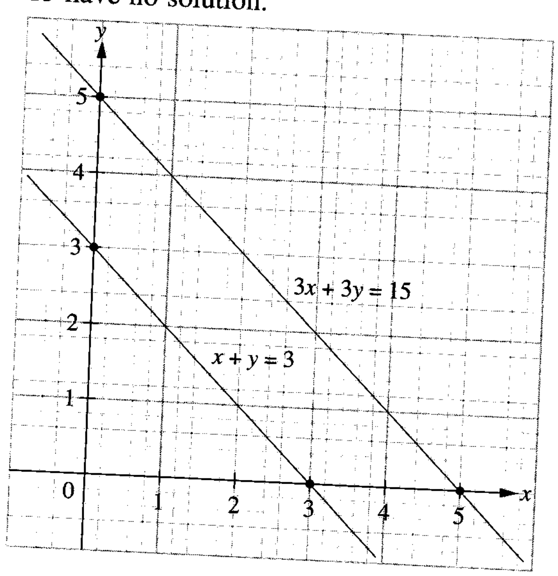
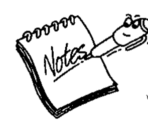


Fig. 9.21

Note: Simultaneous linear equations have no solution if their graphs drawn on the same rectangular plane are parallel.



Dividing $3x + 3y = 15$ throughout by 3, we obtain $x + y = 5$. Comparing $x + y = 5$ and $x + y = 3$, we notice the inconsistency in the equations. Is it possible to find two real numbers whose sum is 5 and 3 at the same time?

Exercise 9e

Solve the following simultaneous equations using graphical method:

1. $3x - y = 0$
   $2x - y = 1$
2. $3x + y = 2$
   $2x - y = 3$
3. $x - y = -3$
   $x - 2y = -1$
4. $4x + y = 2$
   $4x + y = -3$
5. $x + 2y = 3$
   $2x + 4y = 6$
6. $3x - 2y = 7$
   $2x + 3y = 9$
7. $x + 4y = 12$
   $4x + y = 18$
8. $3x + 2y = 4$
   $5x + y = 2$
9. $3x - 4y = 10$
   $5x + 7y = 3$
10. $2x + 5y = 25$
    $3x - 2y = 9$
11. $3x - 2y = 13$
    $2x + 2y = 0$
12. $3x - 4y = 25$
    $4x - 5y = 32$

Applications of Graphs in Practical Situations

Interpretation and Use of Graphs

There is a correspondence between algebra and geometry through the coordinate system. *For example*, through the coordinate system we can convert a point into a pair of numbers and vice versa. Also, a linear equation corresponds to a straight line and a quadratic equation to a parabolic curve. So graphs are the "pictures" of equations. However, sometimes it is easier to use graphs instead of equations to represent relations and to solve problems.

In-Class Activity

Angela, the president of a school photography club, wishes to order some T-shirts with the club's logo and design on them for the members. She goes to Mr Tan's shop to find out the cost of making the T-shirts. Mr Tan, a mathematics enthusiast, shows her a graph displaying the cost ($\$C$) of making N T-shirts.

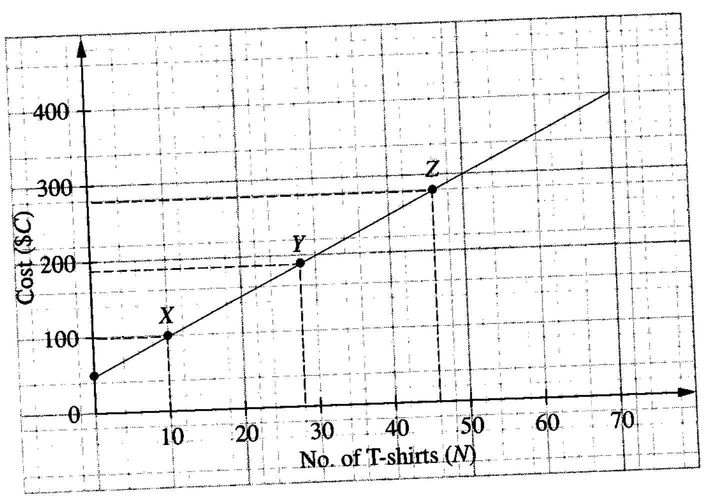


Fig. 9.22

Using the graph, Angela can determine the cost of making a certain number of T-shirts or the number of T-shirts which can be made for a given cost. She reads from the graph that 10 T-shirts cost \$100 (point X), 28 T-shirts cost \$190 (point Y) and 46 T-shirts cost \$280 (point Z).

1. Angela notices that the cost per T-shirt decreases when more T-shirts are ordered and is puzzled by the observation from the graph that 0 T-shirt costs \$50. Discuss with some of your classmates and provide a possible explanation to this problem.

2. In trying to find a connection between the cost and the number of T-shirts, Angela notices that

$$\mathbf{100} = 50 + 5(\mathbf{10}),\ \mathbf{190} = 50 + 5(\mathbf{28}) \text{ and } \mathbf{280} = 50 + 5(\mathbf{46}).$$

With the help of the graph, complete the following table:

No. of T-shirts (N)	10	20	28	36	40	46	50	64
Cost (\$C)	**100** = 50 + 5(**10**)		**190** = 50 + 5(**28**)			**280** = 50 + 5(**46**)		

(a) With the help of the table, write down an equation that connects the cost (\$$C$) to the number of T-shirts (N).

(b) The photography club has 88 members including Angela and she wishes to order a T-shirt for each member. Use the equation in **(a)** to calculate the cost of making 88 T-shirts.

(c) Later, some members decide to order one extra T-shirt each and the new total cost of making the T-shirts is \$650. How many members order an additional T-shirt?

Example 5

The graph in Fig. 9.23 shows that P litres of petrol are consumed by a car to travel D km.

Use the your graph to find

(a) how far the car can travel if it has
   (i) 3 litres of petrol,
   (ii) 5.2 litres of petrol and

(b) the cost of petrol for travelling a distance of 36 km given that 1 litre of petrol costs \$1.40.

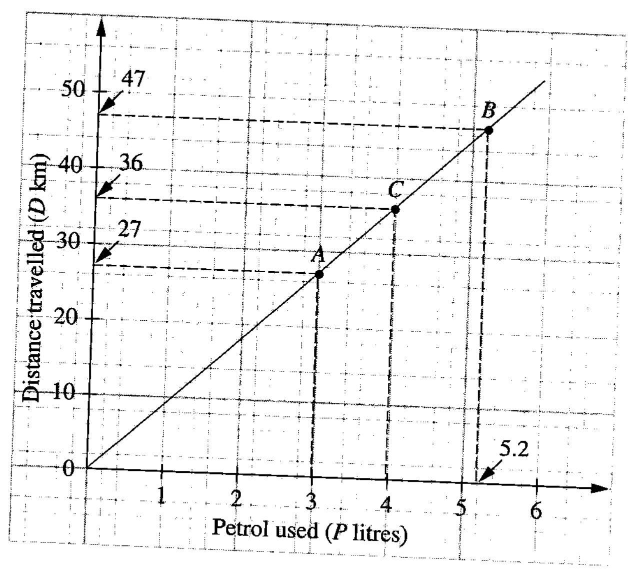


Fig. 9.23

Solution

From the graph,

(a) (i) the car can travel 27 km if it has 3 litres of petrol (point A),
   (ii) the car can travel approximately 47 km if it has 5.2 litres of petrol (point B).

(b) The car uses 4 litres of petrol to travel 36 km (point C).

$\therefore$ the cost of petrol = 4 × \$1.40 = \$5.60.

Conversion Graphs

Conversion graphs are linear graphs connecting two quantities, which are in direct proportion. In the following example, we discuss the graph that enables us to convert between litres and gallons.

Just for fun

A farmer keeps his chickens and rabbits together in a big enclosure. If there are 32 heads and 100 legs altogether, how many chickens and rabbits does the farmer have?

Example 6

The graph in Fig. 9.24 shows the conversion from litres to gallons.

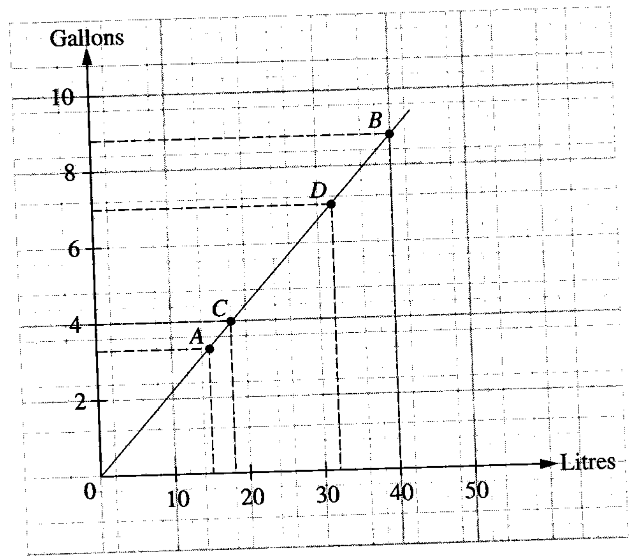


Fig. 9.24

(a) Read from the graph the number of gallons in
(i) 15 litres, (ii) 40 litres.

(b) Read from the graph the number of litres in
(i) 4 gallons, (ii) 7 gallons.

Solution

From the graph,

(a) (i) 15 litres ≈ 3.3 gallons (point *A*)
(ii) 40 litres ≈ 8.8 gallons (point *B*)

(b) (i) 4 gallons ≈ 18 litres (point *C*)
(ii) 7 gallons ≈ 32 litres (point *D*)

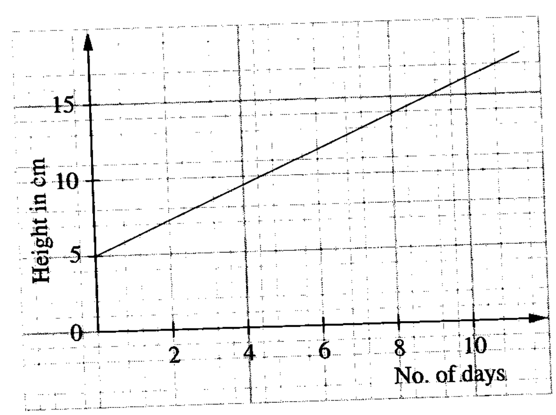
Exercise 9f

1. The graph shows the rate of growth of a certain plant.

Use the graph to find

(a) the height of the plant on the
(i) 4th day of its growth,
(ii) 9th day of its growth and

(b) the day on which the plant is about 10.5 cm tall.

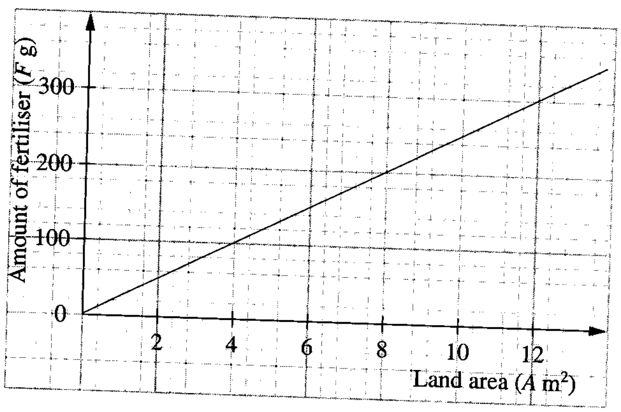


2. The graph below shows that F g of fertiliser is required for a land area of A m^2.

From the graph, find

(a) how many grams of fertiliser are needed for a land area of
  (i) 6 m^2,
  (ii) 9.6 m^2 and

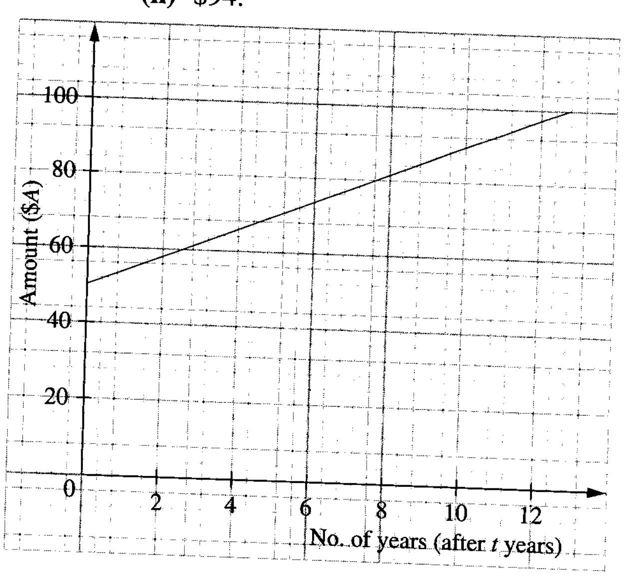
(b) how much land area 280 g of fertiliser will be sufficient for.



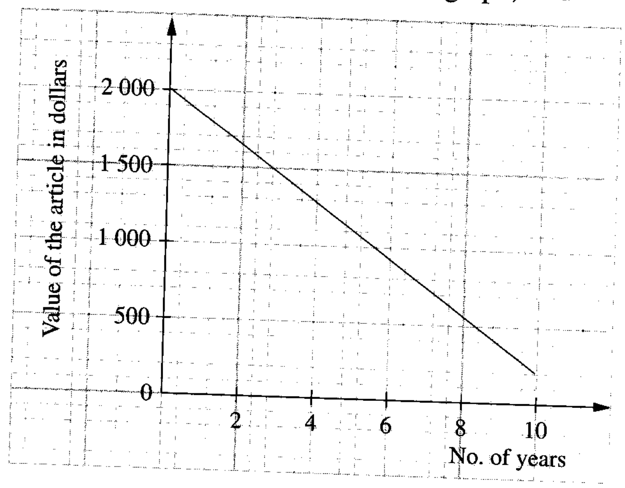
3. The graph below shows that \$50 will amount to \$$A$ after t years if it is invested at 8% simple interest per annum. Use the graph to find

(a) what \$50 will amount to after
  (i) 5 years,
  (ii) 8 years and

(b) after how many years will \$50 amount to
  (i) \$62,
  (ii) \$94.



4. The graph below shows that an article was purchased for \$2 000 and has a scrap value of \$200 after 10 years. From the graph, find



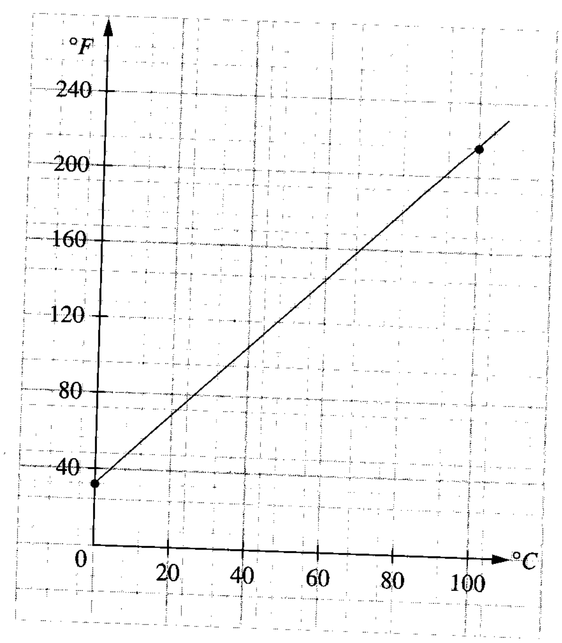
(a) the value of the article after
  (i) 3 years, **(ii)** 8 years and

(b) when the value of the article will depreciate to \$1 100.

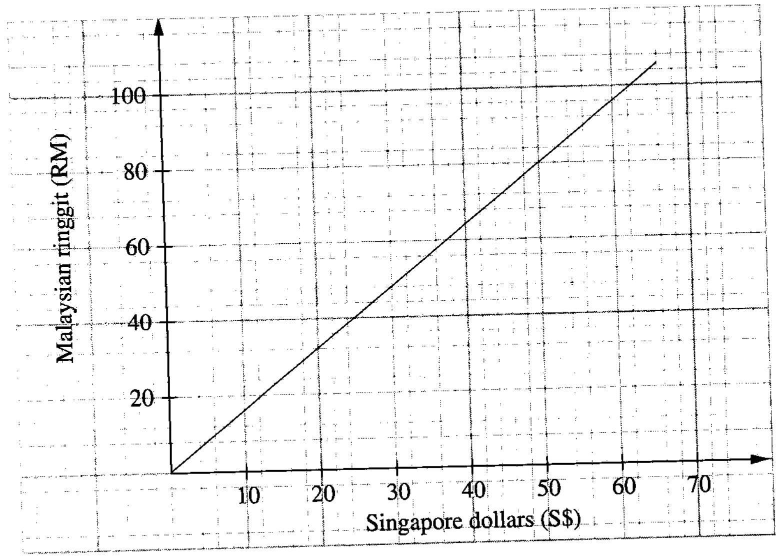
5. The graph below shows the conversion from degrees Celsius (°C) to degrees Fahrenheit (°F). Use the graph to

(a) change each of the following to °F,
  (i) 4°C, **(ii)** 42°C,
  (iii) 80°C.

(b) change each of the following to °C,
  (i) 68°F, **(ii)** 100°F,
  (iii) 180°F.



6. The graph shows the conversion from Singapore dollars (S$) to Malaysian ringgit (RM) on a certain day in 1999.



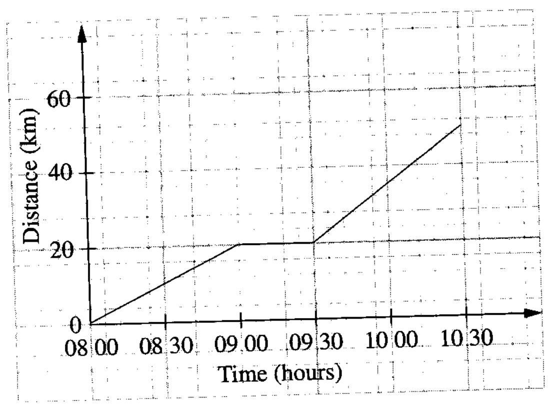
Use the graph to

(a) change each of the following to RM,
(i) S$15 **(ii)** S$34 **(iii)** S$49 **(iv)** S$54,

(b) change each of the following to S$,
(i) RM32 **(ii)** RM56 **(iii)** RM70 **(iv)** RM94.

Travel Graphs

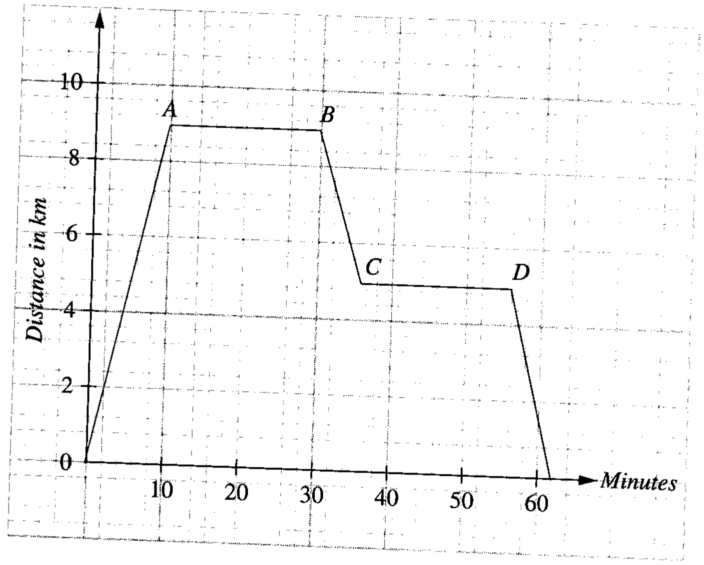
We can use a travel graph to show a journey undertaken by a cyclist. Suppose a cyclist starts a 50-km journey at 08 00 hours. At 09 00 hours, his bike gets a puncture and he spends half an hour repairing it. He then continues his journey and reaches his destination at 10 30 hours. A graph of this journey can be plotted and is shown below.



From the first section of the graph, we can see that the cyclist covers a distance of 20 km in the first hour (08 00 h to 09 00 h). The horizontal section of the graph (09 00 h to 09 30 h) indicates that he stops for half an hour to repair his bike. The third section of the graph is steeper than the first section. This indicates that he cycles at a faster speed from 09 30 h to 10 30 h. He takes $2\frac{1}{2}$ hours to travel 50 km. Thus his average speed for the whole journey is $50 \div 2\frac{1}{2} = 20$ km/h.

Example 7

A technician in a computer firm drives from his workshop to service a customer's computer. On his way back, he stops to service another customer's computer. The travel graph below shows his journey.



(a) How long did he take to service each computer?
(b) How far was his first customer from his workshop?
(c) What was his average speed when he returned from his second customer to his workshop?

Solution

(a) He took 20 minutes to service the first computer (AB). He took 20 minutes to service the second computer (CD).

(b) His first customer was 9 km from his workshop (point A).

(c) Distance travelled = 5 km

Time taken = 6 minutes = $\frac{1}{10}$ hour

$\therefore$ average speed = $5 \div \frac{1}{10} = 5 \times 10 = 50$ km/h

Just for fun

In a housing estate, there are 1 000 married couples. Two-thirds of the husbands who are taller than their wives are also heavier. Three-quarters of the husbands who are heavier than their wives are also taller. If there are 120 wives who are taller and heavier than their husbands, how many husbands are taller than their wives?

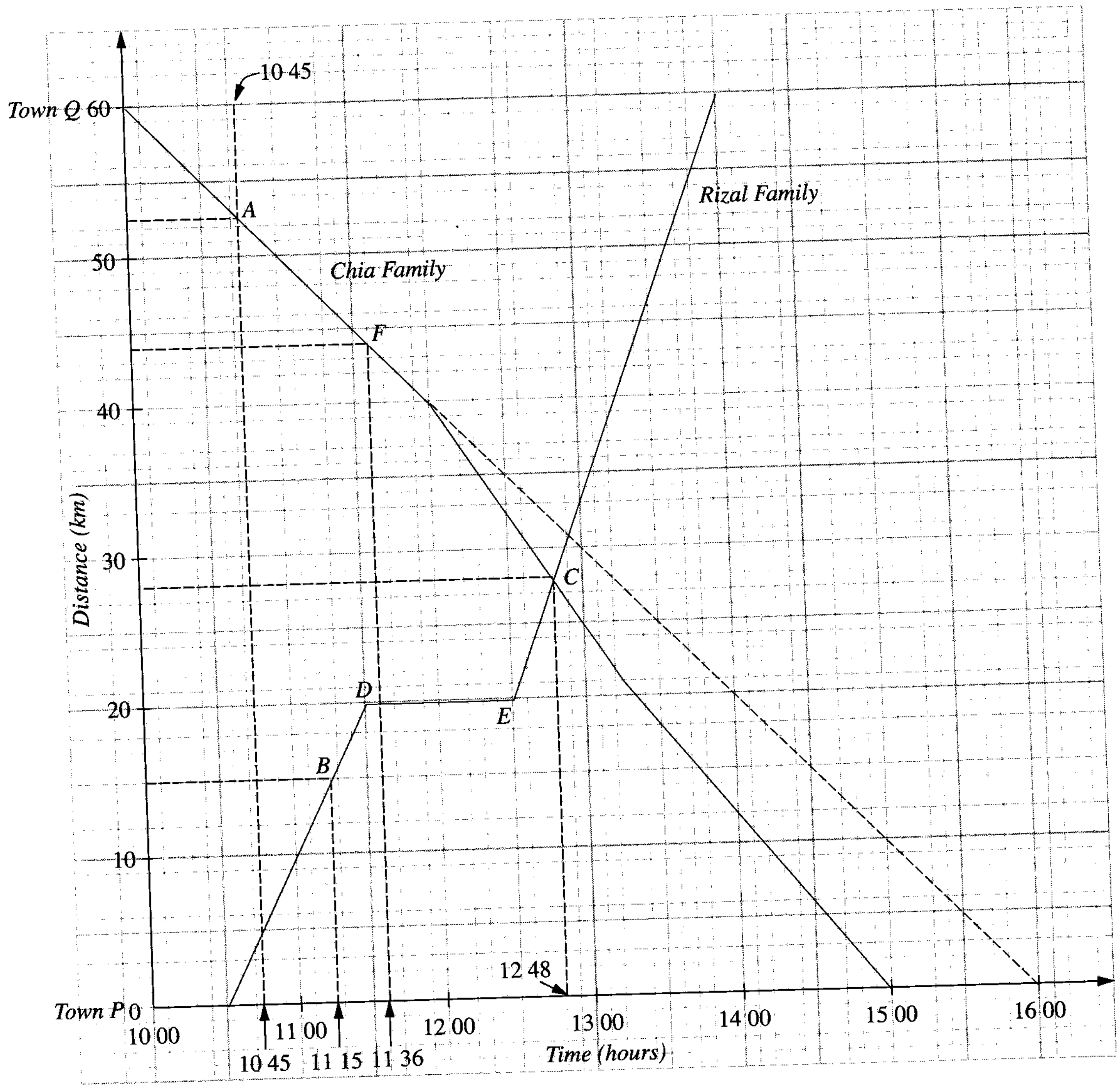
The CD, The Business of Graphs, from the DMS provides many activities on applications of linear graphs like conversion graphs, distance-time graphs and graphs used in practical situations. Explore them.

Example 8

The following travel graphs show the journeys of the Chia family and the Rizal family. The Chia family set off from Town Q to Town P. Half an hour later, the Rizal family set off from Town P to Town Q.

(a) How far is Town P from Town Q?
(b) How far did each family travel in the first 45 minutes of their respective journeys?

(c) *When and where did the two families meet?*
(d) *Which family took a longer time to complete their journey and how much longer?*
(e) *Which family rested during their journey? At what time and for how long did they rest?*
(f) *How far from their destination was the Chia family at 11 36 h?*
(g) *What was the average speed of the Rizal family for the whole journey?*
(h) *At what time would the Chia family reach Town P if they travelled at the initial constant speed throughout their journey?*



Solution

(a) Town P and Town Q are 60 km apart.

(b) The Chia family travelled 7.5 km in the first 45 minutes (point A).

The Rizal family travelled 15 km in the first 45 minutes (point B).

(c) The two families met at 12 48 h, 28 km from Town P (point C).

(d) The Chia family took 5 hours to travel from Town Q to Town P (10 00 h to 15 00 h). The Rizal family took $3\frac{1}{2}$ hours to travel from Town P to Town Q (10 30 h to 14 00 h).

$\therefore$ the Chia family took $\left(5 - 3\frac{1}{2}\right)$h $= 1\frac{1}{2}$ h longer to complete the journey.

(e) The Rizal family took a rest at 11 30 h for 1 hour (DE).

(f) The Chia family was 44 km from Town P at 11 36 h (point F).

(g) Average speed of the Rizal family $= \dfrac{60}{3\frac{1}{2}}$

$= 17\frac{1}{7}$ km/h

(h) If the Chia family travelled at the initial constant speed, the travel graph should be a straight line obtained by producing the first section of the original travel graph. The produced section is the dotted line shown in the diagram.

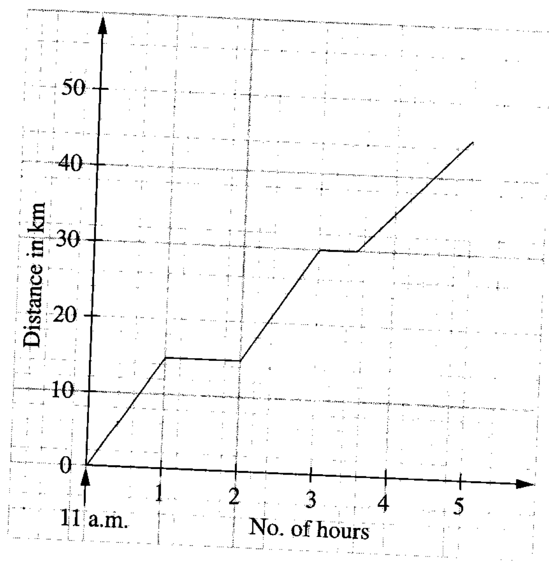
From the graph, the Chia family would reach Town P at 16 00 h if they travelled at a constant speed throughout their journey.

Exercise 9g

1. A cyclist begins his journey of 45 km at 11 a.m. The graph shows his journey.

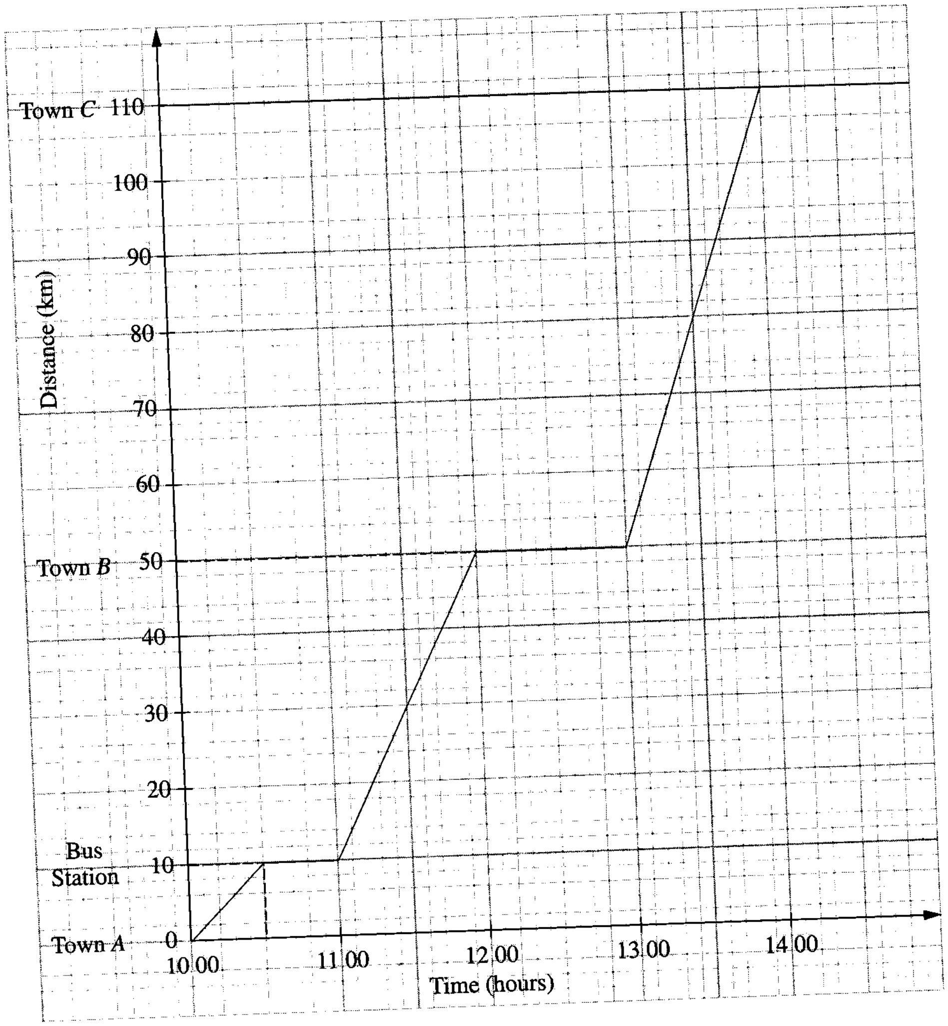
Find

(a) the time he arrived at his destination.
(b) the distance he travelled in the first 3 hours,
(c) the time when he was 15 km from his destination,
(d) how far he was from his destination at 1.30 p.m,
(e) the total time he spent resting.

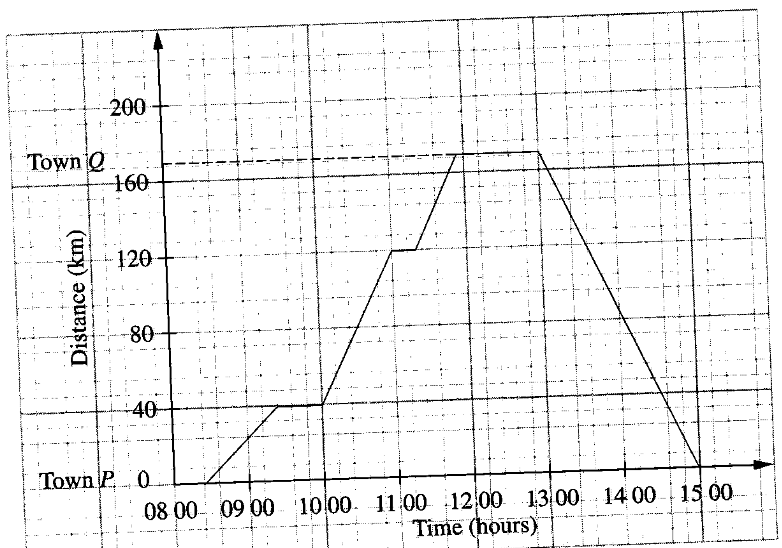


2. A man leaves Town A at 10 a.m. to travel for Town C, 110 km away. First, he cycles to the bus station, leaves his bicycle there and waits for a bus to take him to Town B. Then, from Town B he travels to Town C in his friend's car. The travel graph on the next page shows his journey.

(a) How long does he take to cycle to the bus station?
(b) How far does he cycle?
(c) What is his cycling speed?
(d) What is the speed of the bus?
(e) What is the distance from Town B to Town C?
(f) At what time are the man and his friend 30 km from Town C?



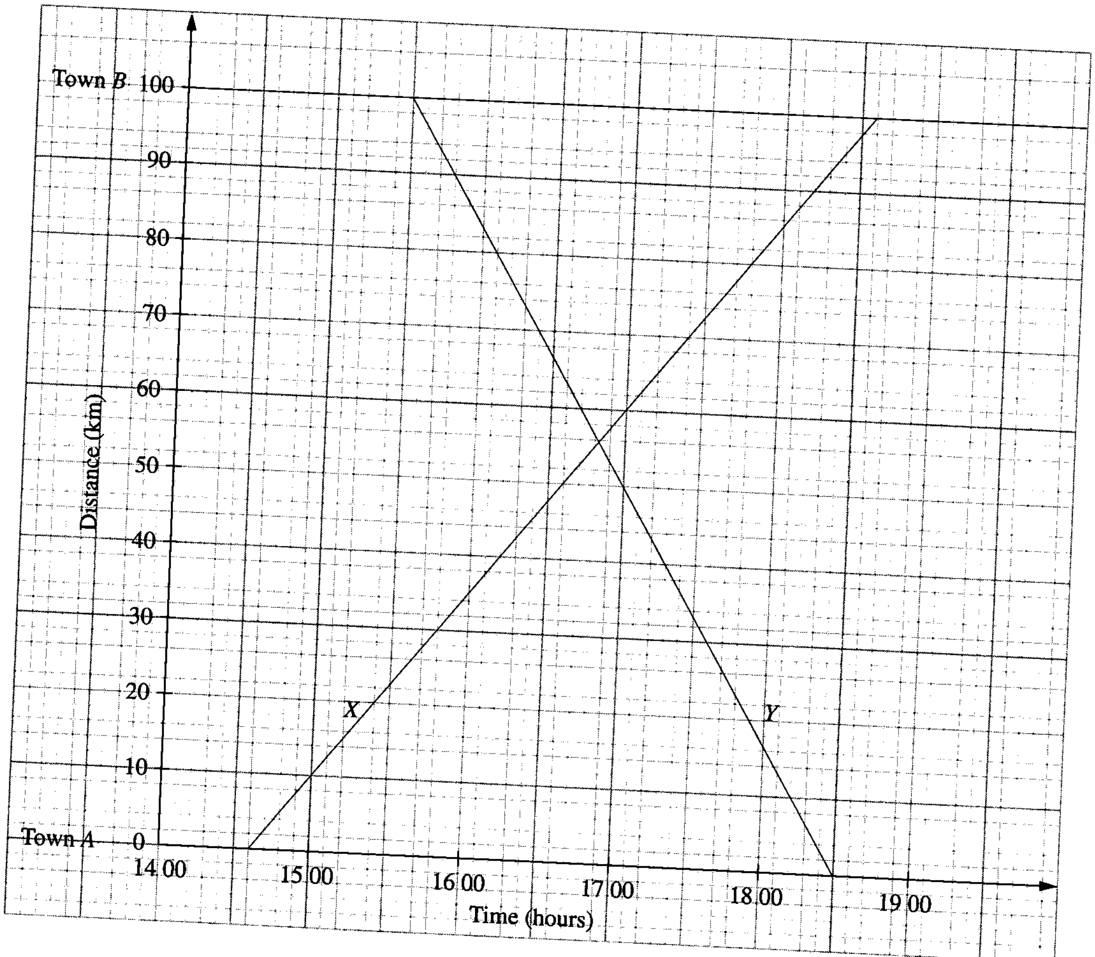
3. A salesman leaves Town P for Town Q. The travel graph shows his journey.



Find

(a) the time he left Town P,
(b) the distance he travelled before his first rest,
(c) his average speed between 10 00 h and 11 00 h,
(d) how long he stayed in Town Q,
(e) how long he took to reach Town P from Town Q,
(f) his average speed for the return journey,
(g) the total distance he travelled,
(h) the number of hours he was away from Town P,
(i) the average speed for the whole journey.

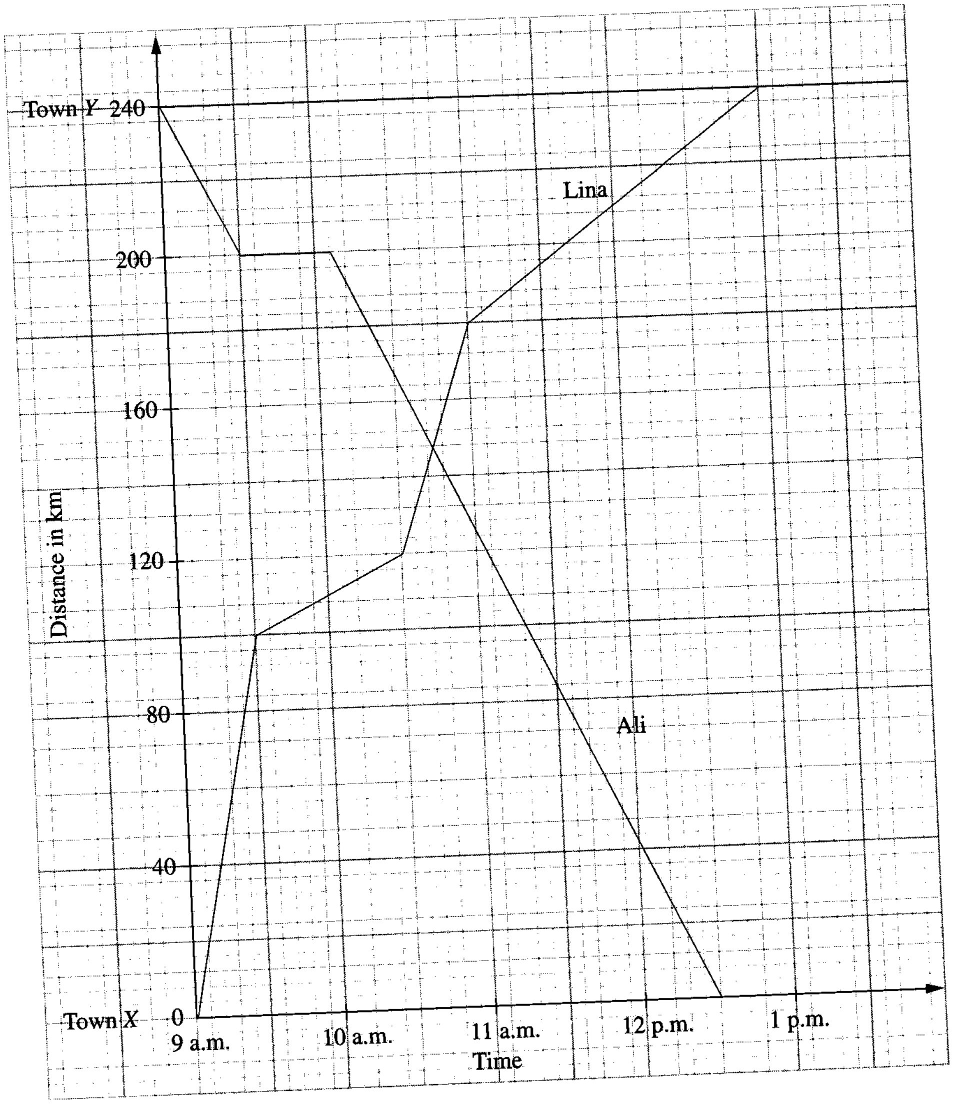
4. The diagram below shows the travel graphs of two men, X and Y, who travel from Town A and Town B respectively.



(a) When did X leave Town A and what was his speed for the journey?
(b) When did Y leave Town B and what was his speed for the journey?
(c) How far from Town B were X and Y when they passed each other?
(d) Find the time when they passed each other.
(e) Who took a longer time to complete the journey and by how much longer?
(f) What is the distance between the two towns?
(g) How far was X from Town A at 4.12 p.m.?
(h) How far was Y from Town B at 5.36 p.m.?

5. At 9 a.m., Lina left Town X by car for Town Y which was 60 km away. At the same time, Ali left Town Y by car for Town X. The travel graphs show their journeys.

(a) Find the average speed of Lina from 9.30 a.m. to 11 a.m.
(b) Find the distance travelled in the first two hours by Lina.
(c) When did Lina and Ali pass each other?
(d) How far were they from Town Y when they passed each other?
(e) How far did Lina travel while Ali was resting?
(f) Who took a longer time to complete the journey and by how much longer?
(g) What was Lina's average speed for the whole journey?



Drawing of Graphs

The examples below show how a graph can be drawn with given information.

Example 9

A spring stretches when a weight is attached to it. The table below shows the different lengths of the spring when different weights are attached to it.

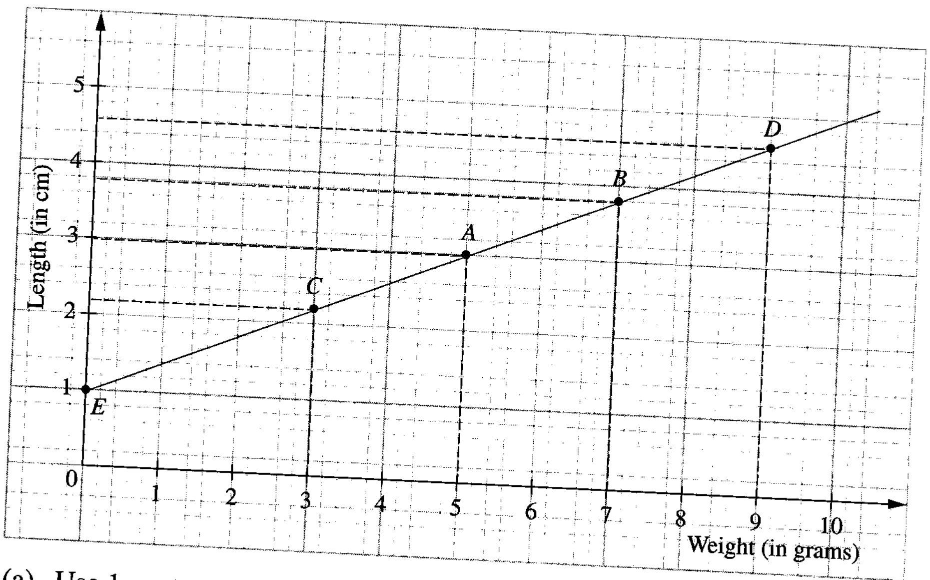
Weight (g)	*0*	*2*	*4*	*6*	*8*	*10*
Length (cm)	*1.0*	*1.8*	*2.6*	*3.4*	*4.2*	*5*

(a) Draw a graph representing these results using appropriate scales.
(b) Use your graph to find the attached weight if the spring is
(i) 3 cm long, *(ii) 3.8 cm long.*

(c) From your graph, find the length of the spring if the attached weight is
(i) 3 g, *(ii) 9 g.*

(d) What is the original length of the spring when no weight is attached to it?

Solution



(a) Use 1 cm to represent 1 g on the horizontal axis and 1 cm to represent 1 cm on the vertical axis. Plot the graph using the table of values given in the previous page. The graph is shown above.

(b) (i) If the spring is 3 cm long, the weight attached is 5 g (point *A*).
(ii) If the spring is 3.8 cm long, the weight attached is 7 g (point *B*).

(c) (i) If the attached weight is 3 g, the length of the spring is 2.2 cm (point *C*).
(ii) If the attached weight is 9 g, the length of the spring is 4.6 cm (point *D*).

(d) The original length of the spring with nothing attached is 1 cm (point *E*).

Example 10

Draw a conversion graph for acres to hectares given that 24.7 acres = 10 hectares. Use your graph

(a) to convert to hectares
(i) 10 acres,
(ii) 37 acres.

(b) to convert to acres
(i) 7 hectares,
(ii) 22 hectares.

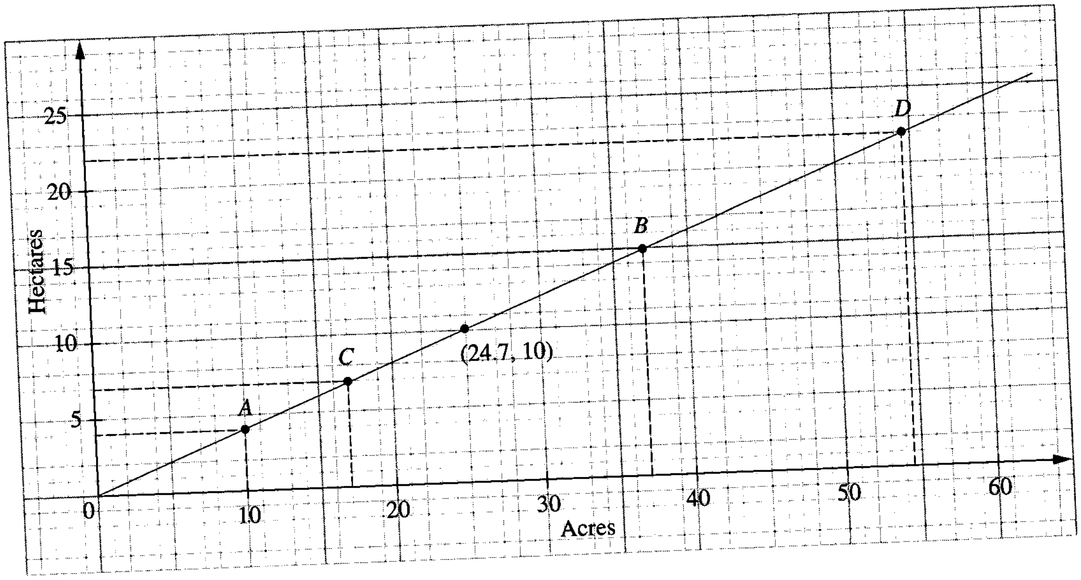
Solution

Use a horizontal scale of 2 cm to represent 10 acres and a vertical scale of 1 cm to represent 5 hectares.

Since 24.7 acres = 10 hectares, we plot the point (24.7, 10) on the graph paper. By joining the origin to this point, we obtain the conversion graph.

Just For Fun

Ahmad, Bala and Chong-lin compete with one another in a bicycle race. Ahmad reaches the finishing point 12 minutes ahead of Bala whereas Bala's time is 3 minutes better than Chonglin's time. It is found that Ahmad's average speed for the whole journey is 5 km/h better than Bala's whereas Bala's average speed for the same journey is 1 km/h better than Chonglin's. Find the length of the whole journey.



From the graph,

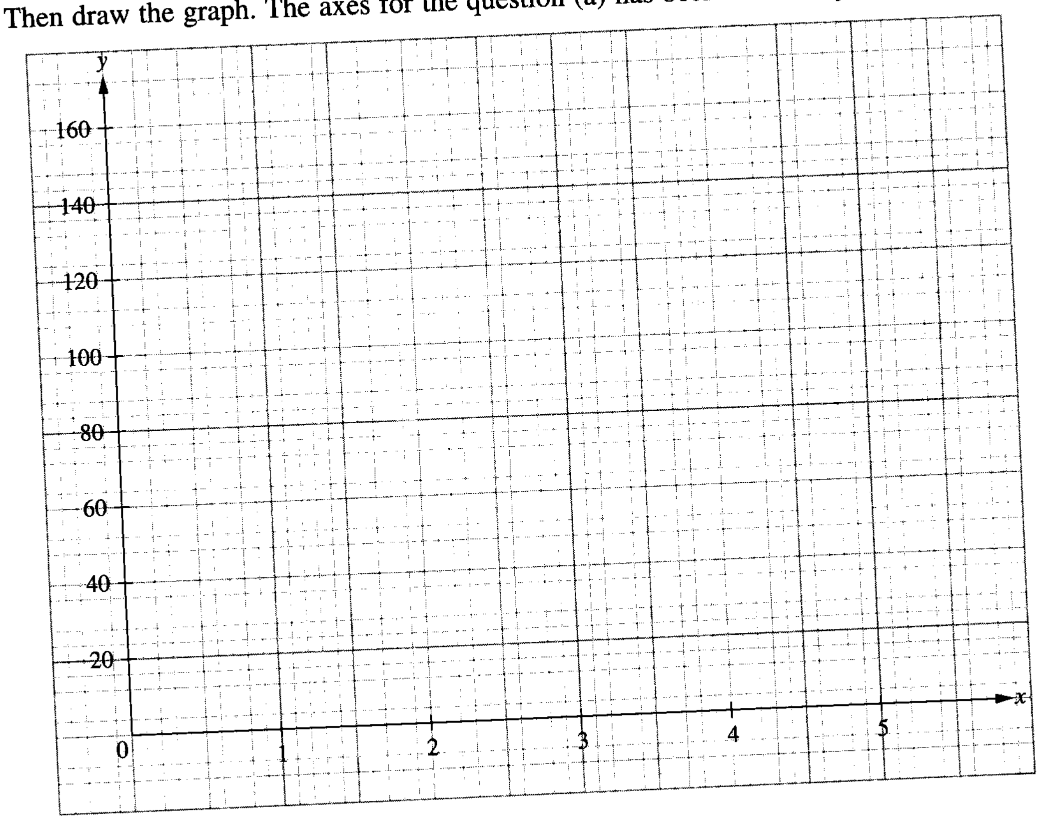
(a) (i) 10 acres ≈ 4 hectares (point A) (ii) 37 acres ≈ 15 hectares (point B)

(b) (i) 7 hectares ≈ 17 acres (point C) (ii) 22 hectares ≈ 54.5 acres (point D)

Think: Why is the origin on the graph?

Exercise 9h

1. For each of the following tables, draw suitable horizontal and vertical axes using the information given. Then draw the graph. The axes for the question (a) has been done for you.



(a)

x	0	1	2	3	4
y	30	60	90	120	150

Horizontal axis: 2 cm to 1 unit.
Vertical axis: 1 cm to 20 units.

(b)

x	0	1	2	3	4	5	6
y	0	30	60	90	120	150	180

Horizontal axis: 2 cm to 1 unit.
Vertical axis: 1 cm to 10 units.

(c)

x	0	2	4	6	8	10
y	0	40	80	120	160	200

Horizontal axis: 1 cm to 1 unit.
Vertical axis: 1 cm to 20 units.

(d)

x	0	10	20	30	40	50
y	0	2.2	4.4	6.6	8.8	11

Horizontal axis: 1 cm to 5 units.
Vertical axis: 1 cm to 1 unit.

(e)

x	10	90	170	250	330	410	490
y	0	14	28	42	56	70	84

Horizontal axis: 2 cm to 50 units.
Vertical axis: 1 cm to 10 units.

2. The speed, V, of a car after it has accelerated uniformly for t seconds is shown in the table below:

Time (t) in seconds	0	5	10	15	20	25	30
Speed (V) in km/h	20	35	50	65	80	95	110

(a) Using suitable scales, draw the graph representing the results.
(b) Use your graph to find the speed of the car after
(i) 7 seconds, **(ii)** 13 seconds, **(iii)** 28 seconds.
(c) From your graph, find how many seconds later the car will reach a speed of
(i) 26 km/h, **(ii)** 53 km/h, **(iii)** 89 km/h.

∗3. The measurements from an experiment were recorded in the following table:

x (seconds)	0	10	20	30	40	50
y (metres)	1 500	1 200	900	600	300	0

Using suitable scales, draw a graph illustrating the results of the experiment. Use your graph to answer the following questions:
(a) Find the value of y when x is
(i) 5, **(ii)** 18, **(iii)** 25, **(iv)** 45, **(v)** 47.
(b) Find the value of x when y is
(i) 1 455, **(ii)** 990, **(iii)** 480, **(iv)** 390, **(v)** 270.

∗4. The table below shows the telephone charge, $$C$, on making n telephone calls.

n	0	10	20	30	40	50	60
$$C$	50	53	56	59	62	65	68

Using suitable scales, draw a graph relating the number of telephone calls to the telephone charge.
(a) From the graph, find the amount to be paid if
(i) 15, **(ii)** 35, **(iii)** 58 calls are made.

(b) Find the number of calls made if the bill is
(i) \$55.40, **(ii)** \$63.50.

(c) Paul made 54 calls and received a bill for \$61.20. Was the bill correct? If not, what should have been the amount on the bill?

∗5. Given that a car can travel 12 km for every litre of petrol, copy and complete the following table:

No. of litres	1	2	3	4	5	6	7	8	9
No. of kilometres	12								

(a) Draw a graph using the above table of values.
(b) Use your graph to find the amount of petrol used in journeys of 42 km, 56 km and 90 km.
(c) Use your graph to find the distance the car can travel on 2.5 litres, 4.8 litres and 8.5 litres of petrol.

∗6. Given that 4 litres of water run into a bath in 8 seconds, copy and complete the following table:

Time (seconds)	8	16	24	32	40
Volume of water (litres)	4				

(a) Draw a graph using the table of values above.
(b) Use your graph to find the volume of water in the bath after
(i) 14 seconds, **(ii)** 20 seconds, **(iii)** 36 seconds.
(c) Use your graph to find the time taken to fill the bath with
(i) 3 litres, **(ii)** 13 litres, **(iii)** 19 litres of water.

∗7. It is given that S\$44 is equivalent to RM100 in 2000. Draw a graph to show the relationship between Singapore dollars and Malaysian ringgit up to RM150. Use your graph to find the conversion of the following:
(a) Singapore dollars into Malaysian ringgit.
(i) S\$10 **(ii)** S\$26 **(iii)** S\$50
(b) Malaysian ringgit into Singapore dollars.
(i) RM30 **(ii)** RM88 **(iii)** RM135

∗8. Draw a graph to convert examination marks out of 120 to percentages, taking 1 cm to represent 10 marks on the horizontal axis and 1 cm to represent 10% on the vertical axis. From your graph, read off as accurately as possible,
(a) the percentages equivalent to 35, 50, 86 and 104 marks.
(b) the marks out of 120 corresponding to 45%, 60%, 75% and 89%.
(c) A pupil must score at least 80% of the total mark to obtain grade *A*. Use your graph to find the minimum mark a pupil must score out of 120 marks to get grade *A*.

∗9. In a city, electricity charges are calculated as follows:
A fixed charge of \$20 plus 30¢ per unit of electricity used.
Taking 2 cm to represent 50 units on the horizontal axis and 2 cm to represent \$20 on the vertical axis, draw a graph to show the charges up to a maximum of 400 units. From your graph, estimate
(a) the cost of using the following units of electricity:
(i) 110 **(ii)** 235 **(iii)** 79 **(iv)** 315,
(b) the number of units of electricity used if the bill is:
(i) \$47 **(ii)** \$66.50 **(iii)** \$95 **(iv)** \$123.50.

Problem Solving Involving Linear Graphs

Linear graphs can often be used to solve problems in daily life.

Example 11

A lorry leaves Town A at 10.00 a.m. and travels at a constant speed of 40 km/h towards Town B. Half an hour later, a bus leaves Town A and travels towards Town B at a constant speed of 56 km/h.

(a) At what time will the bus catch up with the lorry and how far will they be from Town A?
(b) If the bus arrives at Town B 6 minutes ahead of the lorry, what is the distance between Town A and Town B?

Solution

Strategy 1: Use an equation

(a) Let t denote the number of hours taken by the lorry until the bus catches up with it. Since the bus leaves Town A half an hour later, it takes $\frac{1}{2}$ hour less than the lorry. This leads to the following table:

	Speed (km/h)	Hours travelled	Distance travelled
Lorry	40	t	$40t$
Bus	56	$t - \frac{1}{2}$	$56\left(t - \frac{1}{2}\right)$

The bus catches up with the lorry when they have travelled the same distance.

Thus,
$$56\left(t - \frac{1}{2}\right) = 40t$$
$$56t - 28 = 40t$$
$$16t = 28$$
$$t = 1\frac{3}{4} \text{ hours} = 1 \text{ hour } 45 \text{ minutes}$$

The bus catches up with the lorry at 11.45 a.m. and they are $40 \times 1\frac{3}{4} = 70$ km from Town A.

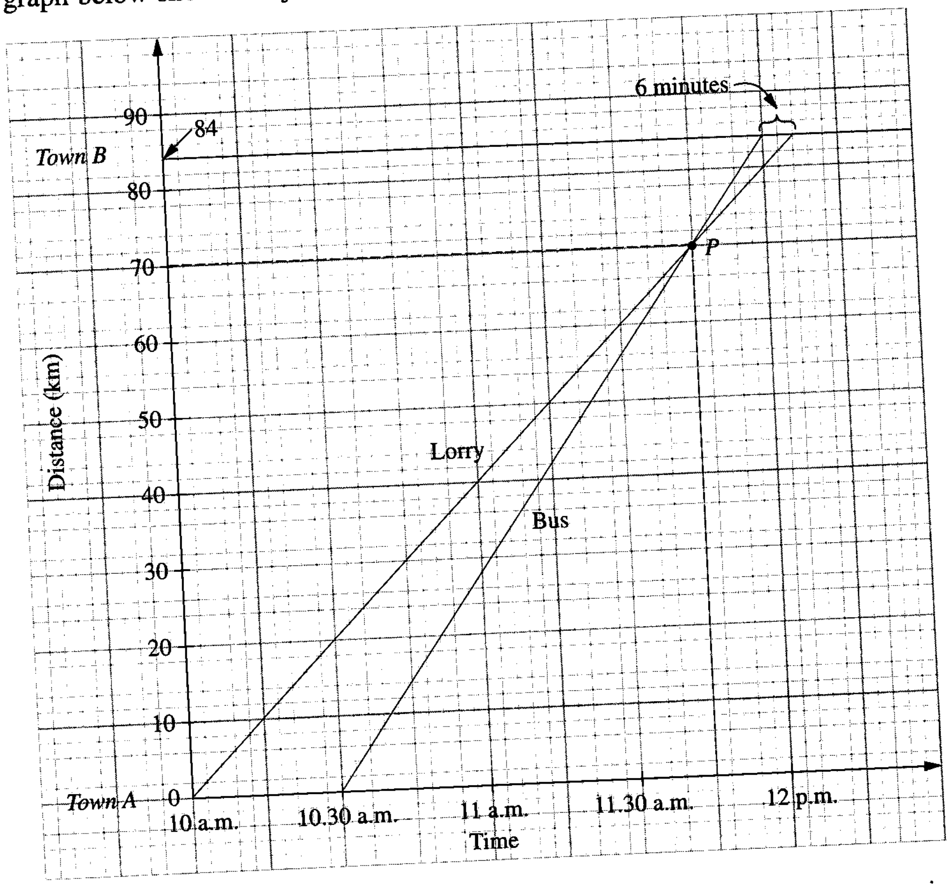
(b) Let x denote the number of hours, after the meeting of the two vehicles, taken by the lorry to arrive at Town B. Since the bus arrives at Town B 6 min or $\frac{1}{10}$ hour earlier, the corresponding time taken by the bus is $\left(x - \frac{1}{10}\right)$ hours.

$$56\left(x - \frac{1}{10}\right) = 40x$$
$$16x = \frac{56}{10}$$
$$x = \frac{7}{20}$$

Thus, the distance between Town A and Town B is $\left(70 + 40 \times \frac{7}{20}\right)$ km or 84 km.

Strategy 2: Use a graph

The travel graph below shows the journeys of the lorry and the bus.



Can you explain how the two lines shown in the travel graph were obtained?

(a) From the graph, the bus catches up with the lorry at 11.45 a.m. and they are 70 km from Town A (point P).

(b) From the graph, when the two vehicles are 6 minutes apart, they are 84 km from Town A. Thus, the distance between Town A and Town B is 84 km.

Example 12

A housewife decides to make cookies for sale. The fixed costs are \$400. The cost of making each cookie is \$1, and each cookie sells for \$1.50.

(a) How many cookies must be made and sold in order to break even?
(b) What is the loss if only 400 cookies are made and sold?
(c) What is the profit if 1 500 cookies are made and sold?

Solution

Strategy 1: Use an equation

(a) Let x denote the number of cookies that must be made and sold to break even.
Expenses = $\$(400 + x)$ and income = $\$1.5x$

To break even, expenses = income

i.e., $400 + x = 1.5x$

$x = 800$

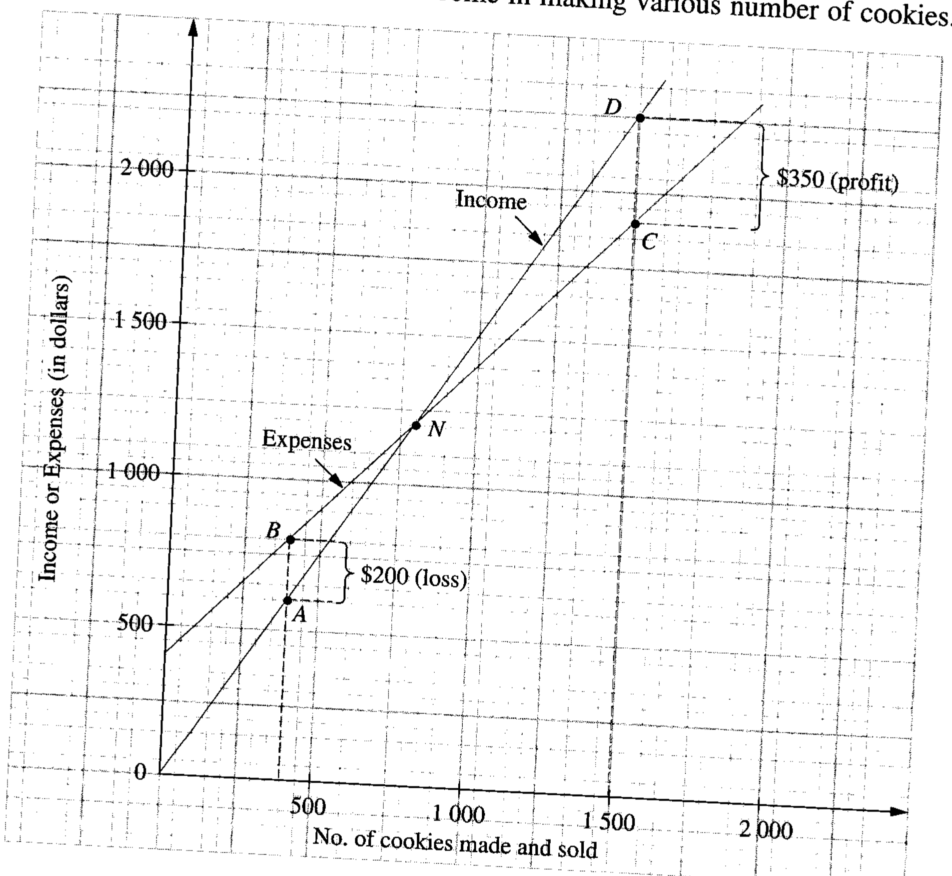
Thus 800 cookies must be made and sold in order to break even.

(b) Expenses in making 400 cookies = \$400 + 400 × \$1 = \$800
Income from selling 400 cookies = 400 × \$1.50 = \$600
The loss is \$(800 – 600) = \$200.

(c) Expenses in making 1 500 cookies = \$400 + 1500 × \$1 = \$1 900
Income from selling 1 500 cookies = 1 500 × \$1.50 = \$2 250
The profit is \$(2 250 – 1 900) = \$350.

Strategy 2: **Use a graph**

The graph below shows the expenses and income in making various number of cookies.



Can you explain how the two lines shown in the above graph were obtained?

From the graph,

(a) the lines representing the expenses and income intersect at $N = 800$. Thus 800 cookies must be made to break even;

(b) the expenses (point B) is \$200 more than the income (point A) when 400 cookies are made. Thus, the loss is \$200;

(c) the expenses (point C) is \$350 less than the income (point D) when 1 500 cookies are made. Thus, the profit is \$350.

Exercise 9i

1. Two towns P and Q are 35 km apart. Peter starts cycling from P towards Q at 12 p.m. at 20 km/h until he is 16 km from P, when he changes speed so that he arrives at Q at 2 p.m. John leaves Q at 12.30 p.m. and cycles to P at a constant speed of 26 km/h. Find
   (a) the time when Peter and John meet,
   (b) Peter's speed in the last part of the journey,
   (c) the time when John reaches P.

2. Mr Lim, an artist, wishes to make ceramic pieces for sale. The fixed costs are \$40. The cost of producing each ceramic piece is \$8, and each sell for \$18.
   (a) Mr Lim will break even when revenue from sales is equal to the production cost. How many pieces must he sell in order to break even?
   (b) Mr Lim makes a profit when the revenue from sales exceeds production cost. At least how many pieces must he sell to make a profit?
   (c) What is Mr Lim's loss if only two ceramic pieces are produced and sold?
   (d) What is his profit if ten ceramic pieces are produced and sold?

3. In a city, electricity charges are calculated as follows:
   Scheme A: A fixed charge of \$30 plus 40 cents per unit of electricity used.
   Scheme B: A flat rate of 60 cents per unit of electricity used.
   (a) Find the charges based on the two schemes for 50 units of electricity used.
   (b) For how many units of electricity used will the charges be the same for both schemes?

Summary

1. A Cartesian plane consists of two axes, the **x-axis** and the **y-axis**, intersecting at right angles at the **origin** O.

2. An ordered pair (a, b) locates a point P in the Cartesian plane, a being the **x-coordinate** and b the **y-coordinate** of P.

3. A graph is a drawing which shows the relationship between numbers or quantities.

4. In many practical situations, information is presented in the form of a graph. We can interpret and use graphs such as travel graphs and conversion graphs to solve problems.

5. Graphs of **linear equations** are straight lines.

6. **(a)** Graphs of equations of the form $y = c$ are straight lines parallel to the x-axis.
   (b) Graphs of equations of the form $x = a$ are straight lines parallel to the y-axis.
   (c) Graphs of equations of the form $y = mx$ pass through the origin.
   (d) Graphs of equations of the form $y = mx + c$ cut the y-axis at the point $(0, c)$.

7. Simultaneous linear equations can be solved graphically by finding the coordinates of the intersection point of the linear graphs representing the linear equations.

Review Questions 9

1. Draw the graphs of the following equations:

   (a) $x = -4$ (b) $y = 6$ (c) $5x - 3y = 45$ (d) $3x + 4y = 8$

2. Use graphical method to solve the following simultaneous equations:

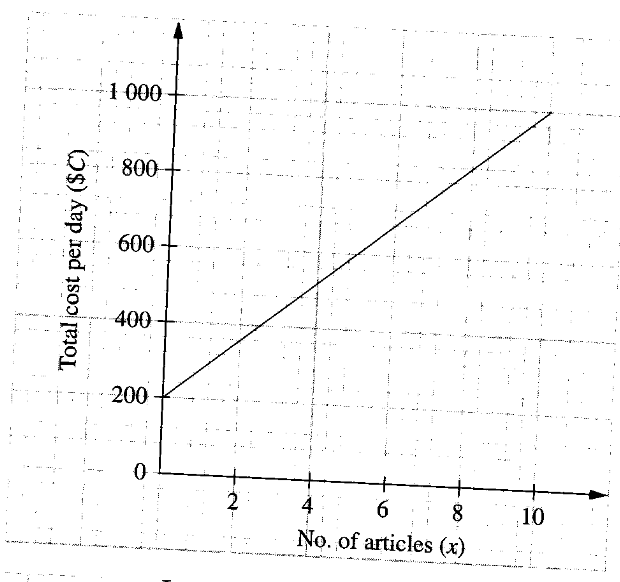
   (a) $x + y = 0$, $3x - 2y = 5$
   (b) $x + 2y = 0$, $5x = 4y + 14$
   (c) $3x - 2y = 3$, $5x - 4y = 4$
   (d) $x - 2y = 2$, $8y = 4x - 8$
   (e) $y = \frac{1}{4}x + 3$, $4y - x = 8$

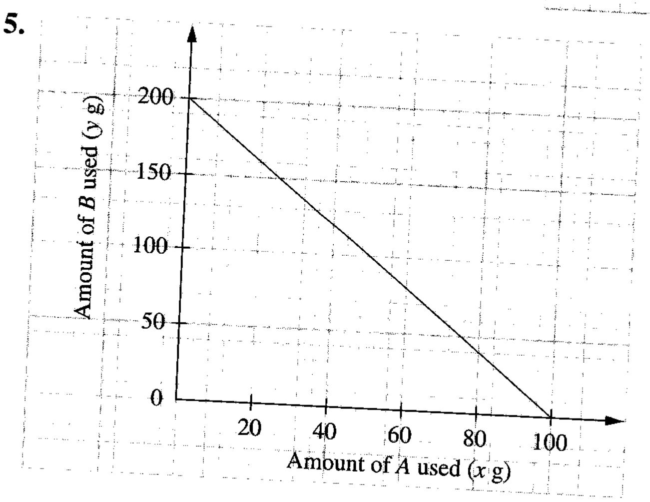
*3. Given that $3x - 5y = 22$ and $4x - 3y = 33$, find graphically the value of $2x + y$.

4. The graph shows the total cost per day ($\$C$) for an output of x articles per day.

   (a) Use your graph to find the total cost per day for a daily output of
   (i) 3 articles, (ii) 8 articles.

   (b) On a particular day, the total cost was $600. How many articles were produced on that day?



5. 

In a nutrition experiment, a special diet is prepared for the experimental animals using two food mixes A and B. Mix A contains 20% protein and mix B 10%. The graph shows the combination of each mix which will provide exactly 20 g of protein. From the graph, find

(a) the amount of A needed to mix with 145 g of B to provide exactly 20 g of protein,

(b) the amount of B needed to mix with 68 g of A to provide exactly 20 g of protein.

6. Given that P denotes the perimeter of a square of side l, copy and complete the following table:

l	5	10	15	20	25
P	20				

(a) Draw a graph using the table of values above.

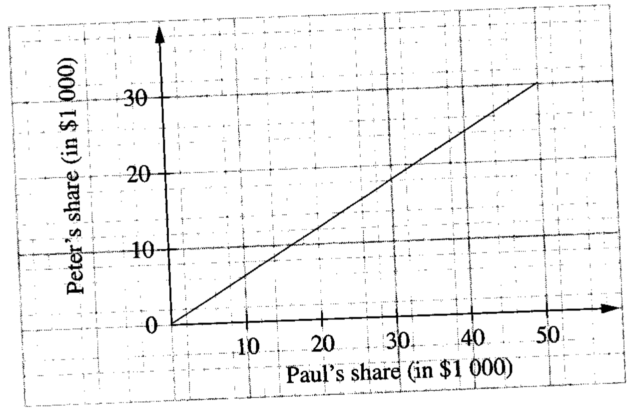
(b) From the graph, find

  (i) the perimeter of a square of side 8 cm,

  (ii) the perimeter of a square of side 14 cm,

  (iii) the length of the side of a square of perimeter 84 cm.

7. Peter and Paul set up a business and they agree to share the profit in the ratio 3 : 5. The graph shows the amounts Peter and Paul will receive depending on the profit made.



Use the graph to find

(a) Peter's share if Paul receives $33 000,

(b) Paul's share if Peter receives $12 000,

(c) the total profit if Paul receives $43 000.

8. On a Saturday, Ahmad left home on his bicycle at 8 a.m. and travelled 4 km at a speed of 8 km/h directly to his school. After staying in school for CA for 2 hours, he returned home by the same route in 20 minutes.

(a) Draw a travel graph with distance travelled on the y-axis and time on the x-axis.

(b) From the graph, find

  (i) the time at which Ahmad arrived home,

  (ii) the speed at which he travelled on the return journey.

✲9. A train leaves Town A at 09 00 and travels towards Town B which is 80 km from Town A at 30 km/h. After travelling 45 km, the train stops for half an hour. Then it completes the journey to Town B at 70 km/h.

(a) (i) Using a scale of 4 cm to represent 1 hour for time taken from 09 00 to 12 00 on the horizontal axis and 2 cm to represent 10 km for distance on the vertical axis, draw the travel graph for this journey. Find the arrival time of the train in Town B.

  (ii) By adding another line to your travel graph, find the arrival time of the train in Town B if the train completes its journey at a constant speed of 30 km/h without stopping.

(b) A second train leaves Town B at 09 30 and travels at a constant speed to Town A, arriving at 11 15.

  (i) On the same axes, draw the travel graph for this journey.

  (ii) Use your graph to find the time and the distance from Town A at which the two trains pass each other.

10. The following graph shows the amount received by a business firm from sales of articles and the expenses incurred in running the business each day.

(a) Find (i) the expenses,

  (ii) the total value of sales,

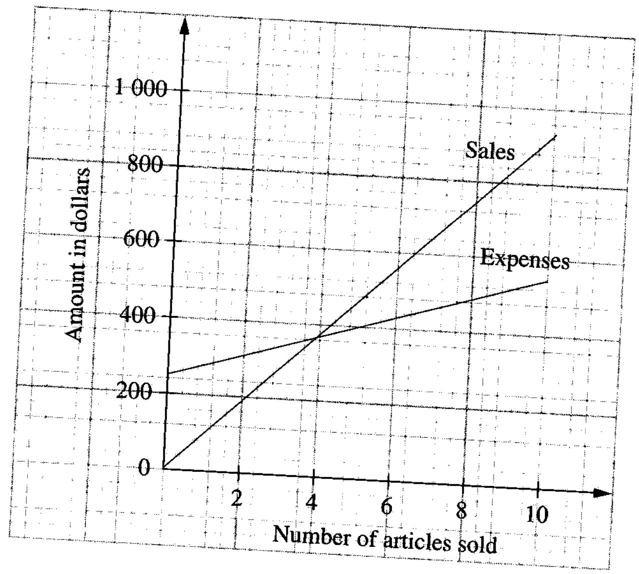
  (iii) the profit,

on the day in which 7 articles were sold.

(b) How many articles must be sold each day to break even?

(c) How much profit is made on the day in which 10 articles were sold?

(d) What happens if only two articles were sold on a particular day?



11. A company pays a salesman $$M$ per week. The amount is made up of a basic wage of $50 plus $10 for each of the n articles he sells. Copy and complete the following table:

n	0	5	10	15	20	25	30	35
M	50	100						

(a) Draw a graph using the table of values above.

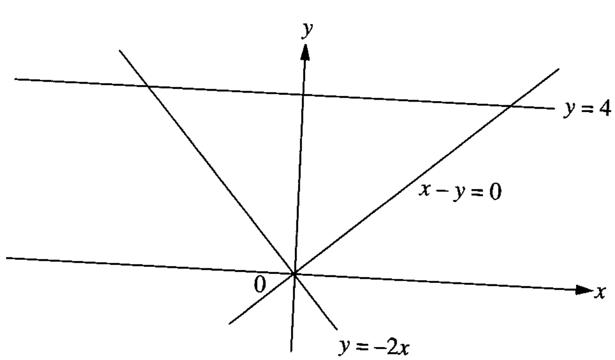
(b) From the graph, find the amount of money the salesman received in a week when he sold
(i) 9 articles, **(ii)** 22 articles, **(iii)** 31 articles.

(c) From the graph, find the number of articles he sold when he received
(i) $180, **(ii)** $240, **(iii)** $390.

Problem Solving

1. Find the value of x such that the three points $A(-1, 1)$, $B(3, 7)$ and $C(x, -2)$ lie on the same straight line.

2. Find the area of the shaded region in the diagram.



3. **(a)** Using the same scales and axes, draw the following straight lines:

   $y = 2, \quad y = \frac{2}{3}x, \quad x + y = 8$

   (b) From your graph, write down the coordinates of the points of intersection of the three lines.

   (c) Find the area of the triangle bound by the three lines.

4. In an examination, the highest score was 70 marks and the lowest score 20 marks. The marks of all the candidates were later changed so that the highest score became 85 and the lowest score 10. Draw a graph from which the revised marks of all the candidates can be read off. Use your graph to find

   (a) the revised mark of a candidate whose original score was 40,

   (b) the original score of a candidate whose revised mark was 58.

5. Towns A and B are 220 km apart. A bus leaves Town A at 08 30 for Town B at a speed of 30 km/h. It arrives at Town B after stopping for 40 minutes at Town C which is 120 km from Town A. A motorist starts from Town B at 09 00 and travels towards Town A at a speed of 40 km/h. Find graphically when and at what distance from Town A they will meet.

Exploration

1. Two cars make a return journey between two towns 100 km apart. The first car does the outward journey at an average speed of 80 km/h and the return journey at an average speed of 60 km/h. The second car does its journey both ways at an average speed of 70 km/h. Will there be a difference in the travelling time for the two cars?

2. Two cyclists travelling in opposite directions approach each other. One is moving at 20 km/h and the other at 18 km/h.

   A bird which flies at an average speed of 40 km/h starts from the first cyclist, flies to the second cyclist and then returns to the first cyclist, travelling to and fro within the space between the two approaching cyclists continuously. If the two cyclists meet after $\frac{1}{2}$ hour, find the total distance the bird has travelled.

3. The monthly cost of running a kindergarten consists of a fixed amount and a variable amount which depends on the number of children attending the kindergarten classes. It costs $3 750 to run classes for 50 children and $4 500 to run classes for 80 children. Find graphically the fixed cost for running the kindergarten and the cost per child.

4. A motorist, X, travelling at a uniform speed of 48 km/h, passes a village at 6 a.m. Another motorist, Y, travelling at a uniform speed of 66 km/h, passes the same village at 6.30 a.m. Draw straight line graphs to show the journeys of X and Y between 6.30 a.m. and 9 a.m., taking 4 cm to represent 1 hour on the time axis and 2 cm to represent 20 km on the distance axis. Deduce from your graph the time at which both motorists are at the same distance from the village.

CHAPTER 10

Graphs of Quadratic Functions

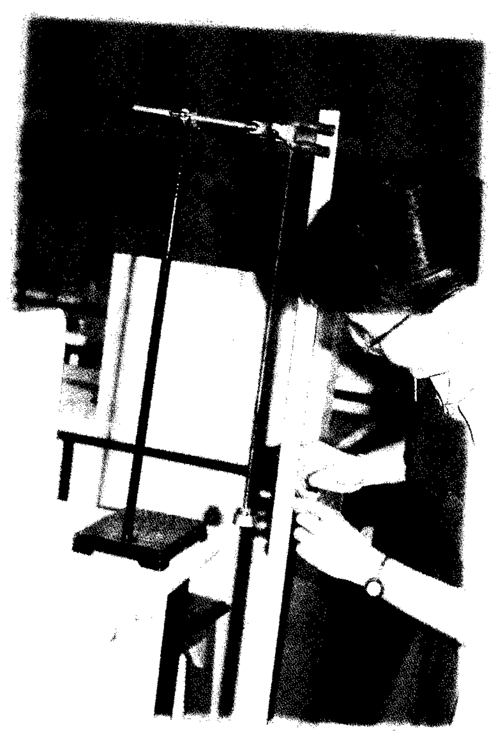
In this chapter, you will learn how to

- △ plot quadratic graphs;
- △ solve quadratic equations graphically.

Preliminary Problem

The picture shows the set-up of an experiment to find the relationship between the extension of the length of a spring and the weight attached to it. From the results of this experiment, a graph can be plotted.

What other experiments can you think of involving two variables?



Quadratic Graphs

In Chapter 9, we learnt that the graph of a linear equation in two variables such as $y = 2x + 1$ is a straight line. In this chapter, we shall consider the graph of a quadratic equation in two variables, say, $y = x^2 - x + 1$. We shall see that unlike linear graphs, quadratic graphs involve curves.

In-Class Activity

You may work in pairs.

1. The following table of values represents a certain equation in two variables.

x	−4	−3	−2	−1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

(a) Examine each ordered pair (x, y) carefully.
  (i) What do you notice about the relationship between x and y?
  (ii) Write down a formula for the equation, expressing y in terms of x.

(b) (i) Construct a pair of axes on a piece of graph paper using 2 cm to represent 1 unit on the x-axis from $x = -5$ to $x = 5$ and 2 cm to represent 5 units on the y-axis from $y = -20$ to $y = 20$.
  (ii) Plot the points of the equation using the table given.
  (iii) Join the points with a curve.
  (iv) Can you describe the shape of the curve?

2. (a) Copy and complete the table below for the equation in two variables represented by the graph in Fig. 10.1.

x	−3	−2	−1	0	1	2	3
y							

(b) (i) Do you notice that for each ordered pair (x, y), the value of y is the result of multiplying the square of the value of x by 2?
  (ii) Write down a formula for this equation, expressing y in terms of x.

(c) Plot the graph of this function on the same pair of axes constructed in 1 (b)(i).

(d) Compare the two graphs.

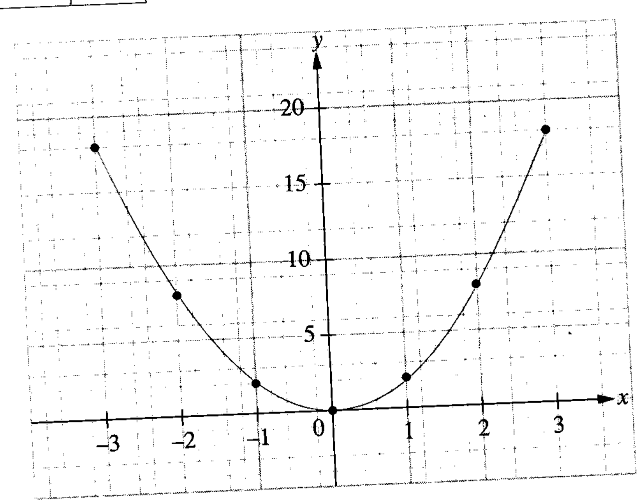


Fig. 10.1

3. (a) Copy and complete the table of values for the equation in two variables $y = \frac{1}{2}x^2$.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y											

(b) Plot the graph of this equation on the same pair of axes constructed in 1 (b)(i).
(c) Compare the three graphs.

4. How do you expect the graphs of each of the following equations:

$y = 3x^2$, $y = 4x^2$, $y = 5x^2$, $y = \frac{1}{3}x^2$, $y = \frac{1}{4}x^2$, $y = \frac{1}{5}x^2$

to behave in relation to the three graphs you have drawn so far? Investigate.

5. Fig. 10.2 shows the graphs of $y = x$ and $y = -x$ on the same axes.
   (a) What is the relationship between the two lines?
   (b) Do you agree that every point on the line $y = x$ has a mirror image with respect to the x-axis on the line $y = -x$?
   (c) Do you notice the symmetry about the x-axis formed by the lines?

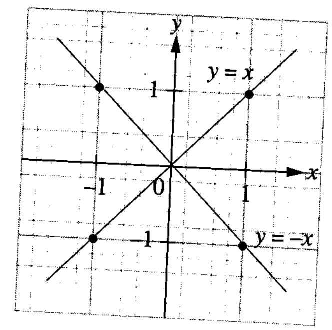


Fig. 10.2

6. Using the ideas discussed in question 5, add the graphs of $y = -x^2$, $y = -2x^2$ and $y = -\frac{1}{2}x^2$ to the same pair of axes constructed in 1(b)(i).

Quadratic Equations in Two Variables of the Form $y = ax^2$ $(a \neq 0)$

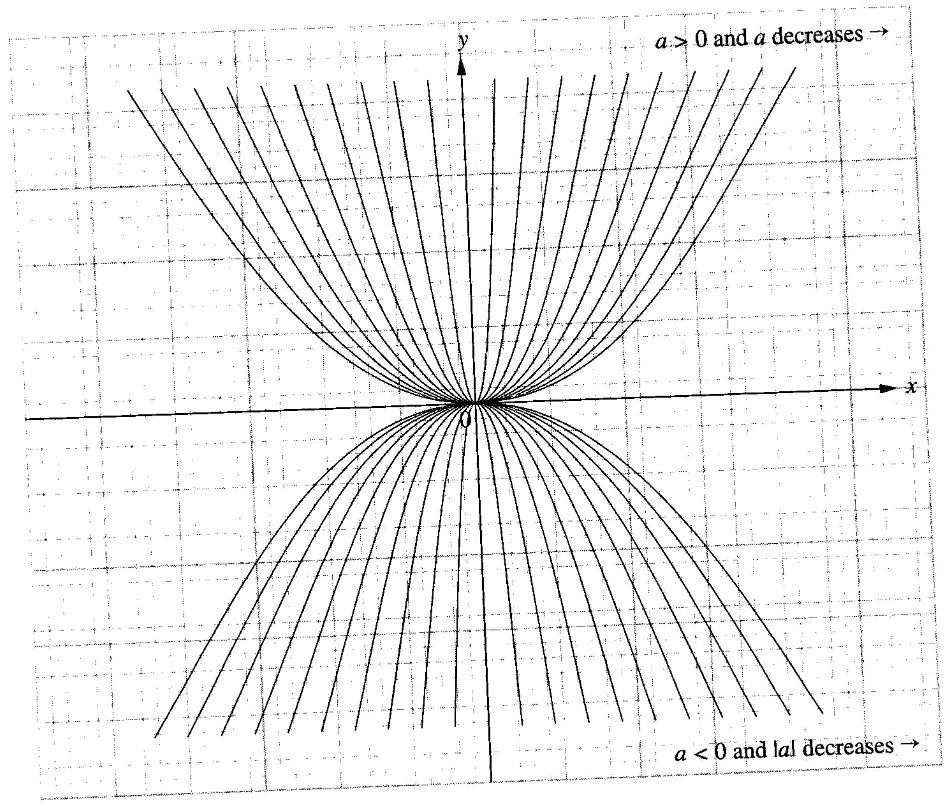
The results from the In-Class Activity can be summarised below.

1. The graphs of $y = ax^2$ $(a \neq 0)$ pass through the origin.
2. The y-axis is the line of symmetry.
3. (a) When a is positive, each graph has a lowest point (the origin) and opens upwards indefinitely.
   (b) The smaller the value of a, the wider the graph opens.
4. (a) When a is negative, each graph has a highest point (the origin) and opens downwards indefinitely.
   (b) The smaller the value of a, the wider the graph opens.

Note: When $a = 0$, what is the graph?

Just for Fun

Six coins are placed in a straight line with their 'heads' facing upwards. The task is to turn the coins such that you end up having 6 coins with their 'tails' up under the condition that you turn 5 coins in each round. What is the minimum number of rounds required to do this? How many rounds will you need to do this if you have 52 coins instead?



Graphs of General Quadratic Equations in Two Variables

Mr Lin wants to pour 12 litres of water equally into two containers. However, he has only two measuring cans of capacity 9 litres and 5 litres with him. How is he to obtain the two equal amounts of water accurately by using the measuring cans?

The general form of a quadratic equation is $y = ax^2 + bx + c$ where a, b and c are real numbers and a is not equal to zero.

Example 1

Draw the graph of $y = x^2 + 1$ for $-4 \leqslant x \leqslant 4$. Find, from the graph,

(a) the value of y when $x = 2.5$,
(b) the values of x when $y = 15$,
(c) the equation of the line of symmetry of $y = x^2 + 1$.

Solution

A table of values for both x and y is set up as shown below:

x	−4	−3	−2	−1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$y = x^2 + 1$	17	10	5	2	1	2	5	10	17

The graph obtained is a curve as shown in Fig. 10.3.

The CD, *The Business of Graphs*, from the DMS has an interesting section on drawing and reading quadratic graphs. Go through the tutorials and activities. Use the Equation Plotter in the CD to plot a few graphs. You can also use the open tools, Graphmatica or Winplot, to explore the shapes of quadratic graphs.

From the graph,

(a) when $x = 2.5$, y is approximately equal to 7.2,

(b) when $y = 15$, x is approximately equal to 3.8 or –3.8.

(c) the y-axis is the line of symmetry and its equation is $x = 0$.

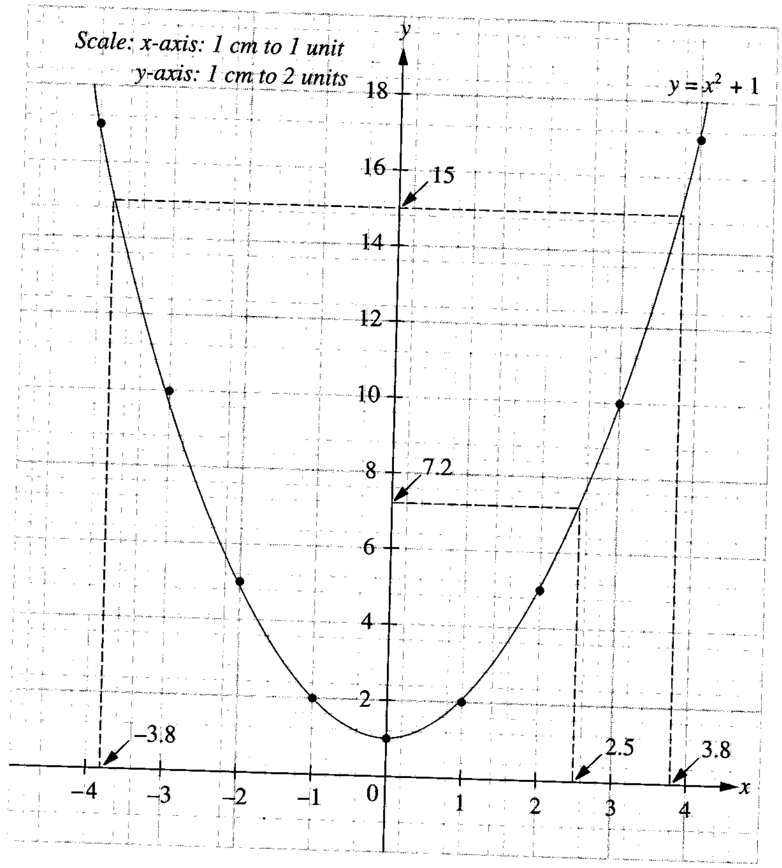


Fig. 10.3

Example 2

Draw the graph of $y = 3 + 2x - x^2$ for $-3 \leqslant x \leqslant 5$. From the graph, find

(a) the greatest value of y,

(b) the values of y when $x = -1.9$, 2.7 and 4.3.

Solution

The table shows the values of y for $-3 \leqslant x \leqslant 5$.

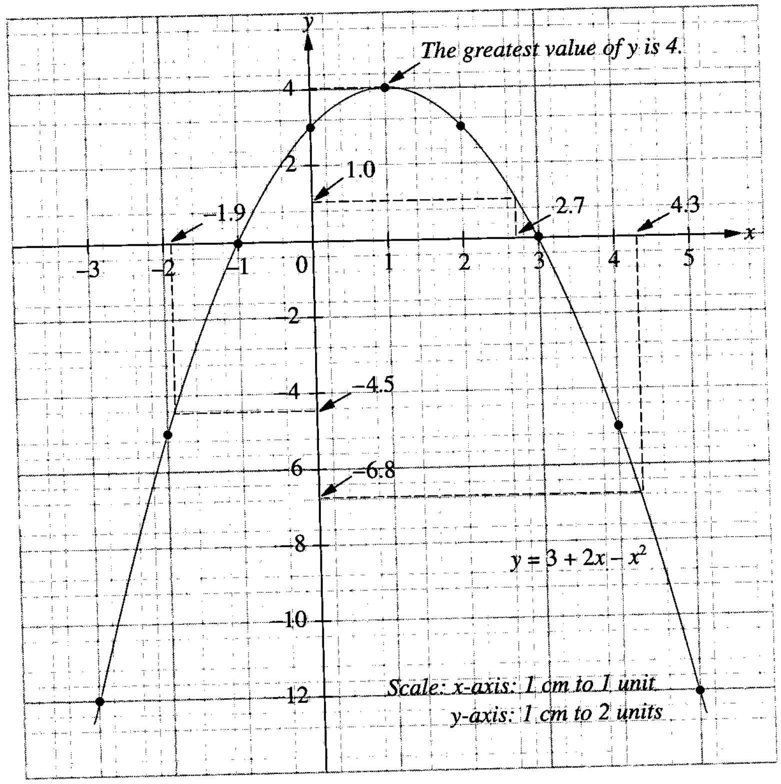
x	–3	–2	–1	0	1	2	3	4	5
y	–12	–5	0	3	4	3	0	–5	–12

(a) The greatest value of y is 4.

(b) When $x = -1.9$, $y \approx -4.5$.

When $x = 2.7$, $y \approx 1.0$.

When $x = 4.3$, $y \approx -6.8$.



Example 3

Draw the graph of $y = x^2 + 2x + 2$ for $-3 \leqslant x \leqslant 3$. From the graph, find

(a) the smallest value of y and the corresponding value of x,
(b) the values of y when $x = -2.7$, 1.4 and 2.3.

Solution

The table below gives the values of y for $-3 \leqslant x \leqslant 3$.

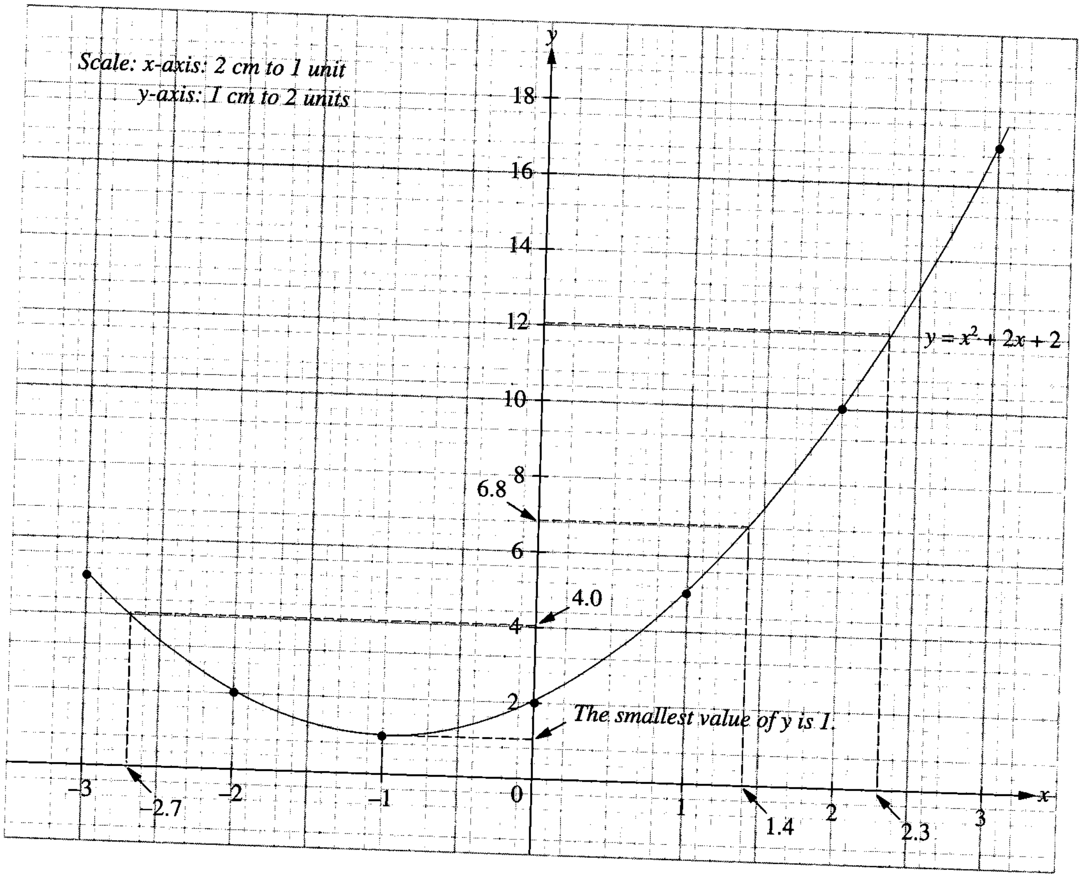
x	−3	−2	−1	0	1	2	3
y	5	2	1	2	5	10	17

(a) The smallest value of y is 1 and it occurs when $x = -1$.

(b) When $x = -2.7$, $y \approx 4.0$.

When $x = 1.4$, $y \approx 6.8$.

When $x = 2.3$, $y \approx 12.0$.



⋆Exercise 10a

1. Copy and complete the following table which gives the values of $y = x^2 + 4$ for $-4 \leqslant x \leqslant 4$.

x	−4	−3	−2	−1	0	1	2	3	4
y	20		8					13	

Using 2 cm to represent 1 unit on the x-axis and 1 cm to represent 2 units on the y-axis, draw the graph of $y = x^2 + 4$. Find the equation of the line of symmetry.

2. Copy and complete the following table which gives values of $y = 3x - 2x^2$ for $-2 \leqslant x \leqslant 3$.

x	−2	$-1\frac{1}{2}$	−1	0	1	2	$2\frac{1}{2}$	3
y		−9		0		−2		

Using 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = 3x - 2x^2$. Is there a line of symmetry?

3. Copy and complete the table which gives values of $y = x^2 - 5x + 4$ for $0 \leqslant x \leqslant 4\frac{1}{2}$.

x	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
y	4			$-1\frac{1}{4}$	−2			$-1\frac{1}{4}$		$1\frac{3}{4}$

Using 4 cm to represent 1 unit on both axes, draw the graph of $y = x^2 - 5x + 4$.

4. **(a)** Given that $y = 2 - 3x - x^2$, copy and complete the following table:

x	−5	−4	−3	−2	−1	0	1	2	3
y	−8			4		2	−2		

(b) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = 2 - 3x - x^2$ for $-5 \leqslant x \leqslant 3$.

(c) Use your graph to find
  (i) the values of x when $y = -3, -8$ and -13,
  (ii) the values of y when $x = -4.5, -3.2$ and 1.6.

5. (a) Given that $y = x^2 - 2x$, copy and complete the following table:

x	–2	–1	0	1	2	3	4
y	8		0	–1			

(b) Using a scale of 2 cm to 1 unit on each axis, draw the graph of $y = x^2 - 2x$ for $-2 \leqslant x \leqslant 4$.

(c) Use your graph to find
  (i) the values of x when $y = -\frac{1}{2}$, 2 and 5,
  (ii) the values of y when $x = -1.8, 2.2$ and 3.5.

(d) Find the equation of the line of symmetry of the curve.

6. (a) Given that $y = x^2 + 3x - 4$, copy and complete the following table:

x	–6	–5	–4	–3	–2	–1	0	1	2
y	14							0	

(b) Using 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = x^2 + 3x - 4$ for $-6 \leqslant x \leqslant 2$.

(c) Use your graph to find
  (i) the values of x when $y = -2$, 5 and 8,
  (ii) the values of y when $x = -5.5, -2.7$ and 1.6.

7. Construct tables of values for the following equations. Plot their graphs using suitable scales.
  (a) $y = x^2 - 4x + 3$ $(-4 \leqslant x \leqslant 4)$
  (b) $y = 3x^2 + 3x - 5$ $(-2 \leqslant x \leqslant 3)$
  (c) $y = -3 + 2x - x^2$ $(-4 \leqslant x \leqslant 4)$
  (d) $y = 25 + 4x - 3x^2$ $(-3 \leqslant x \leqslant 5)$

8. Construct tables of values for each equation below, then draw all their graphs on the same pair of axes using suitable scales.
  (a) $y = x^2$ $(-3 \leqslant x \leqslant 3)$
  (b) $y = x^2 + 3$ $(-3 \leqslant x \leqslant 3)$
  (c) $y = x^2 - 3$ $(-3 \leqslant x \leqslant 3)$
  How are the graphs related to one another?

9. Construct tables of values for each equation below, then draw all their graphs on the same pair of axes using suitable scales.
  (a) $y = -x^2$ $(-3 \leqslant x \leqslant 3)$
  (b) $y = -(x - 2)^2$ $(-1 \leqslant x \leqslant 5)$
  (c) $y = -(x + 2)^2$ $(-5 \leqslant x \leqslant 1)$
  How are the graphs related to one another?

Graphical Solution of a Quadratic Equation

When a given quadratic equation cannot be solved by the factor method, the graphical method provides an alternative.

Example 4

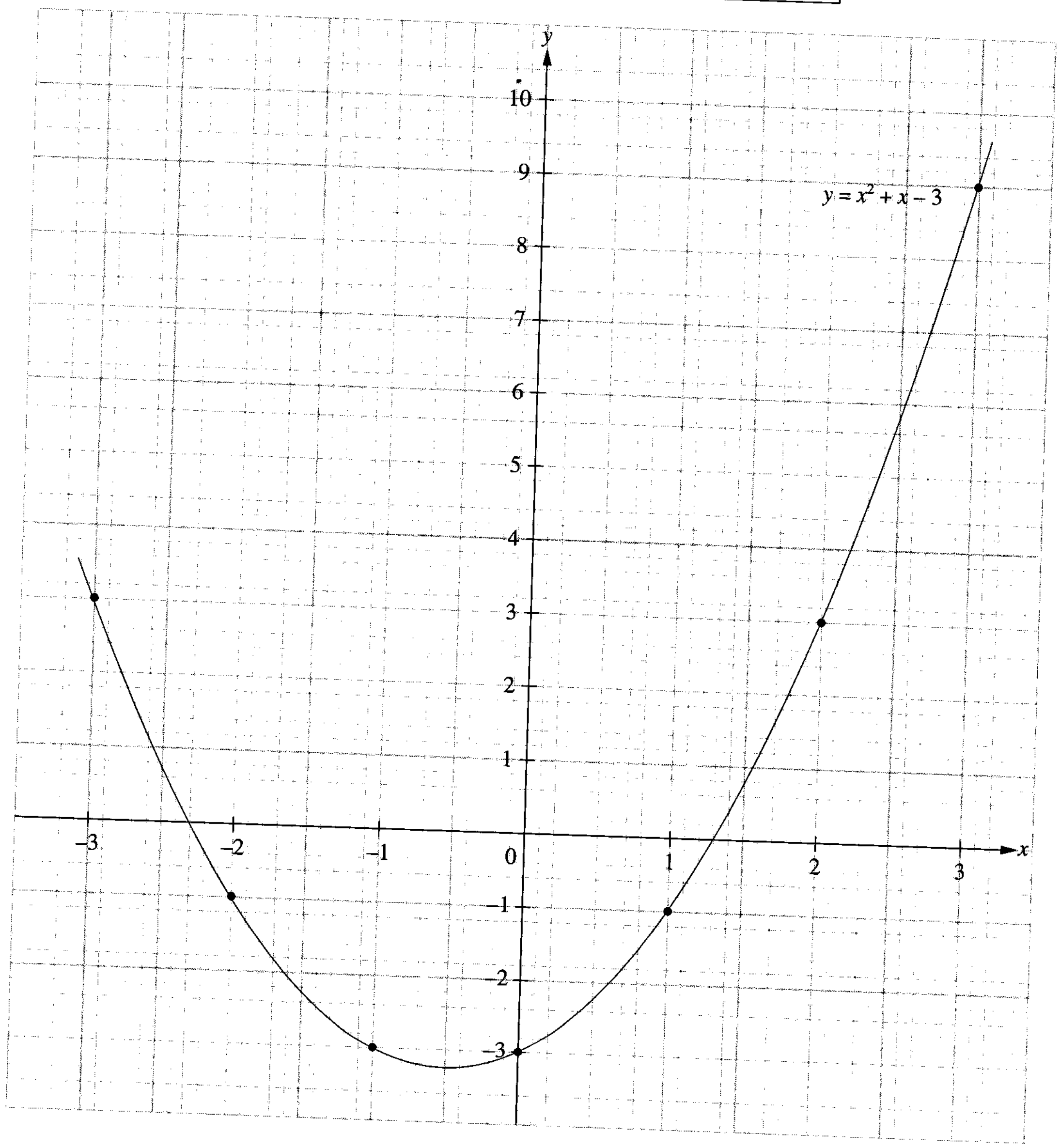
Solve the equation $x^2 + x - 3 = 0$ graphically.

Solution

Draw the graph of $y = x^2 + x - 3$.

The table below gives the values of y for $-3 \leqslant x \leqslant 3$.

x	–3	–2	–1	0	1	2	3
y	3	–1	–3	–3	–1	3	9



The graph $y = x^2 + x - 3$ cuts the x-axis at points where $x \approx -2.3$ and $x \approx 1.3$.

Thus, $x^2 + x - 3 = 0 \Rightarrow x \approx -2.3$ or $x \approx 1.3$.

Problem Solving Involving Quadratic Graphs

Many problems lead to quadratic equations. Graphical method is particularly useful in solving problems involving quadratic equations.

Example 5

Peter makes and sells handmade toys. He finds that if a batch of x toys is made, where $1 \leqslant x \leqslant 14$, the cost per toy \$y is given by $y = x^2 - 18x + 110$.

(a) Draw the graph of $y = x^2 - 18x + 110$ from $x = 0$ to $x = 14$, using 1 cm to represent 2 units on the x-axis and 1 cm to represent 20 units on the y-axis.

(b) Use the graph to write down the number of toys in a batch such that the cost per toy is

(i) a minimum,

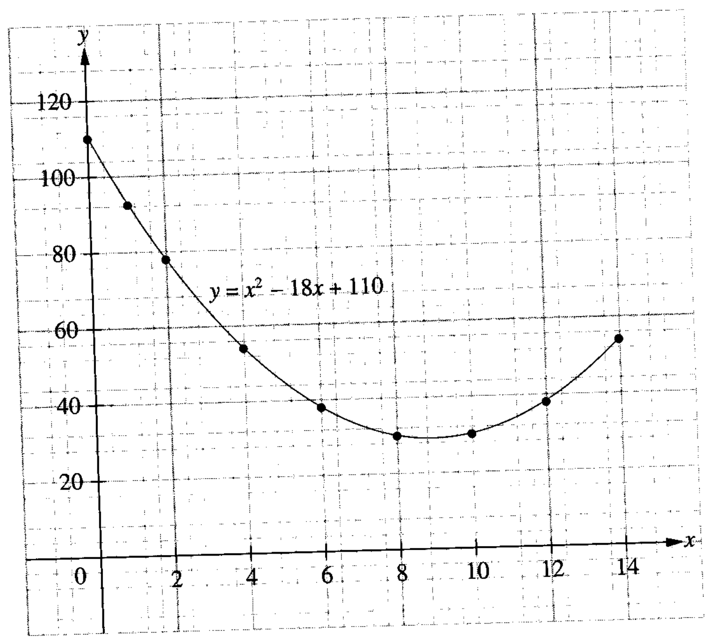
(ii) less than \$60.

Solution

The table below shows the values of y for values of x between 0 and 14.

x	0	1	2	4	6	8	10	12	14
y	110	93	78	54	38	30	30	38	54

(a) The graph of $y = x^2 - 18x + 110$ is drawn as shown below.



(b) From the graph,

(i) y is minimum when $x = 9$.

Thus, the number of toys in a batch that will make the cost per toy minimum is 9.

(ii) $y < 60$ when $x \geqslant 4$.

Thus, the number of toys in a batch that will make the cost per toy less than \$60 are 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14.

Exercise 10b

1. (a) Draw the graph of the equation $y = 2x^2 - 8x$ for $-1 \leqslant x \leqslant 5$, using a scale of 1 cm to 1 unit on the x-axis and 1 cm to 2 units on the y-axis.
   (b) Hence solve the equation $2x^2 - 8x = 0$.

2. (a) Draw the graph of the equation $y = x^2 - 5x + 3$ for $0 \leqslant x \leqslant 5$, taking 4 cm to represent 1 unit on each axis.
   (b) Hence solve the equation $x^2 - 5x + 3 = 0$.

3. (a) Draw the graph of the equation $y = 2x^2 + 3x - 5$ for $-4 \leqslant x \leqslant 3$, taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis.
   (b) Hence solve the equation $2x^2 + 3x - 5 = 0$.

4. (a) Draw the graph of the equation $y = 12 - 2x - 3x^2$ for $-4 \leqslant x \leqslant 4$. Take 2 cm to represent 1 unit on the x-axis and 2 cm to represent 5 units on the y-axis.
   (b) Hence solve the equation $12 - 2x - 3x^2 = 0$.

5. An object sliding down a slope has travelled a distance, s metres, in time, t seconds, where $s = 4t + t^2$.
   (a) Draw a graph to show the distances covered up to 5 seconds.
   (b) Find
      (i) the distance traveled after 2.6 seconds,
      (ii) the time taken to travel 30 metres.

6. A ball rolling on an uneven slope with an initial speed of 10 m/s and is moving at v m/s after t second, where $v = 2t^2 - 8t + 10$.
   (a) Draw the speed-time graph of the ball for the first 5 seconds of the motion.
   (b) Find
      (i) the speed of the ball when it had been moving for 3.8 seconds,
      (ii) its minimum speed,
      (iii) the time at which the ball was moving at 6 m/s.

7. Mary makes and sells handmade handbags. She finds that if a batch of x handbags is made, where $1 \leqslant x \leqslant 14$, the cost per handbag \$$y$ is given by $y = x^2 - 16x + 100$.
   (a) Draw the graph of $y = x^2 - 16x + 100$ for $0 \leqslant x \leqslant 14$, using 1 cm to represent 1 unit on the x-axis and 2 cm to represent 10 units on the y-axis.
   (b) Use the graph to write down the number of handbags in a batch that will make the cost per handbag
      (i) a minimum,
      (ii) less than \$70.

Summary

1. The general form of a quadratic equation is $\boldsymbol{y = ax^2 + bx + c\ (a \neq 0)}$.

2. The quadratic graph of $y = ax^2 + bx + c$ $(a \neq 0)$ has a lowest point when a is positive. It has a highest point when a is negative.

3. In solving the quadratic equation $ax^2 + bx + c = 0$ graphically, the solution is given by the points at which the graph of $y = ax^2 + bx + c$ crosses the x-axis.

Review Questions 10

1. Copy and complete the following table which gives values of $y = 5x - x^2$ for $-\frac{1}{2} \leqslant x \leqslant 5\frac{1}{2}$.

x	$-\frac{1}{2}$	0	1	2	3	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$
y	$-2\frac{3}{4}$	0			6		$2\frac{1}{4}$		$-2\frac{3}{4}$

Using 2 cm to represent 1 unit on each axis, draw the graph of $y = 5x - x^2$. Find the equation of the line of symmetry.

2. **(a)** Given that $y = x^2 - 4$, copy and complete the following table:

x	−3	−2	−1	0	1	2	3	4	5
y		0	−3		−3		5		

(b) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = x^2 - 4$ for $-3 \leqslant x \leqslant 5$.

(c) From your graph, find
- **(i)** the values of y when $x = -0.5$, 1.5 and 3.5,
- **(ii)** the values of x when $y = -2$, 6 and 8.

3. **(a)** Given that $y = x^2 - 2x + 1$, copy and complete the following table:

x	−4	−3	−2	−1	0	1	2	3	4
y	25			4			1	4	

(b) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 2 units on the y-axis, draw the graph of $y = x^2 - 2x + 1$ for $-4 \leqslant x \leqslant 4$.

(c) From your graph, find
- **(i)** the values of x when $y = 3$, 8 and 14,
- **(ii)** the values of y when $x = -2.4$, 0.2 and 3.7.

4. **(a)** Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = 10 - x - x^2$ for values of x such that $-4 \leqslant x \leqslant 3$.

(b) Use your graph to find the values of
- **(i)** x when $y = -1$, 5 and 8,
- **(ii)** y when $x = -2.2$, 1.6 and 2.5.

5. **(a)** Using 2 cm to represent 1 unit on each axis, draw the graph of $y = x^2 - 2x - 5$ for values of x such that $-2 \leqslant x \leqslant 4$.

(b) From your graph, find
- **(i)** the values of x when $y = -3$, 0 and 2,
- **(ii)** the values of y when $x = -1.5$, 0.2 and 2.6.

6. Draw the graph of $y = \frac{1}{5}(13x - x^2)$ for $0 \leqslant x \leqslant 14$ taking 1 cm to represent 1 unit on both axes.

(a) Find the x-coordinate of the point on the curve when $y = 5$.

(b) Find the y-coordinate of the point on the curve when $x = 6.5$.

7. Draw the graph of the equation $y = 3x^2 - 7x + 1$ for $-1 \leqslant x \leqslant 4$ and hence solve the equation $3x^2 - 7x + 1 = 0$.

8. By drawing suitable graphs, solve the following equations.

(a) $2x^2 + 5x - 5 = 0$

(b) $7 + 2x - 2x^2 = 0$

9. Draw the graph of the equation $y = 3x^2 + 4x - 8$ and use your graph to solve the equation $3x^2 + 4x = 8$.

10. Draw the graph of $y = 2x^2 - 3x - 6$ and use your graph to solve the equation $6 + 3x - 2x^2 = 0$.

11. A factory finds that its daily profit, y dollars, is related to x, the number of items it produces daily where $y = -x^2 + 90x$.

(a) Draw the graph of $y = -x^2 + 90x$.

(b) Use your graph to find

(i) the number of items the factory must produce daily in order to maximize the profit,

(ii) the maximum profit.

12. Fandi kicks a soccer ball vertically upwards. The height, h metres, of the ball after t seconds is given by

$$h = 27t - 6t^2.$$

By drawing a suitable graph, find

(a) the maximum height of the ball above the ground,

(b) the time required for the ball to reach the ground again,

(c) the shortest time taken to reach a height of 20 metres.

Problem Solving

1. Draw the graph of $y = x^2$ for $-4 \leqslant x \leqslant 4$, taking 2 cm to represent 1 unit on each axis. Use your graph to find

(a) $\sqrt{6.2}$ (b) $\sqrt{10.5}$ (c) $(-3.6)^2$ (d) 2.6^2.

2. Draw the graphs of $y = -x^2$ and $y = 2x - 3$ on the same axes, taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, for $-5 \leqslant x \leqslant 3$.

Use your graphs to find 4.8^2 and $\sqrt{6}$.

Write down the values of x at the points of intersection of the two graphs and find the equation for which these values of x are the solutions.

CHAPTER 11

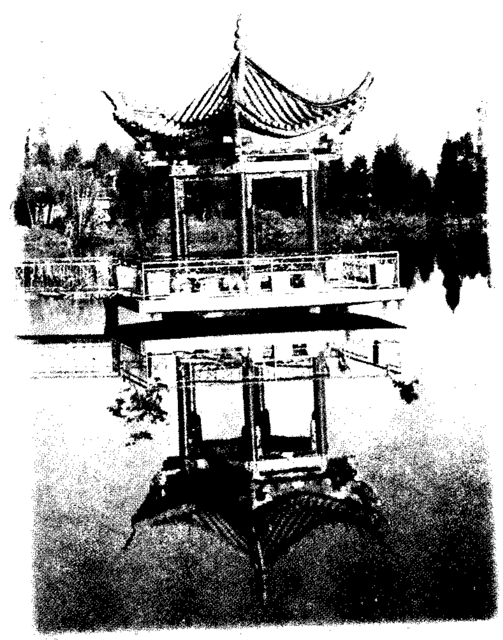
Motion Geometry

In this chapter, you will learn how to

- reflect a simple plane figure in horizontal or vertical lines;
- rotate a simple plane figure;
- translate a simple plane figure;
- enlarge a simple plane figure.

Preliminary Problem

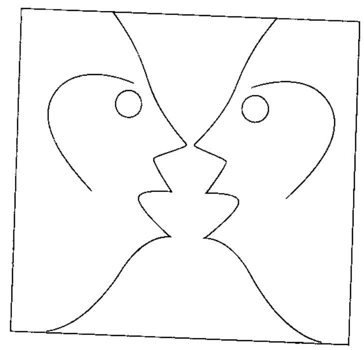
The picture provides an example of reflection in nature. The beautiful pavilion is reflected in the placid surface of the lake. The reflected pavilion is called the mirror image of the actual pavilion. The water surface acts as a mirror line or **line of reflection** or axis of symmetry. If you flip the original pavilion about the line of reflection, would the mirror image and the original match? Can you associate reflection with the flip movements?



Look around you and see whether you can observe any other reflections occurring in nature.

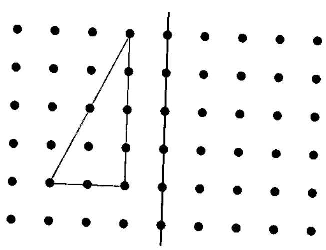
Reflection

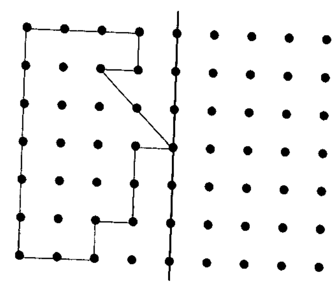
In Chapter 4, we studied line symmetry which is one of the most important properties of reflection. The line of reflection acts as a mirror. Look at the following picture:

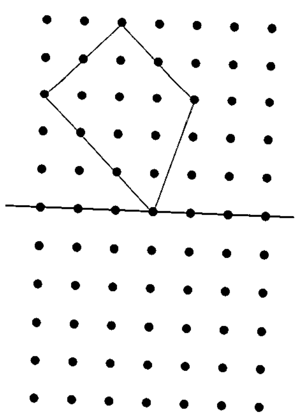


Identify the line of reflection. If you flip over the part of the picture on the left along the line of reflection, will the shape on the right overlap that on the left?

We can actually associate reflection as a flip movement about the line of reflection or axis of symmetry. Flip each of the following diagrams about the line of reflection with a geoboard.







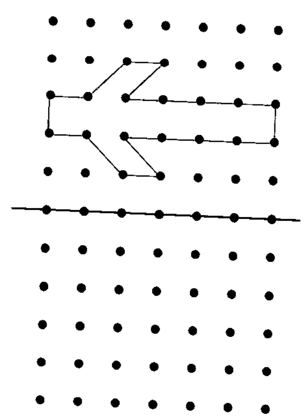


Fig. 11.1(a) on the next page shows the quadrilateral $ABCD$ undergoing a reflection in the y-axis to produce the image $A'B'C'D'$. Fig. 11.1(b) shows the figure $GHIJKL$ undergoing a reflection in the x-axis to produce the image $G'H'I'J'K'L'$.

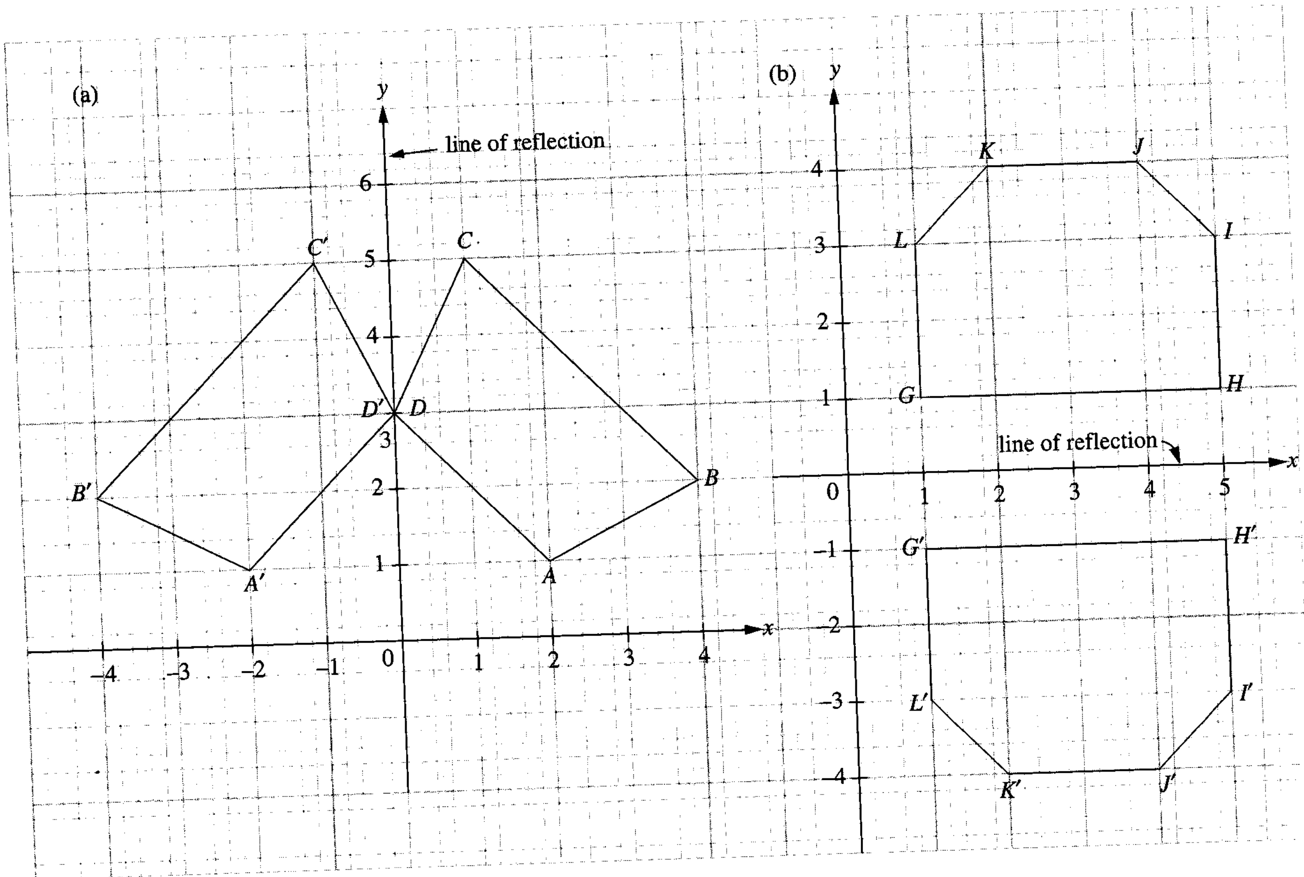
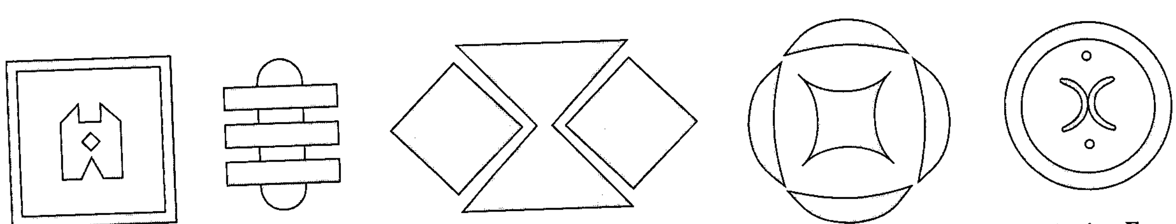


Fig. 11.1

Notice that under reflection, there is no change of shape or size of the figure and those points on the mirror line undergo no change in reflection; point D and its image in Fig. 11.1(a) are the same. Point D is called an **invariant point**, a point that does not undergo any change in a transformation. We say that $ABCD$ has been **mapped** onto $A'B'C'D'$ under reflection in the y-axis and $GHIJKL$ has been mapped onto $G'H'I'J'K'L'$ under reflection in the x-axis.

In-Class Activity

The following are some designs sent in for a company design logo competition. Can you draw a line or lines of reflection through each of them so that one part is the mirror image of the other?



Design A **Design B** **Design C** **Design D** **Design E**

Look around you and collect two company logos or company brand names. Bring these to the class and show your friends the lines of reflection you can identify in each of them.

Construction of the Mirror Image under Reflection

Fig. 11.2 shows a triangle ABC which is to be reflected in the line of reflection m. The construction steps are as follows:

(1) From A, draw a perpendicular line to N on m and produce it beyond N.

(2) With N as centre and radius AN, mark off a point A' on AN produced so that $AN = NA'$. A' is now the image of A under reflection in the line m.

(3) Repeat the above procedure for points B and C to produce B' and C'.

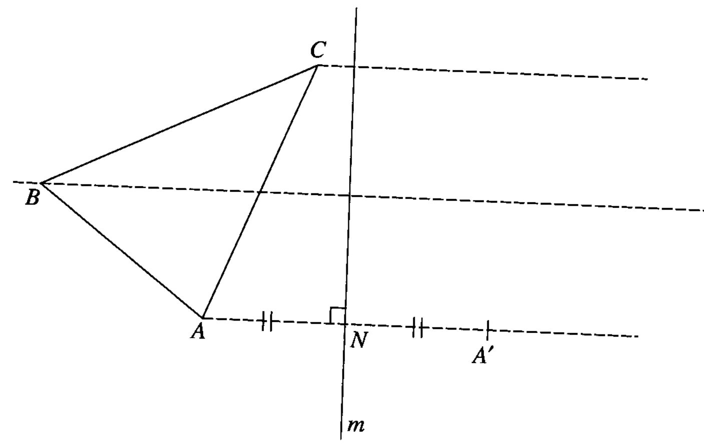


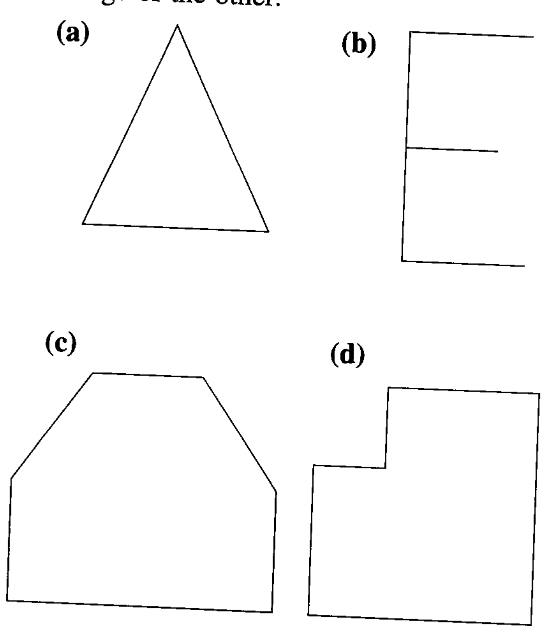
Fig. 11.2

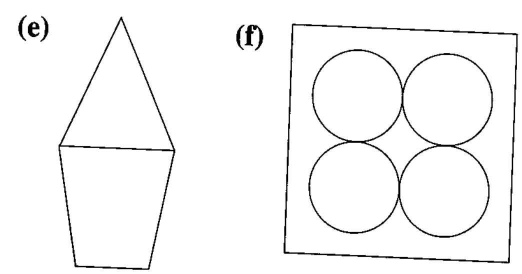
(4) Join $A'B'$, $B'C'$ and $A'C'$ to obtain the mirror image of ABC under reflection in the line m.

Note: Reflection in a real mirror only works from one side. If we stand behind a mirror, no image is produced. In mathematics, we think of the line of reflection as a double-sided mirror that reflects both ways.

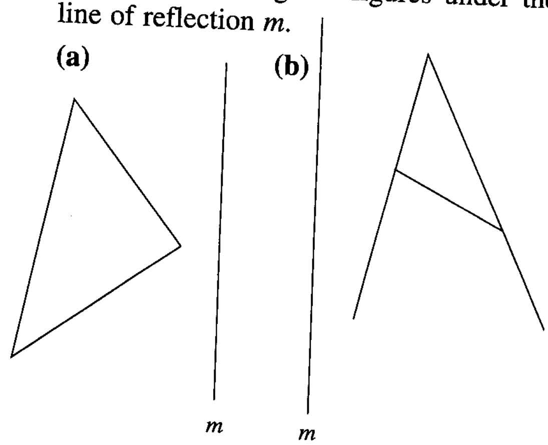
Exercise 11a

1. Each of the following diagrams has at least one line of reflection. Copy the diagrams and draw a line or lines of reflection through each of them so that one part is the mirror image of the other.

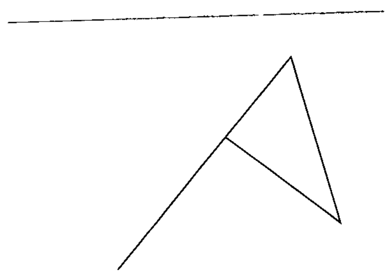




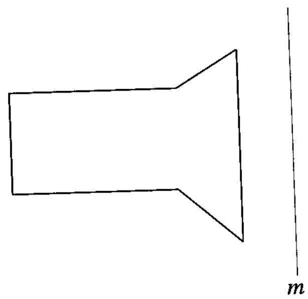
2. Copy each of the following and construct the image of the given figures under the line of reflection m.



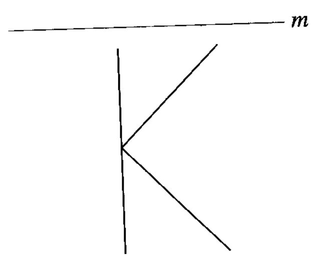
(c)



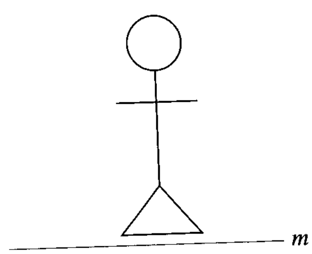
(d)



(e)



(f)



3. Write down the coordinates of the images of each of the following points under a reflection in the x-axis.

(a) (3, 4) **(b)** (2, –3) **(c)** (–2, 5)
(d) (–3, –6) **(e)** (7, 1) **(f)** (8, –4)
(g) (–8, –2) **(h)** (p, q)

4. Write down the coordinates of the images of each of the following points under a reflection in the y-axis.

(a) (5, 3) **(b)** (11, –1) **(c)** (–4, 9)
(d) (–1, –12) **(e)** (1, 8) **(f)** (–4, 5)
(g) (6, –7) **(h)** (p, q)

5. State the coordinates of the reflection of the point (3, 4) in each of the following lines:

(a) $x = 2$, **(b)** $x = 3$, **(c)** $x = 7$,
(d) $x = -4$.

6. State the coordinates of the reflection of the point (2, –4) in each of the following lines:

(a) $y = 0$, **(b)** $y = 3$, **(c)** $y = -4$,
(d) $y = 5.5$.

7. Reflect the point $A(3, -4)$ in the x-axis and then in the line $y = 3$. What are the coordinates of the final image?

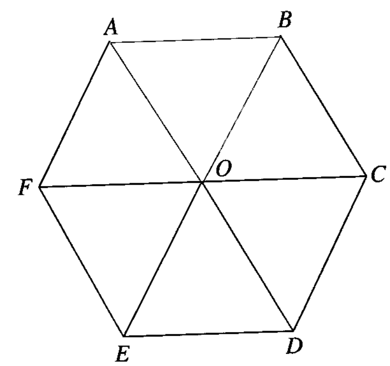
8. Reflect the point $A(3, -4)$ in the line $y = 3$ and then in the x-axis. What are the coordinates of the final image? Is your answer the same as that obtained in question 7?

9. Reflect the point $B(5, 6)$ in the line $x = 2$ and then in the line $x = -3$. What are the coordinates of the final image?

10. Reflect the point $B(5, 6)$ in the line $x = -3$ and then in the line $x = 2$. What are the coordinates of the final image? Is your answer the same as that obtained in question 9? What conclusions can you draw from questions 7 to 10?

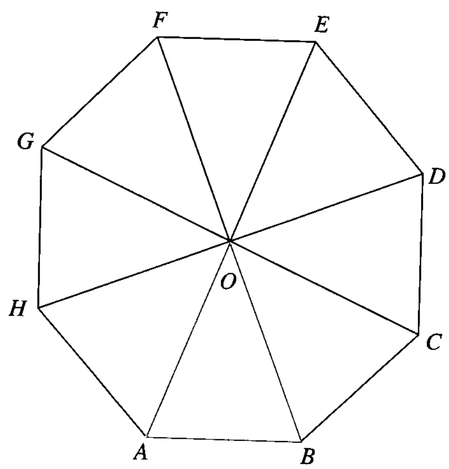
11. *ABCDEF* is a regular hexagon whose centre is O. Name the triangle onto which $\triangle AOB$ will be reflected in the line

(a) OB,
(b) CF,
(c) OA.



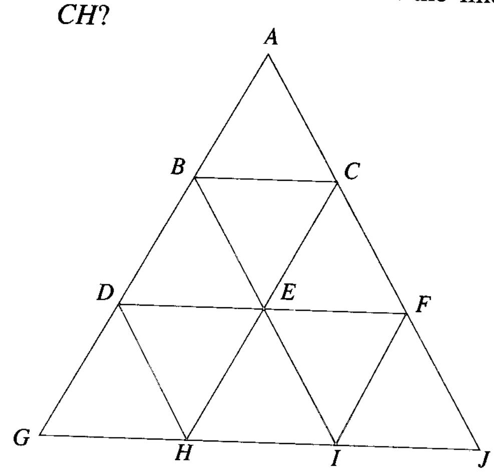
12. The following figure, *ABCDEFGH*, is a regular octagon whose centre is O. Name the triangle onto which $\triangle AOB$ will be reflected in the line

(a) OB,
(b) OC,
(c) OD,
(d) AE.



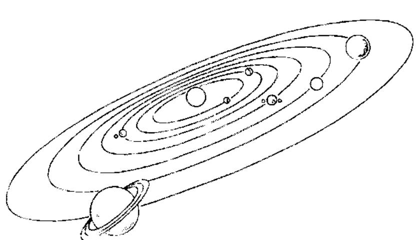
13. The figure on the right shows 9 small equilateral triangles stacked together to form a large equilateral triangle AGJ. What is the image of

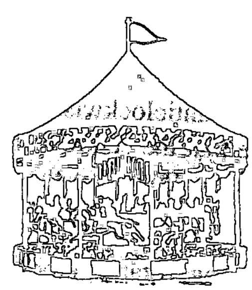
(a) $\triangle ABC$ under a reflection in the line BC,
(b) $\triangle DEH$ under a reflection in the line BI,
(c) rhombus $CEIF$ under a reflection in the line BI,
(d) trapezium $CHIF$ under a reflection in the line CH,
(e) $\triangle DEH$ under a reflection in the line CH?



Rotation

Many objects and apparatus in this world depend on rotation for their existence or their operation. The hands of the clock, the merry-go-round, the opening and closing of doors, the spinning action of a top and the turning of the wheels of a car are examples of rotation in action. Rotation is also indicated by such natural phenomenon as the movement of cyclones, whirlpools and planets. Can you name some other examples of rotation in action around you?



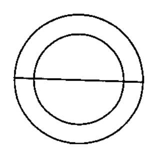


Look at the pictures above. Can you identify the centre of rotational symmetry or the centre of rotation for each picture?

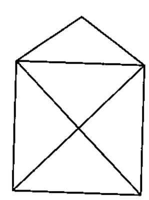
Just For Fun

Draw each figure without lifting your pen from the paper or retracing any line.

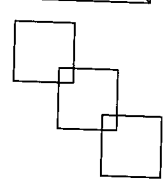
1.



2.

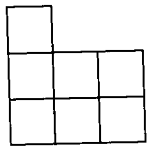


3.



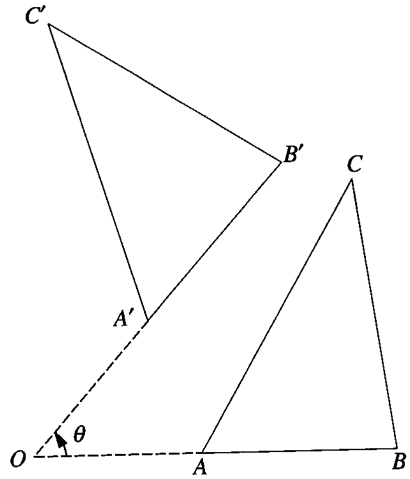
Just for fun

Arrange 20 identical toothpicks as shown.

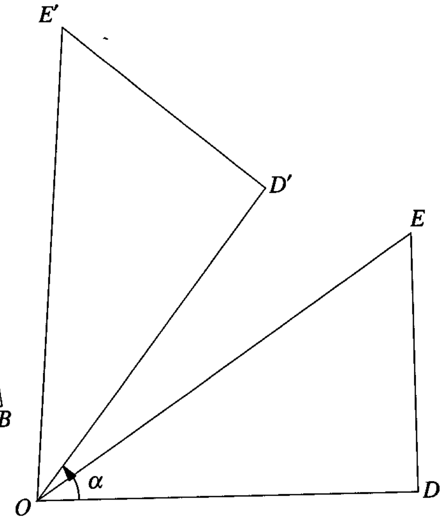


You are allowed to shift the position of any 3 so as to form a figure with 5 squares of the same size.

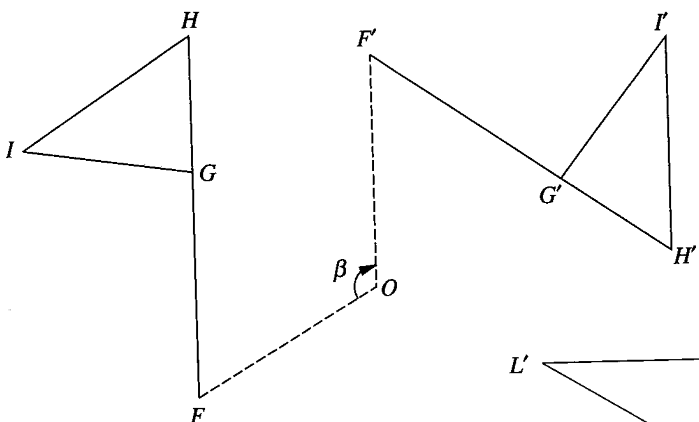
The following diagrams show the rotations of several figures respectively about the point O. This point is called the centre of rotation.



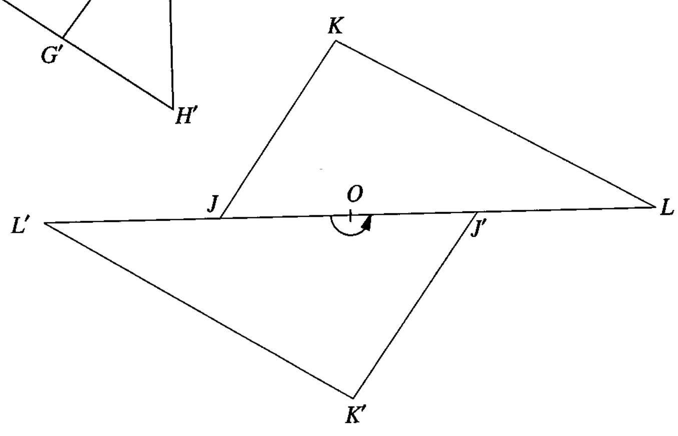
(a)



(b)



(c)



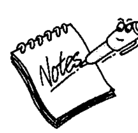
(d)

In Fig. (a), $\triangle ABC$ is rotated through an angle θ anticlockwise about O to obtain the image $\triangle A'B'C'$.

Fig. (b) shows $\triangle ODE$ being rotated anticlockwise through an angle α about O to obtain $\triangle OD'E'$. Notice that the centre of rotation O lies on the triangle and it undergoes no change. Point O is therefore an invariant point.

Fig. (c) shows the flag $FGHI$ being rotated clockwise through an angle β about O to obtain the image $F'G'H'I'$.

Fig. (d) shows $\triangle JKL$ being rotated through 180° about the point O which lies on the side JL to give the image $\triangle J'K'L'$.

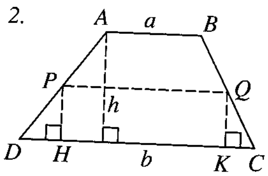


In the case of a rotation of 180°, which is sometimes called a half-turn, the direction can either be clockwise or anti-clockwise.

Investigate

1. The number 6 009 has rotational symmetry. Find another number that has this same property.

2.



ABCD is a trapezium in which AB = a units, DC = b units and the height between the parallel sides AB and DC is h units. P and Q are mid-points of AD and BC respectively. By considering suitable rotations, show that the area of the trapezium is given by the formula, $\frac{1}{2}h(a + b)$.

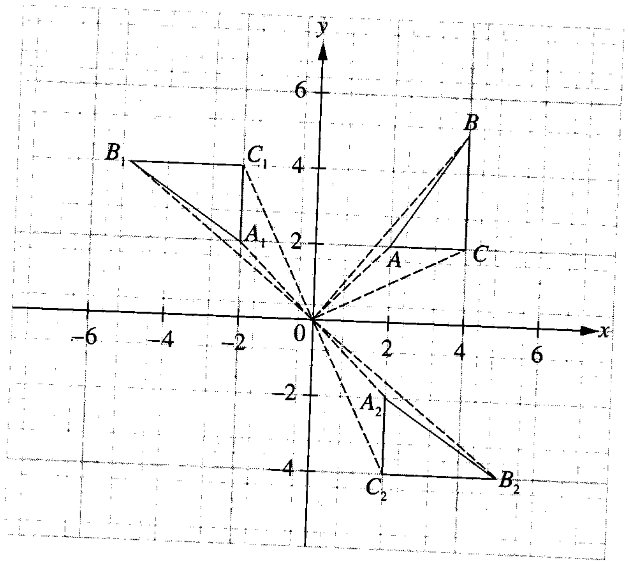


Fig. 11.3

Fig. 11.3 shows the triangle ABC being rotated through 90° anticlockwise about the origin to triangle $A_1B_1C_1$. Triangle $A_2B_2C_2$ is the result of the rotation of triangle ABC through 90° clockwise about the origin. The origin is the centre of rotation or point of rotation. From the graphs, we can find the following coordinates: $A(2, 2)$, $B(4, 5)$, $C(4, 2)$. Can you write the coordinates of A_1, B_1, C_1 and A_2, B_2, C_2?

Notice that in a rotation, every point of the original figure is rotated through the same angle about the centre of rotation. In rotation, there is no change in the shape and size of the figure. If the centre of rotation lies on the figure, then it is the only invariant point.

Construction Steps to Rotate a Figure about a Point

Fig. 11.4 shows $\triangle ABC$ being rotated through 90° anticlockwise about the point O (centre of rotation). The construction steps are as follows:

(1) Join O (the centre of rotation) to A.

(2) From OA, measure 90° in an anticlockwise direction and draw the line Ox.

(3) Using a pair of compasses and with O as centre and radius OA, mark off a point A' on Ox.

(4) Repeat steps 1, 2 and 3 to obtain points B' and C'.

(5) Join $A'B'$, $B'C'$ and $A'C'$ to get the image figure $\triangle A'B'C'$.

Notes

After obtaining points A' and B', we can also use the fact that rotation preserves shape and size to obtain the point C'. Do you think you can do this?

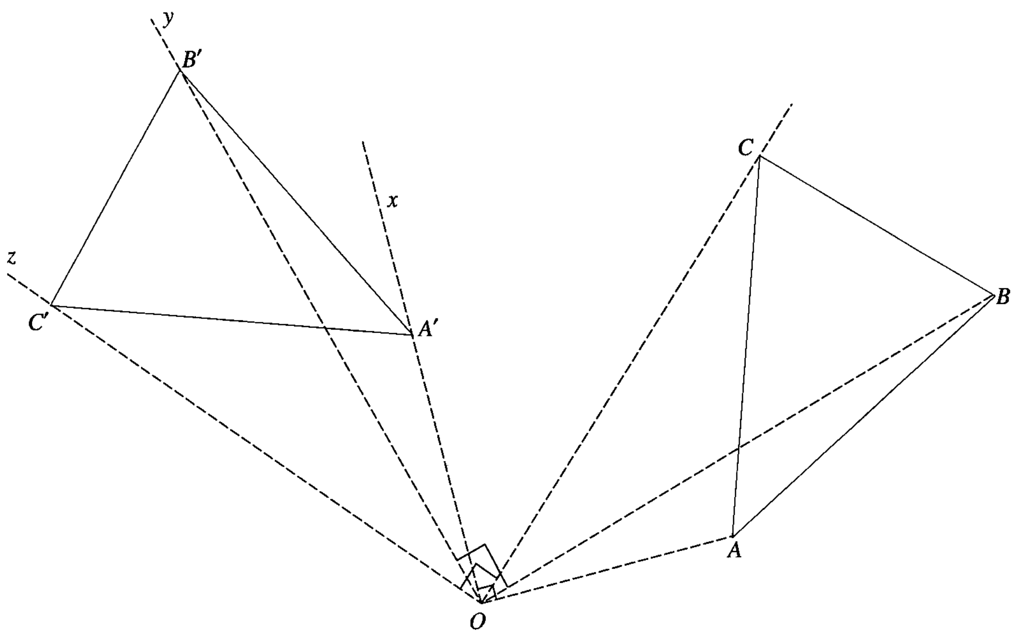
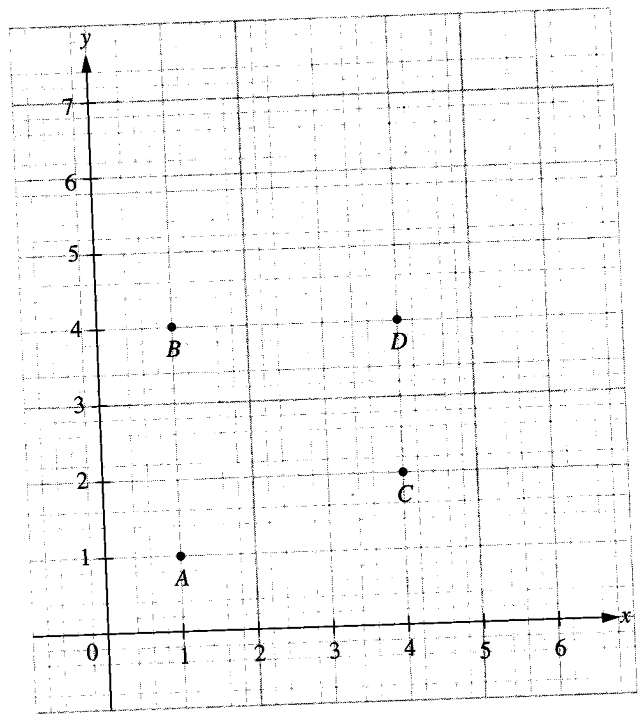


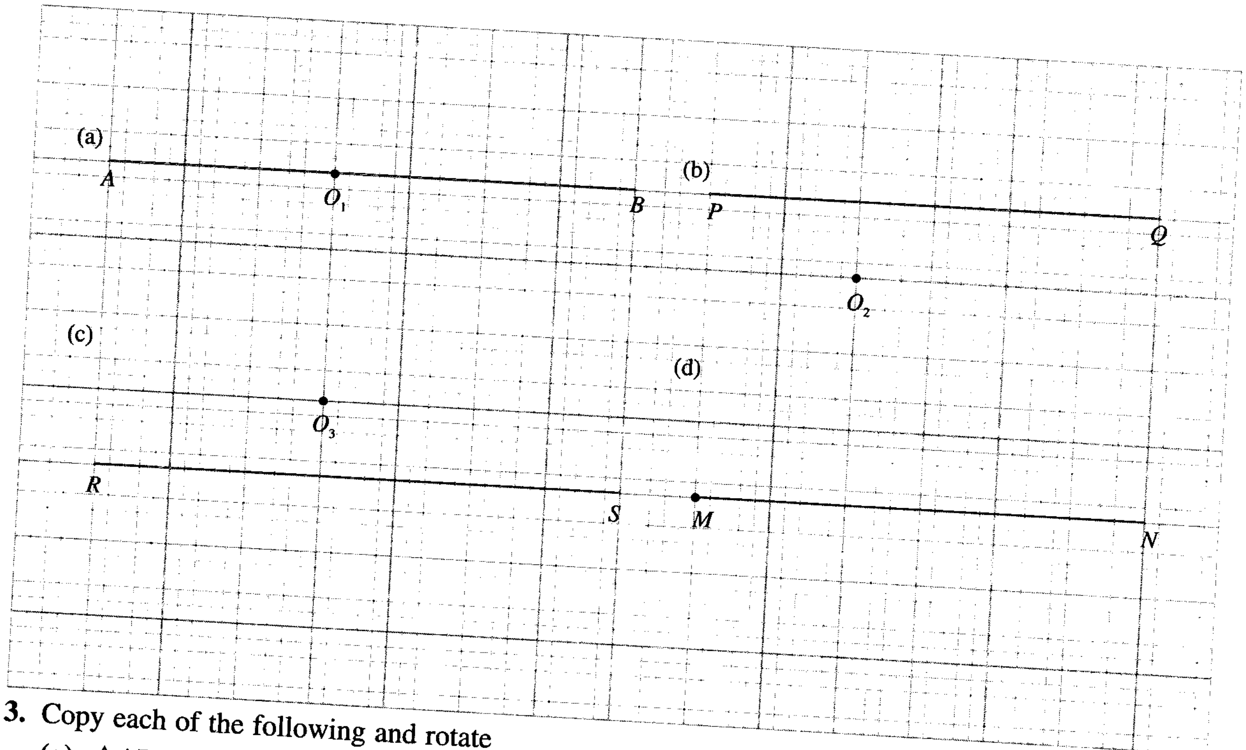
Fig. 11.4

Exercise 11b

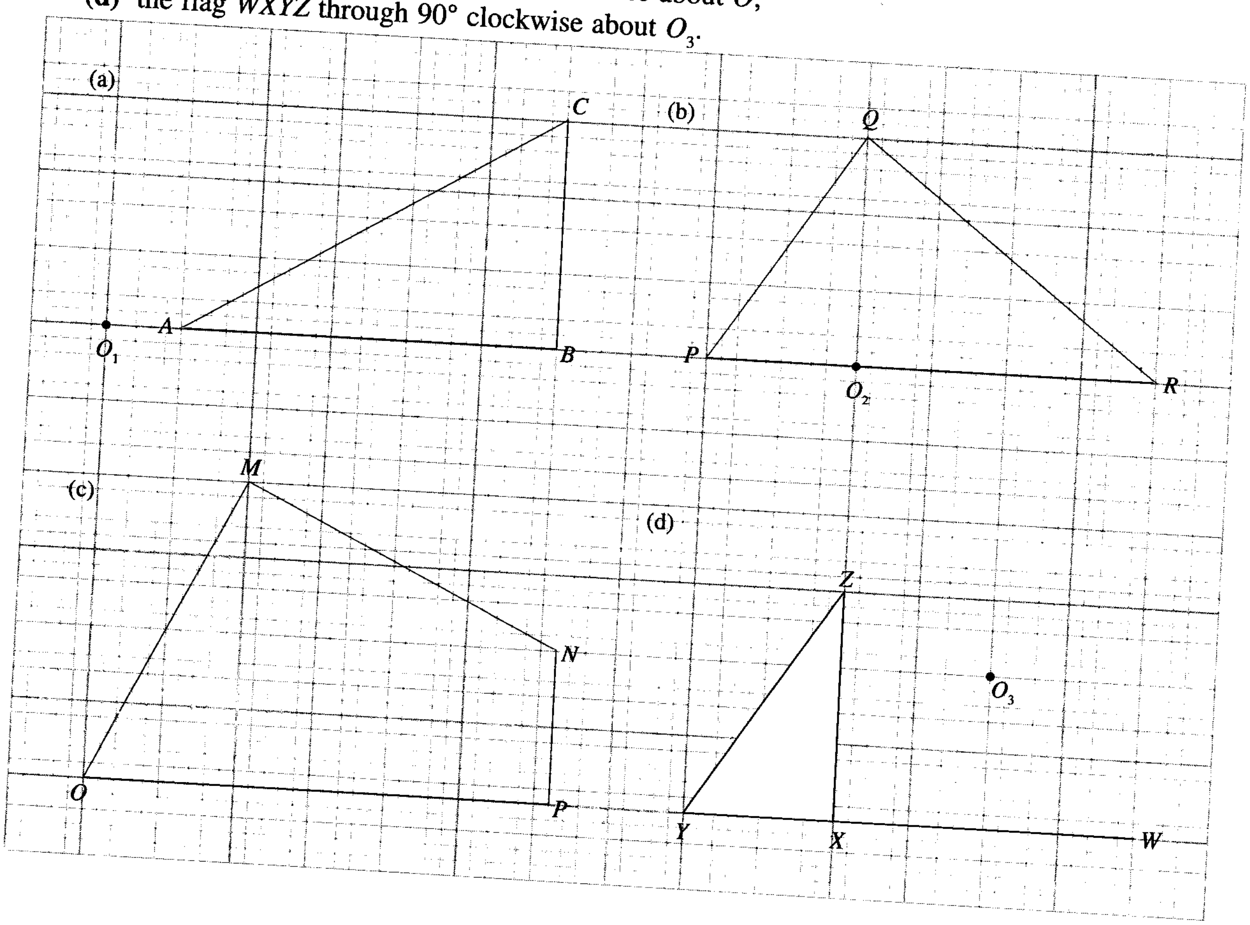
1. The figure on the right shows four points: $A(1, 1)$, $B(1, 4)$, $C(4, 2)$ and $D(4, 4)$. Find the coordinates of the image of the point
   - **(a)** B under a 90° clockwise rotation about point A,
   - **(b)** C under a 90° anticlockwise rotation about the point D,
   - **(c)** B under a 90° clockwise rotation about the point D,
   - **(d)** D under a 180° rotation about the point C,
   - **(e)** A under a 180° rotation about the point B,
   - **(f)** A under a 270° anticlockwise rotation about the point C,
   - **(g)** C under a 270° clockwise rotation about the point B,
   - **(h)** D under a 270° anticlockwise rotation about the point B.



2. Copy each of the following and rotate
   - **(a)** AB through 180° about O_1,
   - **(b)** PQ through 90° anticlockwise about O_2,
   - **(c)** RS through 90° clockwise about O_3,
   - **(d)** MN through 90° anticlockwise about M.



3. Copy each of the following and rotate
   (a) $\triangle ABC$ through 180° about O_1,
   (b) $\triangle PQR$ through 90° anticlockwise about O_2,
   (c) quadrilateral $OMNP$ through 90° clockwise about O,
   (d) the flag $WXYZ$ through 90° clockwise about O_3.



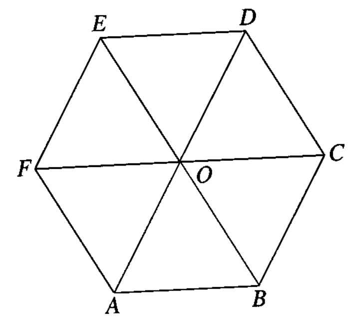
4. Using graph paper, construct the images of the following points under an anticlockwise rotation of 90° about the origin.

(a) (1, 1) **(b)** (5, –3) **(c)** (–2, 6) **(d)** (–4, –6) **(e)** (p, q)

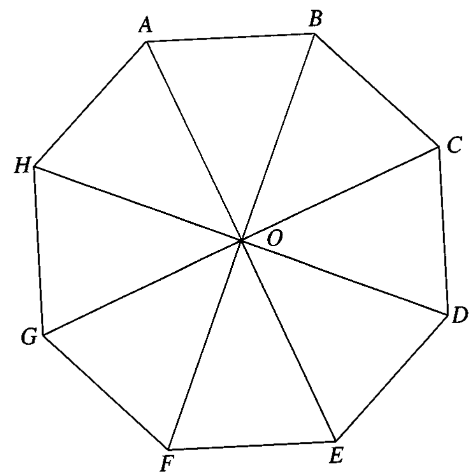
5. Draw on graph paper the triangle ABC with vertices $A(2, 2)$, $B(4, 2)$ and $C(2, 4)$. Rotate $\triangle ABC$ through 90° anticlockwise about the point (0, 1). What are the images of A, B and C?

6. Find the coordinates of the image of the point (3, 5) under a rotation of 180° about the point (3, 0).

7. Find the coordinates of the image of the point (6, 6) under
   (a) a clockwise rotation of 90° about (2, 4),
   (b) an anticlockwise rotation of 90° about (2, 4).



∗8. $ABCDEF$ is a regular hexagon whose centre is O. What is the image of $\triangle AOB$ when it is rotated through
   (a) 60° anticlockwise about O,
   (b) 240° anticlockwise about O,
   (c) 60° clockwise about O,
   (d) 60° anticlockwise about F,
   (e) 60° clockwise about C,
   (f) 180° about O?



∗9. $ABCDEFGH$ is a regular octagon whose centre is O. What is the image of $\triangle AOB$ when it is rotated through
   (a) 180° about O,
   (b) 90° anticlockwise about O,
   (c) 90° clockwise about O,
   (d) 315° anticlockwise about O?

Translation

In Fig. 11.5, the ink bottle is moved, without turning, along a straight line from position A to position B. We say that the ink bottle has been translated from A to B.

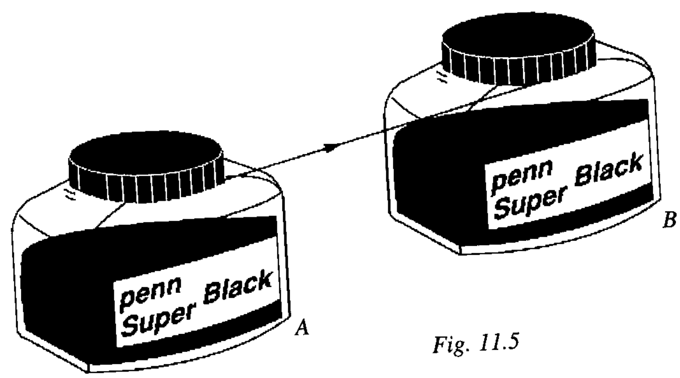
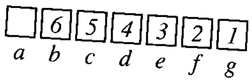


Fig. 11.5



Six counters which occupy six out of seven empty spaces are arranged as shown in the diagram above. Find out the minimum number of moves you have to make in order to reverse the order of the counters (i.e., 123456) given that:

1. *a counter can skip one space and land on the space after that,*
2. *a counter can switch its place with the one next to it.*

For example, counter 5 or counter 6 can be moved to space a.

In Fig. 11.6, $\triangle ABC$ is moved from its original position to $\triangle A'B'C'$, i.e., $\triangle ABC$ is translated to $\triangle A'B'C'$.

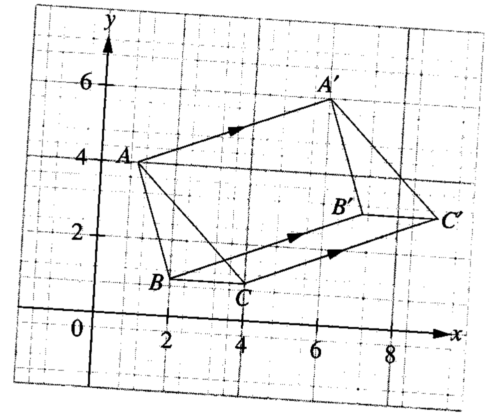
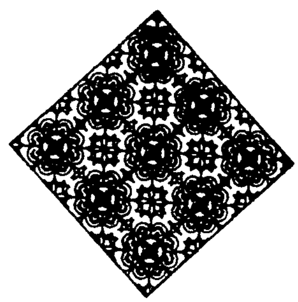
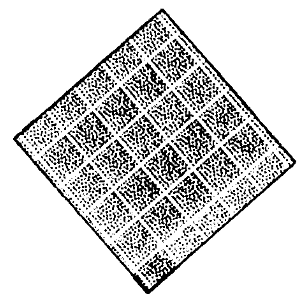


Fig. 11.6

In nature, translation is exhibited by the sliding motion of flowing rivers and moving glaciers. Man has invented many mechanical tools to speed up the movement of objects from one place to another. Translation can be illustrated by the moving of escalators, lifts, conveyor belts and MRT trains. Other examples of translation include curtain prints, batik prints, wallpaper, carpet designs and floor tiles.





Look for other examples of translation in nature. Bring to class pictures or photographs that show or make use of translation.

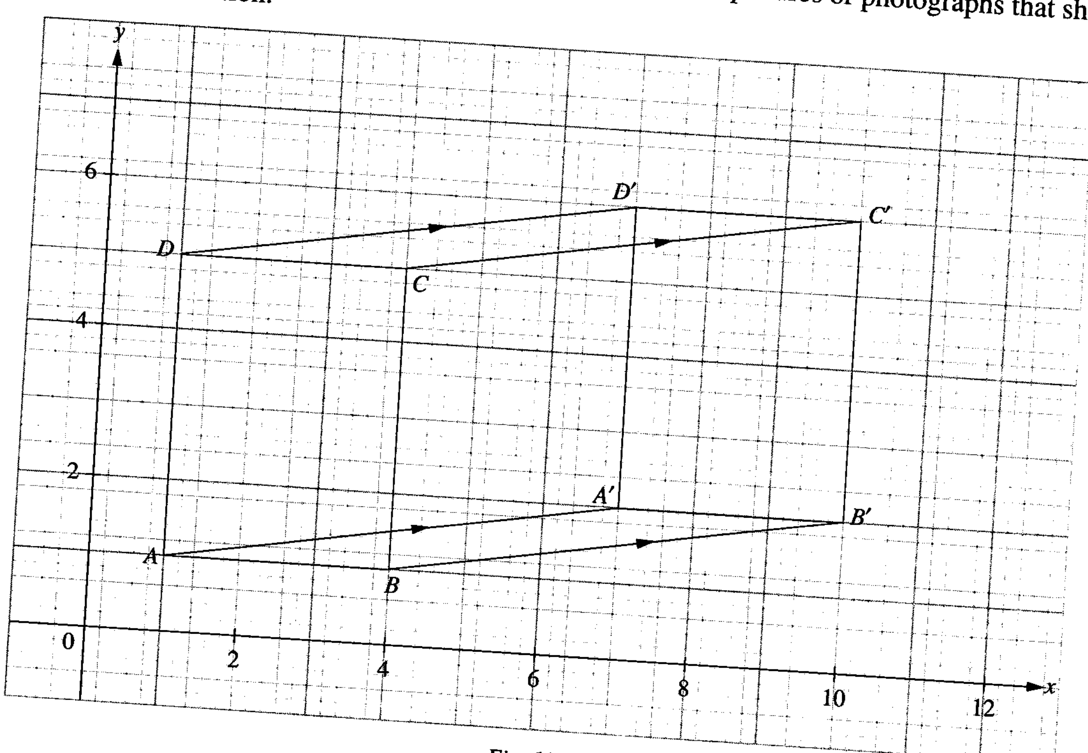


Fig. 11.7

Fig. 11.7 shows an example of translation in the x-y plane where rectangle $ABCD$ is mapped onto rectangle $A'B'C'D'$.

From Fig. 11.7, we notice that

(1) a translation is a transformation of the plane in which all the points are moved the same distance in the same direction, i.e., $AA' \parallel BB' \parallel CC' \parallel DD'$ and $AA' = BB' = CC' = DD'$;

(2) a translation is a rigid transformation in which the lengths and angles remain unchanged or invariant, i.e., $AB = A'B'$, $BC = B'C'$, $CD = C'D'$ and $AD = A'D'$ and $\hat{A} = \hat{A}'$, $\hat{B} = \hat{B}'$, $\hat{C} = \hat{C}'$ and $\hat{D} = \hat{D}'$;

(3) under a translation, there is no change in the shape and size of the figure;

(4) rectangle $ABCD$ has been moved 6 units in the positive x-direction and 1 unit in the positive y-direction.

Just For Fun

(a) 1-kg bags of sugar are stored in 10 different containers labelled A, B, C, D, E, F, G, H, I and J. It is found that one of the containers contains only 950-g bags of sugar. With one weighing, can you detect which container it is?

(b) Suppose all the bags of sugar are stored in 50 different containers with only one container having 950-g bags of sugar. How many weighings do you need in order to detect the correct container?

Example 1

Using a scale of 1 cm to represent one unit on both axes, draw a quadrilateral with vertices A(1, 1), B(5, 2), C(4, 4), and D(2, 4). Find the image of ABCD under a translation of 5 units in the positive x-direction and 2 units in the negative y-direction.

Solution

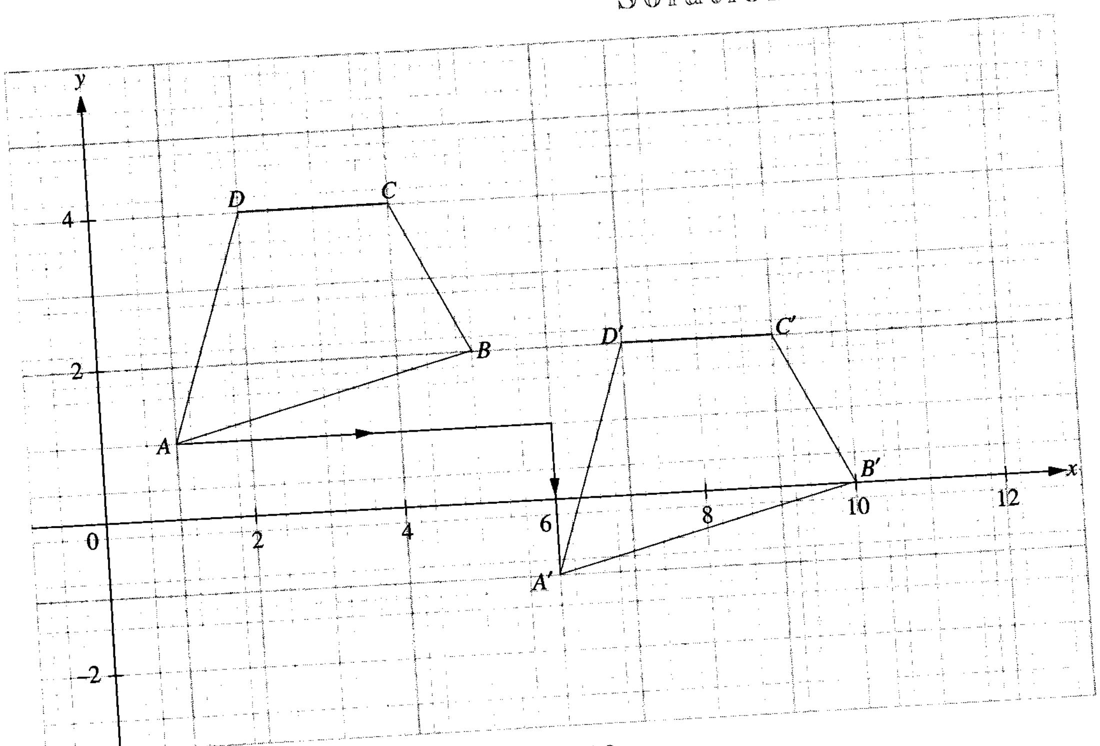


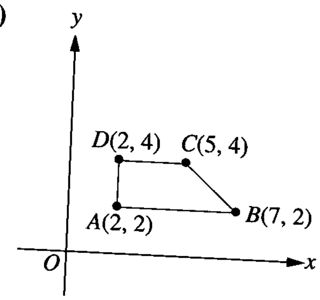
Fig. 11.8

Fig. 11.8 shows $ABCD$ and its image $A'B'C'D'$ when $ABCD$ is translated 5 units in the positive x-direction and 2 units in the negative y-direction.

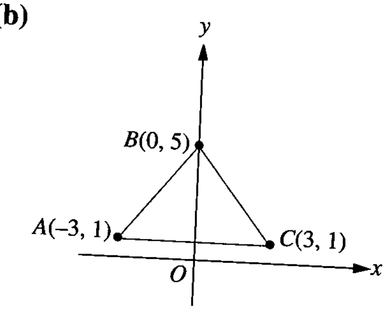
Exercise 11c

1. Copy the following figures on graph paper and draw the images under a translation represented by 2 units in the positive x-direction and 3 units in the positive y-direction.

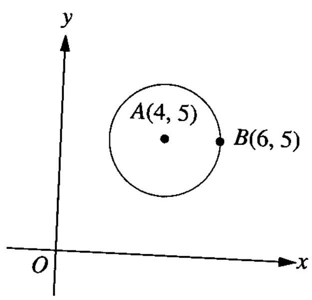
(a)



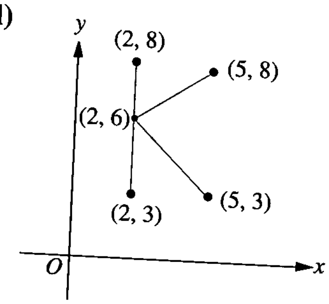
(b)



(c)



(d)



2. Using a scale of 1 cm to represent 1 unit, plot the triangle ABC whose vertices are $A(1, 3)$, $B(3, 4)$ and $C(4, 2)$. Find the new coordinates of $\triangle ABC$ under a translation of 4 units in the positive x-direction and 5 units in the negative y-direction. Plot the image figure of $\triangle ABC$ on the same graph.

3. A quadrilateral has vertices (0, 0), (2, 1), (3, 2) and (3, 5). Give the coordinates of its vertices under the translation of 5 units in the positive x-direction and 7 units in the negative y-direction.

4. $\triangle PQR$ is mapped onto $\triangle P'Q'R'$ by a translation of 1 unit in the negative x-direction and 3 units in the positive y-direction. If $\triangle PQR$ has vertices $P(1, 1)$, $Q(1, 3)$ and $R(3, 6)$, find the coordinates of the vertices of $\triangle P'Q'R'$.

5. A is the point (6, –3) and B is the point (5, 2). If a translation T maps A onto the origin, describe this translation. Find the image of B under this translation. Another translation maps B onto the origin. Find this translation and the image of A under this new translation.

6. Under a translation T, the image of the point (2, 6) is (1, 2). What is the image of (7, –2) under T?

7. Under a translation T, the point (3, 7) is mapped onto (7, 1). Given that T maps (7, 1) onto (h, k), find h and k.

8. Under a translation T, the point $P(4, 5)$ maps onto the point $Q(2, 2)$. Under T, the point Q maps onto the point R. State the coordinates of R.

9. Two translations P and Q are such that they map the origin onto the points (3, –1) and (–2, 4) respectively. Give the images of the points $A(5, 3)$ under P and under Q.

10. The translation T maps the point (2, –1) onto the point (7, 3). Find the coordinates of the image of the triangle ABC under T where A is (8, 4), B is (2, 3) and C is (–2, 5).

11. Find the coordinates of the image of the quadrilateral $A(1, 1)$, $B(2, 6)$, $C(6, 4)$ and $D(5, 2)$ under a translation of 4 units in the positive x-direction and 2 units in the negative y-direction. What would be image of the new quadrilateral if it undergoes another translation of 2 units in the positive x-direction and 6 units in the positive y-direction?

12. Find the coordinates of the image of the quadrilateral $A(1, 1)$, $B(2, 6)$, $C(6, 4)$ and $D(5, 2)$ under a translation of 2 units in the positive x-direction and 6 units in the positive y-direction. What would be the image of the new quadrilateral if it undergoes another translation of 4 units in the positive x-direction and 2 units in the negative y-direction?

Compare your results with question 11. Is translation commutative?

★13. Let A denote the translation of 3 units in the positive x-direction and 4 units in the positive y-direction and B denote the translation of 3 units in the negative x-direction and 3 units in the positive y-direction. Describe the translations C and D such that

(a) $AB = C$, **(b)** $AD = B$.

(The transformation AB represents a transformation B followed by a transformation A).

Enlargement

Investigate

Draw a triangle ABC such that AB = 6 cm, BC = 8 cm and AC = 4 cm. By using enlargements, construct a square PQRS inside the triangle such that PQ is on BC, R on AC and S on AB.

Fig. 11.9 shows two similar triangles ABC and $A'B'C'$. $\triangle A'B'C'$ is an enlargement of $\triangle ABC$. We say that $\triangle ABC$ is transformed onto $\triangle A'B'C'$ by an enlargement with centre O and scale factor $\frac{OA'}{OA}$.

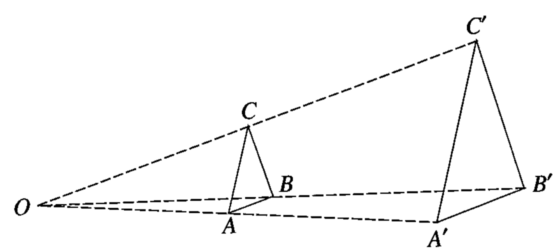


Fig. 11.9

There are many examples of enlargement in our surroundings. An example is the enlargement of a photograph. Also, when your teacher uses an overhead projector to project a picture on a transparency, he is enlarging the picture. The slide projector is another one of Man's mechanical inventions used to enlarge an object. Can you identify other examples of enlargement in our environment?

Do you consider inflating a bicycle tyre tube an enlargement? What about inflating a balloon or a swimming float for children? Do you consider growing up a form of enlargement?

Construction Steps to Enlarge a Figure

Fig. 11.10 shows a triangle ABC being enlarged 3 times (scale factor 3) to triangle $A_1B_1C_1$ with the origin as the centre of enlargement or point of enlargement.

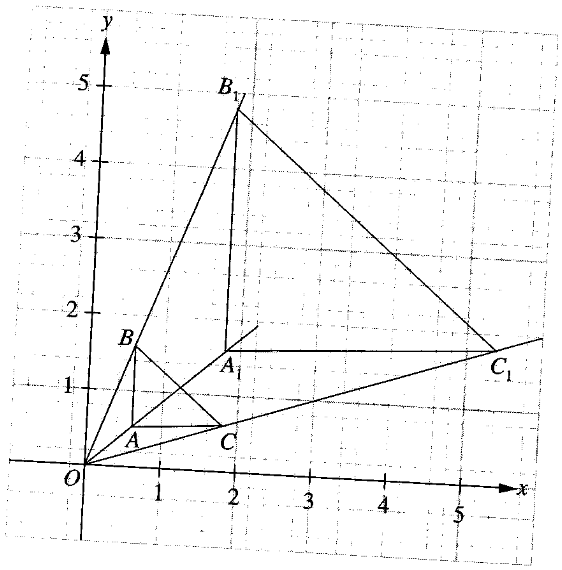


Fig. 11.10

The following steps are taken for the construction:

(1) Join the point of enlargement O to A and produce OA.

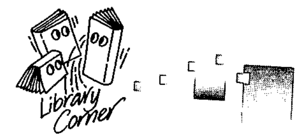
(2) From O, mark off a distance equal to 3 times the length of OA on OA produced to get the point A_1.

(3) Repeat the above procedure for points B and C to get B_1 and C_1.

(4) Join A_1B_1, B_1C_1 and A_1C_1 to get the enlarged figure $A_1B_1C_1$.

Note: In reflection, rotation and translation, there is no change in the shape and size of the image figure. In enlargement, there is no change in the shape of the figure but the size of the image changes.

How many times is the image enlarged as compared to the original triangle in Fig. 11.10? (You may wish to compare this with what you learned in Chapter 2.) Have all the points in the triangle moved?

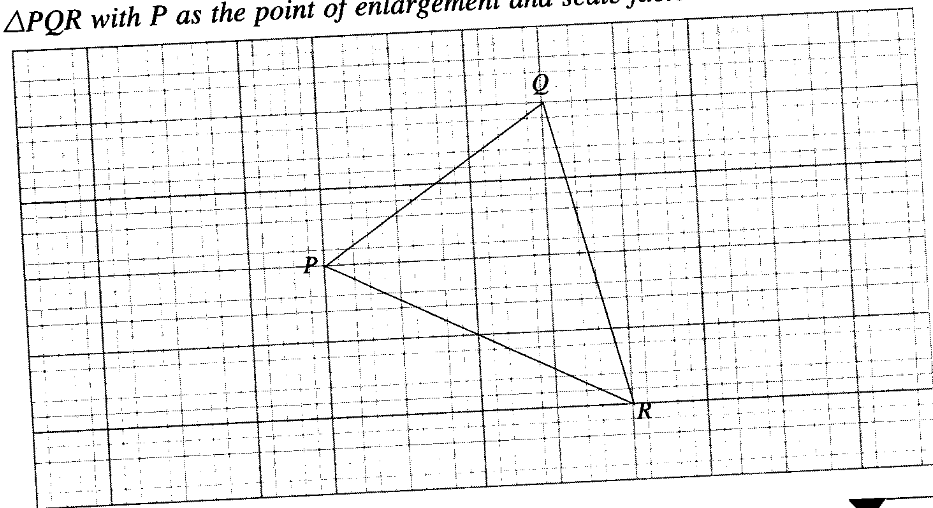


A pantograph is an instrument that enables one to draw enlarged or reduced figures. Pantographs are commercially available in the market. Can you make your own pantograph? Find out how a pantograph works.

The open tool, Geometer's Sketch Pad, is useful for exploring transformations in two dimensions like reflection, rotation and enlargement.

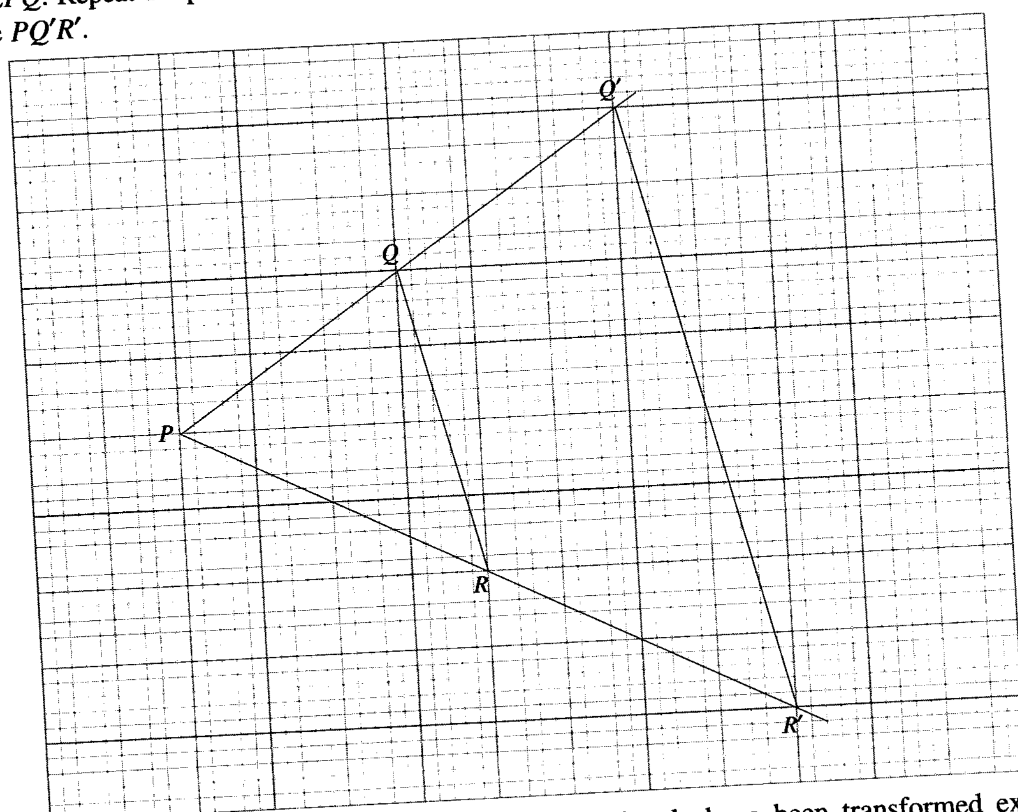
Example 2

Enlarge $\triangle PQR$ with P as the point of enlargement and scale factor 2.



Solution

Produce PQ. With P as centre, use your compasses to mark the point Q' on PQ produced such that $PQ' = 2PQ$. Repeat the procedure for the point R to get R'. The diagram below shows the enlarged triangle $PQ'R'$.



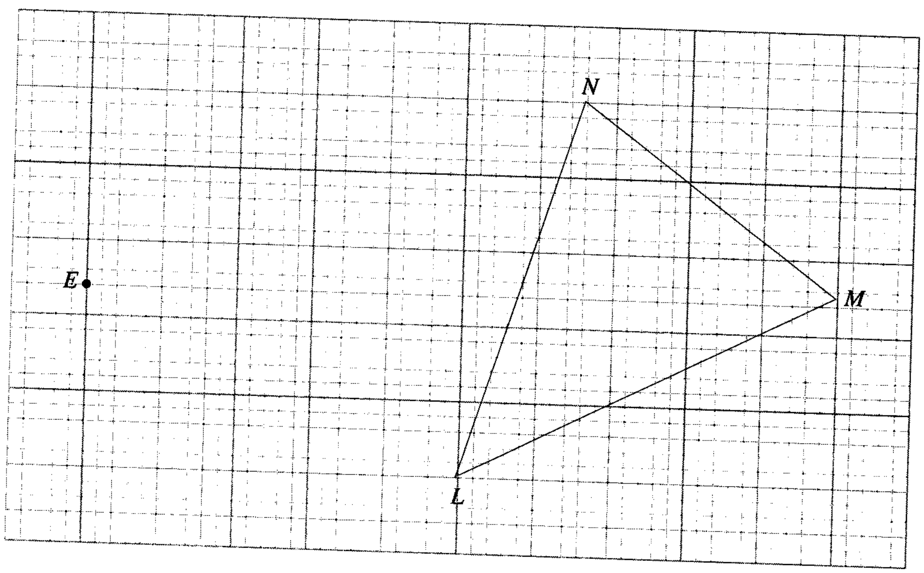
Notice from this example that all the points in the triangle have been transformed except the point P which is the only invariant point.

So far, we only associate enlargement with making the image figure larger than the original. Do you consider deflating a balloon an enlargement?

In mathematics, we consider enlargement a dilating movement. Do you know that when you enter a dark place, the pupils of your eyes will dilate (increase in size) to enable more light to enter so that you can see things more clearly? When you go from a dark place into a brightly lit place, do you know that the pupils of your eyes will tend to reduce in size? You may be surprised to know that reduction is considered a form of enlargement in mathematics.

Example 3

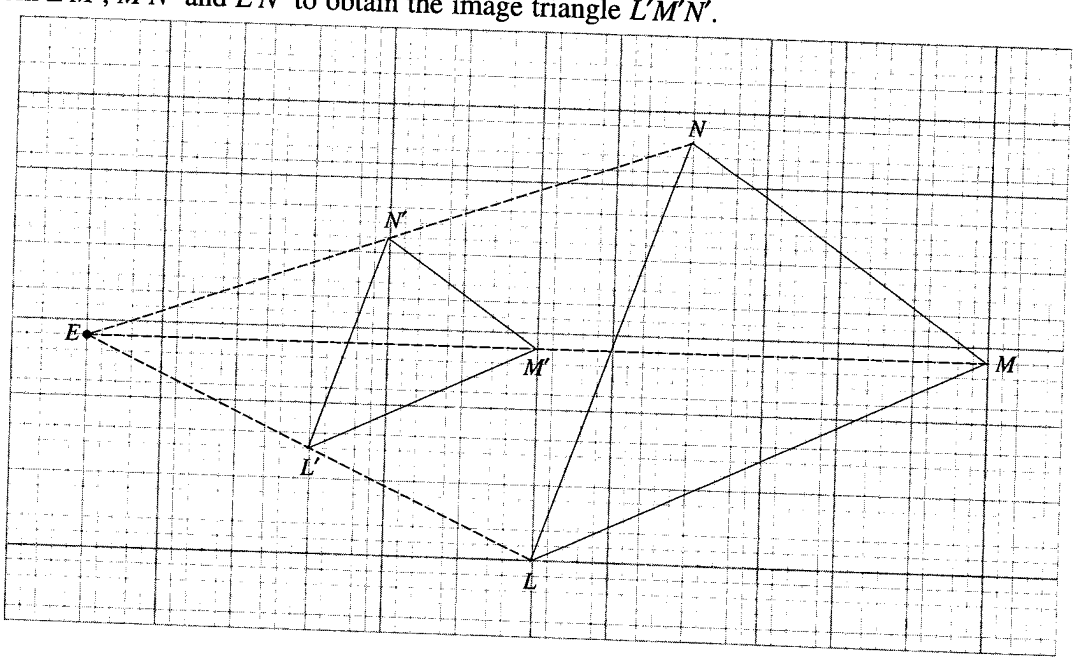
Enlarge $\triangle LMN$ with E as the point of enlargement and scale factor $\frac{1}{2}$.



Solution

The construction steps taken are as follows:

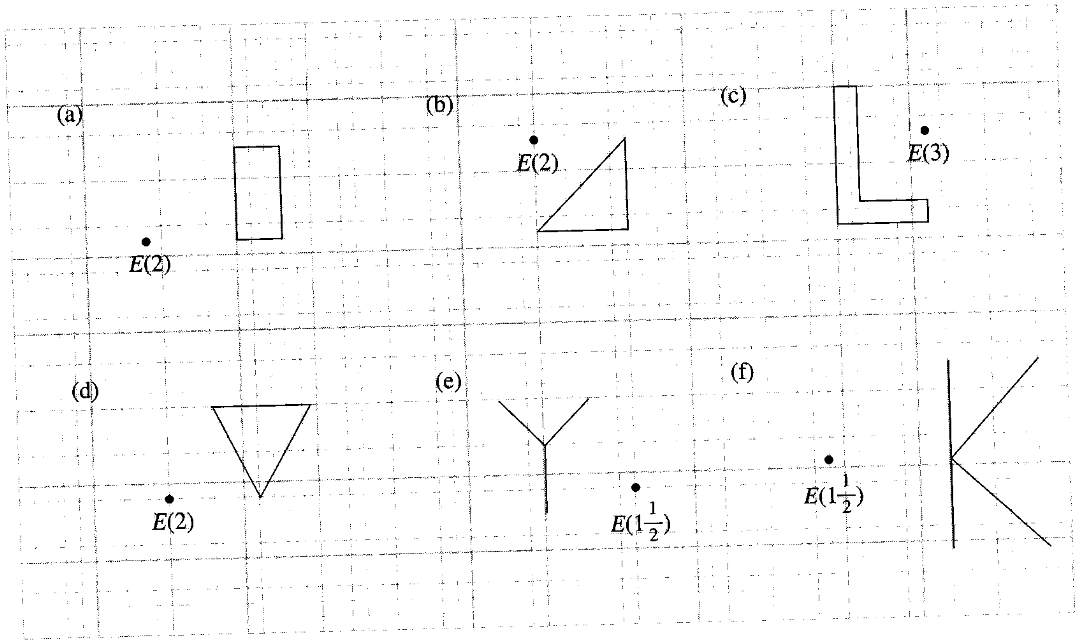
(1) Join E to L.
(2) With E as centre, use your compasses to mark off the point L' on EL such that $EL' = \frac{1}{2}EL$.
(3) Repeat the above procedure for points M and N to get points M' and N'.
(4) Join $L'M'$, $M'N'$ and $L'N'$ to obtain the image triangle $L'M'N'$.



Exercise 11d

1. Explore your surroundings to find examples of enlargement being put into use. Discuss your observations in class.

2. Enlarge the following figures with centre of enlargement E and scale factors each indicated in brackets.



3. Draw on graph paper the triangle ABC with vertices $A(2, 1)$, $B(2, 5)$ and $C(4, 2)$. Enlarge $\triangle ABC$ with the origin as the point of enlargement and scale factor 2.

4. Draw on graph paper the triangle PQR with vertices $P(2, 2)$, $Q(5, 3)$ and $R(3, 5)$. Enlarge $\triangle PQR$ with Q as the point of enlargement and scale factor 3.

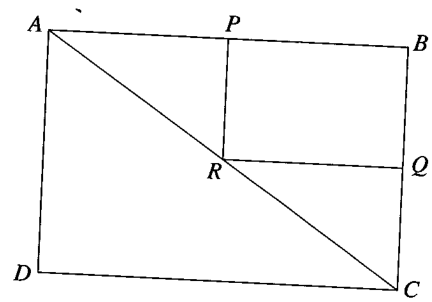
5. Draw on graph paper the quadrilateral $PQRS$ with vertices $P(2, 2)$, $Q(7, 2)$, $R(6, 6)$ and $S(4, 6)$. Enlarge $PQRS$ with $E(4, 4)$ as the centre of enlargement and scale factor $1\frac{1}{2}$.

6. The vertices of $\triangle ABC$ are $A(1, 1)$, $B(3, -1)$ and $C(0, 0)$. $\triangle ABC$ is enlarged to $\triangle PQR$ with $E(4, 4)$ as the centre of enlargement and scale factor $\frac{1}{2}$. Find the coordinates of P, Q and R.

7. The coordinates of $\triangle ABC$ are $A(2, 1)$, $B(7, 1)$ and $C(4, 4)$. $\triangle ABC$ is mapped onto $\triangle APQ$ by an enlargement of scale factor 2.
   (a) State the centre of enlargement.
   (b) Find the coordinates of P and Q.

8. The vertices of $\triangle PQR$ are $P(1, 2)$, $Q(2, 6)$ and $R(8, 1)$. $\triangle PQR$ is mapped onto $\triangle LMN$ by an enlargement centre $E(2, 4)$ and scale factor $\frac{1}{2}$. Find the coordinates of L, M and N by construction.

9. In the figure, $ABCD$ is a rectangle and P and Q are the mid-points of AB and BC respectively.



(a) $\triangle APR$ is mapped by an enlargement centre A and scale factor 2. Name the image figure.

(b) $ABCD$ is mapped by an enlargement centre B and scale factor $\frac{1}{2}$. Name the image figure.

(a) In a pile of 81 coins, one of them is a counterfeit and it weighs more than the others. What is the minimum number of weighings needed before you can isolate the counterfeit coin?

(b) If the same counterfeit coin is placed in a pile of 200 coins, what is the minimum number of weighings needed before you can isolate it?

Combining Transformations

Earlier in the chapter, we studied each transformation on its own. Now we shall examine the effects of combining some of these transformations.

Symbols are often used to represent transformations. We shall adopt the following notations:

If T represents a translation and R a rotation, then TR represents a rotation followed by a translation and RT represents a translation followed by a rotation. Similarly, if E represents an enlargement and M a reflection, then EM represents a reflection followed by an enlargement and ME represents an enlargement followed by a reflection. MM (normally written as M^2) represents a reflection followed by another reflection. Likewise E^2 (EE) represents an enlargement followed by another enlargement.

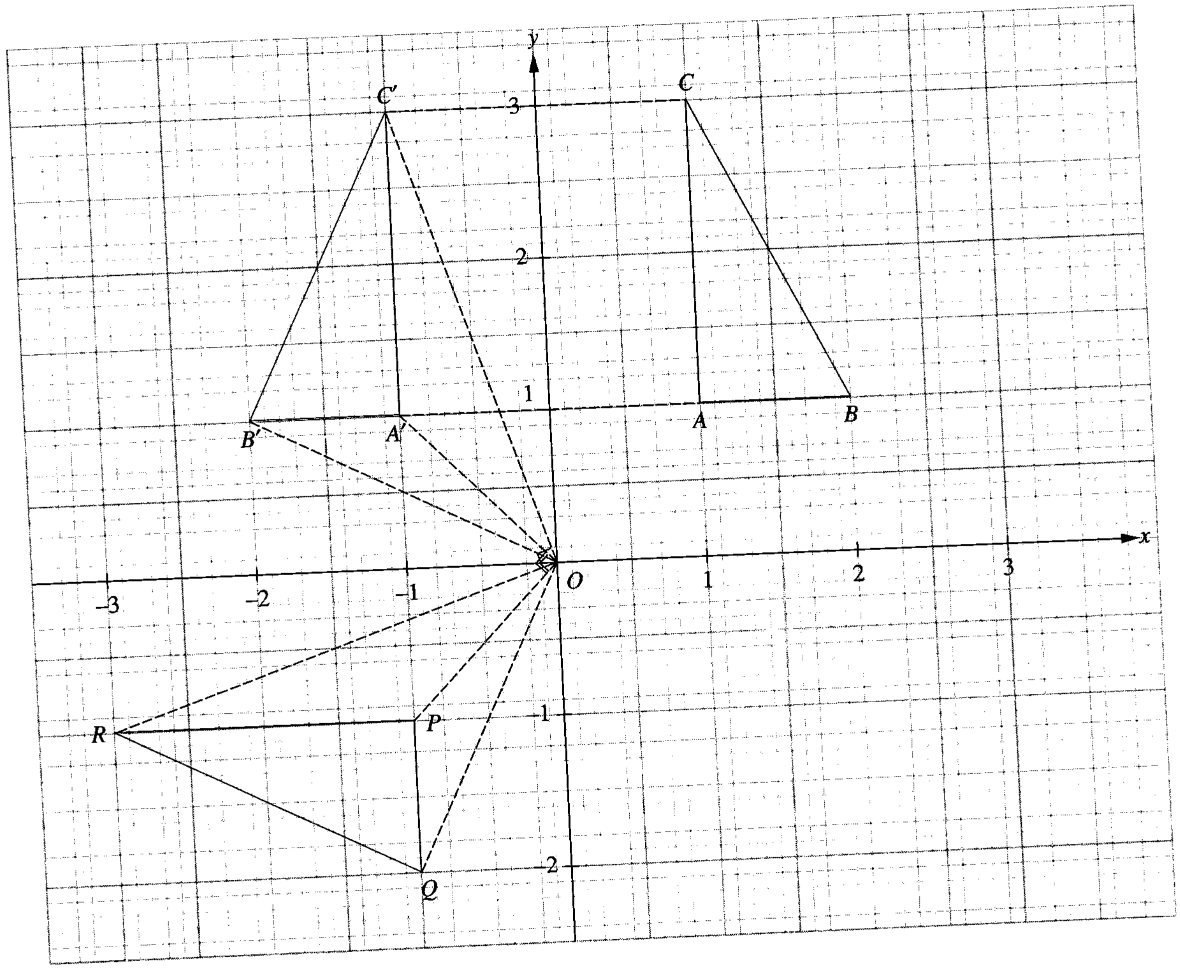
Example 4

Plot $\triangle ABC$ whose vertices are $A(1, 1)$, $B(2, 1)$ and $C(1, 3)$ on a sheet of graph paper. $\triangle ABC$ is reflected in the y-axis followed by a rotation of 90° anticlockwise about the origin to obtain $\triangle PQR$. Plot $\triangle PQR$ on the same graph paper.

Solution

$\triangle ABC$ is reflected in the y-axis to $\triangle A'B'C'$. $\triangle A'B'C'$ is then rotated through 90° anticlockwise about O to obtain $\triangle PQR$ whose coordinates are $P(-1, -1)$, $Q(-1, -2)$ and $R(-3, -1)$.

Would you obtain the same result if $\triangle ABC$ is first rotated through 90° anticlockwise about O and then reflected in the y-axis? Check your answer by doing the transformations on graph paper.



Example 5

Using a scale of 1 cm to represent 1 unit on each axis, draw x- and y-axes for $-5 \leqslant x \leqslant 5$ and $-5 \leqslant y \leqslant 5$.

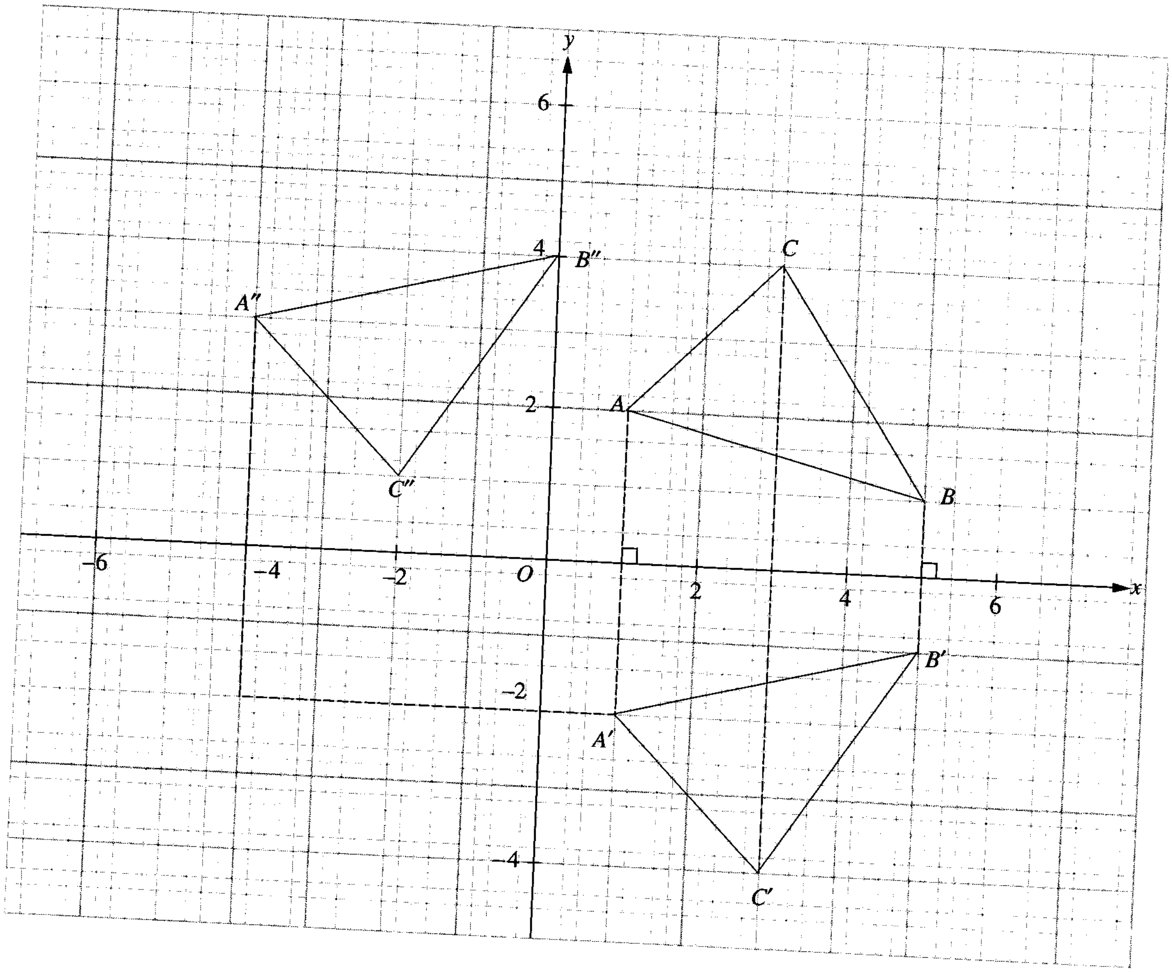
(a) The vertices of $\triangle ABC$ are $A(1, 2)$, $B(5, 1)$ and $C(3, 4)$. Draw and label $\triangle ABC$.

(b) $\triangle ABC$ undergoes a double transformation: a reflection in the x-axis (M), followed by a translation (T) of 5 units in the negative x-direction and 5 units in the positive y-direction. Plot the image of $\triangle ABC$ under TM.

Solution

$\triangle ABC$ is reflected in the x-axis to give the image $\triangle A'B'C'$. $\triangle A'B'C'$ is then translated to $\triangle A''B''C''$ under T. The coordinates of $\triangle A''B''C''$ are $A''(-4, 3)$, $B''(0, 4)$ and $C''(-2, 1)$.

Would you obtain the same result under MT? Check your answer by drawing the image of $\triangle ABC$ under MT.



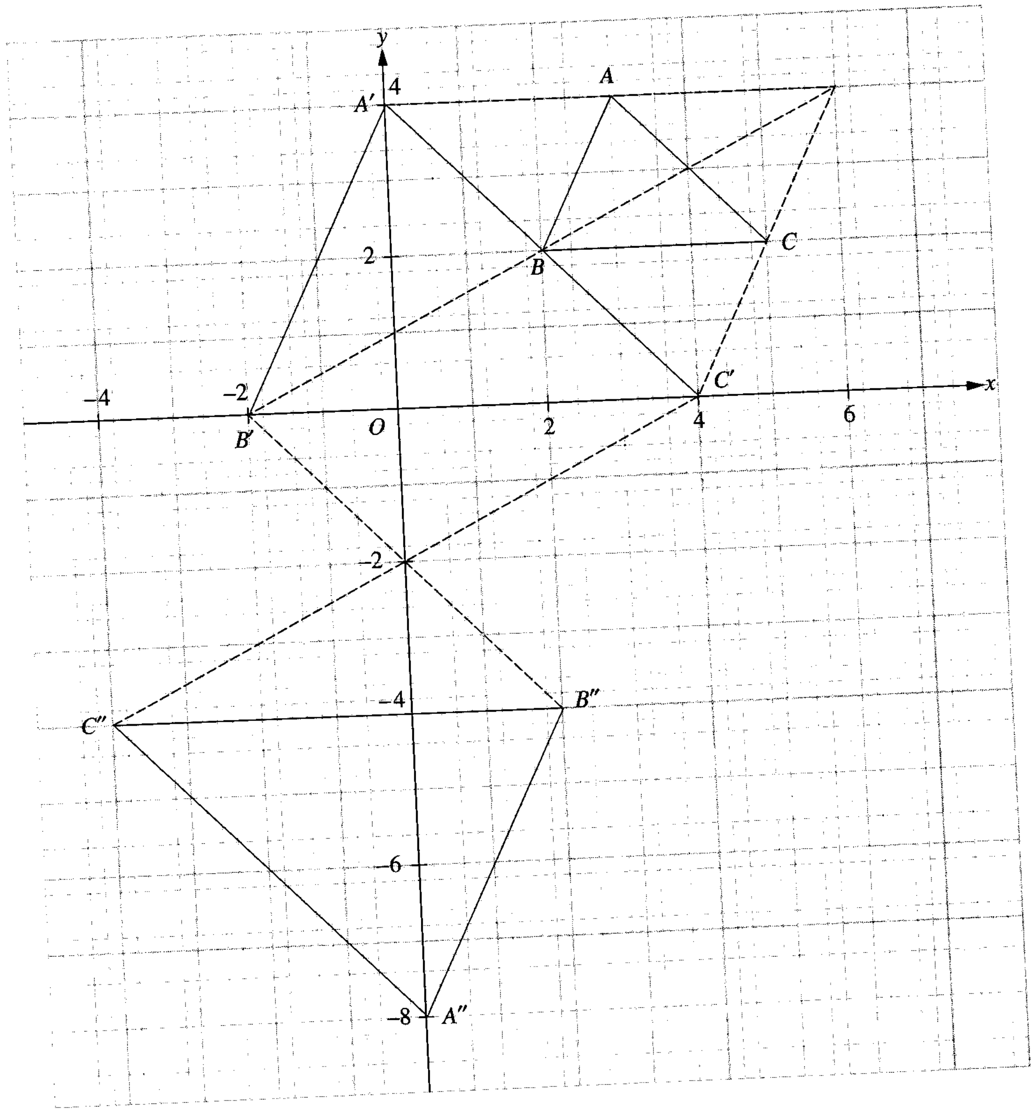
Example 6

Using a scale of 1 cm to 1 unit on each axis, draw the x- and y-axes for $-4 \leqslant x \leqslant 6$ and $-8 \leqslant y \leqslant 4$. The vertices of a triangle ABC are A(3, 4), B(2, 2) and C(5, 2). Draw and label △ABC. The transformation E is an enlargement, centre (6, 4) and scale factor 2 and the transformation R is a rotation of 180° about (0, −2). Draw and label the image of △ABC under the transformation RE.

Solution

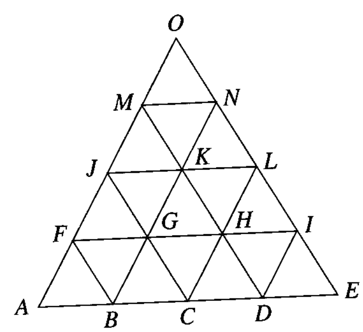
$\triangle A''B''C''$, whose coordinates are $A''(0, -8)$, $B''(2, -4)$ and $C''(-4, -4)$, is the result of the transformation *RE*.

Would you expect the same result for *ER*?



Example 7

The figure shows an equilateral triangle AEO of sides 8 cm each being sub-divided into 16 smaller equilateral triangles of sides 2 cm each.



(a) Describe a single transformation that will map $\triangle AFB$ onto $\triangle AOE$.

(b) Describe a single transformation that will map $\triangle OMN$ onto $\triangle CGH$.

(c) Describe a single transformation that will map trapezium FGKJ onto HGBC.

(d) Describe two successive transformations that will map $\triangle AFB$ onto $\triangle IHD$.

(e) Describe two successive transformations that will map $\triangle OMN$ onto $\triangle CJL$.

Solution

(a) $\triangle AFB$ is transformed onto $\triangle AOE$ by an enlargement with centre A and scale factor 4.
(b) $\triangle OMN$ is transformed onto $\triangle CGH$ by a reflection in the line JL.
(c) Trapezium $FGKJ$ is transformed onto $HGBC$ by a 180° rotation or half turn about the point G.
(d) $\triangle AFB$ is first reflected in the line FB to obtain $\triangle GFB$. $\triangle GFB$ is then translated along AE for a distance of 4 cm to obtain $\triangle IHD$.
(e) $\triangle OMN$ is first reflected in the line MN to obtain $\triangle KMN$. $\triangle KMN$ is transformed onto $\triangle CJL$ by an enlargement scale factor 2 and point of enlargement at O.
Can you think of alternative answers to (b), (c), (d) and (e)?

Exercise 11e

1. If R represents a reflection in the y-axis and T a translation of 2 units in the positive x-direction and 4 units in the positive y-direction, find the image of the point (1, 3) under the combined transformation represented by
   (a) RT, **(b)** TR.

2. Find the coordinates of the image of the point (5, 2) under a reflection in the x-axis followed by a clockwise rotation of 90° about the origin.

3. Points are reflected in the y-axis and their images are rotated through 90° anticlockwise about O. Find the coordinates of the final image of the point
   (a) (2, –3), **(b)** (–4, –1).

4. Find the coordinates of the image of (7, –2) under a translation of 4 units in the negative x-direction and 3 units in the positive y-direction followed by a 180° rotation about the origin.

5. E is an enlargement with centre (0, 0) with scale factor 2 and T is a translation represented by 2 units in the positive x-direction and 1 unit in the positive y-direction. Find the final image of the point (2, 1) under
   (a) ET; **(b)** T^2.

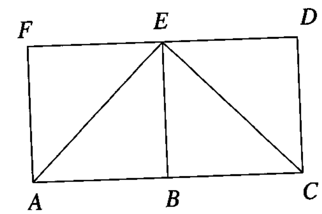
6. The vertices of $\triangle ABC$ are $A(2, 1)$, $B(3, 5)$ and $C(5, 0)$. $\triangle ABC$ is reflected in the x-axis and its image is then rotated through 90° clockwise about the origin. Find the coordinates of the final image of $\triangle ABC$.

7. $ABCD$ is a rectangle whose vertices are $A(2, 1)$, $B(6, 1)$, $C(6, 3)$ and $D(2, 3)$. $ABCD$ is reflected in the x-axis and then rotated through 180° about (0, 0) to $PQRS$. Find the coordinates of the final image $PQRS$.

8. The vertices of the triangle ABC are $A(1, 1)$, $B(1, 3)$ and $C(4, 1)$. Using a scale of 1 cm to represent 1 unit, draw the x- and y-axes for $-10 \leqslant x \leqslant 6$ and $-8 \leqslant y \leqslant 8$. Draw and label $\triangle ABC$.
   (a) $\triangle ABC$ is reflected in the y-axis to obtain $\triangle PQR$. Draw and label $\triangle PQR$.
   (b) $\triangle PQR$ is enlarged with scale factor 2 and point of enlargement at (–1, 0) to $\triangle LMN$. Draw and label $\triangle LMN$.

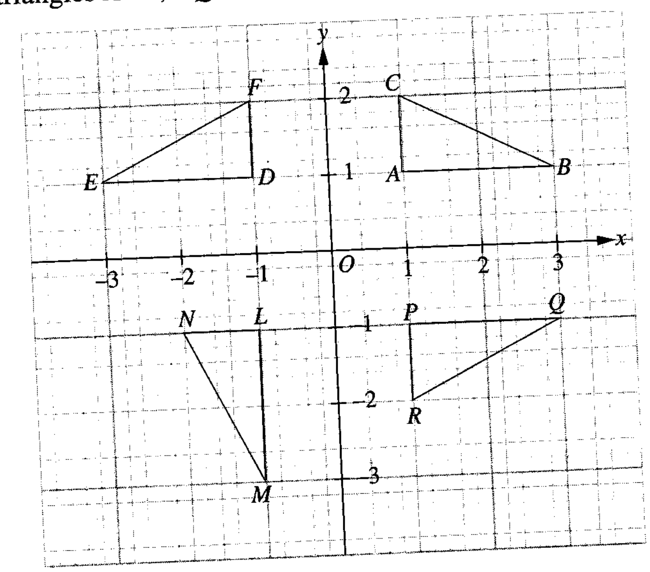
9. Using a scale of 1 cm to represent 1 unit on each axis, draw the x- and y-axes for $-4 \leqslant x \leqslant 8$ and $-10 \leqslant y \leqslant 4$.
   (a) Draw and label the triangle whose vertices are $A(2, 1)$, $B(6, 1)$ and $C(6, 3)$.
   (b) $\triangle ABC$ is mapped onto $\triangle PQR$ by a clockwise rotation of 90° about (0, 1). Draw and label $\triangle PQR$.
   (c) $\triangle PQR$ is enlarged with centre at (0, 0) and scale factor 2 to $\triangle LMN$. Draw and label $\triangle LMN$.

★10. The diagram shows two identical squares $ABEF$ and $BCDE$ with sides of 3 cm respectively.



(a) Describe a single transformation that will map $\triangle ABE$ onto $\triangle CBE$.
(b) Describe a single transformation that will map $ABEF$ onto $BCDE$.
(c) Describe two successive transformations that will map $\triangle AFE$ onto $\triangle CBE$.
(d) Describe a single transformation that will map $\triangle CDE$ onto $\triangle ABE$.

★11. The diagram shows 4 triangles ABC, PQR, LMN and DEF.



(a) Describe a single transformation that will map $\triangle ABC$ onto $\triangle DEF$.
(b) Describe a single transformation that will map $\triangle PQR$ onto $\triangle LMN$.
(c) Describe two successive transformations that will map $\triangle ABC$ onto $\triangle LMN$.
(d) Describe two successive transformations that will map $\triangle PQR$ onto $\triangle DEF$.

Summary

1. For a **reflection**, the original figure and its image are symmetrical about the mirror line. There is no change in the shape and size of the figure under a reflection; however, the orientation of the figure changes. The points on the mirror line are invariant under a reflection.

2. For a **rotation**, we need to specify the centre of rotation and the angle, either clockwise or anticlockwise, to be rotated. In a rotation, there is no change in the shape, size and orientation of the figure. The centre of rotation is the only invariant point in a rotation.

3. A **translation** is the transformation of a plane in which all the points of a figure are moved the same distance in the same direction. There is no change in the shape, size and orientation of the figure in a translation. There is also no invariant point in a translation.

4. For an **enlargement**, we need to specify the centre of enlargement and scale factor. In an enlargement, there is a change in size but shape and orientation are preserved. The centre of enlargement is the only invariant point.

Review Questions 11

1. P is the point (2, –1). Find the coordinates of the image of P under

(a) a reflection in the y-axis;

(b) a 90° anticlockwise rotation about (0, 0);

(c) a translation represented by 1 unit in the positive x-direction and 5 units in the positive y-direction.

2. K is the point (3, –1). Find the coordinates of the image of K under

(a) a reflection in the x-axis followed by a reflection in the y-axis;

(b) a reflection in the y-axis followed by a reflection in the x-axis;

(c) a 90° anticlockwise rotation about (0, 0) followed by a reflection in the y-axis;

(d) a 180° rotation about (2, 0) followed by a reflection in the line $y = 3$.

3. A translation maps the point (–2, 3) onto the point (2, 5) and the point (5, –2) onto the point (x, y). Find the values of x and y.

4. Using a scale of 1 cm to 1 unit on both axes, draw on graph paper the flag formed by the points (1, 1), (1, 2), (1, 3) and (2, 2). Draw the image of the flag under

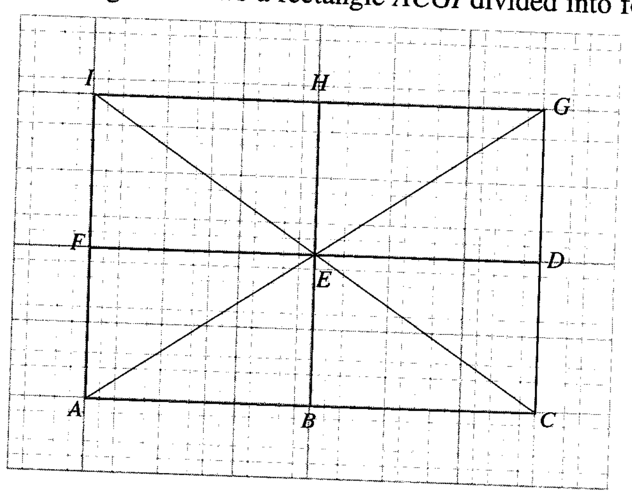
(a) a 90° anticlockwise rotation about (0, 0);

(b) a reflection in the x-axis;

(c) an enlargement of scale factor 2 and centre of enlargement at (0, 0);

(d) a 180° rotation about (0, 0) followed by a reflection in the y-axis.

★5. The diagram shows a rectangle $ACGI$ divided into four smaller equal rectangles.



(a) Describe a single transformation that maps $ABEF$ onto $ACGI$.

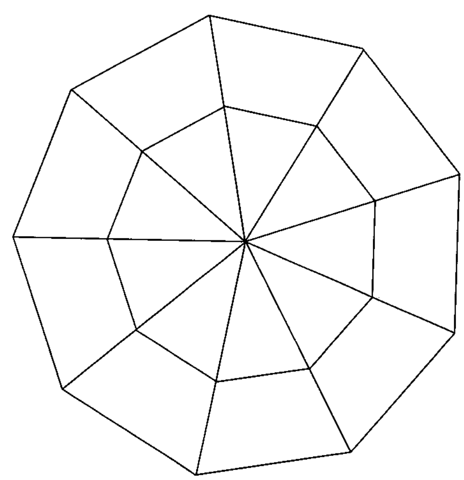
(b) Describe a single transformation that maps $\triangle ABE$ onto $\triangle EDG$.

(c) Describe a single transformation that maps $\triangle ACI$ onto $\triangle GIC$.

(d) Describe two successive transformations that map $\triangle ABE$ onto $\triangle IHE$.

(e) Describe two successive transformations that map $\triangle ACG$ onto $\triangle CAI$.

6. The figure on the right shows two concentric regular nonagons with 18 regions. Shade exactly 6 of the 18 regions so that the resulting figure has no lines of symmetry but has rotational symmetry of order 3.



Problem Solving

1. On graph paper, draw $\triangle ABC$ at $A(3, 1)$, $B(5, 1)$ and $C(5, 2)$. Draw the line $y = x$. $\triangle ABC$ is transformed into $\triangle A_1B_1C_1$ by a reflection in the line $y = x$. $\triangle A_1B_1C_1$ is then transformed into $\triangle A_2B_2C_2$ by a reflection in the x-axis. Draw $\triangle A_1B_1C_1$ and $\triangle A_2B_2C_2$. Describe a single transformation that will map $\triangle ABC$ onto $\triangle A_2B_2C_2$.

2. The transformation P is a reflection in the line $y = 0$ and the transformation Q is a reflection in the line $x = 0$. Describe a single transformation equivalent to

   (a) PQ; **(b)** QP.

3. The transformation P is a 90° clockwise rotation about the origin and the transformation Q is a reflection in the x-axis. Describe a single transformation equivalent to

   (a) PQ; **(b)** QP.

CHAPTER 12

Pythagoras' Theorem

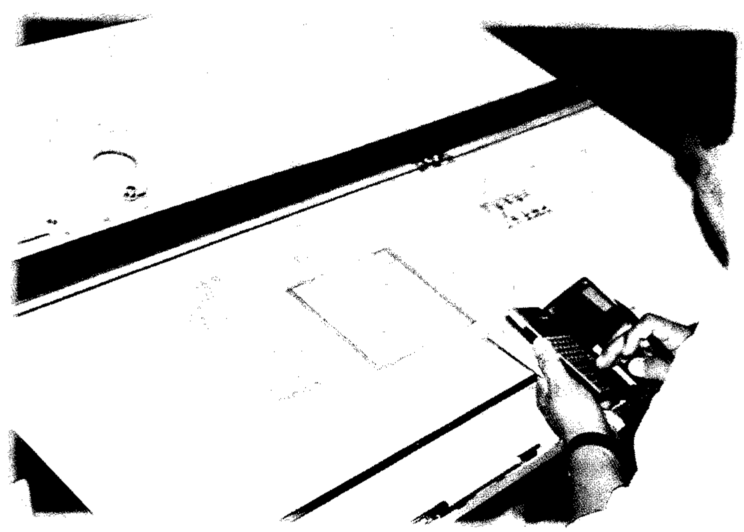
In this chapter, you will learn how to

- find the length of a side of a right-angled triangle using Pythagoras' theorem;
- solve problems involving Pythagoras' theorem.

Preliminary Problem

The picture shows an interior designer busily doing some calculations on the details of his drawing. He will need to make use of Pythagoras' theorem in the course of his work.

Do you know who Pythagoras is?



Pythagoras' Theorem

We always denote the side opposite $\hat{A}$ by a, the side opposite $\hat{B}$ by b and the side opposite $\hat{C}$ by c.

Fig. 12.1 shows a right-angled triangle with $B\hat{A}C = 90°$. The side opposite the right angle A is called the **hypotenuse**. It is the longest side of a right-angled triangle.

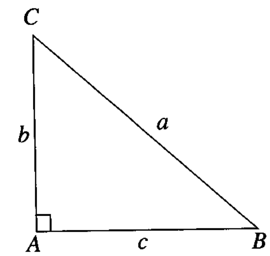


Fig. 12.1

In-Class Activity

Fig. 12.2 shows three right-angled triangles with the lengths of the two shorter sides given (not drawn to scale).

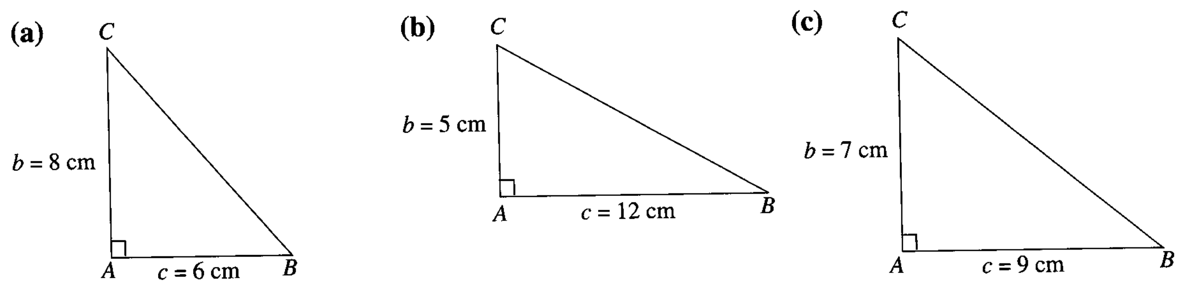


Fig. 12.2

Construct the three triangles in Fig. 12.2 accurately, measure the length of a for each and fill in the following table.

	b	c	a	b^2	c^2	a^2	$b^2 + c^2$
(a)	8	6		64	36		
(b)	5	12		25	144		
(c)	7	9		49	81		

There are many interesting internet sites that provide interactive Java applets for proving the Pythagoras' theorem. (Check with your teacher for the site addresses.) We can also use the open tool, Geometer's Sketch Pad, to establish the proof of this theorem.

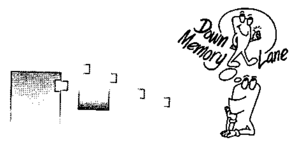
From the results in the table, do you find that $a^2 = b^2 + c^2$? This important relation for the three sides of a right-angled triangle was discovered by a famous Greek philosopher and mathematician, Pythagoras, in the 6th century BC. In honour of his great contribution, this relation is named **Pythagoras' theorem** which states that:

The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.

It's a Fact

A theorem is a fact that has been proved to be true.

i.e., $a^2 = b^2 + c^2$ or $BC^2 = AB^2 + AC^2$ as in Fig. 12.1.



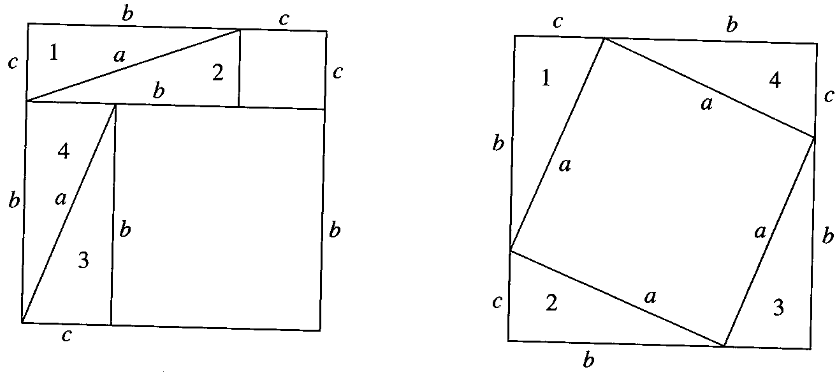
Pythagoras was a Greek philosopher and mathematician of Samos.

In addition to Pythagoras' theorem, some of his other interesting discoveries include:

1. *the musical note produced by a vibrating string of a certain length is exactly one octave lower than the note produced by a string of the same material and half that length,*
2. *other notes in the musical scale can be produced by using certain fractions of the length of the string. For example, a string $\frac{4}{3}$ the length of a C-string produces the note G (one octave lower). The diagram below shows the various musical notes that can be produced for each given fraction of the length of a C-string.*

There are more than 300 proofs of Pythagoras' theorem. We shall examine one of them and learn how to apply it to solve problems.

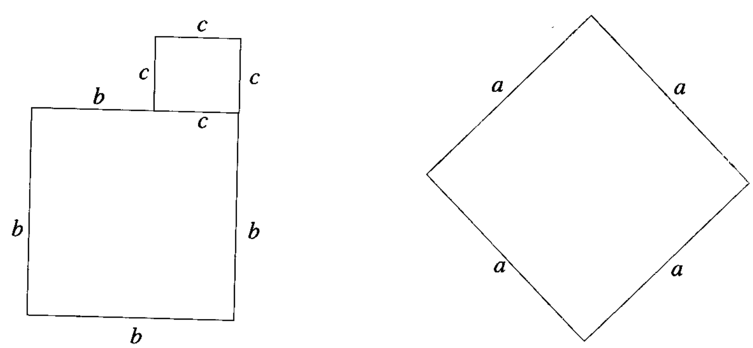
Take eight right-angled cardboard triangles of the same size and arrange them in two designs as shown below:



(a) (b)

Fig. 12.3

Fig. 12.3(a) and Fig. 12.3(b) are squares of sides $(b + c)$ each. Thus the area of Fig. 12.3(a) is equal to the area of Fig. 12.3(b). Now remove the four cardboard triangles to obtain the unshaded regions shown below:



(a) (b)

Fig. 12.4

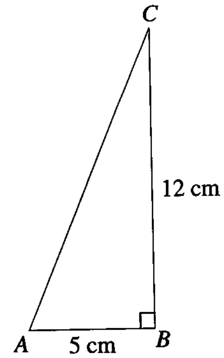
Since the area of the cardboards removed from both figures are equal, the area of Fig. 12.4(a) must be equal to the area of Fig. 12.4(b), i.e., $a^2 = b^2 + c^2$.

The converse of the Pythagoras' theorem is also true, i.e.,

In a triangle with sides a, b and c, if $a^2 = b^2 + c^2$, then the angle facing the side a is a right angle.

Example 1

ABC is a triangle, right-angled at B, AB = 5 cm and BC = 12 cm. Find the length of AC.



Solution

$AC^2 = AB^2 + BC^2$ (Pythagoras' theorem)

$AC^2 = 5^2 + 12^2$

$= 25 + 144$

$= 169$

$AC = \sqrt{169}$ (since AC cannot be negative)

$\therefore \quad AC = 13$ cm

Example 2

Is the triangle whose sides are 15 cm, 36 cm and 39 cm, right-angled?

Solution

A triangle is right-angled if the square of the longest side is equal to the sum of the squares of the other two sides.

$39^2 = 1\ 521$

$15^2 + 36^2 = 225 + 1\ 296$

$= 1\ 521$

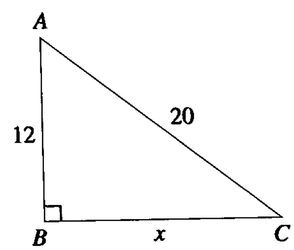
$\therefore \quad 39^2 = 15^2 + 36^2$

$\therefore$ the given triangle is a right-angled triangle.

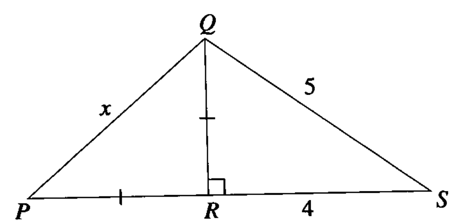
Example 3

Calculate the value of x in each case.

(a)



(b)



Solution

(a) $AB^2 + BC^2 = AC^2$

$12^2 + x^2 = 20^2$

$x^2 = 20^2 - 12^2$

$= 400 - 144$

$= 256$

$x = \sqrt{256}$

$\therefore \quad x = 16$

(b) $QR^2 + RS^2 = QS^2$

$QR^2 + 4^2 = 5^2$

$QR^2 = 5^2 - 4^2$

$= 25 - 16$

$= 9$

$QR = \sqrt{9}$

$\therefore \quad QR = 3$

$QR^2 + PR^2 = PQ^2$

$3^2 + 3^2 = x^2$

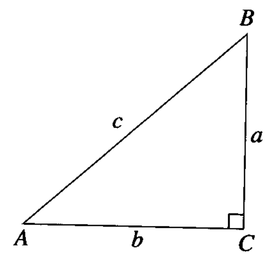
$x^2 = 18$

$\therefore \quad x = 4.24$ (correct to 3 sig. fig.)

Exercise 12a

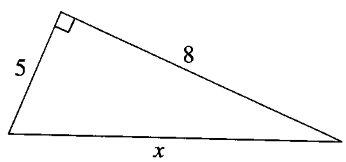
1. Which of the following could be the lengths of the sides of a right-angled triangle?
   - **(a)** 8, 15, 17 **(b)** 7, 25, 26 **(c)** 9, 12, 15 **(d)** 24, 45, 51
   - **(e)** 24, 26, 10 **(f)** 9, 17, 14 **(g)** 0.15, 0.2, 0.25 **(h)** $\frac{6}{13}, \frac{8}{13}, \frac{10}{13}$
   - **(i)** 75, 23, 71 **(j)** 0.8, 0.9, 1.2

2. In the figure, $\hat{C} = 90°$. For each of the following, find the length of the unknown side.
   - **(a)** $a = 9$ cm, $b = 12$ cm **(b)** $a = 15$ m, $b = 8$ m
   - **(c)** $b = 12$ m, $c = 13$ m **(d)** $a = 7$ m, $c = 25$ m
   - **(e)** $a = 40$ cm, $c = 41$ cm **(f)** $a = 21$ cm, $b = 20$ cm
   - **(g)** $a = 35$ cm, $c = 37$ cm **(h)** $a = 11$ cm, $c = 61$ cm
   - **(i)** $a = 28$ cm, $c = 53$ cm **(j)** $a = 33$ cm, $b = 56$ cm
   - **(k)** $a = 4$ m, $b = 7$ m **(l)** $a = 6$ m, $c = 11$ m
   - **(m)** $c = 9$ cm, $b = 6$ cm **(n)** $a = 6.7$ cm, $b = 5.5$ cm
   - **(o)** $c = 9.4$ cm, $a = 4.6$ cm **(p)** $a = 14$ cm, $c = 19$ cm

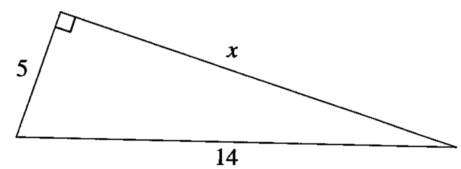


3. Calculate the value of x in each case.

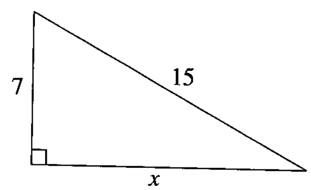
(a)



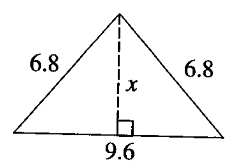
(b)



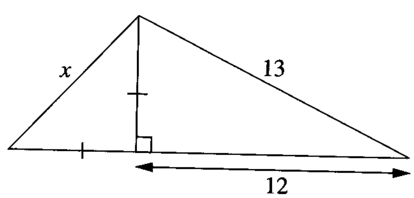
(c)



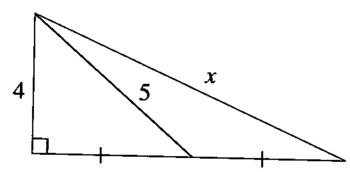
(d)



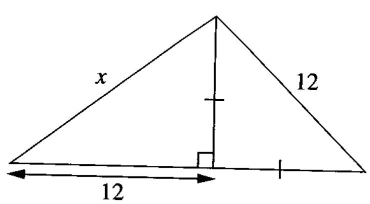
(e)



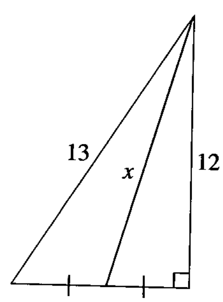
(f)



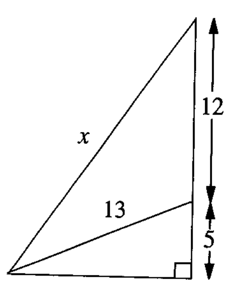
(g)



(h)

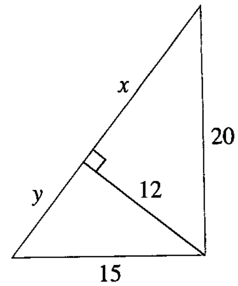


(i)

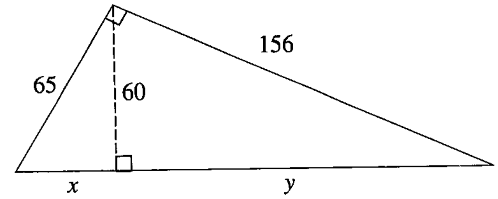


4. Find the values of x and y.

(a)



(b)

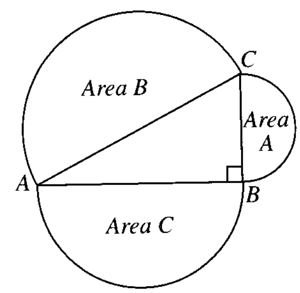


Applications of Pythagoras' Theorem

The following examples involve applying Pythagoras' theorem in solving problems.

Semicircles are drawn on the sides of the right-angled triangle ABC as shown below.

Is Area B = Area A + Area C?



Example 4

How high up the wall is a 10-m ladder if one of its ends on the ground is 2 m from the wall?

Solution

In the diagram, BC is the height that the ladder will reach.

$AC^2 = AB^2 + BC^2$

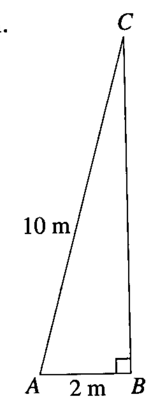
$10^2 = 2^2 + BC^2$

$BC^2 = 10^2 - 2^2$

$= 100 - 4 = 96$

$\therefore \quad BC = \sqrt{96} = 9.80$ m (correct to 3 significant figures)

$\therefore$ the ladder will reach a height of 9.80 m.



Example 5

Each side of a square field ABCD is 50 m long. Find the length of the diagonal of the field.

Solution

$BD^2 = BC^2 + CD^2$

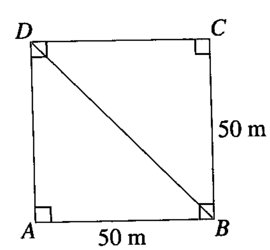
$= 50^2 + 50^2$

$= 2\ 500 + 2\ 500$

$= 5\ 000 \text{ m}^2$

$\therefore \quad BD = \sqrt{5\ 000}$

$= 70.7$ m (correct to 3 significant figures)

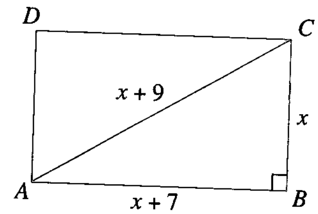


How can you check if $x = 8$ is the correct answer?

Example 6

The diagonal of a rectangular field ABCD is $(x + 9)$ m and the sides are $(x + 7)$ m and x m. Find the value of x.

Solution



$$AC^2 = AB^2 + BC^2$$
$$(x + 9)^2 = (x + 7)^2 + x^2$$
$$x^2 + 18x + 81 = x^2 + 14x + 49 + x^2$$
$$x^2 - 4x - 32 = 0$$
$$(x + 4)(x - 8) = 0$$
$$x = -4 \quad \text{or} \quad x = 8$$ (We cannot have a negative length.)
$$\therefore \quad x = 8$$

Exercise 12b

1. Find the length of the longest line segment that can be drawn on a rectangular board 3.07 m by 2.24 m.

2. A 5-m long ladder is placed against a wall with its foot 1.8 m from the wall. Find how far up the wall the ladder reaches.

3. A post 47 m high is supported by wires attached to its top and to a point on level ground, 18 m from the foot of the post. Find the length of each wire.

4. $\triangle ABC$ is right-angled at B, and D is a point on BC. If $AD = 18$ cm, $BD = 9$ cm and $CD = 4$ cm, find AC.

5. The diagonals of a rhombus are of lengths 16 cm and 12 cm. Find the length of its sides.

6. The lengths of the sides of a triangle are x cm, $(x + 1)$ cm and $(x + 2)$ cm. Determine x so that this triangle is a right-angled triangle.

7. The sides of a rectangle are 24 cm and 15 cm. Calculate the length of its diagonal.

8. The sides of a rectangular swimming pool are 50 m and 30 m. What is the length between the opposite corners?

9. How far from the wall must you place a ladder of length 12 m, if the ladder is to touch a point 10 m above the ground?

10. A man runs diagonally across from one corner of a rectangular field 80 m by 60 m to the opposite corner in a straight line at a speed of 9 metres per second. Find the time taken for him to complete the run.

11. ABC is an equilateral triangle in which $BC = 2$ cm. Find the perpendicular distance from A to BC.

12. One diagonal of a rhombus is 24 cm. Find the length of the other diagonal if each side of the rhombus measures 13 cm.

13. $PQRS$ is a rectangle in which $PQ = 10$ cm and $PS = 6$ cm. T is a point on PQ such that RST is an isosceles triangle whose equal sides are RS and ST. Find RT.

☆14. $PQRS$ is a rectangle in which $PQ = 9$ cm and $PS = 6$ cm. T is a point on PQ such that $PT = 7$ cm and RV is the perpendicular from R to ST. Calculate ST and RV.

15. In a parallelogram $ABCD$, the diagonal AC is at right angles to AB. If $AB = 12$ cm and $BC = 13$ cm, find the area of the parallelogram.

16. A rectangle is 7 cm wide, and the length of each diagonal is 25 cm. Find the length of the rectangle and its area.

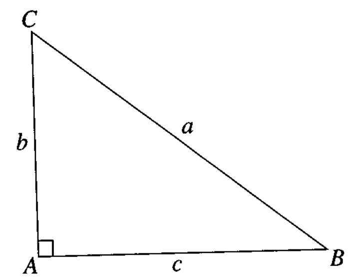
Summary

1. Pythagoras' theorem

For a right-angled triangle ABC,

$AB^2 + AC^2 = BC^2$

i.e., $c^2 + b^2 = a^2$

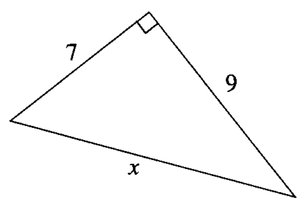


2. The converse of the Pythagoras' theorem states that if $a^2 = b^2 + c^2$, then the triangle with sides a, b and c is a right-angled triangle, with the angle facing the side a being a right angle.

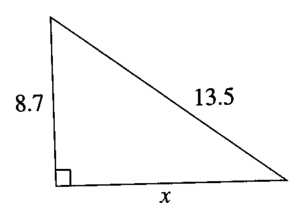
Review Questions 12

1. Calculate the value of x in each of the following:

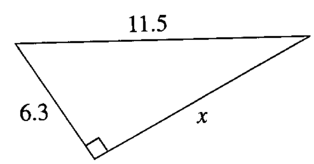
(a)



(b)



(c)



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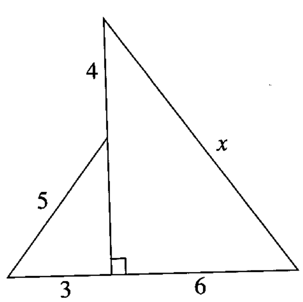

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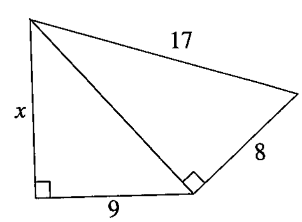

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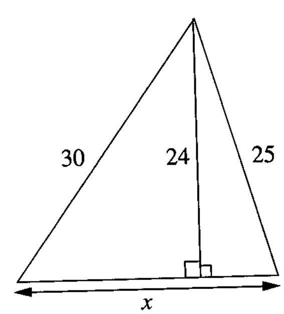


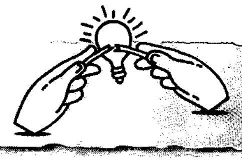
2. The sides of a rectangle are 14 cm and 25 cm. Calculate the length of the diagonal.

3. The diagonal of a square is 42.5 cm. Calculate the perimeter and area of the square. (Leave your answers correct to 1 decimal place.)

4. The length of the sides of a rhombus is 52 cm. One of its diagonals is 48 cm. Find the length of the other diagonal and the area of the rhombus. (Leave your answers correct to 1 decimal place.)

5. Find the distance from the mid-point of one side of a square of side 10 cm to either end of the opposite side.

6. PQR is an isosceles triangle such that $QP = QR = 13$ cm and the altitude $PS = 12$ cm. Find the length of PR.

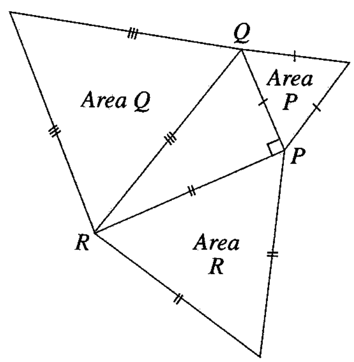


Problem Solving

1. Given a triangle ABC where $(BC)^2 = 370$ units, $(AC)^2 = 74$ units and $(AB)^2 = 116$ units, calculate the area of the triangle.

   [Hint: $370 = 9^2 + 17^2$, $74 = 5^2 + 7^2$ and $116 = 4^2 + 10^2$]

2. Equilateral triangles are drawn on the sides of the right-angled triangle PQR as shown below. Is Area Q = Area P + Area R?



Revision Exercise III No. 1

1. Solve the following equations:
   (a) $2(x - 3) = 8 - 3(x - 2)$
   (b) $\frac{1}{3}x = \frac{1}{7}(90 - x)$
   (c) $\frac{2(x - 1)}{3} - \frac{5(x - 3)}{6} = \frac{3}{4}$
   (d) $3[(x - 2) - (2x - 1)] = 2[(2x + 1) - (x + 2)]$

2. A mother is 30 years older than her daughter. Five years ago she was four times as old as her daughter. How old are they now?

3. If $N = L(1 - d)$, find d in terms of N and L. Given that $N = 40$ and $L = 50$, find the value of d.

4. On a piece of graph paper, draw the x and y axes for $-5 \leqslant x \leqslant 5$ and $-4 \leqslant y \leqslant 4$. Draw the triangle PQR with coordinates $P(3, 4)$, $Q(5, 1)$ and $R(-3, 2)$, $\triangle PQR$ is reflected in the y-axis to obtain $\triangle LMN$. Draw $\triangle LMN$ on the same diagram.

5. (a) Find two numbers whose sum is 90 and one-third of the smaller number is equal to one-seventh of the larger number.
   (b) Find the number of years in which \$1 200 will amount to \$1 600 at 10% simple interest per annum.

6. In $\triangle ABC$, $\angle B = 90°$, $AB = 12$ cm and $AC = 13$ cm. Find
   (a) the length of BC,
   (b) the area of $\triangle ABC$.

7. A scale model of a ship is made to a scale of 1 : 80. If the mast of the model is 42 cm, find the height of the actual mast.

8. If the angles of a pentagon are $x°$, $1\frac{1}{2}x°$, $2\frac{1}{2}x°$, $3x°$ and $(2x - 20)°$, find the value of x.

9. Using a scale of 2 cm to represent 1 unit on both the x- and y-axes, draw the graphs of $y = x - 1$ and $x + y = 5$ and hence determine the coordinates of the point of intersection of the two graphs.

10. The following table shows some values of x and y for the graph $y = x^2 - 2x - 3$.

x	−2	−1	0	1	2	3	4	5
y			−3	−4		0	5	

Copy and complete the table above and draw the graph of $y = x^2 - 2x - 3$ for $-2 \leqslant x \leqslant 5$. Draw the line of symmetry in your graph and state its equation. State the solution of the equation $x^2 - 2x - 3 = 0$.

Revision Exercise III No. 2

1. Solve the following equations:
   (a) $3x - 2(3x - 1) + 4(x + 1) = 0$
   (b) $6 - [(3x - 7) - (7x - 3)] = 0$
   (c) $4(3x - 2) = 7x - 3(2x - 1)$
   (d) $6x - [7x - (8x - 19)] = 2$

2. (a) If $7 : 15 = x : 3$, find x.
   (b) A sum of money is divided between two boys, A and B, in the ratio 5 : 3. If B gets \$9, how much will A get?

3. The line $2x + 3y = 12$ cuts the x- and y-axes at the points A and B respectively. Find the coordinates of the points A and B by plotting the graph $2x + 3y = 12$ on a coordinate graph. If C is the point $(0, -2)$, find the area of the triangle ABC.

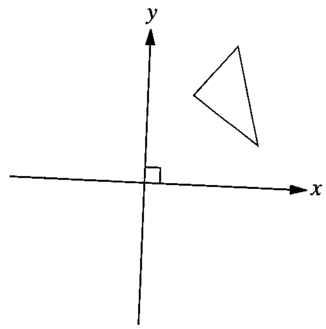
4. Copy and complete the following table of values for $y = 3 + 2x - x^2$.

x	−3	−2	−1	0	1	2	3	4
y	−12			3	4			−5

Plot the graph of $y = 3 + 2x - x^2$ using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 20 units on the y-axis. Use your graph to find
   (a) the greatest value of y,
   (b) the equation of the line of symmetry,
   (c) the values of x when $3 + 2x - x^2 = 0$,
   (d) the values of x when $3 + 2x - x^2 = -2$.

☆5. Factorise (a) $20a^3 - 45a$,
   (b) $a^2 + 6ab + 9b^2 - 1$.

6. Copy and complete the figure to form a symmetrical pattern on both the x- and y-axes.



7. On a certain day in 1999, the exchange rate for RM100 is S$45. Draw a graph to convert Malaysian ringgit to Singapore dollars. Use your graph to find the approximate values of

(a) (i) RM12, (ii) RM38 in S$,
(b) (i) S$42, (ii) S$75 in RM.

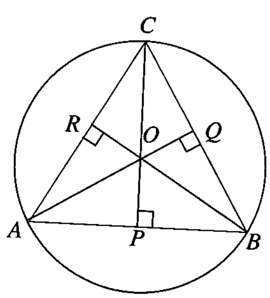
8. Draw the graph of $y = x^2 + 2$ for $-3 \leqslant x \leqslant 3$. Use your graph to find

(a) the value of y when $x = 1.7$,
(b) the values of x when $y = 7.5$.

9. A map is drawn to a scale of 4 cm to 3 km.

(a) Find the scale of the map in the form $1 : n$ where n is an integer.
(b) Two places on the map are 23 cm apart. Find their actual distance.
(c) A forest reserve has an area of 24 cm^2 on the map. Find its actual area.

10.



The diagram above shows a circle, centre O, passing through the points A, B and C. ABC is an equilateral triangle and P, Q and R are the mid-points of AB, BC and AC respectively.

(a) State the number of axis of symmetry of

(i) $\triangle ABC$, (ii) $\triangle OAB$.

(b) Describe a single transformation that will map $\triangle OAP$ onto $\triangle OBP$.
(c) Describe a single transformation that will map $\triangle OAB$ onto $\triangle OBC$.

Revision Exercise III No. 3

1. Make c the subject of the formula $m = \frac{b + 2c}{3}$.

2. If the simple interest on \$6 000 for $3\frac{1}{2}$ years is \$945, find the simple interest on \$5 000 for 8 months at the same rate.

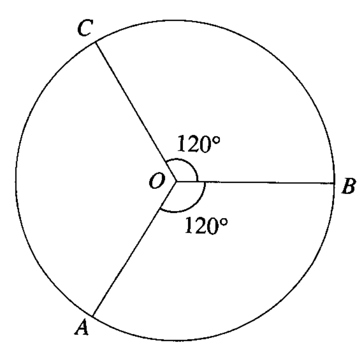
3. Solve the simultaneous equations

$$2x + y = 1,$$
$$4x + 5y = 8.$$

4. Simplify each of the following:

(a) $\frac{x}{5} + \frac{2x - 1}{3}$ (b) $\frac{1}{x} - \frac{2}{x + 2}$

5.



The diagram above shows a circle with 3 equal sectors.

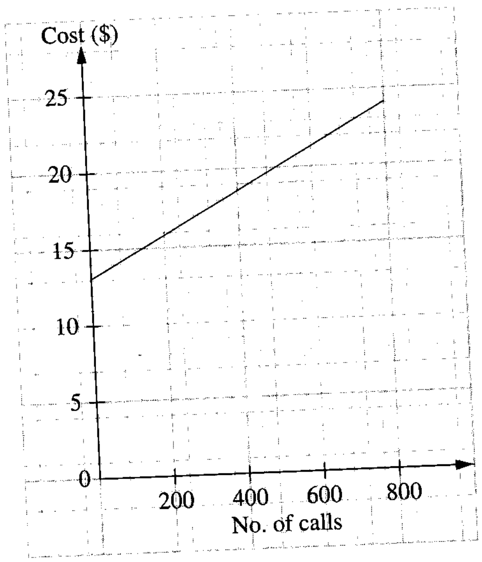
(a) Describe a single transformation that will map sector OAB onto OCB.
(b) Describe a single transformation that will map sector OAC onto OBA.

6. $ABCD$ is a rectangle in which $AB = 16$ cm, $AC = 20$ cm and AE is the perpendicular from A to BD. Find the lengths of AD and AE.

7. A polygon has n sides. Four of its exterior angles are 12°, 25°, 32° and 41° and the remaining $(n - 4)$ exterior angles are each equal to 50°. Find n.

8. The Singapore government gave out \$35 million to 121 000 students in the form of Edusave scholarships and bursaries in 1999. Calculate the average amount received by each student giving your answer correct to the nearest 50 cents.

9. The monthly telephone bill is made up of a basic monthly rental charge plus the number of calls made during the month.



Use the graph above to answer the following questions:

(a) Find the cost of the bill when 460 calls are made.
(b) Find the number of calls made when the monthly telephone bill is \$14.
(c) What is the basic monthly rental charge?

10. Copy and complete the following table for $y = 2x^2 - 9x + 2$.

x	0	1	2	3	4	5	6	7
y	2		−8	−7		7	20	

Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 5 units on the y-axis, draw the graph of $y = 2x^2 - 9x + 2$ for $0 \leqslant x \leqslant 7$.

Use your graph to estimate

(a) the value of y when $x = 4.6$,
(b) the value of x when $y = -5$.

Revision Exercise III No. 4

1. (a) If $4 : x = 9 : 14$, find x.
   (b) A number is divided into two parts in the ratio $7 : 4$. If the larger part is 21, find the number.

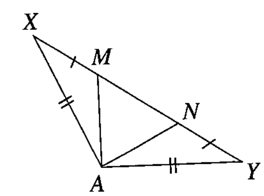
∗2. (a) Multiply $2 - 3x - x^2$ by $1 + 2x - 3x^2$.
   (b) Factorise $a^2 + 2ab + b^2 - c^2$.

3. Simplify (a) $\frac{x}{3} - \frac{2x}{7}$, (b) $\frac{x+1}{x-2} - \frac{1}{x}$.

∗4. Solve the simultaneous equations

$$4x - 5y = 15,$$
$$\frac{x+y}{3} - \frac{x-y}{4} = 1.$$

5. (a)



In the figure shown above, $AX = AY$, $MX = NY$ and $XMNY$ is a straight line. Show that $A\hat{M}N = A\hat{N}M$.

   (b) A rhombus of sides 8 cm each has a diagonal 13 cm long. Find the length of its other diagonal.

6. The area of one face of a cube is 25 cm^2. Find
   (a) the volume of the cube,
   (b) the total length of all the edges of the cube.

7. A, B and C are three points with coordinates (1, 0), (5, 0) and (4, 4) respectively. Draw, on graph paper, the x-axis from −6 to 6 and the y-axis from −6 to 8 using a scale of 1 cm to represent 1 unit on each axis.
   (a) Plot the points A, B and C and join them to form $\triangle ABC$.

(b) $\triangle ABC$ is transformed by a translation represented by 3 units in the negative x-direction and 3 units in the positive y-direction to $\triangle A'B'C'$. Plot $\triangle A'B'C'$ on the same graph.

(c) $\triangle ABC$ is reflected in the y-axis to $\triangle PQR$. Plot $\triangle PQR$ on the same graph.

(d) $\triangle PQR$ is rotated through 90° clockwise about the point (0, 0) to obtain $\triangle LMN$. Plot $\triangle LMN$ on the same graph.

(e) State the coordinates of C', R and N.

8. The relationship between degrees Celsius (°C) and degrees Fahrenheit (°F) is given by the formula $F = \frac{9}{5}C + 32$. Plot a graph of F against C for $0° \leqslant C \leqslant 100°$. Use your graph to find

(a) the value of C when $F = 98$,

(b) the value of F when $C = 50$,

(c) the change in C when F increases from 70 to 120.

9. Choose a suitable scale to draw the graphs of $y = 2x - 8$ and $2y + 3x = 5$. Write down the point of intersection of the two graphs.

10. Copy and complete the following table of values for $y = 3x - x^2$.

x	−2	−1	0	1	2	3	4
y	−10		0		2		−4

Using a scale of 2 cm to 1 unit on both axes, plot the graph of $y = 3x - x^2$ for $-2 \leqslant x \leqslant 4$. Use your graph to answer the following:

(a) What is the greatest value of $3x - x^2$?

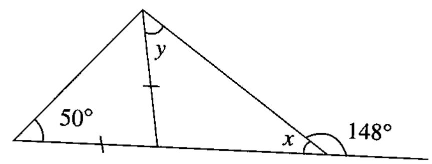
(b) What is the value of y when $x = 2.5$?

(c) What are the possible values of x when $y = -2$?

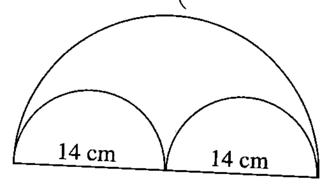
Revision Exercise III No. 5

1. A man invests \$2 000 in a bank paying 5% simple interest per annum and \$3 000 in another bank paying 6% simple interest per annum. What rate per cent does he receive on the investments altogether?

2. (a) Find the angles marked x and y in the figure below.



(b) The figure below shows two identical semicircles inside a larger one. Find the area and perimeter of the shaded portion of the figure. $\left(\text{Take } \pi \text{ to be } \frac{22}{7}.\right)$



✶3. Solve the simultaneous equations

$$3(x - 2) + \frac{y + 5}{2} = 2,$$
$$\frac{x + y}{2} + 3x = 7.$$

4. (a) Factorise $4x^2 + 8xy + 4y^2$ completely.

(b) Given that $x^2 + y^2 = 15$ and $xy = 2$, find the value of $(x - y)^2$.

5. A wire is in the form of a circle of radius 14 cm. If the wire is bent into the shape of a square, find

(a) the perimeter of the square,

(b) the area of the square.

$\left(\text{Take } \pi \text{ to be } 3\frac{1}{7}.\right)$

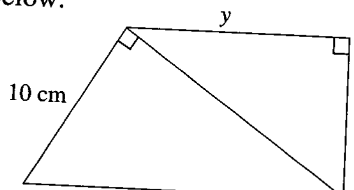
6. Solve the following equations:

(a) $3x^3 - 10x^2 - 8x = 0$

(b) $(2x - 4)(3x + 1) = 12$

7. (a) A ladder 5 m long leans against a vertical wall. Its foot is 1 m away from the base of the wall. What height does the ladder reach?

(b) Calculate the length y in the figure below.



8. Copy and complete the following table for $y = x^2 - 2x - 4$.

x	−3	−2	−1	0	1	2	3	4	5
y		4	−1	−4			−1		11

Using a scale of 2 cm to represent 1 unit on the x-axis, and 2 cm to represent 2 units on the y-axis, plot the graph of $y = x^2 - 2x - 4$ for $-3 \leqslant x \leqslant 5$. Use your graph to answer the following questions:

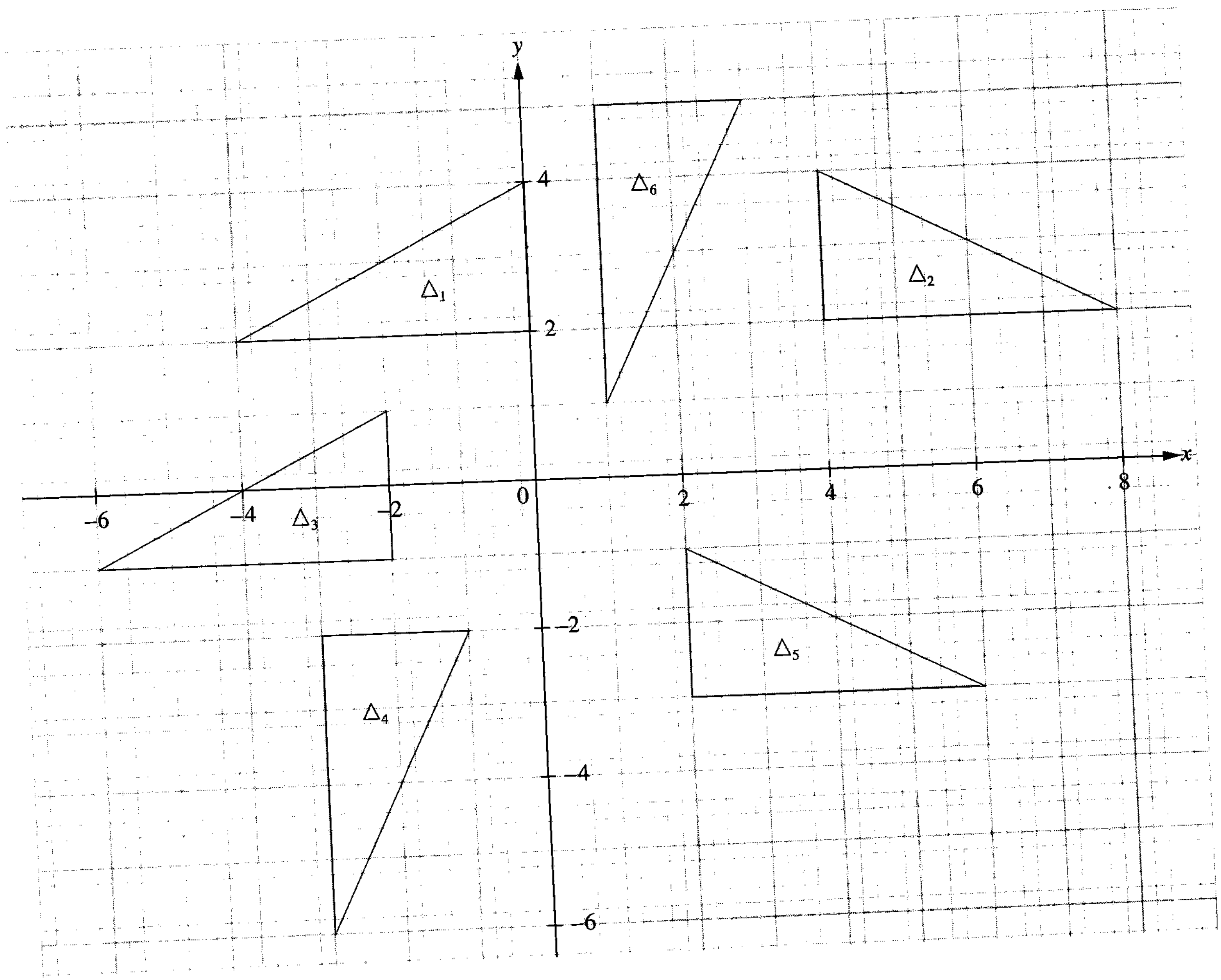
(a) What is the value of y when $x = 1.4$?
(b) What are the values of x when $y = 6$?

9. A car uses 2 litres of petrol for every 25 km it travels. Draw a graph to show the relationship between the distance travelled (d) and the number of litres of petrol used (l). Use your graph to find

(a) the amount of petrol used after travelling 75 km,
(b) the distance travelled after 5 litres of petrol had been used.

10. The diagram below shows congruent triangles. State whether each of the following is true or false.

(a) Δ_1 can be mapped onto Δ_5 by a translation.
(b) Δ_2 can be mapped onto Δ_3 by a translation.
(c) Δ_4 can be mapped onto Δ_6 by a translation.
(d) Δ_4 can be mapped onto Δ_5 by a rotation whose centre is at the origin.
(e) Δ_3 can be mapped onto Δ_5 by a reflection.
(f) Δ_2 is the image of a reflection of Δ_1.
(g) Δ_2 can be mapped onto Δ_4 by a reflection.
(h) Δ_6 can be mapped onto Δ_4 by a translation.
(i) Δ_3 can be mapped onto Δ_5 by an enlargement.



CHAPTER 13

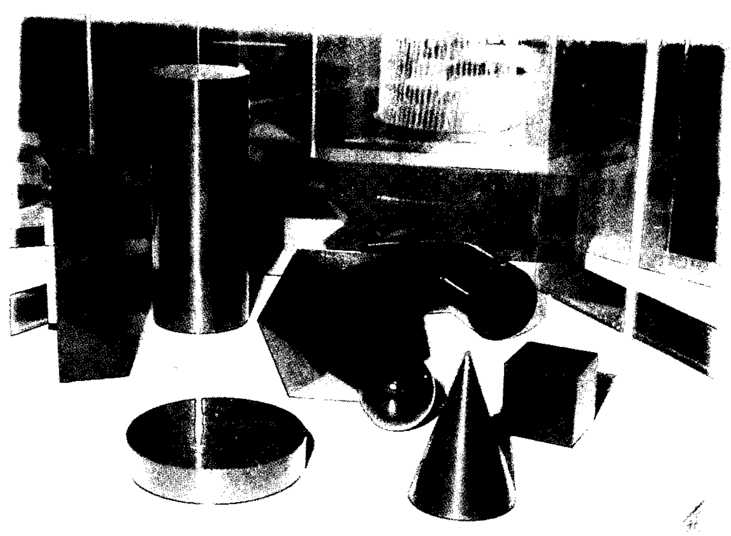
Mensuration

In this chapter, you will learn how to

- find the arc length of a circle by expressing the arc length as a fraction of the circumference of the circle;
- find the area of the sector of a circle by expressing sector area as a fraction of the area of the circle;
- find the area of the segment of a circle;
- find the volume and surface area of a sphere, a pyramid and a cone;
- solve problems involving arc length, sector area of a circle and segment area, and volume and surface area of a sphere, pyramid and a cone.

Preliminary Problem

The geometrical shapes in the picture are common around us. We shall learn how to calculate their surface areas and volumes in this chapter.



Area and Circumference of a Circle

As we learned before, the **area of a circle** = πr^2 and the **circumference of a circle** = $2\pi r$, where r is the radius of the circle.

Note: In the following examples and exercises, where π is not specified, use the value stored in the calculator.

Find out about some of the methods which have been used to calculate the value of π. Write a short essay about your findings.

Example 1

Find the area of a circle whose circumference is 52π cm. Give your answer in terms of π.

Solution

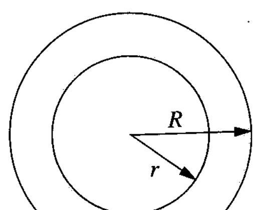
$$\text{Circumference} = 2\pi r = 52\pi$$
$$2r = 52$$
$$r = 26 \text{ cm}$$
$$\text{Area} = \pi r^2$$
$$= \pi(26^2)$$
$$= 676\pi \text{ cm}^2$$

$\therefore$ the area of the circle is 676π cm^2.

William Shanks (1873) spent more than 15 years trying to find the exact value of π. He calculated π to 707 digits! However, his result was accurate only for the first 527 digits.

Example 2

Find the area of the washer shown, given that its outside diameter is 6.4 cm and the diameter of the hole is 3.6 cm. $\left(\textit{Take } \pi \textit{ to be } \frac{22}{7}.\right)$



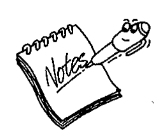
Solution

External radius of ring, $R = \frac{6.4}{2} = 3.2$ cm

Internal radius of ring, $r = \frac{3.6}{2} = 1.8$ cm

$\therefore$ the area of the washer $= \pi R^2 - \pi r^2$

$$= \pi(3.2)^2 - \pi(1.8)^2$$
$$= \pi(3.2^2 - 1.8^2)$$
$$= \pi(7)$$
$$= \frac{22}{7} \times 7$$
$$= 22 \text{ cm}^2$$



π is placed outside the brackets and its numerical value substituted only in the last part of the working.

Exercise 13a

In the following questions, take π to be $\frac{22}{7}$ unless otherwise stated.

1. Below are the areas of some circles. Find their diameters.

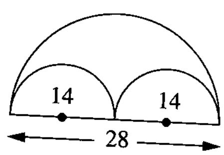
(a) 616 mm^2 **(b)** $779\frac{5}{8}$ m^2
(c) 3 850 cm^2

2. Find the areas of the following rings whose external and internal diameters are

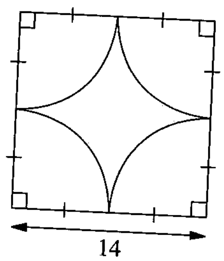
(a) 15 cm and 13 cm,
(b) 1.2 m and 0.9 m,
(c) 40 mm and 33 mm.

3. Find **(i)** the area,
(ii) the perimeter
of the shaded region in each of the following figures, all dimensions being in cm.
(All arcs are either quadrants or semi-circles.)

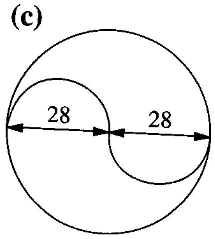
(a)



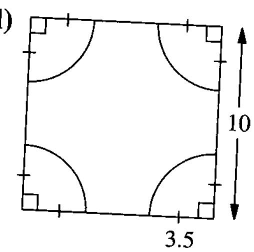
(b)



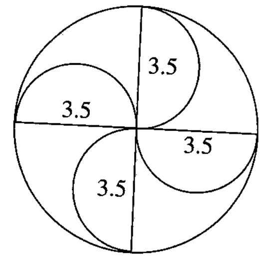
(c)



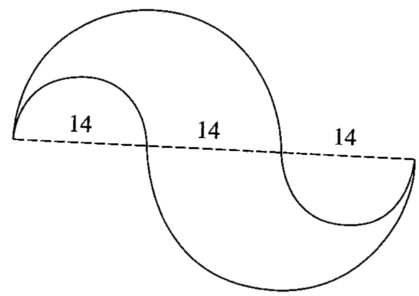
(d)



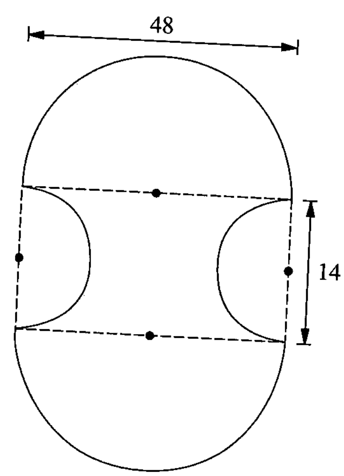
(b)



(c)

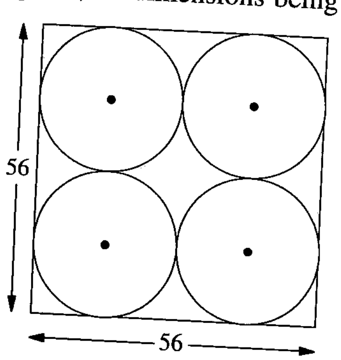


(d)



4. Find the area of the shaded parts in the following figures, all dimensions being in cm.

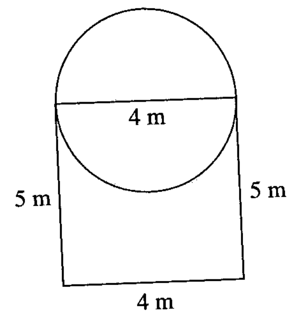
(a)



$\ast$5. The radius of a circular pond is 12 m. Grass grows around the pool, two metres away from its edge. Find, in terms of π, expressions for

(a) the circumference of the circle forming the outer edge of the grass,
(b) the area of the land between the pond and the outer edge of the grass.

6. John decides to paint this figure on his driveway in yellow with black lines. Find
   (a) the area of the shaded region that he will paint,
   (b) the total length that he will paint with black paint.



Length of Arc and Area of Sector

*A **chord** is a straight line joining two points on a curve.*

*An **arc** is a part of a continuous curve.*

*A **sector** is that part of a circle formed between two radii and the circumference.*

Fig. 13.1 shows a circle with centre O. The line PQ is called a **chord**.

PRQ which is part of the circumference is called an **arc**. The arc PRQ is called the **minor arc** and PSQ the **major arc**.

The shaded region enclosed by the chord PQ and the arc PRQ is called the **minor segment** and the unshaded region enclosed by the chord PQ and the arc PSQ is called the **major segment**.

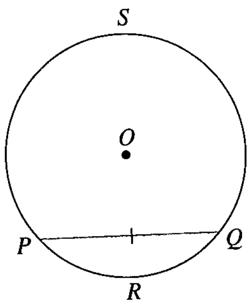


Fig. 13.1

Fig. 13.2 shows a circle with centre O. The part of a circle enclosed by any two radii of a circle and an arc is called a **sector**. The shaded region enclosed by the radii OA, OB and the minor arc AB is called a **minor sector** of the circle. The region enclosed by the radii OA, OB and the major arc ADB is called a **major sector** of the circle.

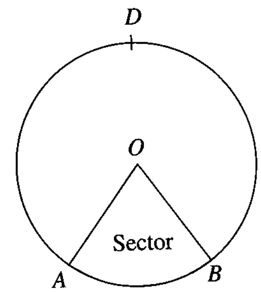


Fig. 13.2

In-Class Activity

You may work in pairs.

1. A **quadrant** is a sector having an angle of 90° at the centre of the circle. A quadrant is a **quarter** of a circle.

   By referring to Fig. 13.3, complete the following.

   (i) $\dfrac{\text{Length of arc } AB}{\text{Circumference of circle}} =$ __________.

   (ii) $\dfrac{\text{Area of sector } OAB}{\text{Area of circle}} =$ __________.

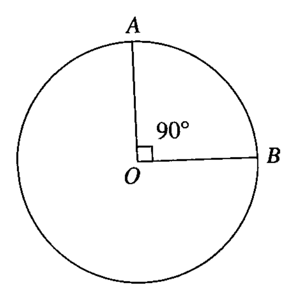


Fig. 13.3

We notice that $\frac{\text{Angle at centre subtended by arc } AB}{\text{Angle subtended by circumference}} = \frac{90°}{360°} =$ ________.

Thus, can we conclude the following?

(i) Length of arc $AB = \frac{90°}{360°} \times$ Circumference of the circle.

(ii) Area of sector $OAB = \frac{90°}{360°} \times$ Area of the circle.

2. A sector with an angle of 180° at the centre of the circle is a **semicircle**. (See Fig. 13.4.) A semicircle is **half** of a circle.

By referring to Fig. 13.4, complete the following.

(i) $\frac{\text{Length of arc } PRQ}{\text{Circumference of circle}} =$ ________.

(ii) $\frac{\text{Area of sector } PRQ}{\text{Area of circle}} =$ ________.

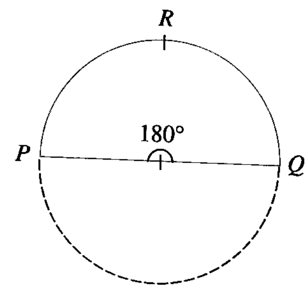


Fig. 13.4

We notice that $\frac{\text{Angle at centre subtended by arc } PRQ}{\text{Angle subtended by circumference}} = \frac{180°}{360°} =$ ________.

Thus, can we conclude the following?

(i) Length of arc $PRQ = \frac{180°}{360°} \times$ Circumference of the circle.

(ii) Area of sector $PRQ = \frac{180°}{360°} \times$ Area of the circle.

3. **(i)** Draw a circle and divide it into three equal sectors.
(ii) What is the angle at the centre of each sector?
(iii) Complete the following.

$\frac{\text{Length of arc of each sector}}{\text{Circumference of circle}} =$ ________. $\frac{\text{Area of each sector}}{\text{Area of circle}} =$ ________.

(iv) Thus, we can conclude that

length of arc of each sector $= \frac{x°}{360°} \times$ Circumference of the circle,

area of each sector $= \frac{x°}{360°} \times$ Area of the circle, where $x =$ ________.

In general, if the angle of a sector is $x°$ (see Fig. 13.5), then

$$\textbf{the length of its arc} = \frac{x°}{360°} \times \textbf{circumference}$$
$$= \frac{x}{360} \times 2\pi r,$$

$$\textbf{the area of its sector} = \frac{x°}{360°} \times \textbf{area of the circle}$$
$$= \frac{x}{360} \times \pi r^2.$$

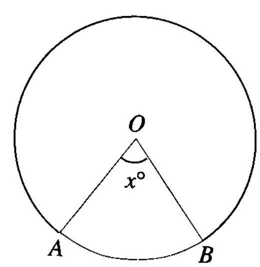
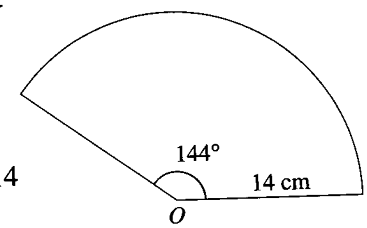


Fig. 13.5

Example 3

Find the area and the perimeter of a sector of a circle with radius 14 cm, given that the angle at the centre of the circle is 144°. $\left(\text{Take } \pi \text{ to be } \frac{22}{7}.\right)$

Solution



Area of the sector $= \frac{144}{360} \times \pi r^2$

$= \frac{144}{360} \times \pi \times 14 \times 14$

$= \frac{2}{5} \times \frac{22}{7} \times 14 \times 14$

$= 246.4 \text{ cm}^2$

Length of the arc $= \frac{144}{360} \times 2\pi r$

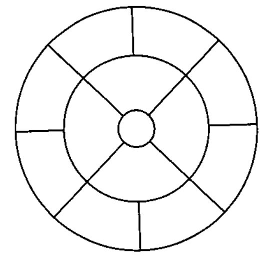
$= \frac{144}{360} \times 2 \times \frac{22}{7} \times 14$

$= 35.2$ cm

$\therefore$ perimeter of the sector $= 35.2 + 14 + 14$

$= 63.2$ cm

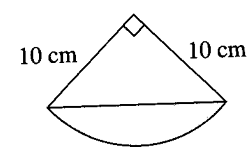
Just for Fun



This diagram shows a map consisting of 13 regions. Find the least number of colours required to colour the map such that the regions with a common border are not coloured with the same colour.

Example 4

The figure shows a quadrant of a circle of radius 10 cm. Find the area of the shaded region and the perimeter of the sector. Give your answer correct to 2 decimal places.



Solution

Area of the quadrant $= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \pi \times 10^2$

$= 78.54 \text{ cm}^2$ (correct to 2 decimal places)

Area of the triangle $= \frac{1}{2} \times 10 \times 10$

$= 50 \text{ cm}^2$

$\therefore$ area of the shaded region $= (78.54 - 50) \text{ cm}^2$

$= 28.54 \text{ cm}^2$

Length of the arc $= \frac{1}{4} \times 2\pi r$

$= \frac{1}{4} \times 2 \times \pi \times 10$

$= 15.71$ cm (correct to 2 decimal places)

$\therefore$ perimeter of the sector $= 15.71 + (2 \times 10)$

$= 35.71$ cm

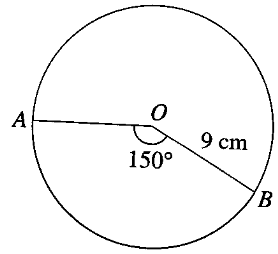
Notes

*The shaded region is called a **segment** of the circle.*

Example 5

In the given figure, O is the centre of a circle of radius 9 cm and $A\hat{O}B = 150°$. Taking π to be $\frac{22}{7}$, find

(a) the length of the major arc,
(b) the area of the major sector.



Solution

(a) Length of the major arc $= \frac{360° - 150°}{360°} \times 2\pi r$

$= \frac{210°}{360°} \times 2 \times \frac{22}{7} \times 9$

$= 33$ cm

(b) Area of the major sector $= \frac{360° - 150°}{360°} \times \pi r^2$

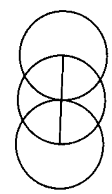
$= \frac{210°}{360°} \times \frac{22}{7} \times 9 \times 9$

$= 148\frac{1}{2}$ cm^2

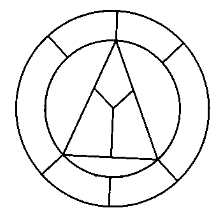
Just For Fun

Which of the two-dimensional figures below can be drawn in one continuous line without going over any part of the line twice, without intersecting the line and without taking the pencil off the paper?

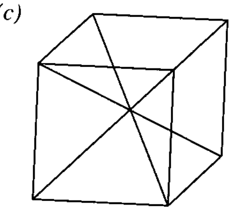
(a)



(b)



(c)



Example 6

(a) An arc of a circle of radius 21 cm has a length of 17.6 cm. Find the angle subtended at the centre of the circle. $\left(\text{Take } \pi \text{ to be } \frac{22}{7}.\right)$

(b) A sector of a circle of radius 12 cm has an area of 128 cm^2. Find the angle subtended at the centre of the circle, giving your answer correct to 1 decimal place.

Solution

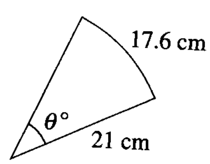
(a) Let the angle subtended at the centre of the circle be $\theta°$.

Length of arc $= \frac{\theta}{360} \times 2\pi r$

$17.6 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 21$

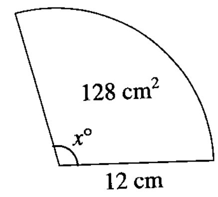
$\frac{11}{30}\theta = 17.6$

$\theta = 48$



$\therefore$ the angle subtended at the centre of the circle is 48°.

(b) Let the angle subtended at the centre of the circle be $x°$.



$$\text{Area of sector} = \frac{x}{360} \times \pi r^2$$

$$128 = \frac{x}{360} \times \pi \times 12 \times 12$$

$$\frac{2\pi}{5}x = 128$$

$$x = \frac{128 \times 5}{2 \times \pi}$$

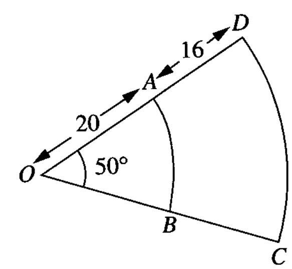
$= 101.9$ (correct to 1 decimal place)

$\therefore$ the angle subtended at the centre of the circle is $101.9°$.

Example 7

In the diagram, AB is an arc of a circle with centre O and radius 20 m. DC is an arc of a circle, centre O and radius 36 m. OAD and OBC are straight lines and $D\hat{O}C = 50°$. Taking π to be 3.142, calculate

(a) the perimeter of the shaded region ABCD,
(b) the area of the shaded region ABCD.



Solution

(a) Length of arc $AB = \frac{50°}{360°} \times 2 \times 3.142 \times 20$

$= 17.46$ m (correct to 2 decimal places)

Length of arc $DC = \frac{50°}{360°} \times 2 \times 3.142 \times 36$

$= 31.42$ m

$\therefore$ perimeter of the shaded region $ABCD = 17.46 + (2 \times 16) + 31.42$

$= 80.9$ m (correct to 1 decimal place)

(b) Area of sector $ODC = \frac{50°}{360°} \times 3.142 \times 36^2$

$= 565.56\ m^2$

Area of sector $OAB = \frac{50°}{360°} \times 3.142 \times 20^2$

$= 174.56\ m^2$ (correct to 2 decimal places)

$\therefore$ area of the shaded region $ABCD = 565.56 - 174.56$

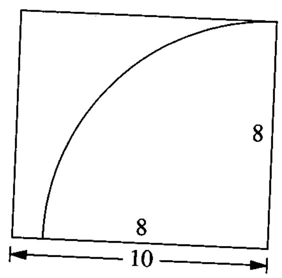
$= 391.0\ m^2$

Exercise 13b

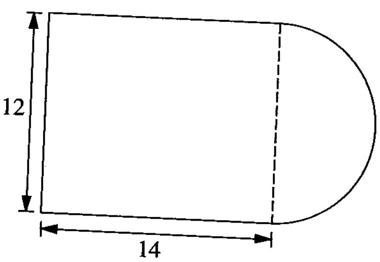
Take $\pi = \frac{22}{7}$ for this exercise unless stated otherwise. (All sectors are either quadrants or semi-circles.)

1. Find **(i)** the perimeter, **(ii)** the area of the shaded regions in the following diagrams. All dimensions are in cm. Take π to be 3.142 and give your answers correct to 1 decimal place.

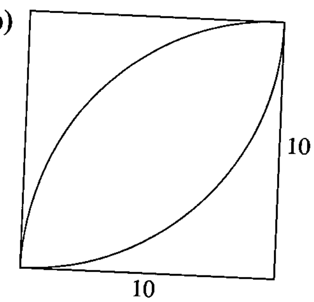
(a)



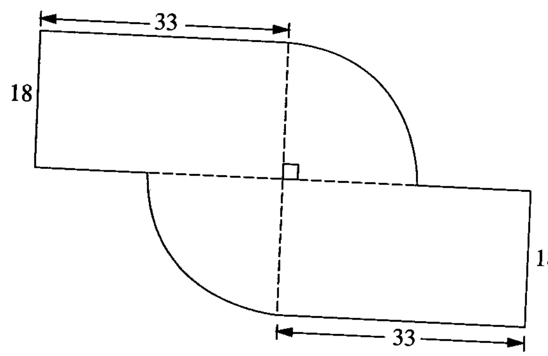
(c)



(b)

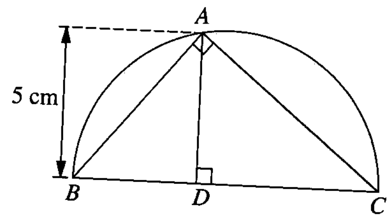


(d)



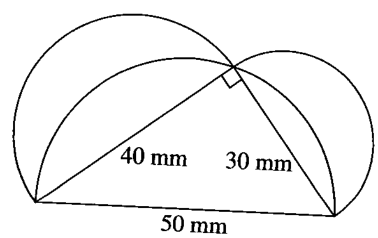
2. Calculate the area of the shaded region in each of the following figures. (Take $\pi = 3.142$ and give your answers correct to 1 decimal place.)

(a)

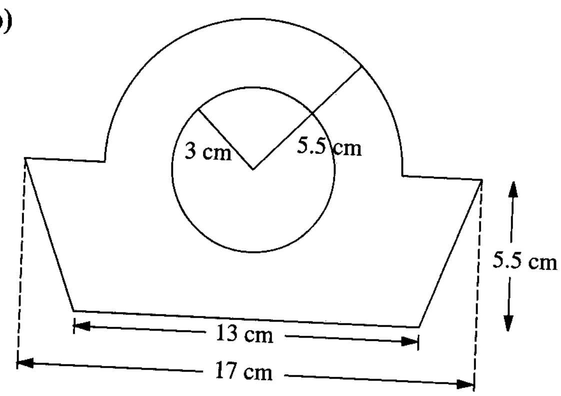


Area of $\triangle ABC = 25\ cm^2$

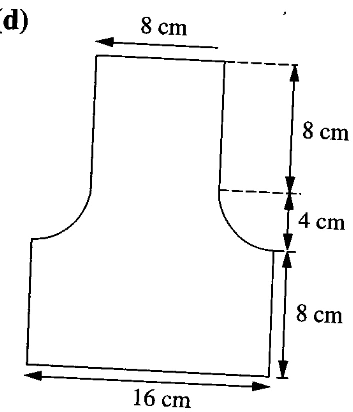
(c)



(b)

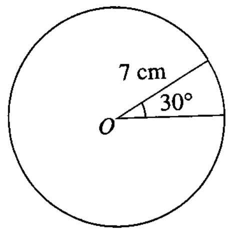


(d)

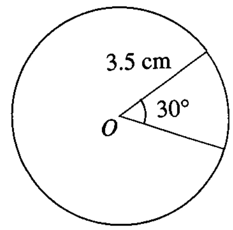


3. For each of the following circles, find **(i)** the perimeter, and **(ii)** the area of the minor sector.

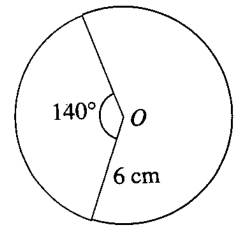
(a)



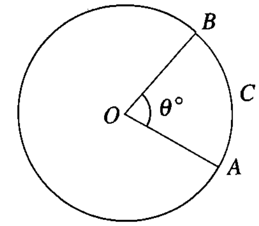
(b)



(c)



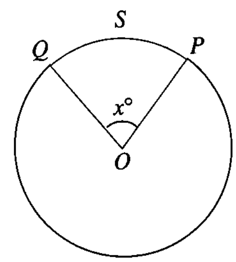
4. The figure shows a circle centre O with $A\hat{O}B = \theta°$. Given that the circumference of the circle is 88 cm, find



(a) the length of arc ACB,
(b) the area of sector $OACB$ when θ is

(i) 45, **(ii)** 60, **(iii)** 99,
(iv) 126, **(v)** 189, **(vi)** 216,
(vii) 288, **(viii)** 342.

5. Given that the area of the circle in the figure is 3 850 cm², find



(a) the area of sector $OPSQ$,
(b) the length of arc PSQ when x is

(i) 36, **(ii)** 54, **(iii)** 84,
(iv) 108, **(v)** 144, **(vi)** 198,
(vii) 252, **(viii)** 324.

6. The radius of a circle is 14 m. Calculate, taking π to be 3.14 and giving each answer correct to the nearest degree, the angle at the centre subtended by an arc of length

(a) 12 m, **(b)** 19.5 m, **(c)** 64.2 m, **(d)** 84.6 m.

7. The diameter of a circle is 18 cm. Calculate, giving each answer correct to the nearest degree, the central angle subtended by the arc of a sector of area

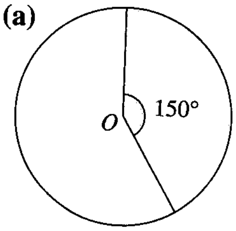
(a) 42.6 cm², **(b)** 117.2 cm², **(c)** 214.5 cm², **(d)** 18.9 cm².

8. Copy and complete the table below for sectors of a circle, giving your answers correct to the nearest whole number.

	Radius	*Angle at centre*	*Arc length*	*Area*	*Perimeter*
(a)	7 cm	72°			
(b)	35 mm				136 mm
(c)		270°		1 848 mm²	
(d)		150°	220 cm		
(e)	14 m		55 m		
(f)		75°		154 cm²	

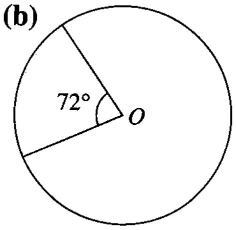
9. Find the radius of each of the following circles, giving your answer correct to the nearest cm. (Take π to be 3.142.)

(a)



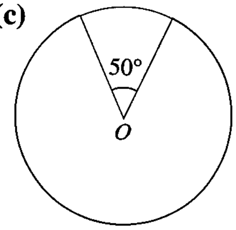
Area of minor sector = 114 cm²

(b)



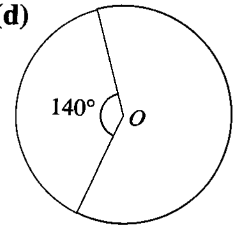
Area of minor sector = 15 cm²

(c)



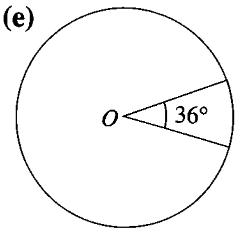
Length of minor arc = 3.06 cm

(d)



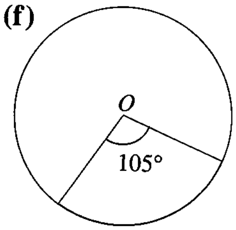
Length of minor arc = 29 cm

(e)



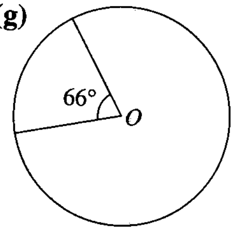
Length of major arc = 85 cm

(f)



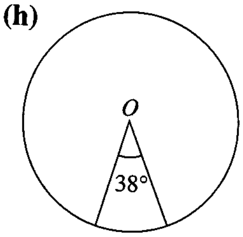
Length of major arc = 27 cm

(g)



Area of major sector = 369 cm²

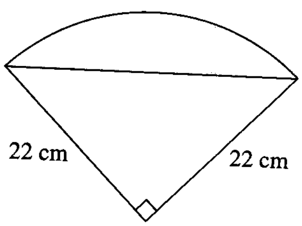
(h)



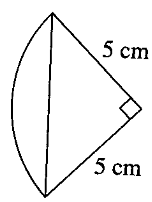
Area of major sector = 45 cm²

10. Find **(a)** the area of each triangle, **(b)** the shaded area of each of the following figures. (Take π to be 3.142.)

(i)



(ii)



∗11. In the diagram, AB is an arc of a circle with centre O and radius r cm. DC is an arc of a circle centre O and radius R cm. OAD and OBC are straight lines and $D\hat{O}C = m°$. Taking π to be 3.142, find

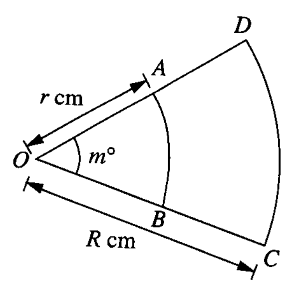
(a) the perimeter of the shaded region $ABCD$,

(b) the area of the shaded region $ABCD$, for each of the following cases:

(i) $r = 10$, $R = 20$, $m = 45$

(ii) $r = 5$, $R = 8$, $m = 120$

(iii) $r = 35$, $R = 49$, $m = 160$



∗12. A piece of wire 32 cm long is bent to form a sector of a circle of radius 6 cm. Find the angle subtended by the wire at the centre of the circle.

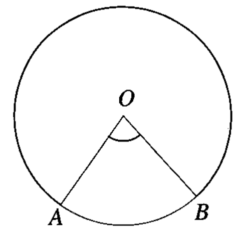
13. The minute hand of a clock is 9.8 cm long. How far does its tip move in 25 minutes?

14. The hour hand of a clock travels through an angle of 45°. If the hour hand is $1\frac{1}{2}$ m long, how far does its tip travel?

☆**15.** A small mat is in the shape of a 225°-sector of a circle with diameter 7 m. Find the area covered by the piece of mat.

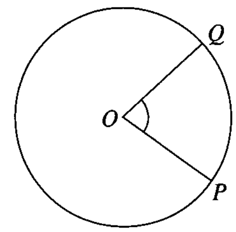
☆**16.** **(a)** In the given figure, the length of the minor arc is $\frac{7}{24}$ of the circumference of the circle. Find $A\hat{O}B$.

(b) Given that the circumference of the circle is 132 cm, find the length of the minor arc AB and the radius of the circle.

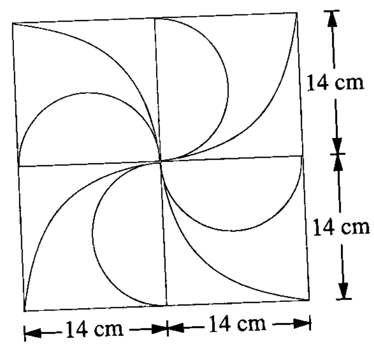


☆**17.** **(a)** In the given figure, the shaded sector POQ is $\frac{5}{18}$ of the area of the whole circle. Find $P\hat{O}Q$.

(b) If the area of the circle is 1 386 cm^2, find the area of the shaded sector POQ and the radius of the circle.



☆**18.** The entire figure shows a pattern made of straight lines, semicircular arcs and arcs of quadrants of circles. Calculate the total length of a metal strip required to make such a pattern.

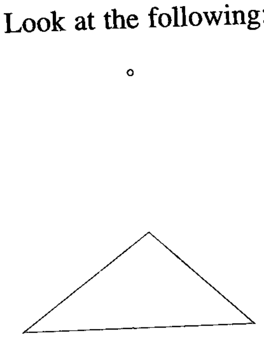


☆**19.** A kite is in the form of a sector of a circle of radius 12 cm. If the perimeter of the kite is 38 cm, calculate the area of the paper used in making the kite.

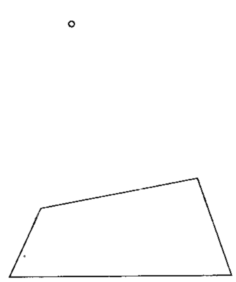
☆**20.** A piece of string 25 cm long goes exactly once round the edge of a plate in the form of a quadrant. Calculate the area of the quadrant.

Pyramids

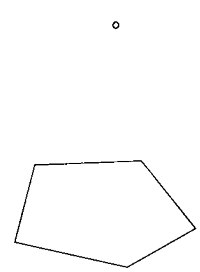
Look at the following:



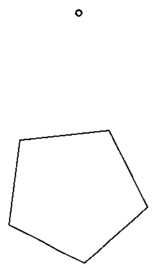
(A) Triangle



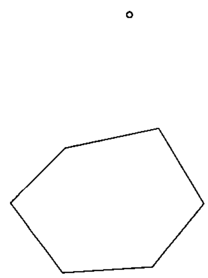
(B) Quadrilateral



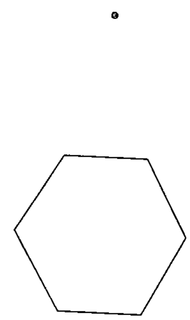
(C) Pentagon



(D) Regular Pentagon

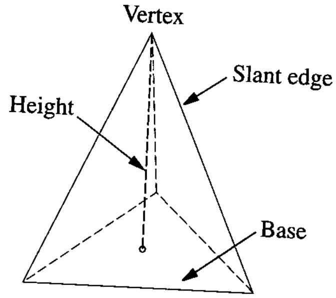


(E) Hexagon

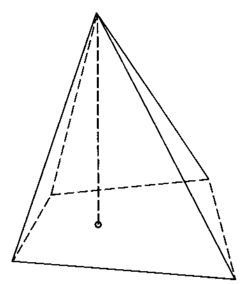


(F) Regular Hexagon

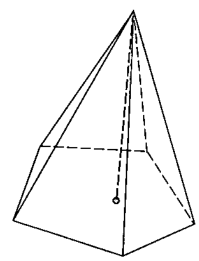
Each polygon shown above is in a plane and the point above the polygon is not located on the same plane. When the corners of the polygon are joined to the point, the resulting figures are called **pyramids**, as shown below.



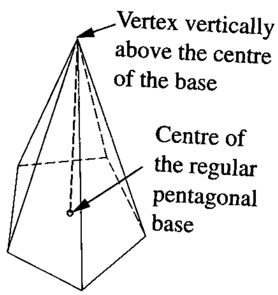
(a)



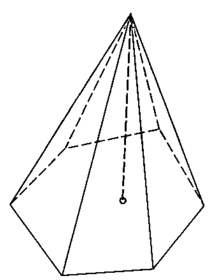
(b)



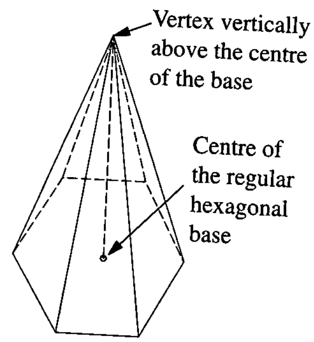
(c)



(d)



(e)



(f)

In other words, a **pyramid** is a solid figure in which one of the faces, the base, is a polygon and the others are triangles which meet at the vertex.

The polygonal region is called the **base** of the pyramid and the point is called the **vertex**. The **height** of the pyramid is the **perpendicular distance** from the vertex to the base of the pyramid. If the pyramid has a regular polygonal base and its vertex is vertically above the centre of the base, then the pyramid is a **right pyramid** (also known as **regular pyramid**). The pyramids (d) and (f) shown above are right pyramids, while pyramids (a), (b), (c) and (e) are not. Why? What kind of triangular base pyramid (a) must have for it to be a right pyramid?

The edges joining the vertex to the corners of the base of a pyramid are called the **slant edges**. In a right pyramid, the slant edges are equal in length.

A triangular pyramid, i.e., a pyramid whose base is a triangle, is also called a **tetrahedron**.

Volume of a Pyramid

Fig. 13.6(a) is a cube of side $2a$ units. The centre of the cube, O, is joined to the corners. The cube is now divided into six regular pyramids, each with one face of the cube as its base and a height of a units. Fig. 13.6(b) shows one such pyramid.

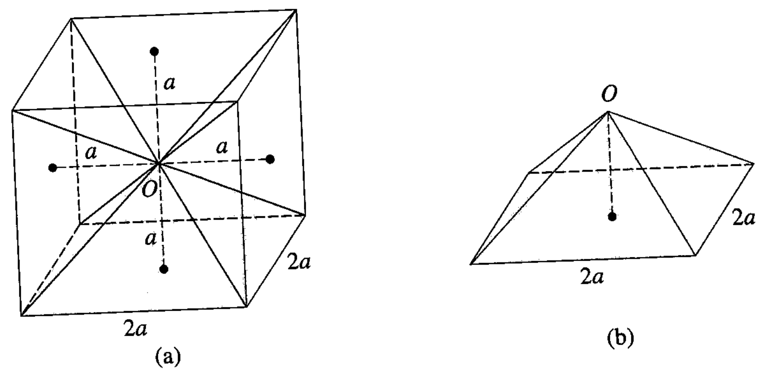


Fig. 13.6

Let the volume of each pyramid be V.

$$\text{Volume of cube} = (2a)^3$$
$$6V = (2a)^3$$
$$V = \frac{1}{6}(2a)^2 \times 2a$$
$$= \frac{1}{3}(2a)^2 \times a$$

Area of square base = $(2a)^2$

Height of pyramid = a

$$\therefore \quad V = \frac{1}{3} \times \text{area of base} \times \text{height}$$

Hence, the volume of a pyramid with a square base = $\frac{1}{3}$ × area of base × height. Is the above formula valid for every pyramid irrespective of the shape of its base?

In-Class Activity

You may carry out the activities together with a classmate.

1. Cut out a cardboard as shown in Fig. 13.7(a) and fold along the dotted lines to obtain an open tetrahedron, as shown in Fig. 13.7(b). Measure its height h.

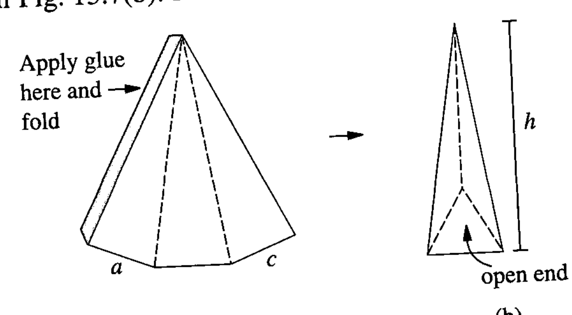


Fig. 13.7

2. Cut out another cardboard as shown in Fig. 13.8(a) and fold along the dotted lines to get a triangular prism with one open end, as shown in Fig. 13.8(b). Ensure that the hollow prism and pyramid you have made have the same base and the same height h.

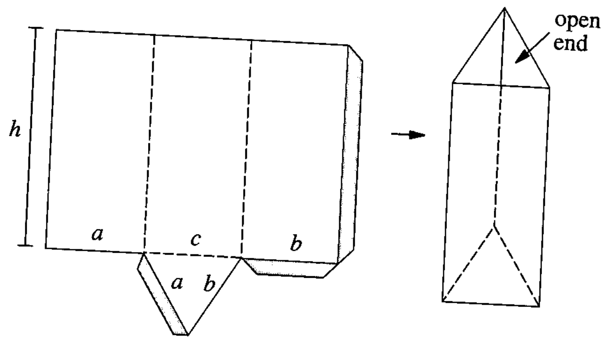


Fig. 13.8

3. Now, turn the pyramid upside down and fill it with sand. Pour the sand into the prism. How many times must you do this before the prism is completely filled?
4. It can be shown in general that the volume of a pyramid is one-third of the volume of a prism with the same base and height.
5. Copy and complete the following:
   Volume of a prism = _______ × _______
   Volume of a pyramid with the same base and height as the prism = _______ × _______ × _______

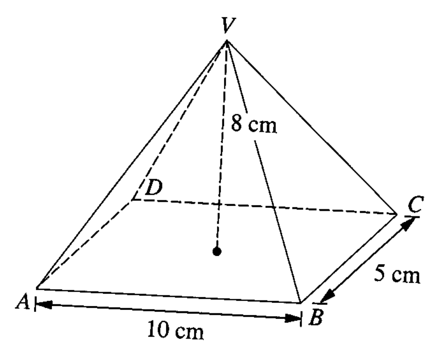
The above suggest that the formula

$$\textbf{volume of pyramid} = \frac{1}{3} \times \textbf{area of base} \times \textbf{height}$$

is valid for every pyramid irrespective of the shape of its base.

Example 8

The figure shows a pyramid with a rectangular base ABCD. Given that AB = 10 cm, BC = 5 cm and that the vertex V is 8 cm vertically above the centre of the base, calculate the volume of the pyramid.



Solution

The volume of the pyramid $= \frac{1}{3} \times \text{area of base} \times \text{height}$

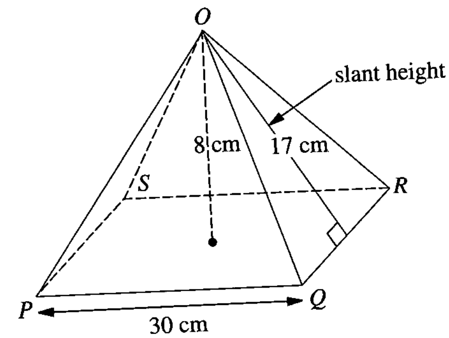
$= \frac{1}{3} \times (10 \times 5) \times 8$

$= 133\frac{1}{3}\ \text{cm}^3$

Example 9

OPQRS is a pyramid whose base is a square of sides 30 cm each. The height of the pyramid is 8 cm and its slant height is 17 cm. Find

(a) the volume of the pyramid,
(b) the total surface area of the pyramid.



Solution

(a) Volume of the pyramid $= \frac{1}{3} \times$ area of base $\times$ height

$= \frac{1}{3} \times 30 \times 30 \times 8$

$= 2\ 400\ cm^3$

(b) Area of $\triangle OQR = \frac{1}{2} \times 30 \times 17 = 255\ cm^2$

Area of the four faces $= 4 \times 255 = 1\ 020\ cm^2$

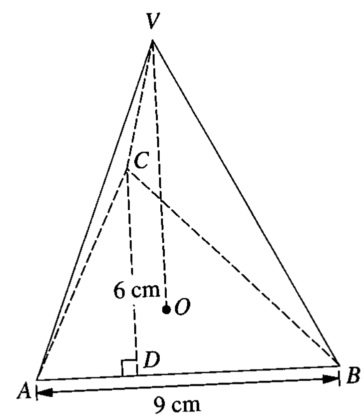
Total surface area of the pyramid = area of base + area of four faces

$= (30 \times 30) + 1\ 020$

$= 1\ 920\ cm^2$

Example 10

VABC is a tetrahedron. Given that AB = 9 cm, CD = 6 cm and the volume of the tetrahedron is 63 cm^3, find the height of the pyramid.



Solution

Area of base = area of $\triangle ABC$

$= \frac{1}{2} \times 9 \times 6$

$= 27\ cm^2$

Volume of pyramid $= \frac{1}{3} \times$ area of base $\times$ height

$63 = \frac{1}{3} \times 27 \times VO$

$9VO = 63$

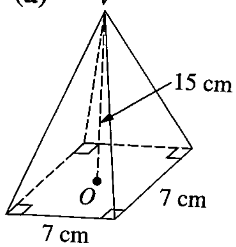
$VO = 7$

$\therefore$ the height of the pyramid is 7 cm.

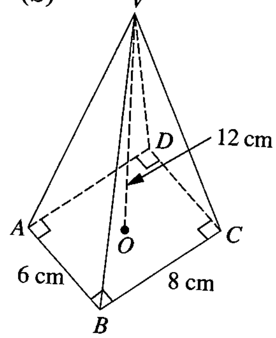
Exercise 13c

1. Calculate the volume of each of the following pyramids where VO is the height.

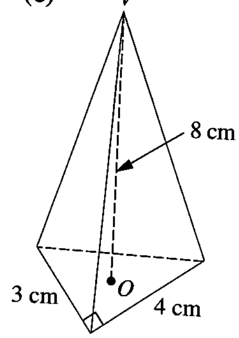
(a)



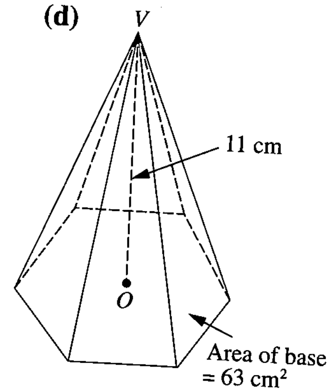
(b)



(c)



(d)



2. The height of the Great Pyramid of Egypt is 150 m and the base is a square of sides 250 m each. Find the volume of the pyramid in m^3.

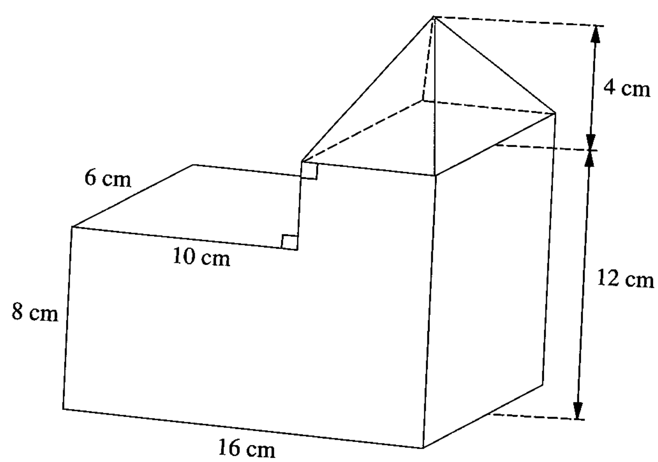
3. Given that the volume of a pyramid with a square base of sides 5 m each is 75 m^3, calculate the height of the pyramid.

4. The height of a pyramid with a square base is 12 cm and its volume is 100 cm^3. Find the length of a side of the square base.

5. A pyramid with a triangular base has a volume of 50 cm^3. If the base and height of the triangle are 5 cm and 8 cm respectively, calculate the height of the pyramid.

6. A solid pyramid of height 40 cm and with a square base of sides 30 cm each is put into a cubical tank of sides 40 cm each. The tank is then filled with water. If the pyramid is removed, find the depth of water in the tank.

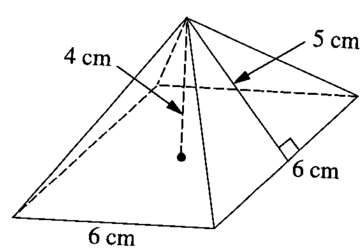
7. Find the volume of the solid shown on the right.



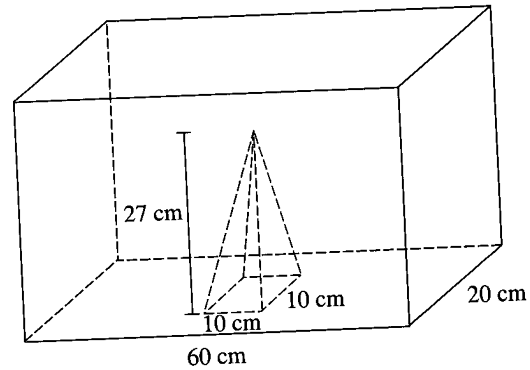
8. A pyramid has a square base of sides 6 cm each and four sides made of isosceles triangles each with a base of 6 cm and a height of 5 cm as shown in the figure. Calculate

(a) the volume,

(b) the total surface area of the pyramid.



9. A rectangular tank has a base 60 cm by 20 cm. A solid metal pyramid with a square base of sides 10 cm each and height 27 cm is placed inside the tank. The tank is then filled with water until it completely covers the pyramid. If the pyramid is removed, calculate the fall in the level of water in the tank.



10. A paper-weight in the form of a pyramid has a square base of sides 5 cm and a height of 3 cm. What is the weight of a dozen such paper weights if 1 cm^3 of nickel, of which they are made, weighs 8.8 g?

Circular Cones

In this chapter, we only study right circular cone.

A **cone** is a solid defined by a closed plane curve (forming the base) and a point (not on the same plane) called the vertex. When the base of a cone is a circle, it is called a circular cone. A right circular cone can be generated by the rotation of the right-angled triangle VOA (in Fig. 13.9) about VO, which represents the height of the cone. The base of the cone is a circle with radius OA. V is the vertex of the cone and VA is the slant height.

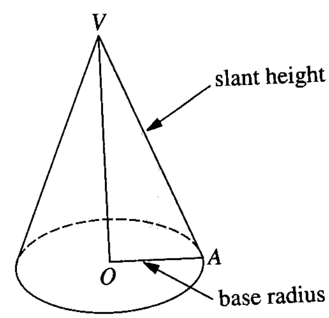


Fig. 13.9

Volume of a Cone

Fig. 13.10 shows that if the number of sides of a polygon increases infinitely, the polygon will finally become a circle.

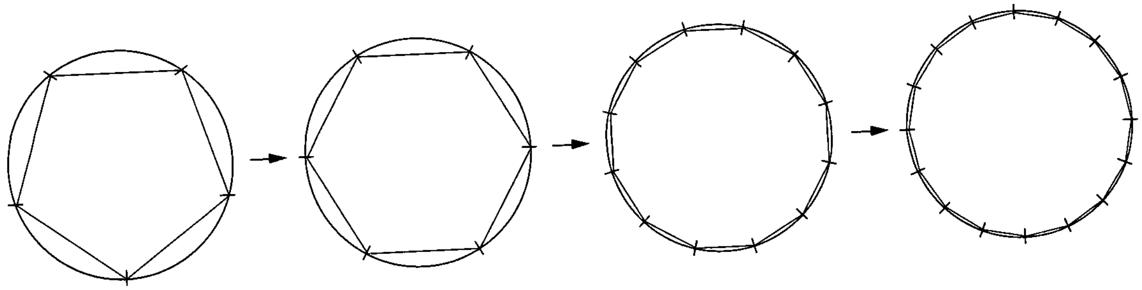


Fig. 13.10

Hence, a right pyramid will eventually become a circular cone if the number of sides of its polygonal base increases infinitely (see Fig. 13.11).

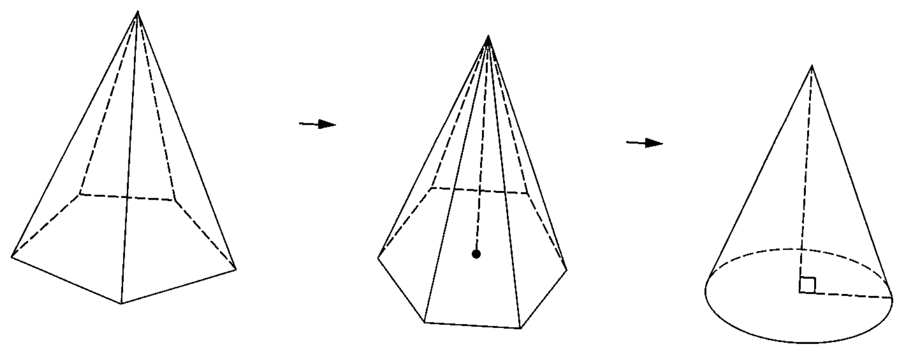


Fig. 13.11

A cone may therefore be considered as a special right pyramid with a circular base. Hence, its volume may also be given by

$$\text{volume of a cone} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

If the cone has a base radius of r units and a height of h units, then

$$\textbf{volume of a cone} = \frac{1}{3}\pi r^2 h$$

Curved Surface Area of a Cone

If we cut along the slant edge VA of a paper cone such as in Fig. 13.12(a) and flatten it out, we will obtain a sector of a circle centre V and radius $VA = l$, as shown in Fig. 13.12(b).

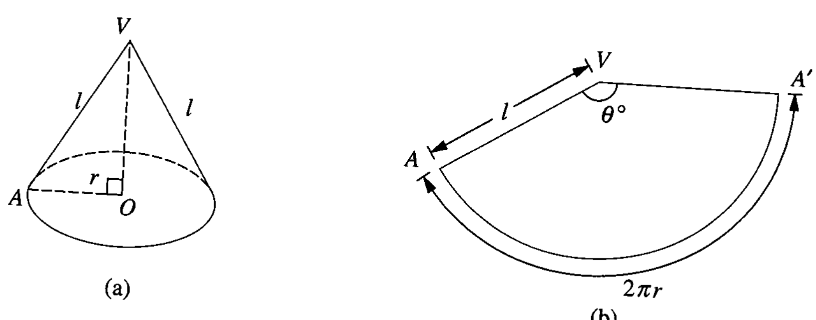


Fig. 13.12

The area of the sector is the area of the curved surface of the cone. The length of the arc AA' in Fig. 13.12(b) is equal to the circumference of the base circle of the cone, i.e., length of arc $AA' = 2\pi r$.

$$\frac{\text{Area of sector}}{\text{Area of circle with radius } l} = \frac{\theta^\circ}{360^\circ}$$

$$\frac{\text{Arc length}}{\text{Circumference}} = \frac{\theta^\circ}{360^\circ}$$

$$\frac{2\pi r}{2\pi l} = \frac{\theta^\circ}{360^\circ}$$

Thus, $$\frac{\text{Area of sector}}{\pi l^2} = \frac{2\pi r}{2\pi l}$$

$$\text{Area of sector} = \frac{2\pi r}{2\pi l} \times \pi l^2$$

$$= \pi r l$$

$\therefore$ area of curved surface of cone = πrl where r is the radius of the base and l is the slant height of the cone.

Alternatively, we may consider the curved surface of the cone to be made up of a large number of small sectors of a circle with radius l where l is the slant height of the cone (see Fig. 13.13(a)).

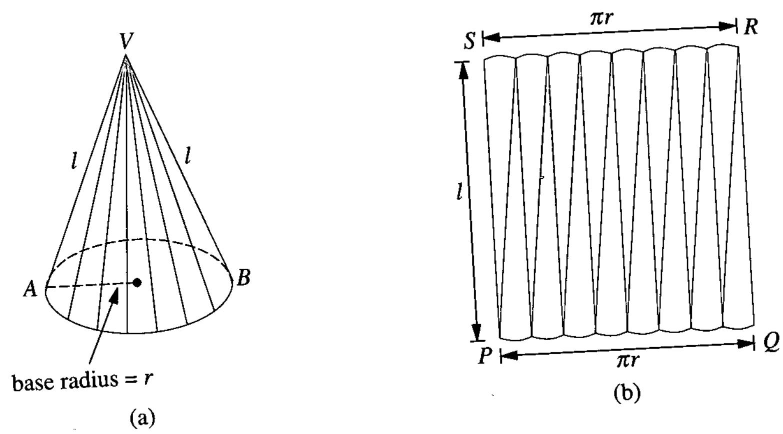


Fig. 13.13

Suppose these sectors are rearranged as shown in Fig. 13.13(b). If the number of small sectors increases infinitely, the figure $PQRS$ will eventually become a rectangle with the base equal to half the circumference of the base of the cone and the height equal to the slant height of the cone.

Thus, the curved surface area of a cone = the sum of the areas of the sectors
$= (\pi r)(l)$
$= \pi rl$

Therefore,

the curved surface area of a cone = πrl

the total surface area of a cone = $\pi rl + \pi r^2$
$= \pi r(l + r)$

Example 11

A cone has a circular base of radius 14 cm, a height of 48 cm and a slant height of 50 cm. Calculate

(a) the total surface area,

(b) the volume of the cone. $\left(\textit{Take } \pi \textit{ to be } \frac{22}{7}.\right)$

Solution

(a) Total surface area of cone $= \pi r(l + r)$

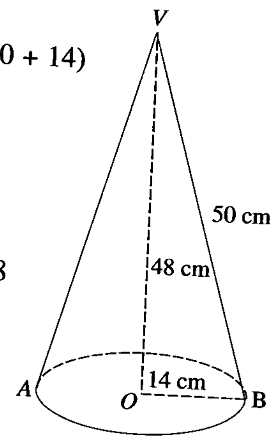
$= \frac{22}{7} \times 14 \times (50 + 14)$

$= 2\ 816 \text{ cm}^2$

(b) Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 48$

$= 9\ 856 \text{ cm}^3$



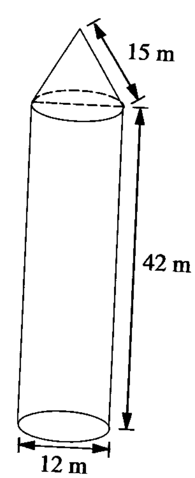
Just For Fun

A cylindrical plastic pail is approximately half-filled with water. Find out how you can determine with certainty whether the amount of water is greater than or lesser than half the capacity of the pail.

Example 12

The figure shows a rocket in the form of a closed cylinder. A cone of the same base radius is attached to the top of the cylinder. Calculate the total surface area of the rocket. $\left(\textit{Take } \pi \textit{ to be } \frac{22}{7}.\right)$

Solution



Area of base of cylinder $= \pi r^2 = \frac{22}{7} \times 6 \times 6 = 113\frac{1}{7} \text{ m}^2$

Area of curved surface of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 6 \times 42$

$= 1\ 584 \text{ m}^2$

Curved surface area of cone $= \pi rl$

$= \frac{22}{7} \times 6 \times 15 = 282\frac{6}{7} \text{ m}^2$

$\therefore$ total surface area of the rocket $= 113\frac{1}{7} + 1\ 584 + 282\frac{6}{7}$

$= 1\ 980 \text{ m}^2$

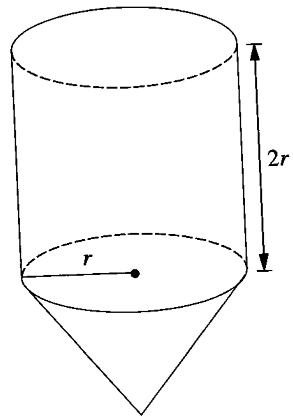
Example 13

The following container is made of a hollow cone of internal radius r cm and a right circular cylinder of the same internal radius and height 2r cm. Given that the height of the cone is three-quarters that of the cylinder and 2.7 litres of water is needed to completely fill the conical part of the container, calculate

(a) the amount of water, in litres, needed to completely fill the container,

(b) the total height of the container.

(Take π to be 3.142.)



Solution

$$\text{Height of cone} = \frac{3}{4} \times \text{height of cylinder}$$
$$= \frac{3}{4} \times 2r = \frac{3}{2}r \text{ cm}$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 \times \frac{3}{2}r$$
$$2\,700 = \frac{1}{2}\pi r^3$$
$$\pi r^3 = 5\,400$$
$$r = \sqrt[3]{\frac{5\,400}{3.142}}$$
$$= 11.98 \text{ cm} \quad \text{(correct to 2 decimal places)}$$

(a) $\text{Volume of cylinder} = \pi r^2 \times 2r$
$= 2\pi r^3 = 2 \times 5\,400$
$= 10\,800 \text{ cm}^3$
$= 10.8$ litres (correct to 3 sig. fig.)

∴ the amount of water needed to completely fill the container
$= 2.7 + 10.8 = 13.5$ litres

(b) The total height of the container $= \frac{3}{2}r + 2r$
$= \frac{7}{2}r$
$= \frac{7}{2} \times 11.98$
$= 41.93$ cm

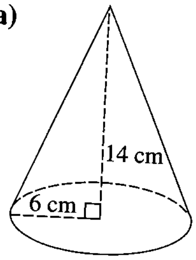
Archimedes was once entrusted with the task of finding out whether the King's crown was made of pure gold. While taking his bath one day, he came up with a solution and was so excited that he dashed out into the street naked shouting "Eureka!" ("I have found it!"). The container that you use in the Science laboratory to measure the volume of an irregular object is known as a Eureka can (aptly named after this incident). Archimedes was so engrossed in his work that when his country was conquered by the Romans, he was still working hard at his mathematics. When a Roman soldier ordered him to leave his desk, Archimedes replied, "Don't disturb my circles." He was killed by that soldier for disobeying orders.

Archimedes' greatest contribution was the discovery that the volume of a sphere is $\frac{2}{3}$ that of a cylinder whose diameter and height are the same as the diameter of the sphere. At his request, a diagram of a sphere in a cylinder like the one in Fig. 13.15(c) was engraved on his tombstone.

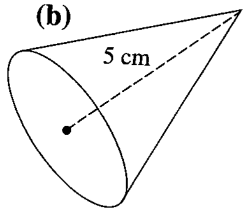
Exercise 13d

1. Find the volume of each of the following cones.

(a)

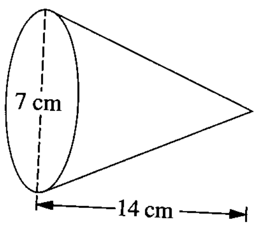


(b)

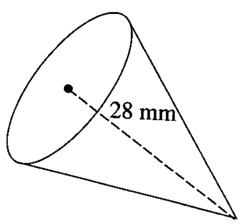


Area of base = 15 cm²

(c)

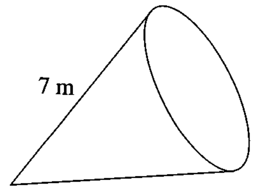


(d)



Circumference of base = 132 mm

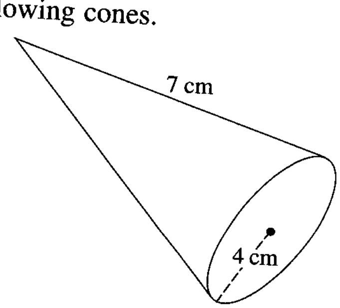
(d)



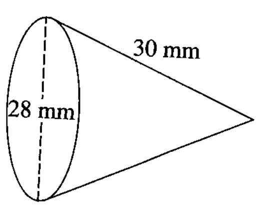
Area of base = 198 m²

2. Find the curved surface area of each of the following cones.

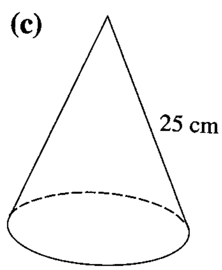
(a)



(b)



(c)



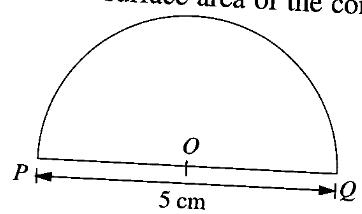
Circumference of base = 132 cm

3. Copy and complete the table below for a right circular cone of base radius r cm, height h cm and slant height l cm.

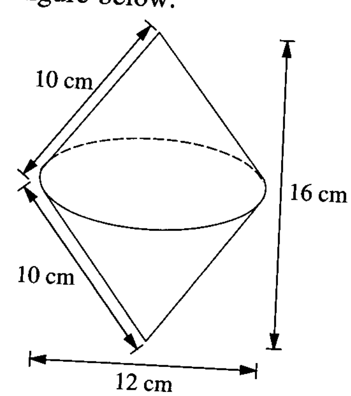
$\left(\text{Take } \pi \text{ to be } \frac{22}{7}.\right)$

	r	h	l	*Base Area (cm²)*	*Volume (cm³)*	*Curved Surface Area (cm²)*	*Total Surface Area (cm²)*
(a)	7	24	25				
(b)	6		10		96π		
(c)		15	17	64π			
(d)	15				$1\,500\pi$		600π

∗4. The semicircle shown is folded to form a right circular cone so that the arc PQ becomes the circumference of the base. Find
- **(a)** the diameter of the base,
- **(b)** the curved surface area of the cone.

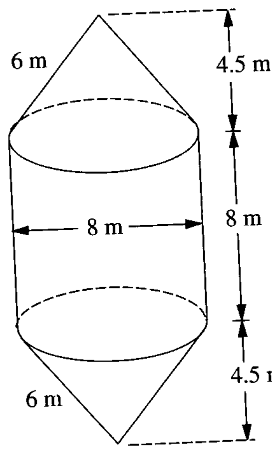


∗5. Find **(a)** the volume,
(b) the total surface area of the solid with conical ends as shown in the figure below.



★6. A conical block of silver has a height of 16 cm and a base radius of 12 cm. How many coins $\frac{1}{6}$ cm thick and $1\frac{1}{2}$ cm in diameter can be made from it?

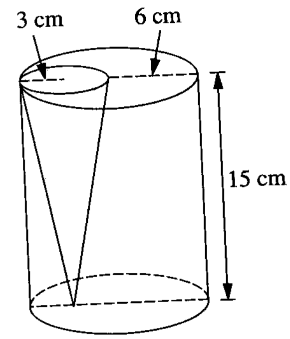
7. Calculate
   (a) the total surface area,
   (b) the volume of the solid cylinder with conical ends as shown below.



8. A solid cone has a circular base of diameter 10 cm and a total surface area of 285 cm^2. Find its slant height.

9. A conical funnel of diameter 23.2 cm and depth 42 cm is full of water. If the water is poured into a cylindrical tin of diameter 16.2 cm, find the least possible height of the tin if it must contain all the liquid.

★10. The figure below shows a cylinder of diameter 12 cm and height 15 cm. A hole in the shape of a cone is bored into one of its ends. If the cone has a diameter equal to half of the diameter of the cylinder, find the volume of the remaining solid.

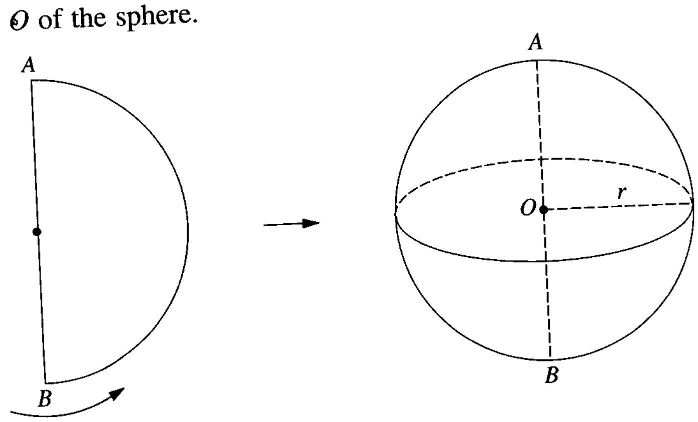


Spheres

Cut out a piece of cardboard in the shape of a semicircle as shown in Fig. 13.14(a). Rotate it using the diameter AB as the axis. A sphere as shown in Fig. 13.14(b) will be generated. If the radius of the sphere is r units, every point on the surface of the sphere will be r units from the centre O of the sphere.

For Your Information

*A **sphere** is a body or space bound by a surface where every point is equidistant from a point within called the centre.*



(a) (b)

Fig. 13.14

If a sphere is cut in half, the two portions are called **hemispheres**.

Volume of a Sphere

Fig. 13.15(a) shows a sphere of radius r units and Fig. 13.15(b) shows a cylinder of base radius r and height $2r$. The sphere is placed in the cylinder as shown in Fig. 13.15(c).

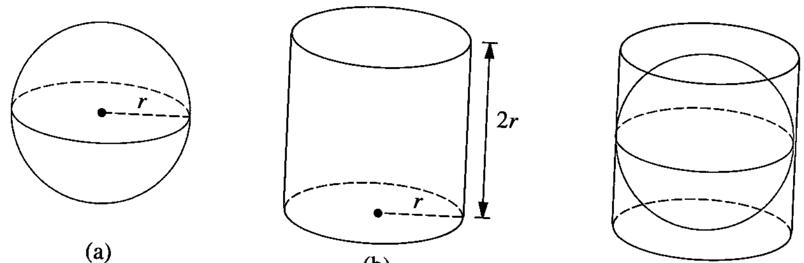


Fig. 13.15

If we fill the cylinder with water and then remove the sphere from the cylinder, it can be shown that the water fills up only $\frac{1}{3}$ of the cylinder.

Thus, the volume of the sphere is $\frac{2}{3}$ the volume of the cylinder having the same diameter as the sphere and height equal to the diameter.

Volume of the cylinder $= \pi r^2 h$

$= \pi r^2(2r)$

$= 2\pi r^3$

Volume of the sphere $= \frac{2}{3} \times 2\pi r^3$

$= \frac{4}{3}\pi r^3$

Volume of a sphere with radius r is given by

$$V = \frac{4}{3}\pi r^3$$

Investigate

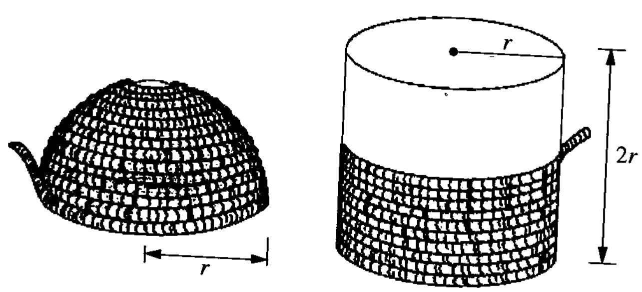
Two spherical watermelons of diameter 30 cm and radius 20 cm are sold for \$5 and \$10 respectively. Which is the better buy? Explain your answer.

Just for fun

A man travels 5 km north, 4 km west and then 5 km south and discovers that he is back to where he began his journey. Can you identify the place where his journey began?

Surface Area of a Sphere

Archimedes, a Greek mathematician (287–212 BC), discovered that the surface area of a sphere is the same as the curved surface area of a cylinder having the same diameter as the sphere and a height equal to the diameter. This can be shown by winding a length of rope round the sphere to cover half the sphere. The rope is then wound round the cylinder as shown on the right.



You will find that the rope covers exactly half the curved surface of the cylinder. Hence, the surface area of a sphere with radius r is equal to the curved surface area of a cylinder with radius r and height $2r$.

Curved surface area of cylinder $= 2\pi rh$

$= 2\pi r(2r)$

$= 4\pi r^2$

Surface area of a sphere with radius $r = 4\pi r^2$

Example 14

A solid sphere has a radius of 3 m. Calculate

(a) its volume, *(b) its surface area.*

Give your answer correct to the nearest whole number, taking π to be $\frac{22}{7}$.

Solution

(a) Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$

$= 113\frac{1}{7}$ m^3

$= 113$ m^3 (correct to the nearest whole number)

(b) Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 3 \times 3$

$= 113\frac{1}{7}$ m^2

$= 113$ m^2 (correct to the nearest whole number)

Example 15

Calculate the mass of 6 000 ball bearings, each of diameter 0.6 cm and made of steel of density 7.8 g/cm^3. Give your answer to the nearest gram, taking π to be $\frac{22}{7}$.

Solution

Volume of each ball bearing $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times \left(\frac{0.6}{2}\right)^3$ cm^3

Volume of 6 000 ball bearings $= \frac{4}{3} \times \pi \times \left(\frac{0.6}{2}\right)^3 \times 6\,000$

$= (8\,000 \times \pi \times 0.027)$ cm^3

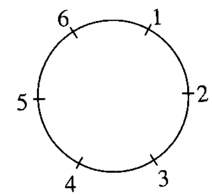
Mass = Volume × Density

$\therefore$ mass of 6 000 ball bearings $= 8\,000 \times \frac{22}{7} \times 0.027 \times 7.8$

$= 5\,295$ g

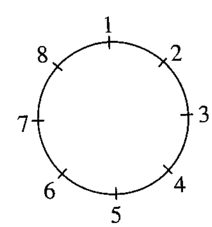
Investigate

1. *Draw a circle and divide its circumference into 6 equal parts. Label the points 1, 2, 3, 4, 5, 6 as shown in the diagram.*



Join 1 to 3, 2 to 4, 3 to 5 and so on until you get a polygon. What other shapes can you see from your diagram?

2. *Draw a circle and divide its circumference into 8 equal parts. Note that the angle subtended at the centre by any one part $= \frac{360°}{8} = 45°$.*



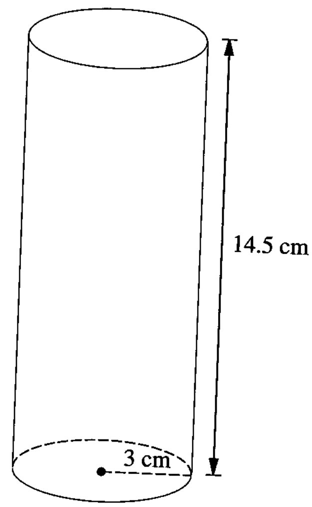
Label the points 1, 2, 3, 4, 5, 6, 7, 8 as shown in the diagram.

Join the points 1 to 3, 2 to 4, 3 to 5 and so on until a polygon is obtained. What other shapes can you see from your diagram?

Repeat the above steps but join points 1 to 4, 2 to 5, 3 to 6 and so on instead. What shapes have you obtained?

Example 16

The internal radius and height of the cylindrical vessel shown are 3 cm and 14.5 cm respectively. Taking π to be 3.142, show that the volume of water it can hold is approximately 410 cm^3. When 300 cm^3 of water is poured into the vessel and 16 identical spheres added, the level of water rises to the brim of the vessel. Assuming that no water is lost by spilling, calculate the radius of each sphere.



Solution

Volume of cylinder = $\pi r^2 h$

$= 3.142 \times 9 \times 14.5$

$= 410.031$

$= 410 \text{ cm}^3$ (correct to 3 significant figures)

Volume occupied by 16 spheres = $410 - 300$

$= 110 \text{ cm}^3$

Volume of 1 sphere = $\frac{110}{16} \text{ cm}^3$

$\frac{4}{3}\pi R^3 = \frac{110}{16}$ where R is the radius of each sphere.

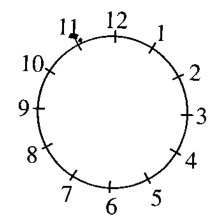
$R^3 = \frac{3 \times 110}{16 \times 4 \times 3.142}$

$R = 1.18 \text{ cm}$ (correct to 2 decimal places)

$\therefore$ the radius of each sphere is 1.18 cm.

3. Draw a circle and divide its circumference into 12 parts (angle subtended at centre between any two parts = $\frac{360°}{12}$ = 30°).

Label the points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.



Join points 1 to 4, 2 to 5, 3 to 6 and so on to obtain a polygon. Repeat the steps in each case but by joining the following points instead:

(a) 1 to 3, 2 to 4, 3 to 5
(b) 1 to 5, 3 to 7, 5 to 9 and so on.
(c) 1 to 5, 2 to 6, 3 to 7 and so on.
(d) 1 to 6, 2 to 7, 3 to 8 and so on.

What shapes do you get in each case?

Example 17

The figure shows a solid made up of a right circular cone fastened on top of a hemisphere of equal radius of 35 cm. Given that the volume of the cone is equal to $1\frac{1}{5}$ the volume of the hemisphere. Find

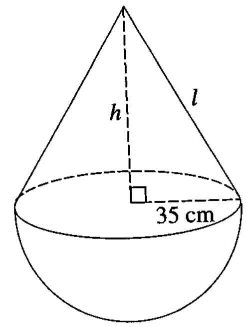
(a) the height of the cone,
(b) the total surface area of the solid in terms of π.

Solution

(a) Let the height of the cone be h cm.

Volume of cone = $\frac{1}{3}\pi(35)^2h$ cm³.

Volume of hemisphere = $\frac{2}{3}\pi(35)^3$ cm³.



$\therefore \quad \frac{1}{3}\pi(35)^2h = \frac{6}{5}\left(\frac{2}{3}\pi(35)^3\right)$

$$h = \frac{\frac{12}{15}\pi(35)^3}{\frac{1}{3}\pi(35)^2} = 84 \text{ cm}$$

(b) Let the slant height of the cone be l cm.

$\therefore \quad l = \sqrt{35^2 + 84^2} = 91$ cm.

Curved surface area of cone = $\pi(35)(91) = 3\ 185\pi$ cm²

Curved surface area of hemisphere = $2\pi(35)^2 = 2\ 450\pi$ cm²

$\therefore$ Total surface area of the solid = $3\ 185\pi + 2\ 450\pi$
$= 5\ 635\pi$ cm².

Exercise 13e

For this exercise, use the value of π stored in the calculator unless stated otherwise.

1. Find the volume and surface area of a sphere with the following radii.
(Take π to be 3.142.)
(a) 3.5 cm **(b)** 10 m **(c)** $\frac{3}{2}$ mm

2. Find the radius and volume of each of the spheres with the following surface areas. (Leave the answers in terms of π if necessary.)
(a) 144π cm² **(b)** $1\ 296\pi$ mm²
(c) 64π m² **(d)** 900π cm²

3. Calculate the radius and surface area of a sphere of volume 850 m³.

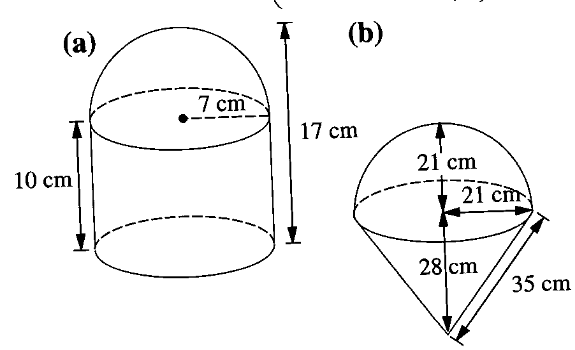
4. A basketball has a surface area of 1 810 cm². Find the radius of the basketball and the volume of air in it.

5. Find the volume of a sphere with the following surface areas:
(a) 49π cm² **(b)** 81π m²

6. Find the radii of Sphere A and Sphere B, given that
(a) Sphere A's volume = $15\frac{3}{16}\pi$ cm³,
(b) Sphere B's surface area = 64π m².

7. Find the volume and surface area of the following solids. $\left(\text{Take } \pi \text{ to be } \frac{22}{7}.\right)$

(a) **(b)**



★**8.** Fifty-four solid hemispheres, each of diameter 2 cm, are melted to form a single sphere. Find the radius of the sphere.

∗**9.** Find the number of steel ball bearings, each of diameter 0.7 cm, that can be made from 1 kg of steel, given that 1 cm^3 of steel weighs 7.8 g.

10. Calculate the mass of 5 000 spherical lead shots each of diameter 3 mm, given that 1 cm^3 of lead weighs 11.4 g.

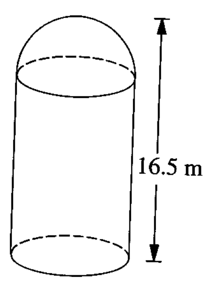
11. A metal container for storing grain is in the form of a circular cylinder. Its height is 8.6 m and its internal radius is 4.2 m. Calculate, giving your answers correct to 3 significant figures,

(a) the total area of the internal surface, excluding the base,
(b) the internal volume of the container.

12. A hollow metal sphere has an internal radius of 20 cm and an external radius of 30 cm. Given that the density of the metal is 7.8 g/cm^3, find the weight of the sphere. (Take π to be 3.14.)

13. A solid metal ball of radius 2 cm is melted and the metal obtained recast to form a solid right circular cone of radius 5 cm. Find the height of the cone.

14. A storage tank in the form of a cylinder has one hemispherical and one flat end. Given that the diameter of the cylinder is 4.7 m and the overall length of the tank is 16.5 m, find the capacity of the tank.



∗**15.** A cylindrical can has a horizontal base of radius 3.4 cm. It contains sufficient water so that when a sphere is placed inside, the water just covers the sphere. If the sphere fits exactly into the can, calculate

(a) the total surface area of the can in contact with the water when the sphere is inside, giving your answer correct to the nearest cm^2,
(b) the depth of the water in the can before the sphere was put in, giving your answer correct to the nearest mm.

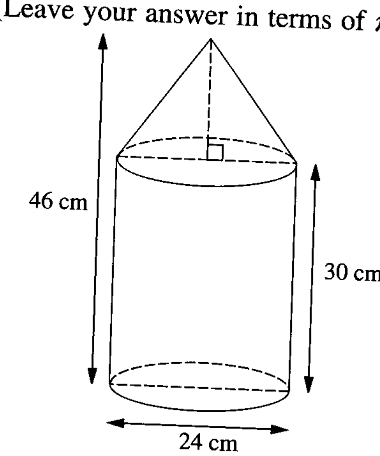
16. A sphere of diameter 25 cm is half-full with acid, all of which is drained into a tall cylindrical beaker 16 cm in diameter. What is the depth of the acid in the beaker?

∗**17.** A cylindrical tin has an internal diameter of 18 cm. It contains water to a height of 13.2 cm. How far will the water level rise when a heavy ball 9.3 cm in diameter is immersed in it?

∗**18.** The diagram below shows a solid consisting of a cone and a cylinder with a common base. Find the

(a) volume of the solid,
(b) total surface area of the solid.

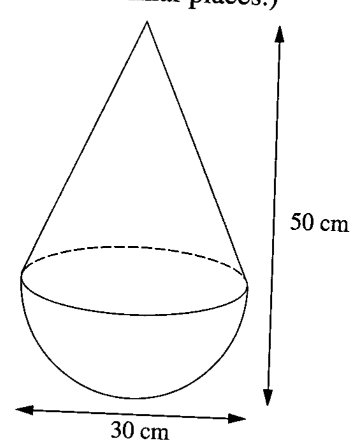
(Leave your answer in terms of π.)



∗**19.** The following diagram shows a solid consisting of a right circular cone fastened to a hemisphere with a common base. Find the

(a) volume of the solid,
(b) total surface area of the solid.

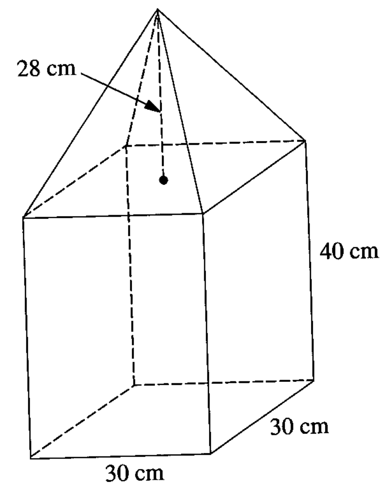
(Take π = 3.142 and leave your answer correct to 2 decimal places.)



★20. The figure shows a solid consisting of a pyramid of height 28 cm fastened to a cuboid of height 40 cm and a square base of sides 30 cm each. Find

(a) the volume of the solid,

(b) the total surface area of the solid (leave your answer correct to 2 decimal places).



Summary

1. For a circle with radius r,

$$\text{area} = \pi r^2$$
$$\text{circumference} = 2\pi r$$

2. For a sector subtending an angle $x°$ at the centre of a circle of radius r,

$$\text{arc length of the sector} = \left(\frac{x}{360}\right) \times 2\pi r$$
$$\text{area of the sector} = \left(\frac{x}{360}\right) \times \pi r^2$$

3. Volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height

4. For a right-circular cone with base radius r cm, slant height l and height h,

$$\text{volume} = \frac{1}{3}\pi r^2 h$$
$$\text{curved surface area} = \pi r l$$
$$\text{total surface area} = \pi r(l + r)$$

5. For a sphere with radius r,

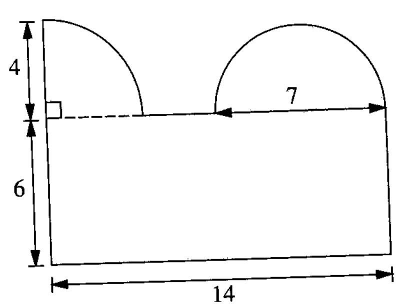
$$\text{volume} = \frac{4}{3}\pi r^3$$
$$\text{surface area} = 4\pi r^2$$

Review Questions 13

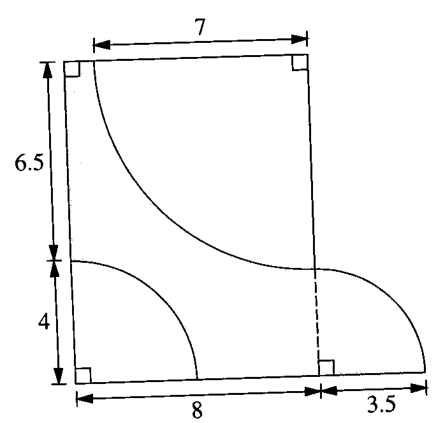
For this exercise, use the value of π stored in the calculator unless stated otherwise.

1. Find the area and perimeter of the shaded portion of each of the following figures. All dimensions are in cm. (Take π to be 3.142.)

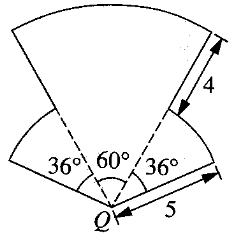
(a)



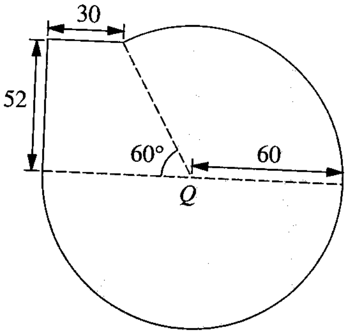
(b)



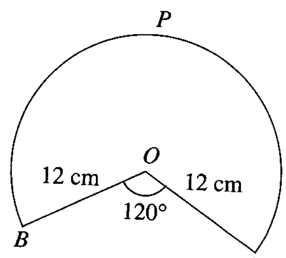
(c)



(d)



2. Write down, without simplification, an expression for the length of the arc BPA. The sector is folded to form the curved surface of a cone by bringing the radii OA and OB together so that the arc APB becomes the circular base of the cone.

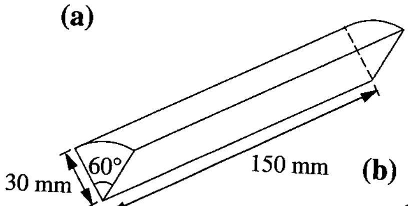


(a) Find the radius of the base of the cone and calculate the curved surface area of the cone.

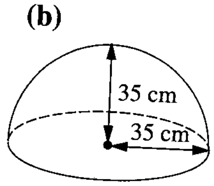
(b) Given that the height of the cone is 8.9 cm, find the volume of the cone.

3. Find the volume and the total surface area of the following solids. (Take π to be 3.142.)

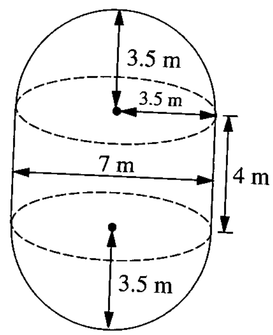
(a)



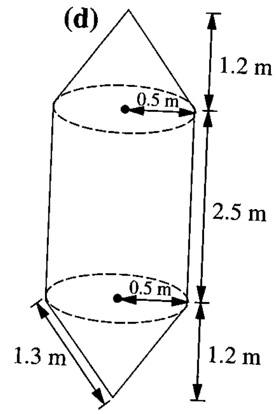
(b)



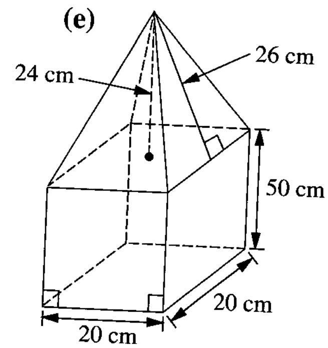
(c)



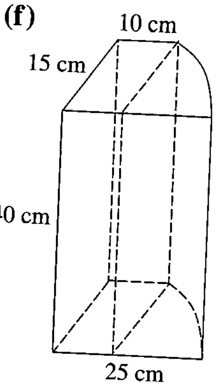
(d)



(e)



(f)



4. Two solid spheres have surface areas 144π cm^2 and 256π cm^2 respectively. They are melted and recast to form a larger sphere. Find the approximate surface area of this sphere in cm^2.

5. A cylinder and a cone have the same height, $2r$, and base diameter, $2r$. A sphere has a diameter $2r$. Find the ratio of the volume of the cylinder to that of the cone and the sphere.

6. The external diameter of a hollow metal sphere is 12 cm and its thickness is 2 cm. Find the radius of a solid sphere made of the same material and having the same weight as the hollow sphere. Given that 1 cm^3 of the metal weighs 5.4 g, find the weight of the sphere in kg, leaving your answer correct to the nearest 0.1 kg.

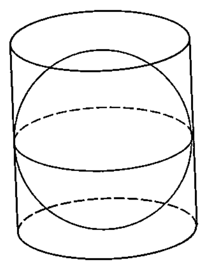
7. In an experiment, a small spherical drop of oil is allowed to fall onto the surface of water so that it produces a thin film of oil covering a large area.

(a) Given that the volume of a drop of oil is 12.5 mm^3, find the number of drops which can be produced by 5 000 mm^3 of oil.

(b) Given that the volume V of a sphere of radius r is $\frac{4}{3}\pi r^3$, express r in terms of V and π. Taking π to be 3.14, calculate the radius of one drop of oil.

8. Fifty-four solid hemispheres, each of diameter 2 cm, are melted to form a solid cone with base diameter 6 cm. Find the height of the cone.

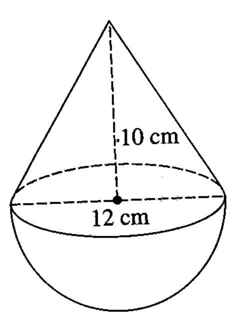
9. The figure shows a sphere of radius r fitting exactly into a cylinder, i.e., the sphere touches the cylinder at the top, bottom and curved surface. Show that the surface area of the sphere is equal to the area of the curved surface of the cylinder. (The surface area of a sphere of radius r is $4\pi r^2$.)



10. A mound of earth is shaped approximately like a right circular cone 6 m high with a base circumference of 30 m. Find the cost of removing it at 99 cents per cubic metre. $\left(\text{Take } \pi \text{ to be } \frac{22}{7}.\right)$

11. Find the cost of painting a hemispherical roof 10 m in diameter at $1.50 per square metre.

⋆12. The figure shows a composite solid consisting of a cone and a hemisphere with a common base. The cone has a height of 10 cm and a base diameter of 12 cm. Find the volume of the whole solid.



⋆13. A solid metal model of a rocket is made by fastening a cone of vertical height 49 cm and base radius 18 cm to a circular cylinder of length 192 cm and radius 18 cm. If the mass of the model is 2 145 kg, calculate the mass of 1 m^3 of the metal in kg, taking π to be $\frac{22}{7}$. $\left(\text{Volume of a cone} = \frac{1}{3}\pi r^2 h.\right)$

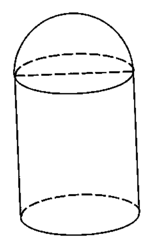
⋆14. A bowl is made by cutting into half a hollow sphere of external diameter 50.8 cm, made of metal 2.54 cm thick.

(a) If the bowl is filled with a liquid of density 31.75 kg/m^3, calculate the total mass of liquid in the bowl.

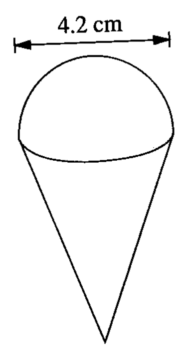
(b) The bowl when empty weighs 97.9 kg. Calculate the density, in kg/m^3, of the metal of which the bowl is made of.

$\left(\text{Volume of a half-sphere of diameter } d \text{ is } \frac{1}{12}\pi d^3.\right)$

⋆15. The diagram shows a solid cylindrical stone pillar whose top is a hemisphere. Given that the pillar is 40 cm in diameter and weighs as much as a solid stone sphere of radius 40 cm, find the height of the pillar.



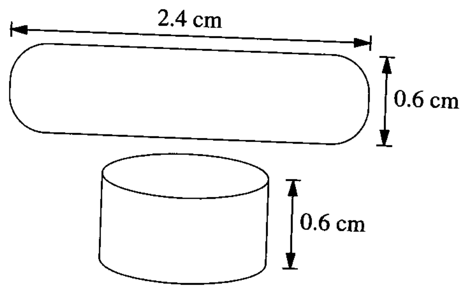
⋆16. The wafer cone shown contains 56 cm^3 of ice-cream filled to the bottom. The diameter of the cone is 4.2 cm, and the top of the ice-cream has the shape of a hemisphere. Find the height of the cone correct to the nearest cm.



⋆17. A vitamin tablet is 2.4 cm long and is in the shape of a cylinder with hemispheres of diameter 0.6 cm attached to both ends. Another vitamin tablet is in the shape of a cylinder of height 0.6 cm.

(a) Find the radius of the cylindrical tablet given that its surface area is equal to that of the first tablet.

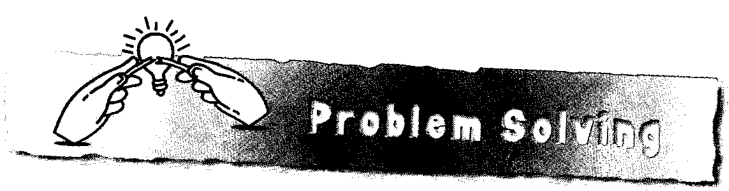
(b) Find the volume of each tablet.



18. Using full-flushes instead of half-flushes on cisterns in the toilet wastes 9.5 litres of water per flush. A school has a population of 1 380, assuming that each person uses the toilet three times in a day and each time full-flush is used, calculate the total amount of water wasted in the course of the year assuming that a year has only 205 school days.

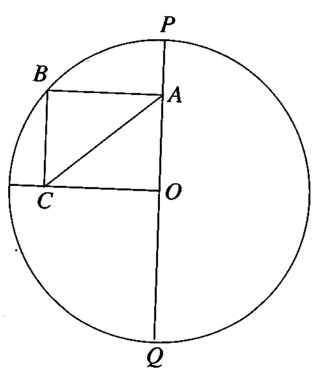
If these wasted water were to be put into cylindrical containers with a base radius of 24 cm and height of 38 cm, calculate the number of such containers that need to be used giving your answer correct to the nearest whole number.

19. Washing dishes under a running tap wastes an average of 155 litres of water per wash. Calculate the total amount of water wasted in a year if a family washes dishes twice a day for a year with 365 days. If this water were to be transferred into small spherical containers of radius 8.5 cm, how many such containers will be needed? Give your answer correct to the nearest whole number.

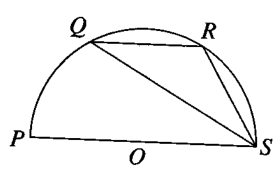


Problem Solving

1. In the figure, PQ is a diameter of the circle and $OABC$ is a rectangle. If $AQ = 14.4$ cm and $AP = 3.2$ cm, find the length of AC.



2. In the figure at the right, $PQRS$ is a semicircle. Given that $PS = 2r$ cm and arc PQ = arc QR = arc RS, calculate the area of the shaded region. Give your answer in terms of π and r.

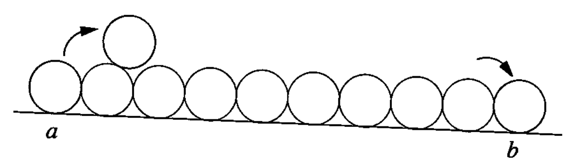


3. A square is inscribed in a circle of radius x cm. Find the area of the square in terms of x.

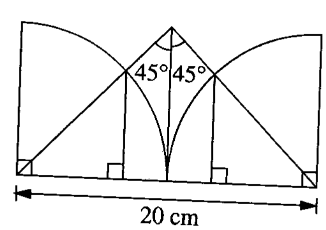
$\left(\text{Take } \pi = \frac{22}{7} \text{ where necessary.}\right)$

4. The surface area of a cube is x cm^2 and its volume is y cm^3. If $x = y$, find the length of a side of the cube.

5. The diagram below shows a circle (shaded) being rolled, without slipping, from position a to position b, across the top of eight identical circles. How many revolutions does it make to reach b?



6. Find the area of the shaded regions in the diagram below:



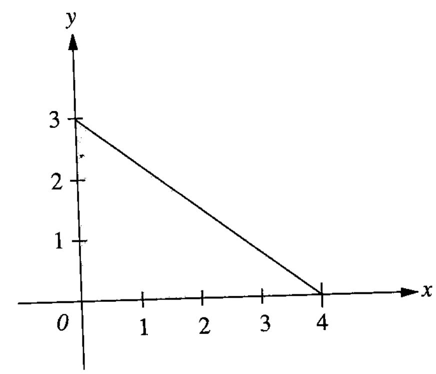
Exploration

1. **(a)** What is the radius and the height of the cone formed when the shaded triangle in the diagram is rotated about

   (i) the x-axis,

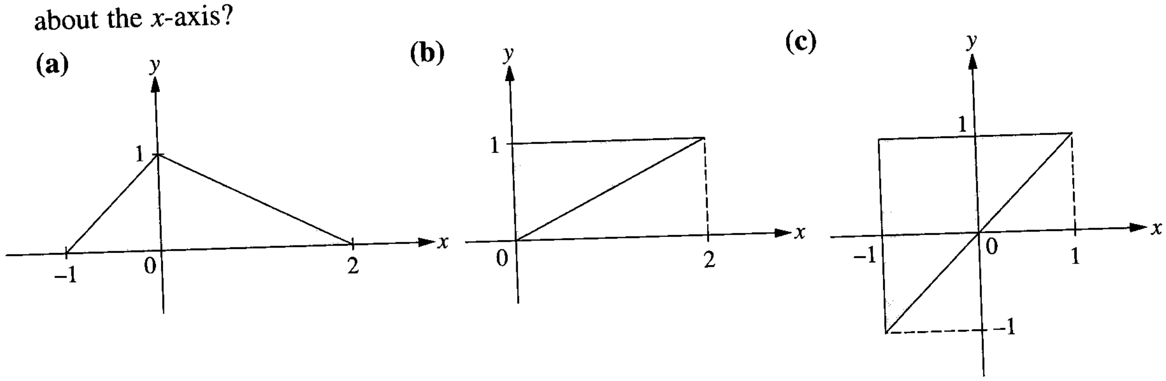
   (ii) the y-axis?

   (b) Do the two cones formed have the same volume? If not, which cone has the bigger volume?



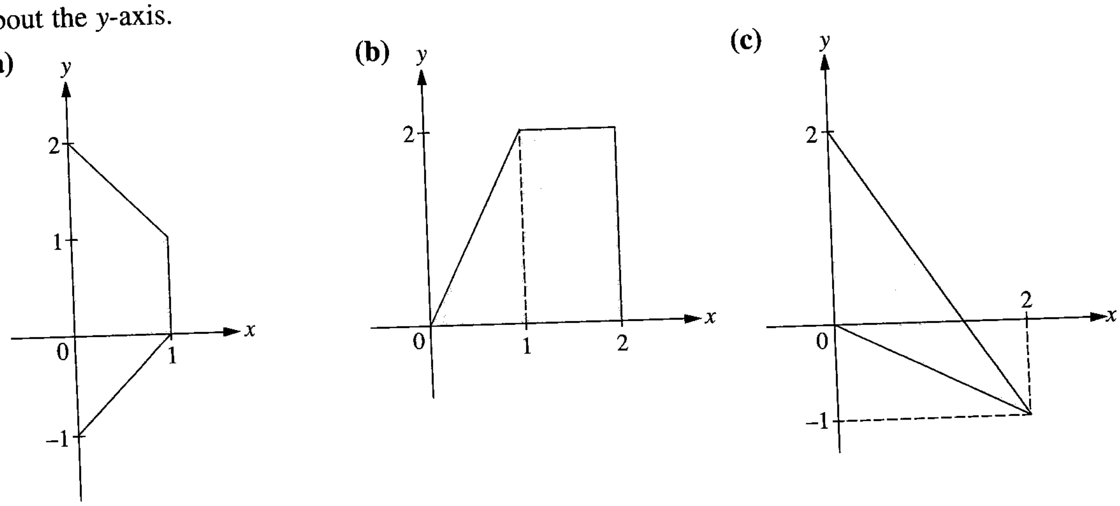
2. What will be the solid formed when the shaded region in each of the following diagrams is rotated about the x-axis?

(a) **(b)** **(c)**



3. Describe the solid formed when the shaded region in each of the following diagrams is rotated about the y-axis.

(a) **(b)** **(c)**



CHAPTER 14

Statistics — Measures of Central Tendency

In this chapter, you will learn how to

- △ find the mean;
- △ find the median;
- △ find the mode

of a set of data.

Preliminary Problem

Do you know that Singaporeans consume around 660 tonnes of rice and 430 tonnes of seafood per day? The Department of Statistics in Singapore collects and classifies various records such as these for administrative reference so that policies and decisions can be better formulated to meet the needs of the country.

The Averages

In Book 1, we learnt how to organise statistical data and present them in a diagram so that they can be understood and important information can be extracted from the data.

In this chapter, we will study ways of obtaining some 'typical values' to represent or describe a whole set of data.

In-Class Activity

Divide the class into two groups for the activity.

1. Measure the height of each member in your group and record the results to the nearest centimetre.

2. Using the set of measurements obtained, find the following:

(a) the greatest height, **(b)** the smallest height,

(c) the height that occurs most frequently,

(d) the middle value by arranging all the heights from the smallest to the largest or vice versa,

(e) the average value by adding up all the heights and dividing by the number of members in your group.

Note: In 2(d), one quick way of finding the middle height is to line up all the members of your group according to the heights in ascending or descending order and pick the height of the middle person from the data. For a group with even number of members, there are two middle persons. Obtain their heights, add them and divide by 2.

3. Now, you are required to choose one of the values obtained in 2. to be a representative value of the different heights of members in your group. Which would be your choice?

(a) Will the extreme values, i.e., the greatest and the smallest values be good representative values? Discuss.

(b) The other three values are commonly used as representative values of a set of data.

(i) The value that occurs most often is called the **mode**. What is the mode of the data of your group?

(ii) The middle value is called the **median**. Write down the median of your data.

(iii) The value obtained by adding all the values and dividing by the number of values is called the **arithmetic mean** or simply the **mean**. What is the mean height of your group?

In statistics, the mean, median and mode are known as **averages**. We expect these values to represent the 'centre' of the distribution of a set of data and thus they are said to be **measures of central tendency**.

The Mean

The **mean** of a set of numbers is the sum of numbers divided by the number of numbers in the set, i.e.,

$$\textbf{mean} = \frac{\textbf{sum of the numbers}}{\textbf{number of numbers}}$$

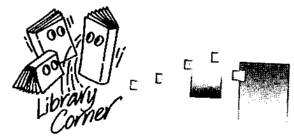
Example 1

A study was made on 5 typical fast-food meals in a certain country. The table below shows the amount of fat, in number of teaspoons, present in each meal. Calculate the mean amount of fat for these 5 fast-food meals.

Fast-food meal	*A*	*B*	*C*	*D*	*E*
Fat (teaspoons)	*14*	*18*	*22*	*10*	*16*

Solution

$$\text{Mean} = \frac{\text{sum of the numbers}}{\text{number of numbers}}$$
$$= \frac{14 + 18 + 22 + 10 + 16}{5}$$
$$= 16$$



The Gross National Product (GNP) per capita of a country is a measure of the country's wealth divided by the population of the country. Find out the GNP per capita for Singapore from 1998 to 2000.

Note that in this case the mean, 16, falls exactly in the 'middle' of the set of 5 numbers representing the measurements of fat in the fast-food meals.

However, not every set of data will have the mean falling exactly in the 'middle'. Also note that in general, the mean of a set of numbers may not be equal to one of the numbers in the set.

Example 2

100 crates of apples imported from a country were inspected. The table below shows the number of bad apples recorded for each crate.

No. of bad apples	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Frequency (no. of crates)	*6*	*9*	*12*	*28*	*20*	*15*	*5*	*2*	*2*	*1*

Find the mean number of bad apples per crate.

Solution

$$\text{Mean} = \frac{\text{total number of bad apples}}{\text{total number of crates}}$$
$$= \frac{\text{number of bad apples} \times \text{frequency}}{\text{total frequency}}$$
$$= \frac{0 \times 6 + 1 \times 9 + 2 \times 12 + 3 \times 28 + 4 \times 20 + 5 \times 15 + 6 \times 5 + 7 \times 2 + 8 \times 2 + 9 \times 1}{6 + 9 + 12 + 28 + 20 + 15 + 5 + 2 + 2 + 1}$$
$$= \frac{341}{100} = 3.41$$

We let x_1 denote the first number and f_1 the corresponding frequency, i.e., $x_1 = 0, f_1 = 6$. In the same manner, we let $x_2 = 1, f_2 = 9$; $x_3 = 2, f_3 = 12$; … $x_{10} = 9, f_{10} = 1$.

$$\text{Mean} = \frac{\text{total number of bad apples}}{\text{total number of crates}}$$

$$= \frac{\text{number of bad apples} \times \text{frequency}}{\text{total frequency}}$$

$$= \frac{x_1 f_1 + x_2 f_2 + \dots + x_{10} f_{10}}{f_1 + f_2 + \dots + f_{10}}$$

$$= \frac{341}{100} = 3.41$$

In general, for a set of numbers $x_1, x_2, \dots, x_n$ occurring with corresponding frequencies $f_1, f_2, \dots f_n$, usually displayed in the form of frequency table

x	x_1	x_2	...	x_n
f	f_1	f_2	...	f_n

The mean of the distribution is given by the formula

$$\textbf{mean} = \frac{x_1 f_1 + x_2 f_2 + \dots + x_n f_n}{f_1 + f_2 + \dots + f_n}$$

Example 3

The table below represents the results of a survey carried out on 60 families.

No. of children	*0*	*1*	*2*	*3*	*4*	*5*
No. of families	*6*	*14*	*18*	*9*	*10*	*3*

Calculate the mean number of children per family.

Solution

It is good practice to set up a table like the one below to show the calculations clearly.

No. of children (x)	*No. of families (f)*	*fx*
0	6	0
1	14	14
2	18	36
3	9	27
4	10	40
5	3	15
	Total = 60	Total = 132

$$\text{Mean} = \frac{\text{Total number of children}}{\text{Total number of families}} = \frac{132}{60} = 2.2$$

$\therefore$ the mean number of children per family = 2.2

Note: The answer does not suggest that every family has 2.2 children. It merely means that the average number of children per family is 2.2.

Exercise 14a

1. Find the mean of the following set of numbers:
   (a) 7, 6, 4, 8, 2, 5, 10
   (b) 63, 80, 54, 70, 51, 72, 64, 66
   (c) 10.8, 11.5, 10.9, 12.5, 11.8, 10.3
   (d) 138, 164, 150, 148, 152, 144, 168, 135, 160
   (e) 109.4, 108.5, 103.1, 111.3, 121.2

2. The number of passengers on buses travelling on a certain route were recorded as shown below.

   29, 42, 45, 39, 41, 38, 37, 38, 43, 40, 36, 35, 32, 38

   Find the mean number of passengers.

3. A bandmaster keeps a record of the number of students turning up for band practice.

   24, 27, 30, 32, 33, 33, 28, 31, 30, 29, 32, 31, 27, 26, 30, 32, 29, 29, 28, 31

   What is the mean number of students turning up for band practice?

4. The data shows the number of kilotonnes (thousands of tonnes) of cheese produced in a certain country each year from 1981 to 2000.

   68, 72, 84, 87, 88, 90, 96, 89, 94, 102, 120, 124, 118, 130, 136, 128, 142, 144, 156, 175

   Find the mean number of kilotonnes of cheese produced each year.

5. The prices (in \$) of various computer books on designing of web pages in a bookshop are given below.

   19.90, 24.45, 34.65, 26.50, 44.05, 38.95, 56.40, 48.75, 29.30, 35.65

   Find the mean price of these books.

6. The mean of six numbers is 41. Three of the numbers are 32, 31 and 42. The remaining three numbers each equals to a.
   (a) What is the sum of the six numbers?
   (b) Find the value of a.

7. The mean of three numbers x, y and z is 6 and the mean of five numbers x, y, z, a and b is 8. Find the mean of a and b.

8. The heights of three plants A, B and C in a garden are in the ratio 2 : 3 : 5. Their mean height is 30 cm.
   (a) Find the height of plant B.
   (b) If another plant D is added to the garden and the mean height of the four plants is now 33 cm, find the height of plant D.

9. A small factory employs 7 experienced and 5 inexperienced workers. The mean monthly wage of these 12 workers is \$700.
   (a) Calculate the total wage bill for the 12 workers.
   (b) Given that the mean wage of the 5 inexperienced workers is \$602, calculate the mean wage of the 7 experienced workers.

10. The mean height of 20 boys and 14 girls is 161 cm. If the mean height of the 14 girls is 151 cm, calculate the mean height of the 20 boys.

11. Find the mean of each of the following distributions:

(a)

x	2	3	4	6	8	9
f	3	8	5	7	5	2

(b)

x	35	45	55	65	75	85	95
f	4	6	10	15	8	5	2

(c)

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
f	40	20	50	30	10	20	60	70

(d)

x	62	77	91	107	122	137
f	2	13	35	36	12	2

12. In a soccer season, a soccer team played 30 matches. The table below shows the distribution of the number of goals scored per match.

No. of goals scored per match	0	1	2	3	4	5	6
No. of matches	6	8	5	6	2	2	1

Calculate the mean number of goals scored per match.

13. The table below shows the marks obtained by 40 pupils in English and Mathematics tests.

Marks	1	2	3	4	5	6	7	8	9	10
English	1	6	14	4	8	2	4	0	1	0
Mathematics	4	1	6	5	10	3	5	3	1	2

Calculate the mean marks for each subject.

The Median

In a survey to find out the living standards of the people in a certain area, the monthly incomes of nine households chosen from the area were obtained:

Household	1	2	3	4	5	6	7	8	9
Income (in dollars)	1 720	1 940	1 960	2 030	2 050	2 250	2 400	2 550	22 250

The mean income of these nine households is \$4 350. This number gives a distorted picture of the standard of living of the people in that area because eight out of nine households have incomes well below it. A more realistic way of illustrating the living standards of the people would be to use the income in the middle. This number, \$2 050, is the **median**. There are as many households having incomes above this number as there are below it.

Definition: (a) The median for an **odd** number of numbers is the middle number when the numbers are arranged in order of increasing magnitude.

(b) The median for an **even** number of numbers is the mean of the two middle numbers when the numbers are arranged in order of increasing magnitude.

Example 4

Seven students from a class participated in a Science quiz. Their scores were 84, 95, 72, 88, 67, 80 and 91. Find the median score.

Solution

First, we arrange the scores in order of increasing magnitude:

$$67, 72, 80, \boxed{84}, 88, 91, 95$$

Since we have an odd number of numbers, the median is the middle score, i.e., 84.

Example 5

Assuming that three more students from the same class participated in the same quiz in ***Example 4*** *and scored 74, 76 and 92 respectively, calculate the mean and median of the 10 scores.*

Solution

$$\text{Mean} = \frac{\text{Total number of marks}}{\text{Total number of students}}$$

$$= \frac{67 + 72 + 80 + 84 + 88 + 91 + 95 + 74 + 76 + 92}{10}$$

$\therefore$ the mean = 81.9

The scores arranged in order of increasing magnitude are:

$$67, 72, 74, 76, \boxed{80}, \boxed{84}, 88, 91, 92, 95.$$

Since we have an even number of scores, the median is the mean of the two middle scores.

The two middle scores are 80 and 84.

$$\therefore \text{ the median} = \frac{80 + 84}{2} = 82$$

The Mode

Often it is useful to know the number which occurs most frequently in a set of numbers. For instance, a person selling drinks in a school canteen would be interested in the brand of drinks which is the most popular so that he can place his orders accordingly and not incur much wastage later. Manufacturers would like to produce their goods (e.g. shoes, shorts, skirts, etc.) in the most popular sizes so as to gain a higher percentage of the market share.

The number which occurs most frequently in a set of numbers is called the **mode** of the set of numbers.

Example 6

The scores of a basketball team in a series of matches are 63, 63, 75, 67, 69, 52, 50, 63, 56, 52. Find the mode.

Solution

The modal score or the mode of the scores is 63 since it appears most often (3 times) among the ten scores.

Example 7

Suppose the basketball team in ***Example 6*** *played two more matches and scored 70 and 52 respectively. Calculate the mean, median and mode of the 12 scores.*

Solution

$$\text{Mean} = \frac{\text{Total score}}{\text{Total number of matches}}$$

$$= \frac{63 + 63 + 75 + 67 + 69 + 52 + 50 + 63 + 56 + 52 + 70 + 52}{12}$$

$$= 61$$

$\therefore$ the mean is 61.

The scores arranged in order of magnitude are:

50, 52, 52, 52, 56, 63, 63, 63, 67, 69, 70, 75.

The two middle scores are both 63.

$\therefore$ the median $= \frac{63 + 63}{2} = 63$

The modes of this set of scores are 52 and 63 since they occur the most frequently at an equal number of times.

A set of numbers is said to be **bimodal** if it contains two modes.

On an island, there are two tribes of people: the Lucky and the Truthful. The Lucky tribespeople always tell lies while the people of the Truthful tribe always tell the truth. One day, a visitor to the island met three persons A, B and C. He was curious to know which tribe each belonged to. He asked A but A spoke so softly that he could not hear. B claimed that he heard what A said and told the visitor, 'He says he belongs to the Lucky tribe.' C quickly turned to B and bluntly told him, 'You are a liar.' Which tribe does A, B and C belong to respectively?

The arithmetic mean is the most widely used measure of central tendency. To get an idea on the use of mean, for example, in ranking basketball players according to their various skills, you can log on to www.nba.com.

It is also worthwhile to visit Singapore statistics website, www.singstat.gov.sg/, where you can find real life data to practice on the calculation of the measures of central tendency.

Example 8

The following table shows the distribution of the number of children in 300 families in a certain city. Find the mean, the median and the mode of the distribution.

No. of children (x)	*No. of families (f)*	*fx*
1	*11*	*11*
2	*79*	*158*
3	*82*	*246*
4	*67*	*268*
5	*20*	*100*
6	*25*	*150*
7	*11*	*77*
8	*5*	*40*
	Total = 300	*Total = 1 050*

Solution

$$\text{Mean} = \frac{\text{Total number of children}}{\text{Total number of families}}$$

$$= \frac{1\ 050}{300} = 3.5$$

∴ the mean is 3.5.

Since there is an even number (300) of families, the median is the mean of the number of children of the two middle families.

The two middle families are the 150th and 151st families.

Note that the sums of the first two numbers and the first three numbers in the frequency column are $11 + 79 = 90$ and $11 + 79 + 82 = 172$ respectively. This indicates that the number of children for each of the two middle families is 3.

So the median is given by $\frac{3 + 3}{2} = 3$.

Since the largest number of families (82) each has 3 children, the mode is 3.

Comparison of the Mean, Median and Mode

The *arithmetic mean* is the most widely used measure of central tendency. It is usually preferred over the median and the mode. It is the most reliable measure provided there are no extreme values in the data because all the values in the data are used in calculating the mean unlike the mode and the median. Whenever the set of data contains extreme values, the median or mode will probably be a better indicator of the whole set of data because they are not influenced by extreme values.

The *median* may be the preferred measure of central tendency for describing economic, sociological and educational data. It is popular in the study of the social sciences because much of the data in the social sciences contain extreme values, as in the set of household incomes we discussed earlier.

The *mode* is more useful in business planning as a measure of popularity that reflects central tendency or opinion. Examples include the drink-seller wanting to know the most popular brand of drinks, manufacturers who want to know the most popular sizes of shoes, shorts, skirts, etc., all of which have been mentioned earlier.

In more advanced statistical work, the *mean* is usually used as a measure of central tendency as it is easy to manipulate mathematically.

Exercise 14b

1. Determine the mean, median and mode of the following sets of numbers.

(a) 10 11 13 11 15 16

(b) 8 11 14 13 14 9 15

(c) 2 5 6 3 7 8 4 12
11 9 10 7 6 8 9 7

(d) 88 93 85 98 102 98 93 104
102 98

(e)

x	0	1	2	3	4	5	6
f	1	2	2	3	4	5	4

(f)

x	2	4	6	8	10	12
f	2	4	10	6	3	1

☆**2.** The following table shows the monthly wages of 27 employees in a certain factory in 1991.

Wages $(x)	670	760	850	960	1 000	1 200
No. of employees (f)	4	9	8	3	2	1

Find **(a)** the mean monthly wage,
(b) the median monthly wage,
(c) the modal monthly wage.

☆**3.** Two dice are tossed 30 times. The sum of the score each time is shown below:

Score (x)	2	3	4	5	6	7	8	9	10	11	12
Frequency (f)	1	1	3	4	6	8	3	2	1	1	0

Find the mean, the median and the mode of the scores.

☆**4.** The table below shows the frequency distribution of the number of spelling mistakes in a composition made by each pupil in a class of 36.

No. of mistakes (x)	0	1	2	3	4	5	6	7
No. of pupils (f)	3	7	10	6	5	3	1	1

Find **(a)** the mean,
(b) the median,
(c) the mode of the distribution.

☆**5.** **(a)** The median of a set of eight numbers is $4\frac{1}{2}$. Given that seven of the numbers are 9, 2, 3, 4, 12, 13 and 1, find the eighth number.

(b) The mean of a set of six numbers is 2 and the mean of another set of ten numbers is m. If the mean of the combined set of sixteen numbers is 7, find the value of m.

✶**6.** Peter and Paul were playing golf. The scores on the first nine holes are shown in the table below:

Hole	1	2	3	4	5	6	7	8	9	**Total**
Peter	3	2	5	7	3	2	2	4	17	45
Paul	4	4	6	8	3	3	2	6	6	42

On the ninth hole, Peter got stuck in a sand trap and lost the game.

(a) Calculate the mean score on the nine holes for each player.

(b) Which player did better on most of the holes? Do the mean scores indicate this?

(c) What were the median scores for both players?

(d) Find the mode of each player's scores.

(e) Which measure of central tendency – the mean, the median or the mode – do you think gives the best comparison of the abilities of Peter and Paul?

☆7. Two classes, each with 21 students were given a physical fitness test. The results of the number of pull-ups performed in 30 seconds were recorded in the tables below:

Class A	*No. of pull-ups*	under 5	6	7	8	9	over 10
	No. of students	3	4	4	7	2	1

Class B	*No. of pull-ups*	under 5	6	7	8	9	over 10
	No. of students	3	7	4	4	2	1

(a) Can you calculate the mean number of pull-ups performed by the students in each class? Why?

(b) Find the median number of pull-ups for the students in each class.

(c) Which would you use, the median or the mode, to compare the results of the two classes?

8. A box contained five cards numbered 1, 2, 3, 4 and 5. A card was drawn from the box, its number noted and then replaced. The process was repeated 100 times and the table below shows the resulting frequency distribution.

Card	1	2	3	4	5
Frequency	21	x	y	18	17

(a) Show that $x + y = 44$.

(b) If the mean of the distribution is 2.9, show that $2x + 3y = 112$.

(c) From (a) and (b), find the value of x and of y, and then state the mode and the median of the distribution.

Summary

1. A set of data can be described by numerical quantities called **averages** or **measures of central tendency**.

2. The three common measures of central tendency are the **arithmetic mean**, the **median** and the **mode**.

3. The **arithmetic mean** or the **mean** of a set of numbers is the sum of numbers divided by the number of numbers in the set.

4. The **median** for an **odd** number of numbers is the middle number when the numbers are arranged in order of magnitude. The median for an **even** number of numbers is the mean of the two middle numbers when the numbers are arranged in order of increasing magnitude.

5. The **mode** is the number that occurs most frequently.

Review Questions 14

1. The table below shows the number of goals scored by each team in a soccer competition.

No. of goals	0	1	2	3	4	5
No. of teams	15	19	8	7	1	0

(a) How many teams scored more than two goals?
(b) Calculate the mean number of goals scored by the teams.
(c) Find the **(i)** modal,
(ii) median
number of goals scored.

***2.** During a one-month period, the number of sick leave days of 100 workers in a factory were recorded as shown in the table below:

No. of sick leave days	0	1	2	3	4
No. of workers	45	32	14	6	3

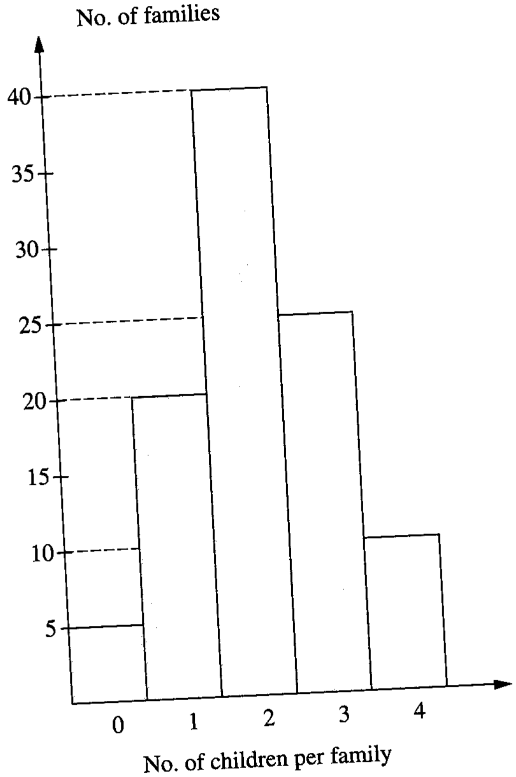
Calculate
(a) the mean,
(b) the median,
(c) the mode of the frequency distribution.

***3.** The table below shows the number of people in each of the 100 cars passing a particular place.

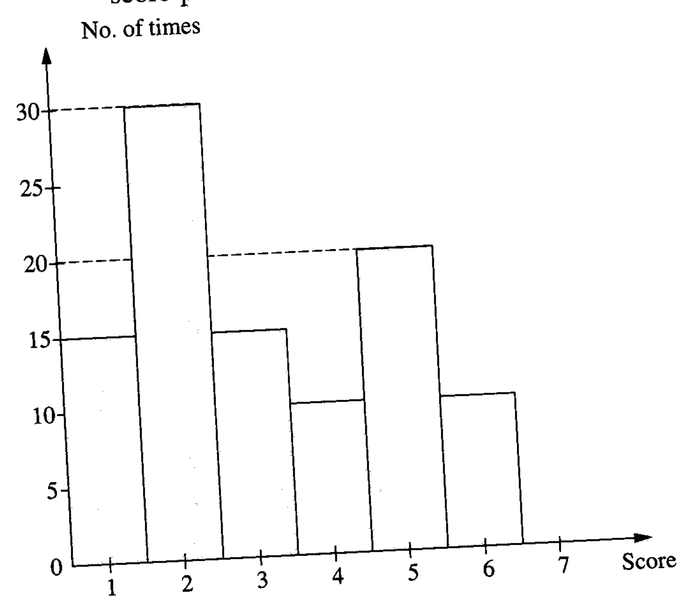
No. of people in each car	1	2	3	4
No. of cars	x	50	y	16

(a) Find the value of $x + y$.
(b) If the mean number of people per car is 2.4, show that $x + 3y = 76$.
(c) Find the values of x and y by solving appropriate equations.
(d) State the modal number of people per car.

***4.** The following diagram illustrates the number of children per family of a sample of 100 families in a certain housing estate.
(a) State the modal number of children per family.
(b) Calculate the mean number of children per family.
(c) Find the median number of children per family.



***5.** The diagram below shows the scores recorded after a man fired 100 shots at a target.
(a) State the modal score per shot.
(b) Calculate the mean score and the median score per shot.

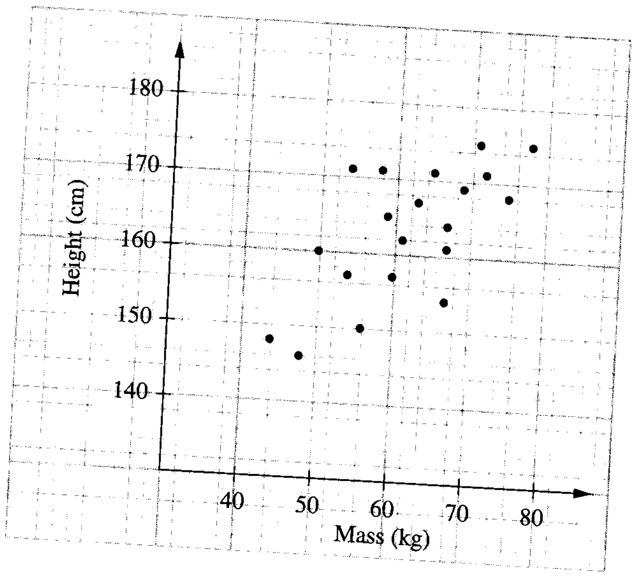


6. Sixty motorists were surveyed to find the amount in complete dollars spent on petrol per month. The results are displayed in a stem and leaf diagram as shown below.

15	2	6	8								
16	3	6	7	9							
17	3	3	4	8	8	9	9				
18	1	3	3	3	4	5	6	8	9	9	
19	1	2	3	4	4	4	5	6	6	9	9
20	2	3	4	5	6	7	7	7	7	8	
21	0	2	5	9	9						
22	1	5	6								
23	1	3	4	7							
24	2	4	8								

Find the mode, the median and the mean of the distribution of the monthly petrol bills of the motorists.

7. The heights and masses of 20 secondary two students are given in the graph.



(a) List the 20 masses in ascending order. Using your list find the
 (i) median,
 (ii) mode,
 (iii) mean.

(b) List the 20 heights in ascending order. Using your list find the
 (i) median,
 (ii) mode,
 (iii) mean.

8. The table below lists the estimated hourly cost for manufacturing workers in some countries in 1999.

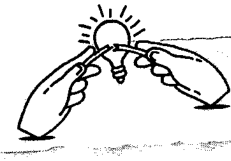
Country	China	Korea	Mexico	Singapore	Malaysia	Indonesia
Hourly cost US$	2.11	4.89	1.52	7.32	2.59	0.09

(a) How many times more expensive is a Singaporean worker's cost per hour as compared to his Indonesian counterpart?

(b) Express the percentage difference of the hourly cost of a Singaporean worker as compared to his Malaysian counterpart taking the hourly cost of a Malaysian worker as the base.

(c) Draw a bar graph to represent the hourly cost of a manufacturing worker in the above countries.

(d) A multinational company employs 32 workers in China, 45 workers in Korea, 65 workers in Singapore, 67 workers in Mexico, 87 workers in Malaysia and 43 workers in Indonesia. Calculate the mean and median hourly cost of the workers employed by the company assuming that the workers in each country earn the same average hourly pay.



Problem Solving

1. Given that nine numbers 16, w, 17, 9, x, 2, y, 7 and z have a mean of 11, find the mean of w y and z.

2. For the set of numbers $-\frac{1}{x^2}, \frac{2}{x}, -\frac{1}{x}, 1, \frac{1}{x}, \frac{1}{x^2}$,
   - **(a)** obtain an expression for the mean,
   - **(b)** find the mode given that $x = 1$,
   - **(c)** obtain an expression for the median
     - **(i)** when $x > 2$,
     - **(ii)** when $0 < x < 1$,
     - **(iii)** when $-1 < x < 0$,
     - **(iv)** when $x < -2$.

Exploration

1. Find out the prices of 5 different brands of cigarettes available in the market and the number of cigarettes in each packet.

2. Peter's father smokes an average of 30 cigarettes per day.
   - **(a)** Using the average price of the 5 brands of cigarettes in **1.**, calculate how much Peter's father can save if he gives up smoking for 1 year.
   - **(b)** Which of the following items do you think Peter's father can buy if he gives up smoking for 1 year?
     - **(i)** Autofocus camera
     - **(ii)** Nicam stereo TV
     - **(iii)** Mini hi-fi system
     - **(iv)** Refrigerator
     - **(v)** All of the above items

   Kicking the habit must be a good idea, huh?

CHAPTER 15

Trigonometrical Ratios

In this chapter, you will learn how to

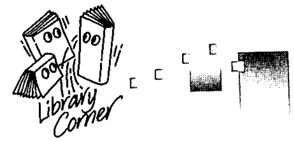
- △ find the length of a side of a right-angled triangle and an angle of a right-angled triangle using the sine, cosine and tangent ratios for acute angles;
- △ solve trigonometrical problems in two dimensions.

Preliminary Problem

The picture shows a surveyor at work using a modern instrument called a theodolite. A surveyor requires a good knowledge of trigonometry to carry out his measurements.

Do you know how trigonometry works?

Trigonometrical Ratios



Besides the three trigonometrical ratios, sine, cosine and tangent, there are three others: secant, cosecant and cotangent. Find out what these three trigonometrical ratios represent.

Trigonometry is a branch of mathematics that deals with the relations of sides and angles of a triangle as well as the relevant functions of any angles.

When we use a ruler or tape measure to measure the thickness of a book, the length of a pen, the height of a table or the width of a classroom, we are making **direct measurements**.

In some situations, direct measurements are difficult, dangerous or even impossible to obtain. For example, it is dangerous for you to climb up the school's flag pole to measure its height. It is difficult and dangerous to obtain the height of a cliff and it is impossible to measure the height of the highest mountain peak in the world, Mount Everest, by direct measurement.

The above problems can be solved by **indirect measurements** with the help of **Trigonometry**. Trigonometry is a powerful tool for making indirect measurements of distance or height. It plays an important role in the field of surveying, navigation, engineering and many other branches of physical science.

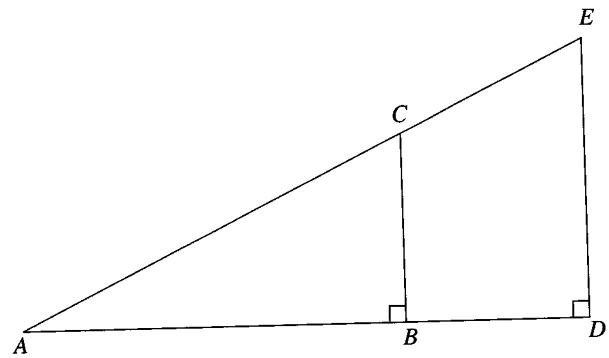


Fig. 15.1

Fig. 15.1 shows two similar triangles ABC and ADE. Measure the lengths of AB, AC, BC, AD, AE and DE. Find the value of the following ratios.

(a) $\frac{BC}{AB}$ and $\frac{DE}{AD}$ **(b)** $\frac{BC}{AC}$ and $\frac{DE}{AE}$ **(c)** $\frac{AB}{AC}$ and $\frac{AD}{AE}$

Do you notice that the ratios of the corresponding lengths of two similar triangles are the same?

In $\triangle ABC$ (Fig. 15.2), with respect to $\hat{A}$, the side BC is called the **opposite side**, the side AB is called the **adjacent side** and the side AC opposite the right angle is called the **hypotenuse**.

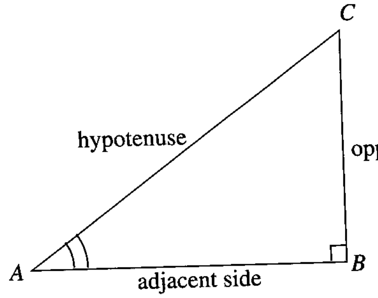
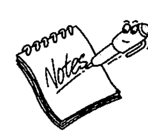


Fig. 15.2



In Fig. 15.2, with respect to $\hat{C}$, AB is the opposite side and BC is the adjacent side. AC still remains the hypotenuse.

For this right-angled triangle ABC,

the constant ratio $\frac{BC}{AC}$ or $\frac{\text{opposite side}}{\text{hypotenuse}}$ is called the sine of angle $\hat{A}$,

the constant ratio $\frac{AB}{AC}$ or $\frac{\text{adjacent side}}{\text{hypotenuse}}$ is called the cosine of angle $\hat{A}$, and

the constant ratio $\frac{BC}{AB}$ or $\frac{\text{opposite side}}{\text{adjacent side}}$ is called the tangent of angle $\hat{A}$.

The following abbreviations are used:

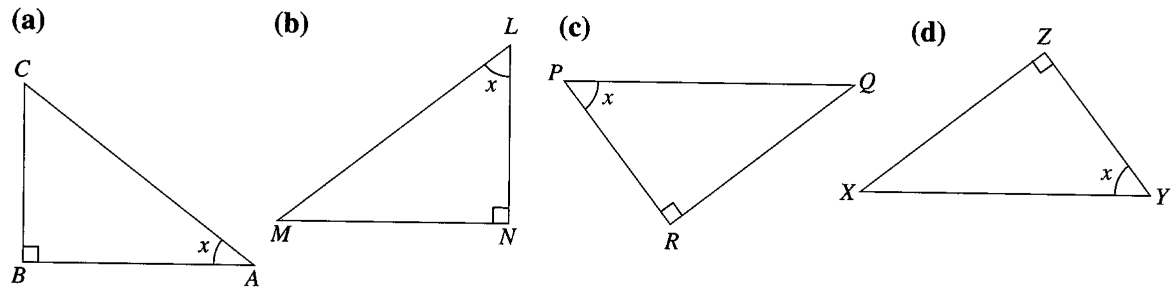
$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AC} \quad \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{AB}{AC} \quad \tan A = \frac{\text{opp}}{\text{adj}} = \frac{BC}{AB}$$

The trigonometrical ratios of an angle are numerical quantities. Each of them represents the ratio of one length to another and therefore do not carry any units.

Exercise 15a

1. For each of the following right-angled triangles, name

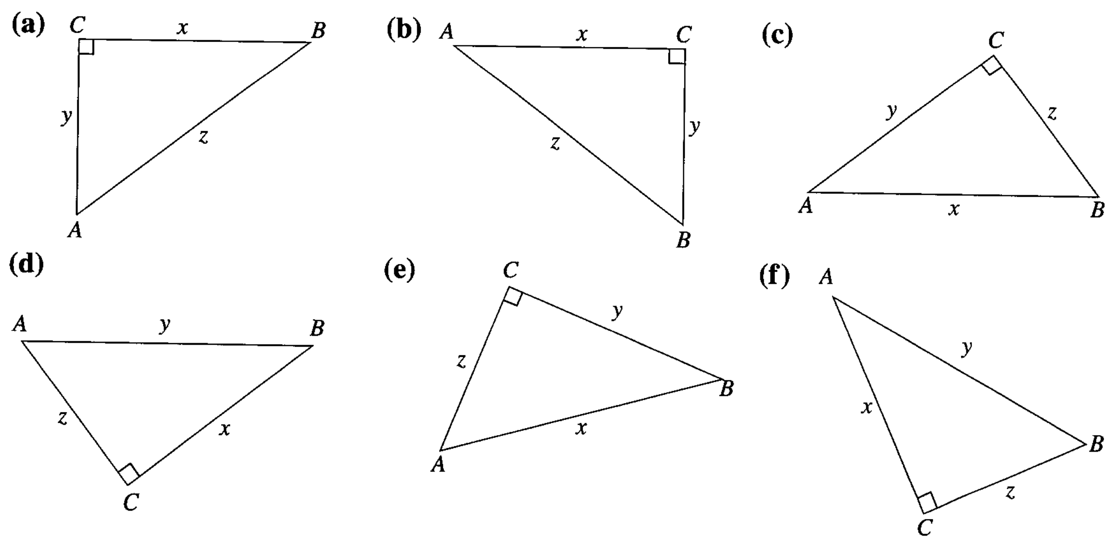
(i) the hypotenuse; **(ii)** the side opposite x; **(iii)** the side adjacent to x.



2. For each of the following right-angled triangles, find

(i) $\sin A$, **(ii)** $\cos A$, **(iii)** $\tan A$, **(iv)** $\sin B$, **(v)** $\cos B$, **(vi)** $\tan B$

in terms of x, y and z.

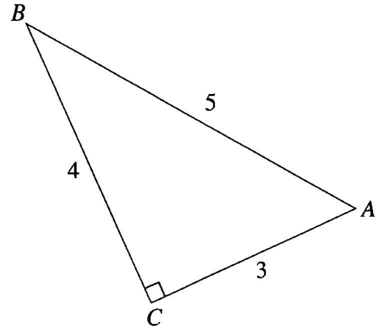


3. For each of the following right-angled triangles, find

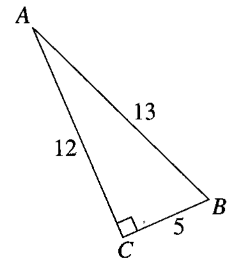
(i) $\sin A$, **(ii)** $\cos A$, **(iii)** $\tan A$, **(iv)** $\sin B$, **(v)** $\cos B$, **(vi)** $\tan B$.

Give your answers as a fraction in its lowest terms.

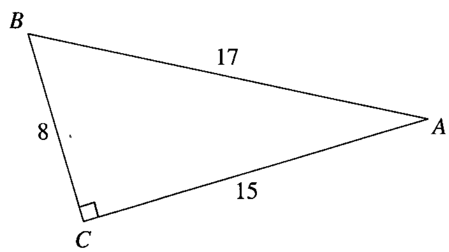
(a)



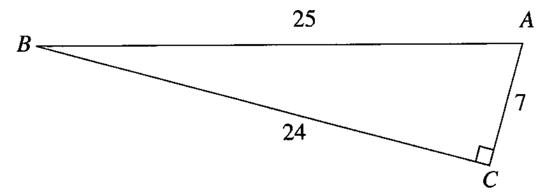
(b)



(c)

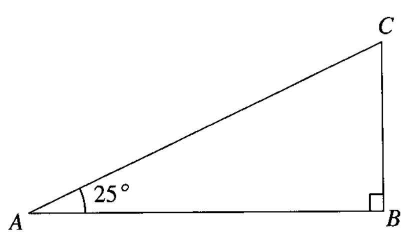


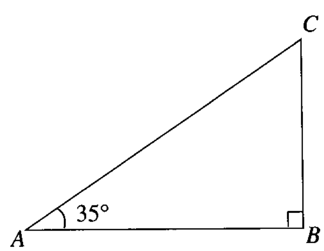
(d)



Values of Trigonometrical Ratios

Construct a right-angled triangle ABC with $\hat{B} = 90°$ and $\hat{A} = 25°$.





Measure BC and AB and then compute the value of $\frac{BC}{AB}$. You will find the answer to be approximately 0.466.

If you construct $\hat{A}$ to be equal to 35°, you will find $\frac{BC}{AB}$ to be approximately 0.7. If you carry on with this process, you will be able to obtain a table of values for tangents from 1° to 90°. Similarly, if you compute $\frac{BC}{AC}$ and $\frac{AB}{AC}$ for values of A from 1° to 90°, you will be able to obtain tables of values for the sine and cosine ratios.

It's a Fact

Trigonometrical ratios can be used to describe many natural phenomena. They are applied in the study of electronics, heat, acoustics, X-rays, light, etc. You will learn about all these when you advance to higher levels of study. Meanwhile, let us get the basics right first.

The open tool, Geometer's Sketch Pad, is a useful tool to explore the values of trigonometrical ratios as the angles vary.

A sample of values of trigonometrical ratios is given below:

Angle	Tangent	Sine	Cosine
50°	1.191 8	0.766 0	0.642 8
51°	1.234 9	0.777 1	0.629 3
52°	1.279 9	0.788 0	0.615 7
53°	1.327 0	0.798 6	0.601 8

You can obtain the values of the trigonometrical ratios using your calculator.

Use of Calculator

It is very easy to use a calculator to find the values of trigonometrical ratios.

Example 7

Find the values of the following, giving your answers correct to 4 significant figures.

(a) $\tan 32°$ *(b)* $\sin 15.3°$ *(c)* $\cos 25.96°$

(d) $2 \sin 37° + 5 \tan 35.6°$ *(e)* $\dfrac{\cos 57°}{\sin 46.5° + \tan 26.4°}$

Solution

As we are computing the values of angles in degrees, set the MODE key to "DEG". The following sequence of keying-in follows the "Direct Algebraic Logic" type of Scientific calculator. Refer to your calculator manual if you are using a different type of calculator.

(a) For tan 32°, press [tan] 32 [EXE] and the answer display is 0.624 869 351 9. Correcting the answer to 4 significant figures, you will get 0.624 9.

(b) For sin 15.3°, press [sin] 15.3 [EXE] and the answer displayed is 0.263 873 05. It is 0.263 9, correct to 4 significant figures.

(c) For cos 25.96°, press [cos] 25.96 [EXE] and the answer displayed is 0.899 099 868. Correct to 4 significant figures, it is 0.899 1.

(d) Press 2 [sin] 37 [+] 5 [tan] 35.6 [EXE] and the answer displayed is 4.783 278 462. It is 4.783, correct to 4 significant figures.

(e) Press [cos] 57 [÷] [(] [sin] 46.5 [+] [tan] 26.4 [)] [EXE] and the answer displayed is 0.445 775 526 3. Correct to 4 significant figures, it is 0.445 8.

In a right-angled triangle, the hypotenuse is the longest side. Thus any trigonometrical ratio which has the hypotenuse as the denominator can never be greater than 1. Those ratios which do not involve the hypotenuse as the denominator are not restricted in value, for either of the two sides which subtend the acute angle may be greater. Therefore, **the sine and cosine of an angle can never be greater than 1** while its tangent can take on any numerical value.

So far, we have dealt only with right-angled triangles with the definition of trigonometrical ratios being confined to acute angles. In Book 3, however, the definition of the trigonometrical ratios will be extended to the case of obtuse angles and to the solution of triangles of any size and shape.

Exercise 15b

1. Use your calculator to find the values of the following trigonometrical ratios. Give your answers correct to 4 significant figures.
   - **(a)** $\tan 46°$ **(b)** $\cos 45°$ **(c)** $\sin 58°$ **(d)** $\tan 25.5°$
   - **(e)** $\sin 75.3°$ **(f)** $\cos 59.4°$ **(g)** $\tan 64.31°$ **(h)** $\sin 86.55°$
   - **(i)** $\cos 77.45°$ **(j)** $\tan 7.08°$ **(k)** $\cos 88.36°$ **(l)** $\sin 2.36°$

2. Use your calculator to evaluate each of the following, giving your answer correct to 4 significant figures.
   - **(a)** $\sin 35° + \cos 49°$ **(b)** $2\cos 42.3° + 3\sin 16.8°$ **(c)** $6\tan 71.3° - 4\cos 14.9°$
   - **(d)** $\tan 16.7° \times \sin 71.6°$ **(e)** $\dfrac{5\tan 61.42°}{2\cos 10.36°}$ **(f)** $\dfrac{4\cos 68.73°}{2\sin 31.81°}$
   - **(g)** $\dfrac{\tan 15.4° + \cos 32.6°}{\sin 78.42°}$ **(h)** $\dfrac{\sin 72.4° - \tan 7.24°}{\cos 16.94°}$ **(i)** $\dfrac{\sin 79.4°}{\cos 78.97° + \tan 36.4°}$
   - **(j)** $\dfrac{\tan 47.9°}{\cos 84.3° - \sin 63.4°}$ **(k)** $\dfrac{\cos 67.4° + \sin 89.4°}{\tan 63.4° \times \cos 15.4°}$ **(l)** $\dfrac{\sin 24.6° \div \cos 62.1°}{\tan 21.4° + \cos 13.9°}$
   - **(m)** $\dfrac{\sin 56.9° - \cos 72.64°}{\tan 15.3° \times \sin 83.4°}$ **(n)** $\dfrac{\tan 49.86° + \sin 17.9°}{\cos 36.5° \div \tan 13.3°}$

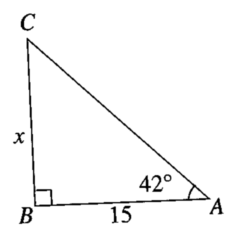
Solving Right-Angled Triangles Using Trigonometrical Ratios

The following examples illustrate the application of the trigonometrical ratios in the solution of right-angled triangles.

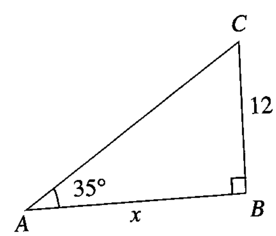
Example 2

Find the value of x in each of the triangles shown, giving your answer correct to 4 significant figures.

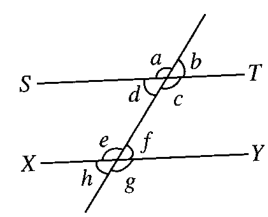
(a)



(b)



*Given that ST//XY, **b** = (9x + 2)° and f = (6x + 23)°, find **a** as given in the diagram below.*



Solution

(a) $\tan B\hat{A}C = \dfrac{\text{opp}}{\text{adj}} = \dfrac{BC}{AB}$

$\tan 42° = \dfrac{x}{15}$

$x = 15\tan 42°$

$= 13.506\ 060\ 66$

$= 13.51$ (4 significant figures)

(b) $\tan 35° = \dfrac{12}{x}$

$x\tan 35° = 12$

$x = \dfrac{12}{\tan 35°}$

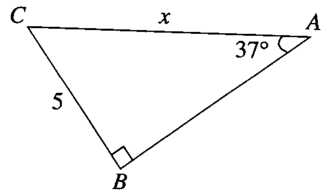
$= 17.137\ 776\ 08$

$= 17.14$ (4 significant figures)

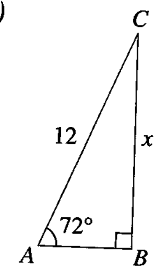
Example 3

Find the value of x in each of the following triangles, giving your answer correct to 4 significant figures.

(a)



(b)



Solution

(a) $\sin B\hat{A}C = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AC}$

$\sin 37° = \frac{5}{x}$

$x = \frac{5}{\sin 37°}$

$= 8.308\ 200\ 706$

$= 8.308$ (4 significant figures)

(b) $\sin 72° = \frac{x}{12}$

$x = 12 \sin 72°$

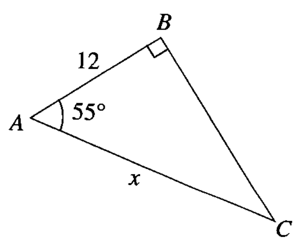
$= 11.412\ 678\ 2$

$= 11.41$ (4 significant figures)

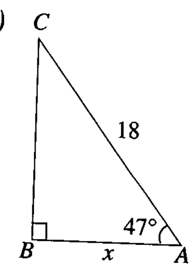
Example 4

Find the value of x in each of the following triangles, giving your answer correct to 4 significant figures.

(a)



(b)



Solution

(a) $\cos B\hat{A}C = \frac{\text{adj}}{\text{hyp}} = \frac{AB}{AC}$

$\cos 55° = \frac{12}{x}$

$x \cos 55° = 12$

$x = \frac{12}{\cos 55°}$

$= 20.92$

(b) $\cos 47° = \frac{x}{18}$

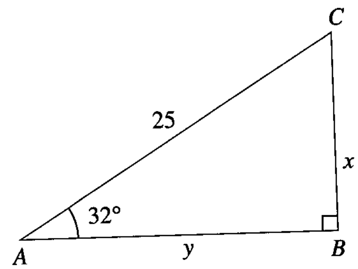
$x = 18 \cos 47°$

$= 12.28$

Example 5

Find the values of x and of y in the given triangle, giving your answer correct to 4 significant figures.

Solution



$\cos B\hat{A}C = \frac{\text{adj}}{\text{hyp}} = \frac{AB}{AC}$

$\cos 32° = \frac{y}{25}$

$y = 25 \cos 32°$

$= 21.20$

After we have obtained the value of y, the value of x can be found using three different methods.

Method I

$\sin 32° = \frac{x}{25}$

$x = 25 \sin 32°$

$= 13.25$

Method II

$\tan 32° = \frac{x}{y} = \frac{x}{21.20}$

$x = 21.20 \tan 32°$

$= 13.25$

Method III

$x^2 + y^2 = 25^2$ (Pythagoras' theorem)

$\therefore \quad x^2 + 21.20^2 = 25^2$

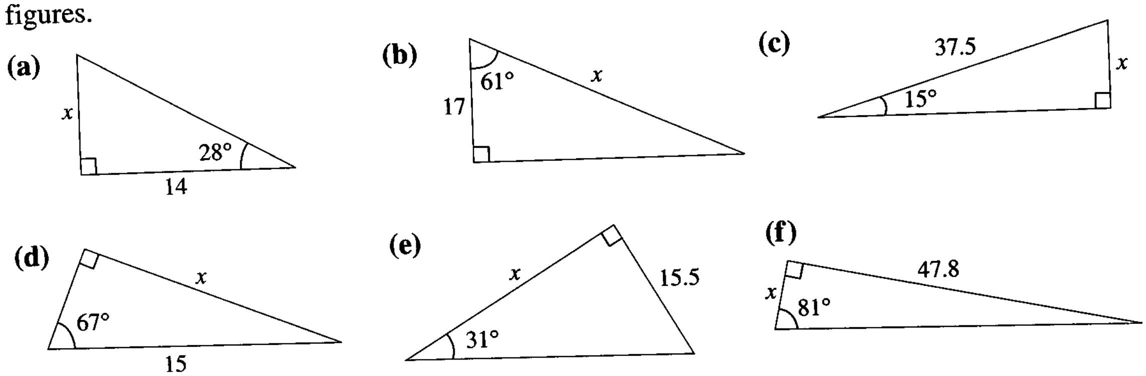
$x^2 = 25^2 - 21.20^2 = 175.56$

$x = \sqrt{175.56} = 13.25.$

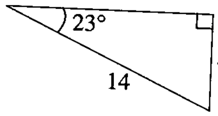
Which method would you prefer? Why?

Exercise 15c

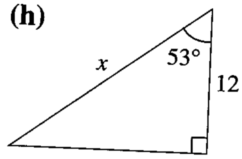
1. Find the value of x in each of the following triangles, giving your answer correct to 4 significant figures.



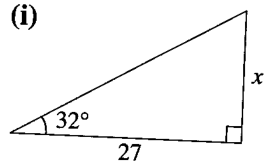
(g)



(h)



(i)



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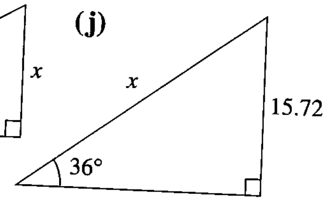
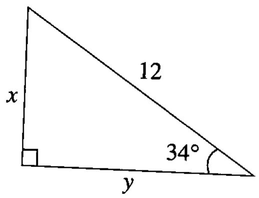
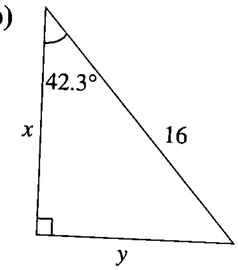


2. Find the values of x and y in each of the following triangles, giving your answers correct to 4 significant figures.

(a)



(b)



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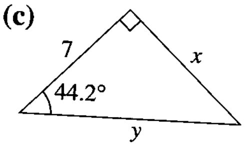

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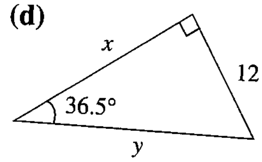

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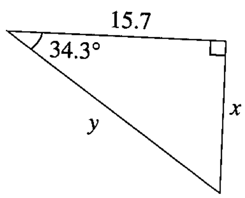

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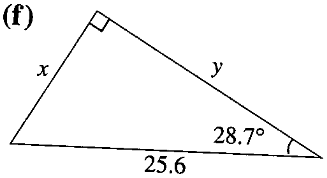

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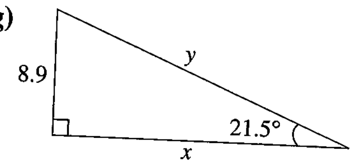

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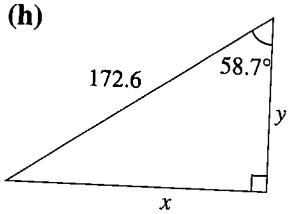

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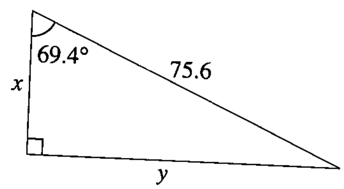

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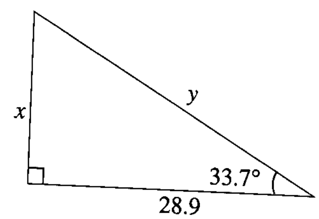

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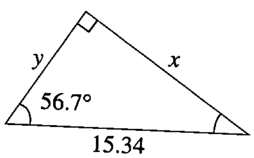

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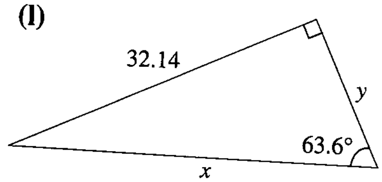
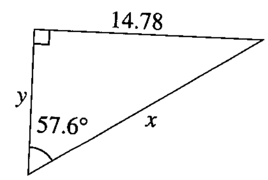
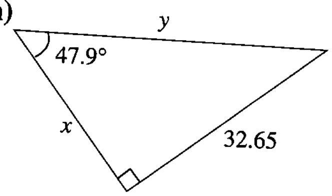


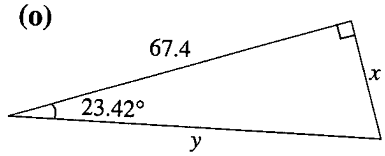
(m)



(n)



(o)



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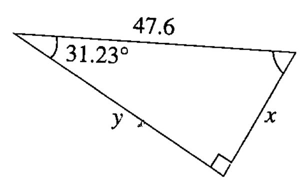
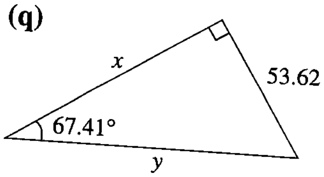
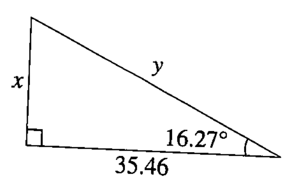


(q)



(r)

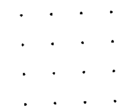


Finding the Value of an Angle with Trigonometrical Ratios

The following examples illustrate the use of trigonometrical ratios to determine the corresponding angles.

Just for Fun

Look at the given diagram.



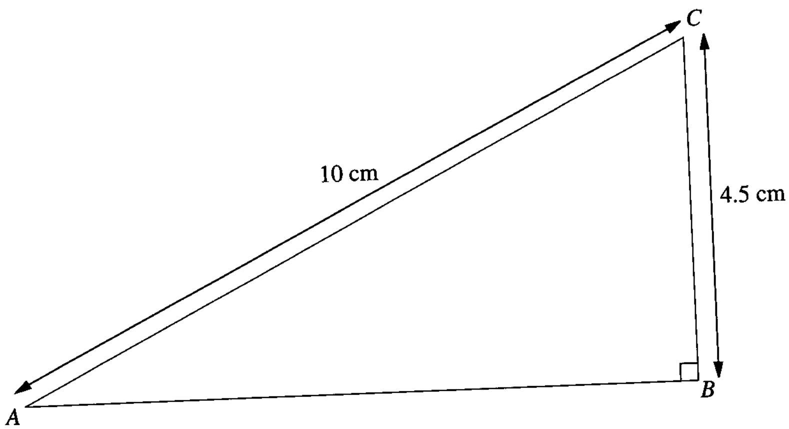
Can you connect all the 16 dots by drawing six straight lines without lifting up your pen at all?

Example 6

Construct an angle whose sine is 0.45 and measure the angle. Use your calculator to find the angle whose sine is 0.45.

Solution

Since $0.45 = \frac{4.5}{10}$, we can construct a right-angled triangle whose hypotenuse is 10 units and one of its sides 4.5 units. Draw two lines perpendicular to each other and right-angled at B. With B as centre and radius 4.5 cm, draw an arc to cut the vertical line at C. With C as centre and radius 10 cm, draw an arc to cut the horizontal line at A. Hence, $\sin B\hat{A}C = \frac{4.5}{10} = 0.45$ and $B\hat{A}C$ is the required angle. By measurement, $B\hat{A}C \approx 27°$.



Set the MODE to 'DEG'. Press [sin⁻¹] 0.45 [EXE] to get 26.743 683 95, i.e., the angle whose sine is 0.45 is 26.743 683 95°. The angle correct to 4 significant figures is 26.74°.

Thus when $\sin B\hat{A}C = 0.45$, we write $B\hat{A}C = \sin^{-1} 0.45 \approx 26.74°$.

Note: There is a slight difference in the values of angle BAC obtained by measurement and that by using calculator because of the approximate nature of measurements.

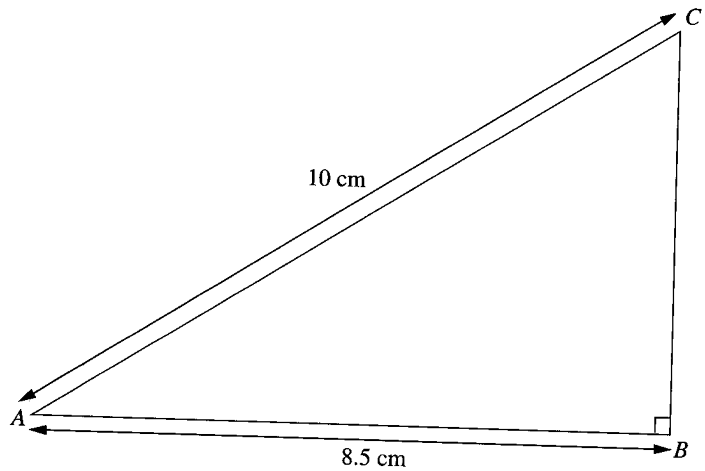
Any three numbers which can be the lengths of a right-angled triangle are known as a "Pythagorean triple". Furthermore, if the three numbers are integers with no common factor except 1, they are known as "primitive Pythagorean triple". Thus (3, 4, 5) is a primitive Pythagorean triple while (6, 8, 10) is just a Pythagorean triple. Try to find 3 sets of primitive Pythagorean triple.

Example 7

Construct an angle whose cosine is 0.85 and measure the angle. Use your calculator to find the angle whose cosine is 0.85.

Solution

Since $0.85 = \frac{8.5}{10}$, a right-angled triangle with hypotenuse 10 units and adjacent side 8.5 units is required. Draw two lines perpendicular to each other and right-angled at B. With B as centre and radius 8.5 cm, draw an arc to cut the horizontal line at A. With A as centre and radius 10 cm, draw an arc to cut the vertical line at C. Hence, $\cos B\hat{A}C = \frac{8.5}{10} = 0.85$ and $B\hat{A}C$ is the required angle. By measurement, $B\hat{A}C \approx 32°$.



Press [cos⁻¹] 0.85 [EXE] to get 31.788 330 62, i.e., the angle whose cosine is 0.85 is 31.788 330 62°. The angle correct to 1 decimal place is 31.8°.

Thus when $\cos B\hat{A}C = 0.85$, we write $B\hat{A}C = \cos^{-1} 0.85 \approx 31.8°$.

Example 8

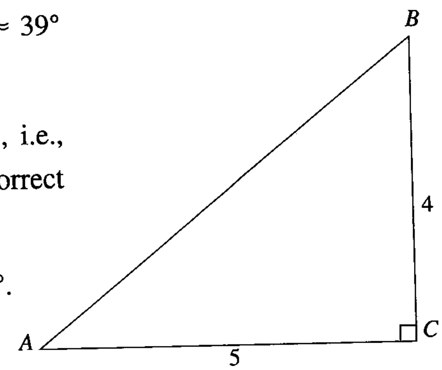
Construct an angle whose tangent is 0.8 and measure the angle with your protractor. Use your calculator to find the angle whose tangent is 0.8.

Solution

Since $0.8 = \frac{8}{10}$, we can construct a right-angled triangle such that the ratio of the opposite side to the adjacent side of the required angle is 8 : 10 or 4 : 5. Draw $AC = 5$ cm, $BC = 4$ cm and $B\hat{C}A = 90°$.

Then $\tan B\hat{A}C = \frac{BC}{AC} = \frac{4}{5} = 0.8$.

Hence $B\hat{A}C$ is the required angle and by measurement $B\hat{A}C \approx 39°$ (correct to the nearest degree).



Press [tan⁻¹] 0.8 [EXE] and the display is 38.659 808 25, i.e., the angle whose tangent is 0.8 is 38.659 808 25°. The angle correct to 1 decimal place is 38.7°.

Thus when $\tan B\hat{A}C = 0.8$, we write $B\hat{A}C = \tan^{-1} 0.8 \approx 38.7°$.

Exercise 15d

1. Construct the angles whose trigonometrical ratios are given below and measure the angles formed.

(a) $\tan A = 1.2$ **(b)** $\tan B = 2.7$ **(c)** $\sin C = 0.7$ **(d)** $\sin D = 0.25$
(e) $\cos E = 0.35$ **(f)** $\cos F = 0.62$

2. Use your calculator to find the following angles whose trigonometrical ratios are given. Give your answers correct to 1 decimal place.

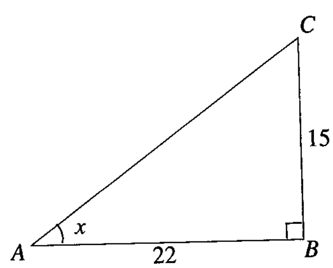
(a) $\tan A = 1.23$ **(b)** $\tan B = 2.56$ **(c)** $\sin C = 0.78$ **(d)** $\sin D = 0.527$
(e) $\cos E = 0.352$ **(f)** $\cos F = 0.725$ **(g)** $\tan G = 0.786$ **(h)** $\tan H = 1.275$
(i) $\sin I = 0.468$ **(j)** $\sin J = 0.867$ **(k)** $\cos K = 0.765$ **(l)** $\cos L = 0.924$
(m) $\tan M = 1.234$ **(n)** $\tan N = 2.137$ **(o)** $\sin O = 0.423$ **(p)** $\sin P = 0.654$
(q) $\cos Q = 0.912\ 4$ **(r)** $\cos R = 0.812\ 3$

More examples on how we can use the values of trigonometrical ratios to find the values of the relevant angles are given below.

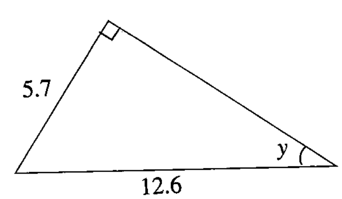
Example 9

Find the angles marked x and y in the figures below. Give your answers correct to 4 significant figures.

(a)



(b)



Solution

(a) $\tan x = \frac{BC}{AB}$

$= \frac{15}{22}$

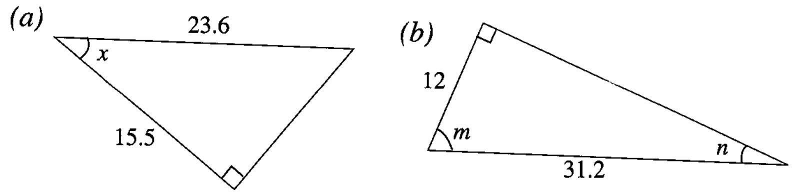
$x = 34.29°$

(b) $\sin y = \frac{5.7}{12.6}$

$y = 26.90°$

Example 10

Find the unknown angles marked in the figures below, giving your answers correct to 4 significant figures.



Solution

(a) $\cos x = \frac{15.5}{23.6}$

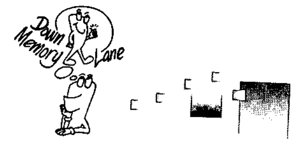
$x = 48.95°$

(b) $\sin n = \frac{12}{31.2}$

$n = 22.62°$

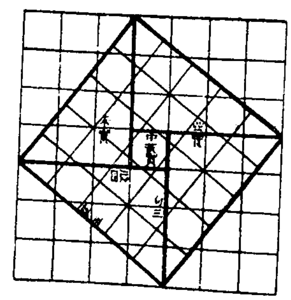
$\therefore \quad m = 180° - 90° - 22.62°$

$= 67.38°$



The Chinese were proficient in the use of Pythagoras' theorem. A diagram which shows this relationship appears in 'Chou Pei Suan Ching' around 3rd century BC. This is considered to be the oldest known demonstration of the validity of this theorem.

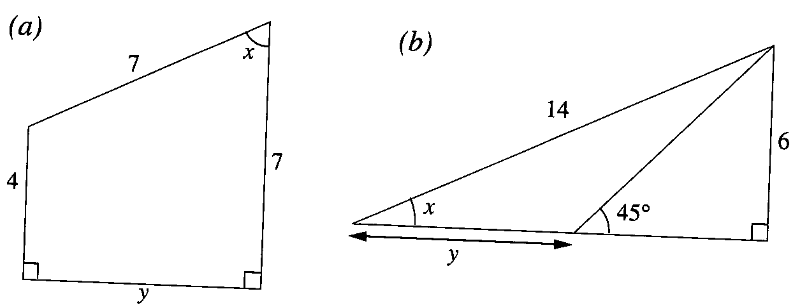
However, it is believed that the Chinese knew about this theorem around 1 100 BC, long before Pythagoras was even born.



Historians researching on the Babylonian study found the discovery of the diagonal of a square (given the measurements of the sides) as 'sufficient proof that the Pythagoras' theorem was known more than a thousand years before Pythagoras.'

Example 11

Find the values of the unknown angles and sides indicated in the following diagrams. Give your answers correct to 4 significant figures.



Solution

(a) Using the notations in the given diagram, $DK = 7 - 4 = 3$ and $AK = BC = y$.

In $\triangle ADK$, $\cos x = \frac{DK}{DA}$

$= \frac{3}{7}$

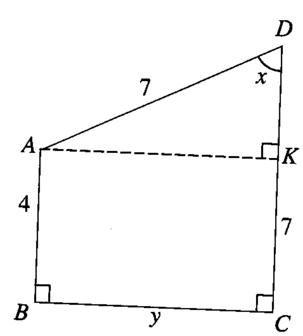
$\therefore \quad x = 64.62°$

$\tan 64.62° = \frac{AK}{DK}$

$= \frac{y}{3}$

$\therefore \quad y = 3 \tan 64.62°$

$= 6.324$

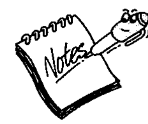


Alternatively, we can use Pythagoras' theorem to find the value of y.

$7^2 = y^2 + 3^2$

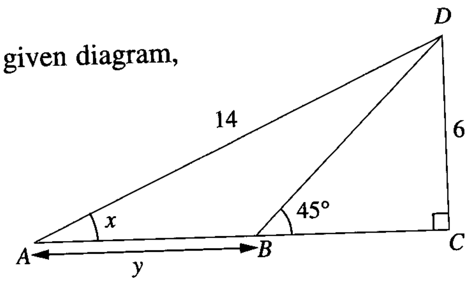
$y^2 = 7^2 - 3^2 = 40$

$y = \sqrt{40} = 6.325$



Can you explain why there is a difference of 0.001 in the two values of y?

(b) Using the notations in the given diagram,



$\sin x = \frac{CD}{AD}$

$= \frac{6}{14}$

$x = 25.38°$

In $\triangle BCD$, $\tan 45° = \frac{CD}{BC}$ **or** Since $\triangle BCD$ is an isosceles $\triangle$, $\therefore BC = 6$

$= \frac{6}{BC}$

$BC = \frac{6}{\tan 45°}$

$\therefore \quad BC = 6$

In $\triangle ACD$, $14^2 = AC^2 + 6^2$

$AC^2 = 14^2 - 6^2 = 160$

$AC = \sqrt{160} = 12.649$

$\therefore \quad y = AC - BC$

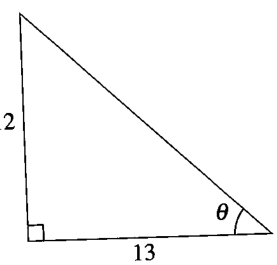
$= 12.649 - 6 = 6.649$

Exercise 15e

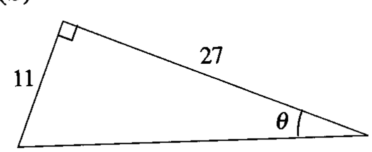
1. Find the value of θ in each of the following triangles, giving your answers correct to 4 significant figures.

(All dimensions are in cm.)

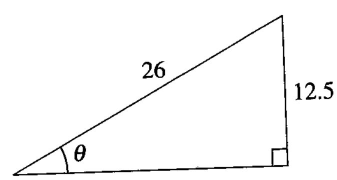
(a)



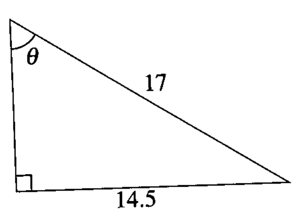
(b)



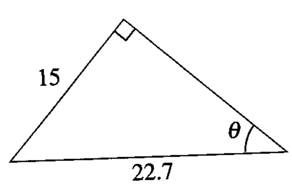
(c)



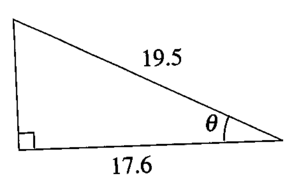
(d)



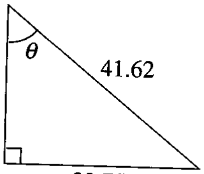
(e)



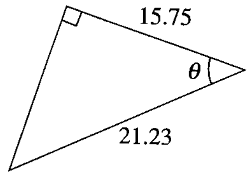
(f)



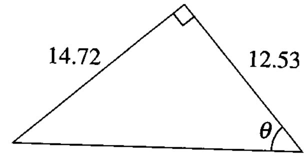
(g)



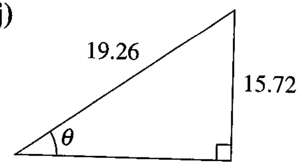
(h)



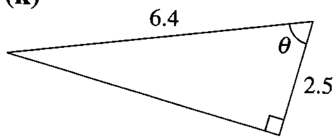
(i)



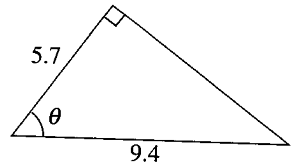
(j)



(k)

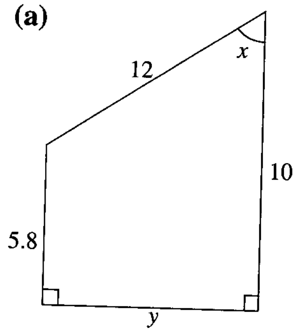


(l)

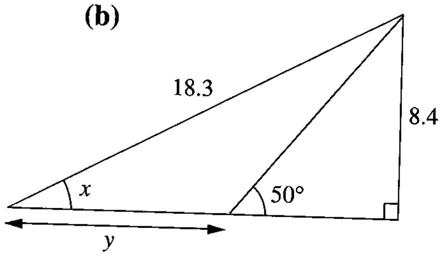


2. Find the unknown angles and sides marked x, y and z in the following triangles. Give your answers correct to 4 significant figures.

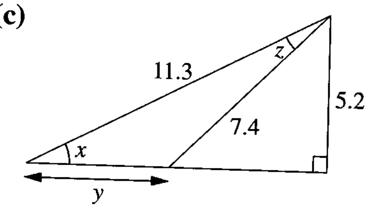
(a)



(b)



(c)



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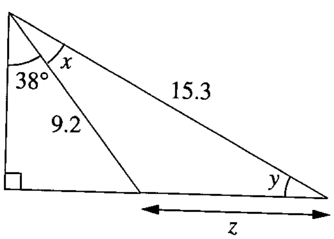
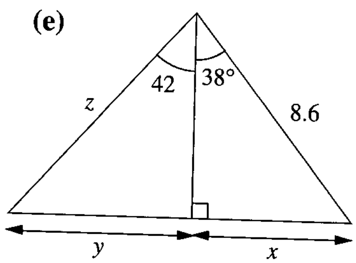
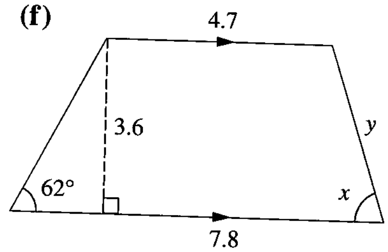


(e)



(f)

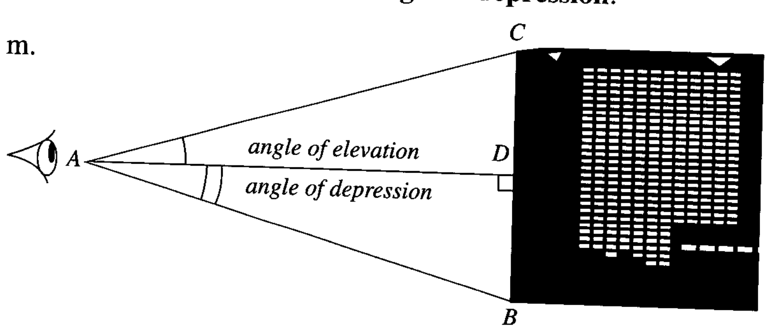


Practical Applications of Trigonometry

Suppose you stand in front of a school block BC and look at point C, the top of the block, and at point D which is at eye level, then the angle CAD is called the **angle of elevation**. It is the angle made between a line from the eye, A, to the object C and the horizontal line AD. If you look at point B, which is at the bottom of the block, the angle DAB is called the **angle of depression**.

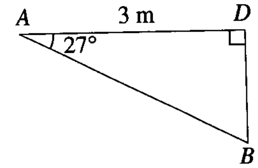
Suppose $BD = 1.5$ m and $AD = 3$ m.

Then $\tan B\hat{A}D = \frac{1.5}{3} = 0.5$.



Using a scale of 1 cm to 1 m, draw the triangle ADB and use a protractor to measure the angle BAD. It is found to be approximately 27°. From the diagram shown, it is found that $\tan B\hat{A}D = \frac{1.5}{3} = 0.5$.

We can use our calculator to find the angle BAD. It is found to be 26.6° (correct to 1 decimal place).

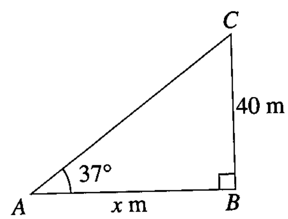


Example 12

The angle of elevation of the top of a tower 40 m high is 37° when seen from a point on level ground. Find the distance of the point from the foot of the tower.

Solution

The diagram shows the tower BC from a point A.



Let $AB = x$ m.

$$\tan 37° = \frac{BC}{AB}$$
$$= \frac{40}{x}$$
$$x = \frac{40}{\tan 37°}$$
$$= 53.08 \quad \text{(correct to 4 significant figures)}$$

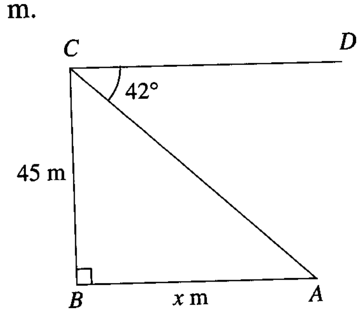
∴ the distance from the point to the foot of the tower is 53.08 m.

Example 13

From the window of a building 45 m above ground level, the angle of depression of a car on level ground is 42°. How far is the car from the building?

Solution

In the diagram, A represents the car, BC represents the height of the building where C is the point of observation and angle ACD is the angle of depression. Let $AB = x$ m.



$$A\hat{C}B = 90° - 42° = 48°$$
$$\tan 48° = \frac{x}{45}$$
$$x = 45 \tan 48°$$
$$= 49.98 \quad \text{(correct to 4 significant figures)}$$

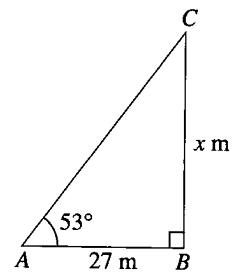
∴ the car is at a distance of 49.98 m from the building.

Example 14

At a point 27 m from the front of a pole, the angle of elevation of the top of the pole is 53°. Find the height of the pole.

Solution

In the diagram, AB and BC represent the length of the shadow and the height of the pole respectively.



Let $BC = x$ m.

$$\tan 53° = \frac{x}{27}$$

$$x = 27 \tan 53°$$

$$= 35.83 \quad \text{(correct to 4 significant figures)}$$

$\therefore$ the height of the pole is 35.83 m.

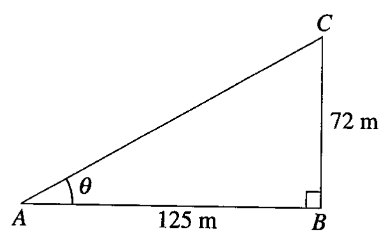
Example 15

A lighthouse is 72 m high. Find the angle of elevation of its top from a point 125 m away on level ground.

Solution

The diagram shows the lighthouse BC of height 72 m at a distance of 125 m from A.

Let the angle of elevation be θ.



$$\tan \theta = \frac{72}{125}$$

$$= 0.576$$

$$\theta = 29.9°$$

$\therefore$ the angle of elevation is 29.9°.

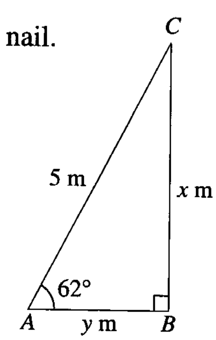
Example 16

A ladder which is 5 m long leans against a nail on the wall. It makes an angle of 62° with the ground. Find the height of the nail above the ground and the distance of the foot of the ladder from the wall.

Solution

In the figure, AC represents a ladder leaning against the wall BC where C is the nail.

Let AB be y m and BC be x m.



$$\sin 62° = \frac{BC}{AC} = \frac{x}{5}$$

$$x = 5 \sin 62°$$

$$= 4.41$$

$$\cos 62° = \frac{AB}{AC} = \frac{y}{5}$$

$$y = 5 \cos 62°$$

$$= 2.35$$

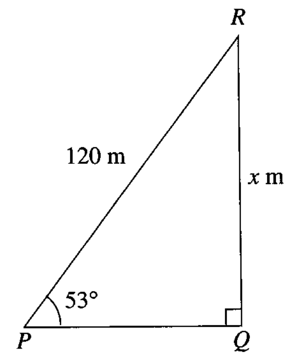
$\therefore$ the nail is at a height of 4.41 m from the ground and the ladder is 2.35 m away from the wall.

Example 17

A kite has 120 m of string attached to it when it flies at an elevation of 53°. How far is it above the hand holding it? (Assume that the string is taut).

Solution

In the figure, PR represents the length of the string, R, the position of the kite and RQ the height of the kite above the hand holding it.



Let RQ be x m.

$$\sin 53° = \frac{x}{120}$$

$$x = 120 \sin 53°$$

$$= 95.84 \quad \text{(correct to 4 significant figures)}$$

$\therefore$ the kite is 95.84 m above the hand holding it.

Exercise 15f

1. What is the angle of elevation of the top of a tree 12.7 m high from a point 23.7 m away on level ground?

2. An isosceles triangle has a vertical angle of 108° and a base 20 cm long. Calculate its altitude.

3. An aerial mast is supported by four wires attached to points on the ground each 57 metres away from the foot of the mast. If each wire makes an angle of 32° with the horizontal, find the height of the mast.

4. If a cone is 8.4 cm high and has a vertical angle of 72°, calculate the diameter of its base.

5. To find the width of a river, a boy places a wooden peg at a point A on one side directly opposite an object B on the opposite bank. From A, he walks 50 m along the bank to a point C. He observes that $A\hat{C}B = 34°$. Calculate the width of the river.

6. The angle of elevation of the top of a post from a point on level ground 38 m away is 33.23°. Find the height of the post.

7. The length and width of a rectangle are 19.2 cm and 12.4 cm respectively. Find the angle between a diagonal and the shorter side of the rectangle.

8. From a point at the same level as the foot of a building 50 metres high, the angle of elevation of its top is found to be 52.83°. Find the distance of the point of observation from the top of the building.

9. The base of an isosceles triangle is 18 cm and its altitude is 12 cm. Find its vertical angle.

10. A church spire 82 metres high casts a shadow 62 metres long. Find the angle of elevation of the sun at that moment.

11. A ladder 6.5 metres long leans against a wall, touching a window sill, and makes an angle of 62° with the ground. Find the height of the window sill above the ground. How far is the foot of the ladder from the foot of the wall?

12. The angle of depression of a boat 65.7 m from the base of a cliff is 28.9°. How high is the cliff? (Give your answer correct to 1 decimal place.)

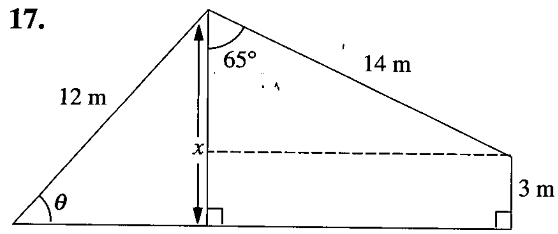
13. A boat is tied by a rope 12 m long to a ring that is 3.5 m above the water. If the rope is taut, find the angle it makes with the water.

14. A kite at the end of a 124-m string makes an angle of 58° with the ground. Find the height of the kite above the hand holding it. (Give your answer correct to 1 decimal place.)

15. A 4-m plank rests against a wall 1.8 m high so that 1.2 m of it projects beyond the wall. Find the angle the plank makes with the wall.

16. Find the distance between the points of a pair of dividers with arms 9.8 cm long, when the angle between the arms is 62°.

17.



Find the values of x and the angle θ in the figure above.

More Examples on Applications of Trigonometry

Trigonometry can be used to solve a variety of practical problems.

Example 18

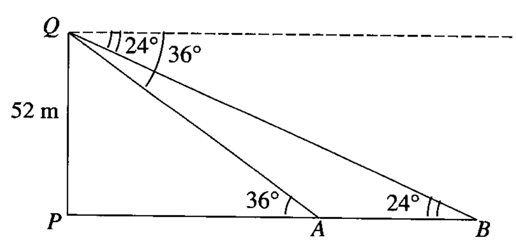
From the top of a cliff 52 m high, the angles of depression of two ships due east of it are 36° and 24° respectively. Find the distance between the ships.

Solution

In the figure, PQ represents the cliff and A and B the ships.

$P\hat{B}Q = 24°$

$P\hat{A}Q = 36°$



In $\triangle PAQ$, $\tan 36° = \frac{PQ}{PA}$

$= \frac{52}{PA}$

$\therefore \quad PA = \frac{52}{\tan 36°}$

$= 71.57$ m

In $\triangle PBQ$, $\tan 24° = \frac{PQ}{PB} = \frac{52}{PB}$

$PB = \frac{52}{\tan 24°}$

$= 116.79$ m

$AB = PB - PA$

$= 116.79 - 71.57$

$= 45.22$ m

$\therefore$ the distance between the ships is 45.22 m.

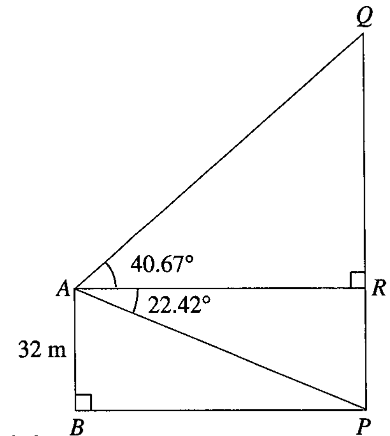
Example 19

From the top of a building 32 m high, the angles of elevation and depression of the top and foot of an aerial mast are 40.67° and 22.42° respectively. Find the height of the mast.

Solution

In the figure, PQ represents the aerial mast and AB the building.

$AB = PR = 32$ m



$$\text{In } \triangle APR, \tan 22.42° = \frac{PR}{AR} = \frac{32}{AR}$$

$$AR = \frac{32}{\tan 22.42°} = 77.56 \text{ m}$$

$$\text{In } \triangle AQR, \tan 40.67° = \frac{QR}{AR} = \frac{QR}{77.56}$$

$$QR = 77.56 \tan 40.67° = 66.64 \text{ m}$$

$$PQ = PR + QR = 32 + 66.64 = 98.64 \text{ m}$$

$\therefore$ the height of the mast is 98.64 m.

Just for fun

1. *Write the numbers 9 to 12 using only four 1's and the operational symbols +, –, ×, ÷.*

   For example,

   $13 = 11 + 1 + 1.$

   $8 = \frac{1}{0.1} - 1 - 1.$

2. *Write down the number 100 using only four 7's.*

3. *Write down the number 100 using the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 in order and using the usual operational symbols.*

   For example,

   $100 = 1 \times 2 \times 3 + 4 + 5 + 6 + 7 + 8 \times 9.$

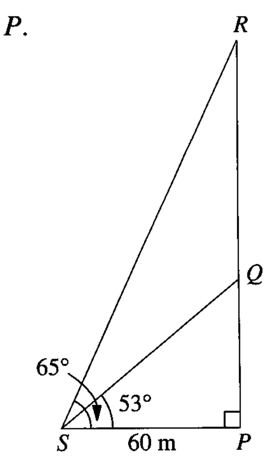
Find at least 3 other ways in which this can be done.

Example 20

A tower stands on top of a cliff. At a distance of 60 m from the foot of the cliff which is at ground level, the angles of elevation of the top of the tower as well as the cliff are 65° and 53° respectively. Find the height of the tower.

Solution

In the diagram, PQ represents the cliff and QR the tower. S is 60 m from P.



$$\text{In } \triangle SPQ, \tan 53° = \frac{PQ}{SP} = \frac{PQ}{60}$$

$$PQ = 60 \tan 53° = 79.62$$

$$\text{In } \triangle SPR, \tan 65° = \frac{PR}{SP} = \frac{PR}{60}$$

$$PR = 60 \tan 65° = 128.67$$

$$QR = PR - PQ = 128.67 - 79.62 = 49.05$$

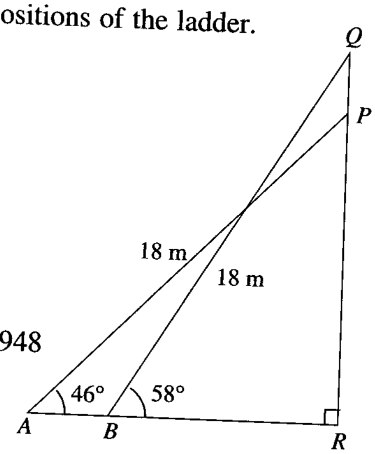
$\therefore$ the height of the tower is 49.05 m.

Example 21

A ladder of length 18 m leans against the lower edge of a window of a building standing on ground level. It makes an angle of 46° with the horizontal. When it leans against the top edge of the window, it makes an angle of 58° with the horizontal. Find the height of the window.

Solution

In the figure, PQ represents the window and AP and BQ the two positions of the ladder.



In $\triangle APR$, $\sin 46° = \frac{PR}{AP}$

$= \frac{PR}{18}$

$PR = 18 \sin 46°$

$= 12.948$

In $\triangle BQR$, $\sin 58° = \frac{QR}{BQ}$

$= \frac{QR}{18}$

$QR = 18 \sin 58°$

$= 15.264$

$PQ = QR - PR$

$= 15.264 - 12.948$

$= 2.32$

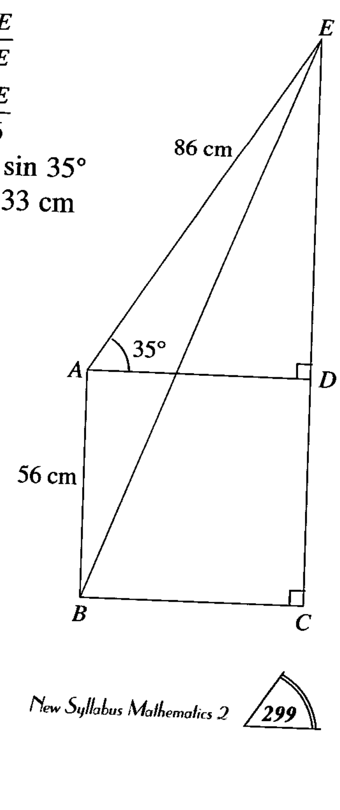
$\therefore$ the height of the window is 2.32 m.

Example 22

In the figure, ABCD is a rectangle in which AB = 56 cm, AE = 86 cm and $E\hat{A}D$ = 35°. Calculate the length of

(a) BC, *(b) DE,* *(c) BE.*

Solution



(a) In $\triangle ADE$, $\cos 35° = \frac{AD}{AE}$

$= \frac{AD}{86}$

$AD = 86 \cos 35°$

$= 70.45$ cm

$AD = BC$

$\therefore$ $BC = 70.45$ cm

(b) In $\triangle ADE$, $\sin 35° = \frac{DE}{AE}$

$= \frac{DE}{86}$

$\therefore$ $DE = 86 \sin 35°$

$= 49.33$ cm

(c) $CE = CD + DE$

$= 56 + 49.33$

$= 105.33$ cm

$BE^2 = BC^2 + CE^2$

$= 70.45^2 + 105.33^2$

$= 16\,058$

$\therefore$ $BE = \sqrt{16\,058}$

$= 126.7$ cm

Exercise 15g

1. From the top of a lighthouse 52 metres high, the angles of depression of two ships due north of it are 42° and 37°. How far apart are the ships?

2. The angle of elevation of the top of a vertical cliff, as seen from a boat 120 m away, is 32°. The angle of elevation of the top of a flagpole at the edge of the cliff, as seen from the boat, is 37°. Find the height of the flagpole.

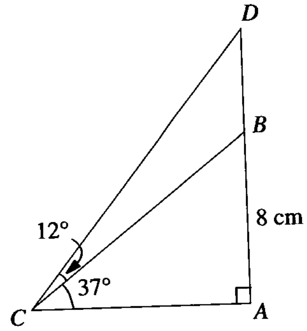
3. H is a point on the ground due east of a building, 72 m high, and K is a point due west of the building. The angles of elevation of the top of the building from H and K are 43° and 54° respectively. Find the actual distance of H and K from the top of the building.

4. Two masts are 20 m and 12 m high. If the line joining their tops makes an angle of 35° with the horizontal, find their distance apart.

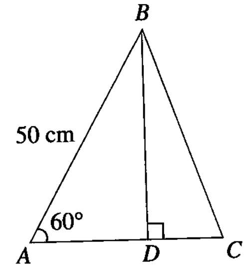
5. Two buildings on level ground are 120 m and 85 m tall respectively. If the angle of elevation of the top of the taller building as observed from the top of the shorter building is 33.9°, find their distance apart.

✶6. A tower stands on top of a cliff. At a distance of 55 m from the foot of the cliff, the angles of elevation of the top of the tower as well as the cliff are 60° and 45° respectively. Find the height of the tower.

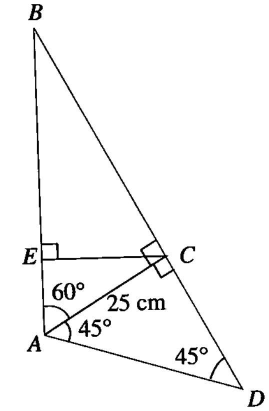
7. For the figure below, calculate BD.



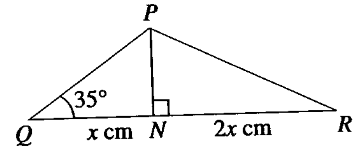
8. In the figure, $B\hat{A}C = 60°$ and BD is perpendicular to AC. If $AB = 50$ cm and $AC = 40$ cm, find **(a)** BD, **(b)** AD, **(c)** DC, **(d)** $B\hat{C}A$.



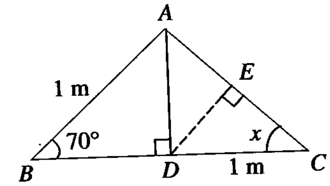
✶9. In the figure, $AC = 25$ cm.
Find the lengths of **(a)** CE, **(b)** BE, **(c)** AD.



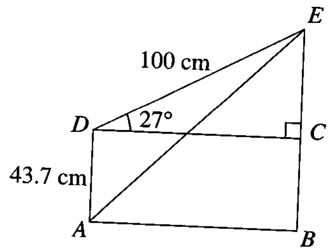
10. In the figure below, N is the foot of the perpendicular from P to the side QR of $\triangle PQR$. If $QN = x$ cm, $NR = 2x$ cm and $P\hat{Q}R = 35°$, write down an expression for PN in terms of x, and hence calculate $P\hat{R}Q$.

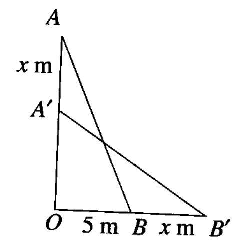


✶11. In the figure below, AD is perpendicular to the base BC of $\triangle ABC$ and DE is perpendicular to AC. The angle ABD is 70° and $AB = DC = 1$ m. Calculate the value of x and hence calculate the length of CE. (C)



12. In the figure, $ABCD$ is a rectangle in which $AD = 43.7$ cm, $DE = 100$ cm and $E\hat{D}C = 27°$. Calculate **(a)** AB, **(b)** EC, **(c)** $E\hat{A}D$.





∗13. A ladder AB, of length 13 m, rests against a vertical wall with its foot on a horizontal floor at a distance of 5 m from the wall. When the top of the ladder slips down a distance x m on the wall, the foot of the ladder moves out x m. Find x.

14. A pendulum 45 cm long swings through a vertical angle of 30°. Find the distance of the altitude through which the pendulum bob rises.

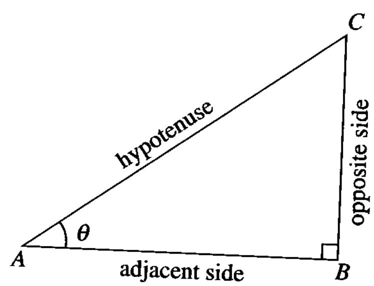
∗15. A tree is x m high. The angle of elevation of its top from a point P on the ground is 23°. From another point Q, 10 m from P and in line with P and the foot of the tree, the angle of elevation is 32°. Find x.

Summary

$$\sin\theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{AB}{AC}$$

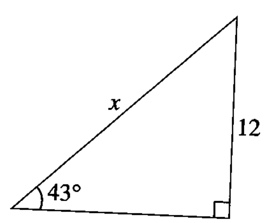
$$\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{BC}{AB}$$



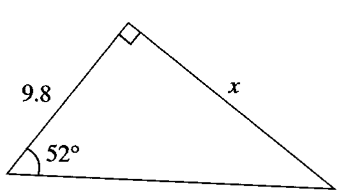
Review Questions 15

1. Find the values of x and y in the following figures. Give your answers correct to 3 significant figures.

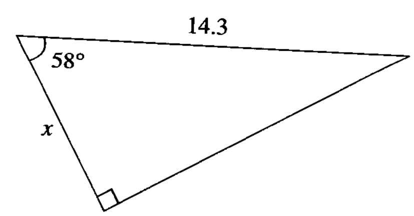
(a)



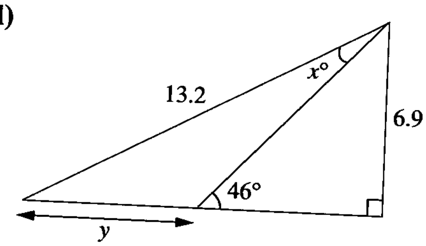
(b)



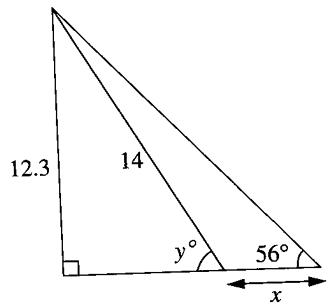
(c)



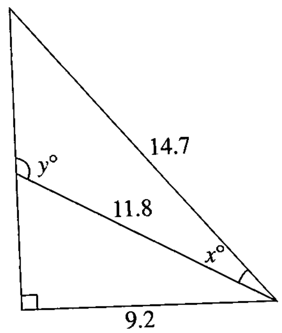
(d)



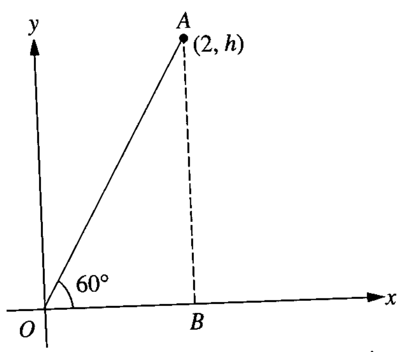
(e)



(f)



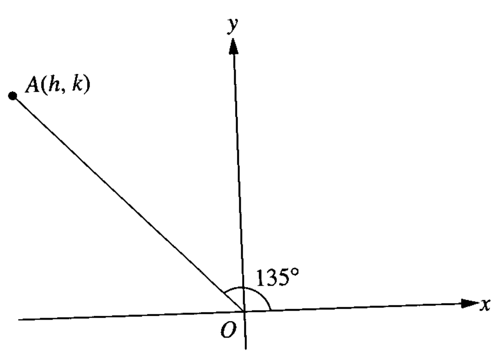
2.



The coordinates of the point A is $(2, h)$ and $A\hat{O}B = 60°$. Calculate

(a) the length of OA, (b) h.

3.

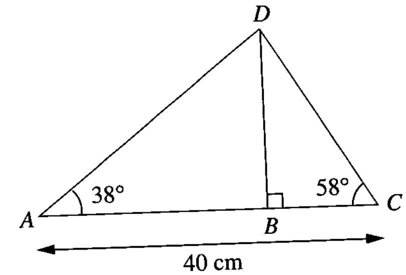


In the diagram, $A\hat{O}x = 135°$ and $OA = 3\sqrt{2}$, find the values of h and k.

4. In $\triangle ABC$ where $A\hat{B}C = 90°$, $AB = 24$ cm and $\sin A\hat{C}B = \frac{3}{5}$, find the value of

(a) AC, (b) BC,
(c) $\cos A\hat{C}B + \tan B\hat{A}C$.

5.



In the diagram, $AC = 40$ cm, $D\hat{A}B = 38°$, $D\hat{C}B = 58°$ and $D\hat{B}C = 90°$. Calculate the lengths of BD and AD.

6. The length of the sides of a rhombus is 21 cm. One of its interior angles is 112°. Calculate the lengths of the diagonals of the rhombus.

7. A ladder 3.4 m long rests against a wall at an angle of 78° to the horizontal. Find the distance of the ladder from the foot of the wall.

8. DEF is a triangle in which $DE = DF = 17$ cm and $EF = 16$ cm. Find the lengths of the heights DM and EN where DM and EN are perpendicular to EF and DF respectively.

✶9. The angle of depression of a boat from the top of a cliff is 24°. When the boat moves a distance of 80 m directly towards the cliff, the angle of depression becomes 32°. Find the height of the cliff.

✶10. The lower edge of a window in a house is 15 m above ground level. The angles of elevation of the top and bottom of the window are 27.4° and 23.25° respectively from a point K on level ground. Find the height of the window.

✶11. The angle of elevation of the top of a building at point A (on level ground) is 62°. At a point 120 m away from A, the angle of elevation is found to be 35°. Find the height of the building.

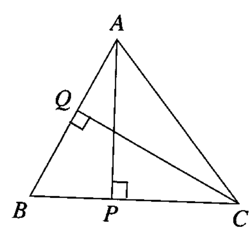
✶12. The angles of elevation of a tower at two places due west of it are 63° and 56°. Given that the foot of the tower and the two points are on ground level and the distance between the two points is 20 m, find the height of the tower.

Problem Solving

1. Given that $\sin x < \cos x$ and $0° < x < 90°$, state a possible value of x. Explain your answer clearly.

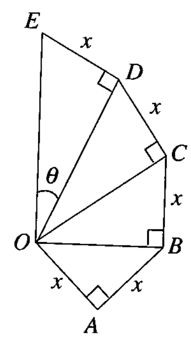
2. If $2 \sin^2 x + 3 = 7 \sin x$, find a possible value of x in the range $0° < x < 180°$.

3. In the figure below, AP and CQ are perpendicular to BC and AB respectively. If $AP : CQ = 3 : 4$, find the numerical value of $\frac{\sin A}{\sin C}$.



4. Given that $\sin^2 x + \cos^2 x = 1$, find the value of $\sin^2 2x + \cos^2 2x$.

5. In the figure, $OA = AB = BC = CD = DE = x$ cm, $O\hat{A}B = O\hat{B}C = O\hat{C}D = O\hat{D}E = 90°$. Find the numerical value of $\cos D\hat{O}E$, i.e., $\cos \theta$.



Revision Exercise IV No. 1

1. If $\frac{c}{y} = k\left(1 + \frac{x}{y}\right)$, express k in terms of c, x and y. Find k where $c = 5$, $x = 1\frac{1}{2}$ and $y = 2\frac{1}{2}$.

2. A car travels 360 km in 5 hours. What is the speed of the car in

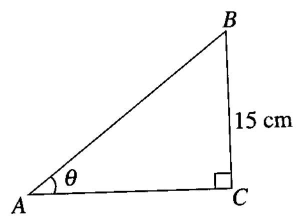
(a) kilometres per hour,
(b) metres per second?

3. Factorise the following expressions:

(a) $x^2 - 21x + 20$
(b) $x^2 + 13x + 36$
(c) $1 + 3x - 5k - 15kx$
∗(d) $12x^2 - 4xy - 5y^2$

4. (a) The volume of a cone is 56.8 cm^3 and its base radius is 14.7 cm. Find its height.

(b)



In the figure, $\sin\theta = \frac{3}{5}$. Find the lengths of AB and AC respectively.

5. For the distribution 2, 5, 3, 6, 7, 3, 6, 5, 6, 4, find

(a) the mode,
(b) the median,
(c) the mean.

6. Simplify

(a) $\frac{(2x - 1)}{3} - \frac{5(x - 3)}{6} - \frac{3}{4}$,
(b) $3[(a - 2b) - (2a - b)] - 2[(2a + b) - (a + 2b)]$,
(c) $\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}$,
(d) $\frac{a - b}{a} - \frac{b - a}{b}$.

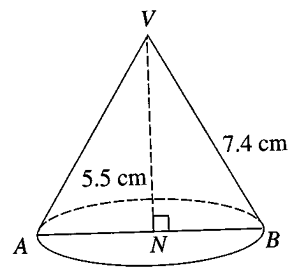
7. (a) A pyramid stands on a square base of sides 5.8 cm each. If the height of the pyramid is 10 cm, find its volume. Give your answer correct to 3 significant figures.

(b) Ten dozen lead spheres, each of diameter 3 cm, are melted and recast into a cylinder 12 cm in diameter. Calculate the height of the cylinder. (Take $\pi = 3.142$.)

8. In the triangle ABC, $\hat{C} = 90°$, $\hat{B} = 58°$ and $BC = 20$ cm. Calculate

(a) the length of AC,
(b) the area of the triangle.

9.



The figure shows a cone of height 5.5 cm and a slant height of 7.4 cm.

(a) Calculate
 (i) the diameter of the base of the cone,
 (ii) the volume of the cone, giving your answer correct to 1 decimal place.

(b) If the cone is made of solid material of density 3.4 g/cm^3, calculate the mass of the cone. Give your answer correct to 1 decimal place.

10. Copy and complete the following table of values for $y = x^2 - 4x - 2$.

x	–1	0	1	2	3	4	5	6
y		–2		–6		–2		10

Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 2 units on the y-axis, draw the graph of $y = x^2 - 4x - 2$.

Use your graph to estimate

(a) the value of y when $x = 3.5$,
(b) the minimum value of $x^2 - 4x - 2$.

Revision Exercise IV No. 2

1. Simplify (a) $\frac{x}{y} + \frac{z^2}{xy} + \frac{y}{x}$,

   (b) $\frac{3x - 2y}{2} - \frac{x + y}{3} - 1$.

2. Solve the following equations:

   (a) $\frac{x + 2}{6} + \frac{x - 2}{2} = \frac{4 - 4x}{12}$

   (b) $\frac{4}{5x} = \frac{1}{10} + \frac{7}{10x}$

   (c) $\frac{3}{x + 3} = \frac{4}{x + 9}$

   (d) $\frac{15}{x - 8} = \frac{6}{20 - x}$

3. In $\triangle ABC$, $\hat{C} = 90°$, $AC = 8$ cm and $BC = 6$ cm.

   (a) Find the length of AB.

   (b) If CD is the perpendicular from C to AB, find the length of CD.

4. If $\tan \theta = 2\frac{2}{5}$ where θ is acute, find the value of

   (a) $5 \cos \theta$, (b) $\sin (90° - \theta)$.

5. (a) A ladder 12 m long leans against a wall. Its foot on the ground is 8 m from the wall.

   (i) Calculate the angle the ladder makes with the ground.

   (ii) How high up the wall does the ladder reach?

   (b) A tower 60 m high has a shadow 75 m long. Find the angle of elevation of the sun.

6. (a) A pyramid stands on a square base of sides 9 cm each. If its height is 15 cm, calculate its volume.

   (b) Find the diameter of a circular cylinder of volume 100 cm^3 and height 6.8 cm. Give your answer correct to 3 significant figures.

7. (a) Find the volume and total surface area of a closed cylinder of height 20 cm and radius 10 cm.

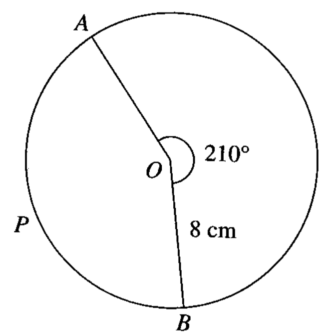
   (b) Find the volume and surface area of a sphere of radius 6.5 cm.

8. The diagram shows a circle centre O and radius 8 cm. Given that the reflex angle AOB is 210°, find in terms of π,

   (a) the area of the circle,

   (b) the length of the minor arc APB,

   (c) the area of the shaded region.



9. The mean of five numbers is 34. Three of the numbers are 29, 26 and 35. If the remaining two numbers are in the ratio 1 : 3, find the numbers.

10. Copy and complete the table of values for $y = x^2 - 2x + 3$.

x	–3	–2	–1	0	1	2	3	4	5
y	18		6		2			11	

Using a scale of 2 cm to represent 1 unit on the x-axis and a scale of 1 cm to represent 1 unit on the y-axis, draw the graph of $y = x^2 - 2x + 3$ for $-3 \leqslant x \leqslant 5$.

   (a) Use your graph to find the value of y when $x = 2.5$ and the values of x when $y = 9$.

   (b) State the equation of the line of symmetry.

Revision Exercise IV No. 3

1. If \$6 560 is divided into three shares in the ratio $1\frac{1}{2} : 2\frac{1}{3} : 3$, how much is the smallest share?

2. (a) If $\frac{a-b}{c-b} = \frac{x}{y}$, make b the subject of the formula. Find the value of b when $a = -1$, $c = 2$, $x = 10$ and $y = 6$.

(b) Solve the equation $(2x + 5)(2x - 5) = (x - 1)(4x + 3)$.

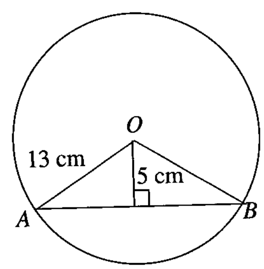
3. If 5 kg of tea and 2 kg of coffee cost \$81 and 1 kg of tea and 4 kg of coffee cost \$54, find the price of 1 kg of tea and 1 kg of coffee.

4. In triangle ABC, $\hat{C} = 90°$ and $AC = 12$ cm. If its area is 48 cm^2, calculate

(a) the length of BC,

(b) $B\hat{A}C$.

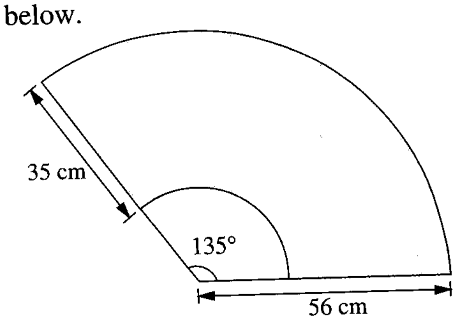
5.



Find the length of AB and the angle AOB in the figure above.

6. (a) Find the volume of a cylindrical steel bar 20 cm long and 3.2 cm in diameter. Find also its total surface area. Give your answer correct to 1 decimal place. (Take $\pi = 3.14$.)

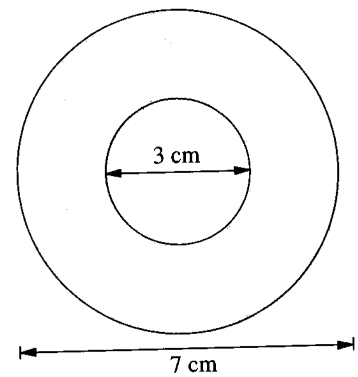
(b) Taking π to be $3\frac{1}{7}$, find the area of the shaded region of the figure shown below.



7. The diagram shows a cylindrical washer of outer diameter 7 cm and inner diameter 3 cm.

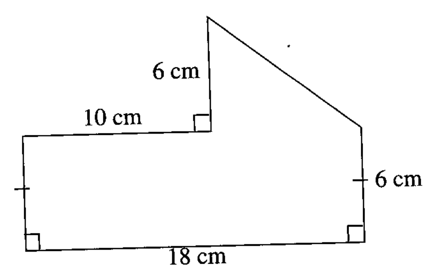
(a) Taking π to be 3.14, calculate the area of the shaded region.

(b) Given that the washer has a thickness of $\frac{1}{4}$ cm and is made of a metal of density 4 g/cm^3, calculate the mass of the washer correct to the nearest gram.



8. The mean of 12 numbers is 8 and the mean of the first 5 numbers is 7.6. Find the mean of the last 7 numbers.

9. The cross-section of a solid block of wood is shown below.



(a) If the length of the piece of wood is 1 m, find its volume.

(b) If 1 cm^3 of wood weighs 0.7 g, find the weight of the block of wood in grams.

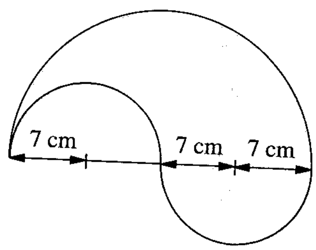
10. (a) A rectangular solid 24 cm by 22 cm by 21 cm is melted down and recast into a solid cone of base radius 28 cm. Find the height of the cone.

(b) Find the surface area of a sphere of diameter 28 cm. $\left(\text{Take } \pi = \frac{22}{7}.\right)$

Revision Exercise IV No. 4

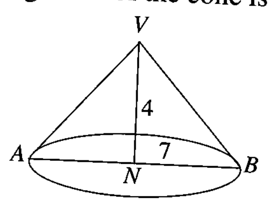
1. (a) When the price of an article is increased by 15%, the increase in the price of the article is $45. Find its original price.
   (b) Find the principal that will earn $221 simple interest in $6\frac{1}{2}$ years at $4\frac{1}{4}\%$ per annum.

2. The diagram consists of three semicircles whose radii are 7 cm, 7 cm and 14 cm.



Calculate, taking $\pi = \frac{22}{7}$,
   (a) the circumference of the shaded region,
   (b) the area of the shaded region.

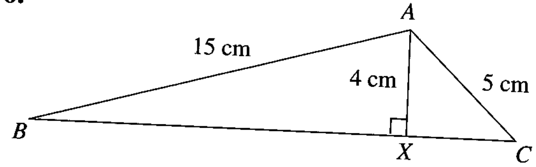
3. In a right circular cone, the base diameter $AB = 7$ cm and the height VN of the cone is 4 cm. Calculate
   (a) the volume,
   (b) the slant side VA of the cone.



4. A rectangular tank 3 m long and 2.4 m wide internally contains 7 200 litres of liquid. Find the depth of the liquid in the tank.

5. From a point 10 m away from the base of a building, the angles of elevation of the top and bottom of a window are 40° and 30° respectively. How tall is the window? (Give your answer correct to 2 significant figures.)

6.



From the figure above, calculate
   (a) the length of BC (correct to 3 significant figures),
   (b) $B\hat{A}X$.

7. Find the area of an isosceles triangle whose base is 30 cm and whose base angle is 40°. Give your answer correct to 1 decimal place.

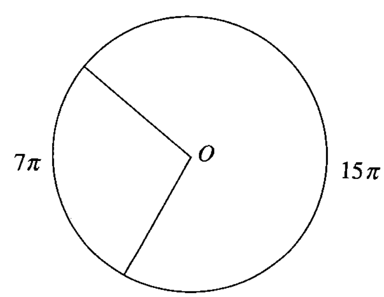
8. The distribution of the length (x cm) of 80 leaves is given in the table below:

x (in cm)	10	11	12	13	14	15	16
Frequency	6	10	20	22	14	6	2

Find (a) the mode,
   (b) the median,
   (c) the mean of the distribution, giving your answer correct to 1 decimal place.

9. In $\triangle ABC$, $\hat{C} = 90°$, $AB = 13$ cm and $AC = 5$ cm. Find the length of BC and the area of $\triangle ABC$.

10. The lengths of the minor arc and the major arc of a circle are 7π cm and 15π cm respectively. Find, taking $\pi = \frac{22}{7}$,
   (a) the radius of the circle,
   (b) the area (shaded region) enclosed by the minor arc and the radii.



Revision Exercise IV No. 5

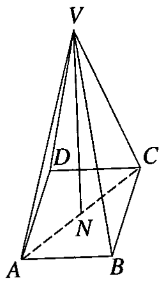
1. The average speed of a car for a 425-km journey was 50 km/h. If the car stopped for 90 minutes on the road, find the average speed of the car while travelling.

2. The commission received by a salesman is $7\frac{1}{2}\%$ on all his sales. If his sales totalled $2 240, find his commission.

3. Given that $s = ut + \frac{1}{2}at^2$, express a in terms of s, u and t. Find the value of a when $s = 10\frac{1}{2}$, $u = 1\frac{1}{2}$ and $t = 3$.

4. A man paid \$51 for 3 shirts and 12 pens. If a shirt costs \$4.50 more than a pen, find the price of each shirt and each pen.

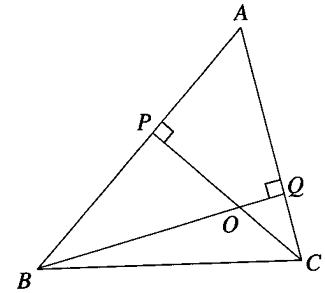
5. A right pyramid has a square base of sides 4 cm each and height 8 cm. Calculate
   (a) its volume,
   (b) $V\hat{A}N$.



6. (a) The volume of a cone is 78.8 cm^3 and its base radius is 4.2 cm. Taking π to be 3.142, find the height of the cone. Give your answer correct to 3 significant figures.
   (b) A cylindrical metal bar of length 2 m and diameter 2 cm is melted to form a circular disc of thickness 1 cm. Find the diameter of the disc.

7. Using a scale of 2 cm to represent 2 units on both axes, plot the graphs of $y = 2x + 2$ and $y + 3x = 10$. Use your graph to find the solution of the simultaneous equations $y = 2x + 2$ and $y + 3x = 10$.

8. In the figure below, $A\hat{P}C = A\hat{Q}B = 90°$. $OP = 20$ cm, $OB = 38$ cm and $OC = 12$ cm. Calculate the lengths of PB and OQ.



9. Using a scale of 1 cm to 1 unit on both axes, draw $\triangle ABC$ with coordinates $A(1, 1)$, $B(3, 1)$ and $C(4, 3)$. $\triangle ABC$ is transformed onto $\triangle PQC$ under an enlargement, centre (4, 3) and scale factor 2. Draw $\triangle PQC$ on the same diagram. $\triangle PQC$ is then transformed onto $\triangle LMN$ under a reflection in the x-axis. Draw $\triangle LMN$ on the same diagram.

10. The following is a set of marks scored by 12 students in a Mathematics test marked out of a total of 20.

    18, 4, 7, 11, 15, 8, 14, 11, 18, 19, 11, 20

    (a) Find the mode.
    (b) Find the value of the median.
    (c) Calculate the mean of the 12 marks.
    (d) Another boy who was absent on that day was given the same test the following day and he scored x marks. With this new mark, the mean now becomes $13\frac{2}{13}$. Find the value of x.

End-of-Year Examination Specimen Paper 1

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** allowed to be used.*

1. Evaluate each of the following, giving your answer in the standard form $A \times 10^n$, where $1 \leqslant A < 10$ and n is an integer:

(a) $(3.4 \times 10^6) \times (4.5 \times 10^{-3})$ [2]

(b) $(6.5 \times 10^5) + (6.4 \times 10^4)$ [2]

2. A semi-circular flower bed has a perimeter of 72 cm. Calculate the area of the flower bed, taking π to be $\frac{22}{7}$. [3]

3. Make x the subject of the formula $y = \frac{nx}{k - hx}$. [3]

4. Solve the following equations:

(a) $3x - (2 - 4x) = 5(x - 11)$ [2]

(b) $\frac{x}{4} - \frac{2x + 3}{5} + 3 = 0$ [2]

5. If a shopkeeper sells a jacket for \$81, he will make a profit of 8%. Find the selling price if he sells it at a gain of 15%. [3]

6. Factorise each of the following:

(a) $px + py - qx - qy$ [2]

(b) $2c^2 - 5cd + 3d^2$ [2]

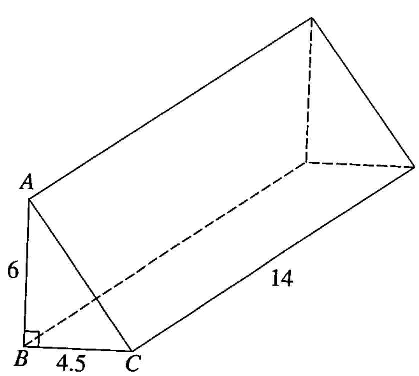
7. Solve the following equations:

(a) $2x^2 + 5x - 3 = 0$ [2]

(b) $16(x - 1)^2 - 9 = 0$ [2]

8. The distance of 25 km between two towns, P and Q, is represented by a line of 5 cm on a map. If the scale of the map is $1 : 5x$, find the value of x. [3]

9.



The figure shows a triangular bar of cross-section ABC. If the length of the bar is 14 cm, $AB = 6$ cm and $BC = 4.5$ cm, find the volume of the bar. [3]

10. Three of the interior angles of a pentagon are 88°, 113° and 139° while the other two are $7x°$ and $13x°$. Find x. [2]

11. Which of the letters B, E, S, T have

(a) just one axis of symmetry? [2]

(b) rotational symmetry of order 2? [1]

12. The following are marks scored by 9 pupils in a Science test marked out of a total of 15:

4, 9, 11, 8, 10, 5, 11, 14, 7

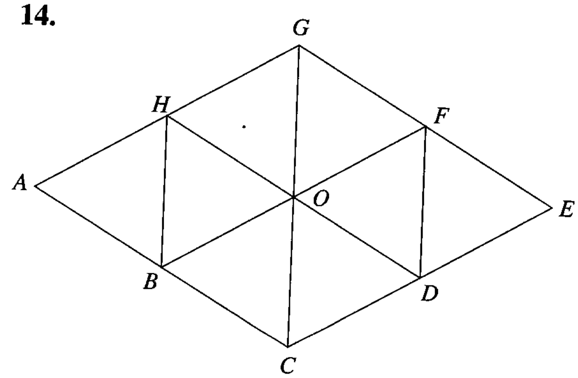
Find **(a)** the mode, [1]

(b) the median, [1]

(c) the mean of this set of marks. [2]

★13. If $x^2 + y^2 = 57$ and $xy = 3$, find the value of $3(x + y)^2$. [3]

14.



The diagram above shows 8 congruent equilateral triangles each of length 3 cm.

(a) Describe a single transformation that will map

(i) $\triangle ABH$ onto $\triangle EDF$, [1]

(ii) $\triangle OBC$ onto $\triangle OFG$, [1]

(iii) $\triangle ABH$ onto $\triangle ACG$, [2]

(iv) the rhombus $ABOH$ onto the rhombus $HOFG$. [1]

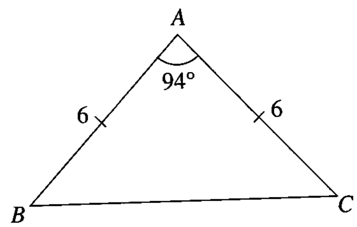
(b) Describe two successive transformations that will map $\triangle DEF$ onto $\triangle HOB$. [2]

Part II (50 marks) **Time: 1 h 15 min**

*Answer **all** the questions. Calculators may be used.*

Section A (22 marks)

1.



In the isosceles triangle ABC, $AB = AC = 6$ cm and $B\hat{A}C = 94°$. Calculate the length of BC. [4]

2. The coordinates of the end-points of a line AB are $A(1, 5)$ and $B(4, 2)$.

(a) Find the image of AB under a reflection in the x-axis. [2]

(b) AB is rotated through 90° anticlockwise about $O(0, 0)$. Find the coordinates of its image. [2]

3. A regular pyramid stands on a square base of sides 8.6 cm each. If the height of the pyramid is 9.2 cm, calculate

(a) the volume of the pyramid, [3]

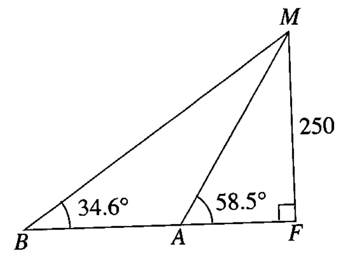
(b) the length of a sloping edge. [4]

Give your answers correct to 4 significant figures.

4. A rectangular tank has a base 2.4 m by 1.8 m. It is being filled with 230 litres of a liquid per minute. Find the depth of liquid in the tank after 8 minutes. If the density of the liquid is 1.25 g/cm³, find the mass of liquid in the tank after 8 minutes. Give your answer in kg. [7]

Section B (28 marks)

5. The following figure shows a man standing on top of a cliff 250 m high. He observes two ships, A and B, and their angles of depression to be 58.5° and 34.6°, respectively. Find the distance between A and B. [7]



6. Copy and complete the table of values for $y = x^2 - 2x - 4$. [1]

x	−2	−1	0	1	2	3	4	5
y	4		−4	−5			4	11

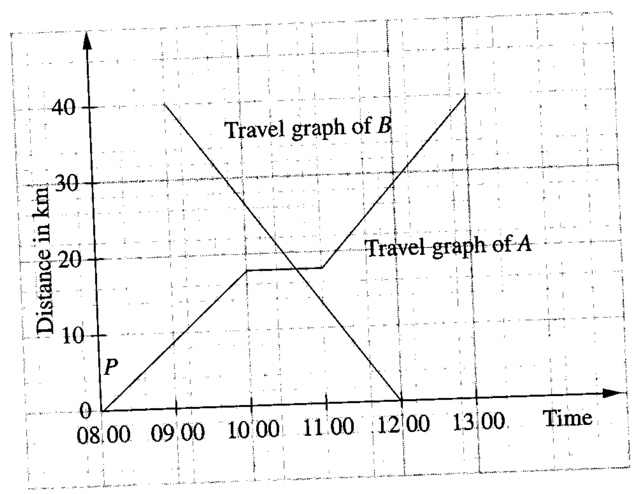
Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 2 units on the y-axis, plot the graph $y = x^2 - 2x - 4$ for $-2 \leqslant x \leqslant 5$. [3]

Use your graph to answer the following:

(a) What is the value of y when $x = 2.4$?

(b) What are the possible values of x when $y = -2$? [2]

7. The travel graphs below show the journeys of two cyclists, A and B. A starts from P at 08 00 and travels towards Q, a distance of 40 km away. B starts from Q at 09 00 and travels towards P.



Find, from the graph,

(a) the speed of B for the whole journey, [1]

(b) the time and how far A and B are from Q when they meet, [2]

(c) when and for how long A took a rest, [2]

(d) how far A is from Q when B reaches P. [2]

8. The Singapore government spent \$4.45 billion on education in 1997. In 1996 the expenditure on education was \$3.77 billion.

(a) Calculate the percentage increase in expenditure on education from 1996 to 1997, giving your answer correct to 2 decimal places. [1]

(b) In 1998 the government spent an additional 29% on education over 1997 figure. Calculate the total amount spent in 1998, giving your answer correct to the nearest million dollars. [2]

(c) If there were a total of 595 000 pupils in 1997, calculate the average amount spent on each pupil in 1997, giving your answer correct to the nearest dollar. [2]

(d) Given that the number of pupils in 1997 was 1.3% more than in 1996, calculate the number of pupils in 1996, giving your answer correct to the nearest 500. [2]

End-of-Year Examination Specimen Paper 2

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** allowed to be used.*

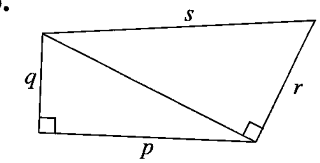
1. Given that $v^2 = u^2 + 2as$, express s in terms of u, v and a. Find the value of s if $u = 2$, $v = 7$ and $a = 4\frac{1}{2}$. [4]

2. The sum of two numbers is 94. If twice the smaller number minus the larger is 26, find the numbers. [4]

3. The coordinates of the point of intersection of the lines $ax + y = 3$ and $x + 2y = b$ are $(2, -3)$. Find a and b. [3]

4. The lengths of the two diagonals of a rhombus are 16 cm and 12 cm respectively. Find the length of one side of the rhombus. [3]

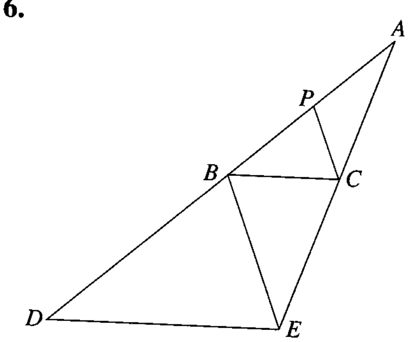
5.



For the figure above, show that

$$s^2 = p^2 + q^2 + r^2.$$ [3]

6.

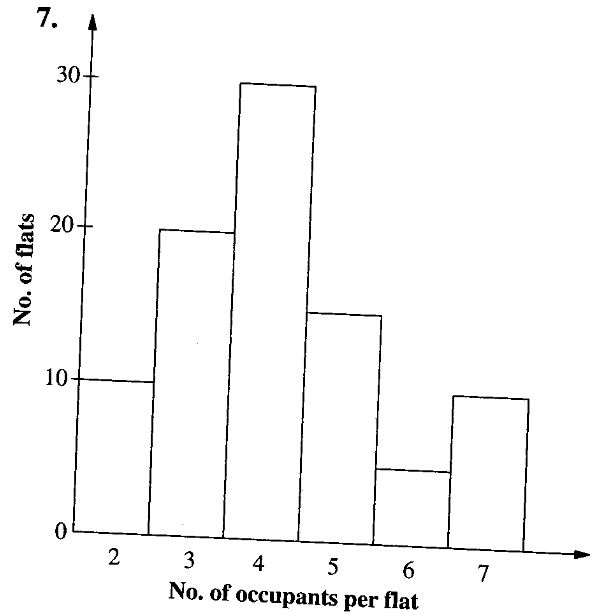


In the diagram above, B and C are the midpoints of AD and AE respectively, while P is the midpoint of AB. Describe a single transformation that will map

(a) $\triangle PBC$ onto $\triangle BDE$, [2]

(b) $\triangle APC$ onto $\triangle ABE$. [2]

7.



The bar chart shows the number of occupants per HDB flat in a particular location.

(a) State the modal number of occupants per flat. [1]

(b) Calculate the mean number of occupants per flat. [2]

(c) If this information is to be represented by a pie chart, find the angle of the sector representing flats with 7 occupants. [2]

8. Simplify the following:

(a) $\frac{1}{x} + \frac{1}{y}$ [1]

(b) $\frac{1}{x+2} - \frac{1}{x+3}$ [2]

(c) $\frac{2}{x+y} + \frac{3}{x-y}$ [2]

9. Expand the following:

(a) $(2x + y)(3x - 5y)$ [2]

(b) $(2x - 3y)(2x + y - 4)$ [2]

∗**10.** Solve the equation $x^3 - 5x^2 + 6x = 0$. [3]

11. If $\cos\theta = \frac{5}{13}$ where θ is an acute angle, find the value of

(a) $3\tan\theta$, (b) $2\sin\theta$. [4]

12. A rectangular tank 50 cm long and 40 cm wide contains 100 litres of water. Find the depth of the water. [3]

13. Factorise the following:

(a) $36x^2 - 49y^2$ [2]

(b) $12a^2 - 31a - 15$ [1]

∗(c) $h^2 - 3hk - 54k^2$ [2]

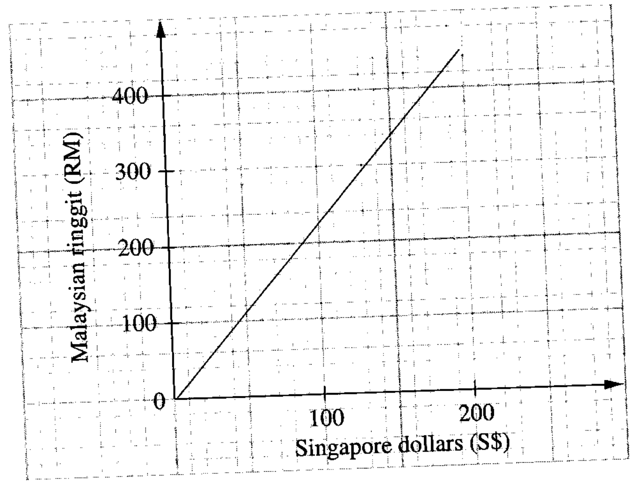
Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used.*

Section A (22 marks)

1. The vertical angle of an isosceles triangle is 54° and the height is 32 cm. Find the length of the base of the triangle and its area. [4]

2. The diagram shows the conversion graph of Malaysian ringgit and Singapore dollars in a certain day in 1999. Use the graph to answer the following:

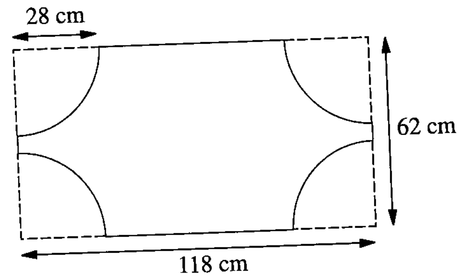


(a) How many Malaysian ringgit could you get for S$200? [1]

(b) How many Singapore dollars could a tourist get for RM240? [1]

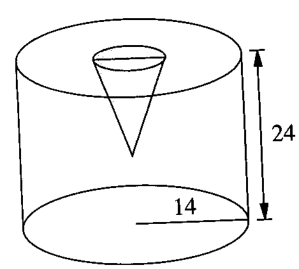
(c) How many Malaysian ringgit could one get for S$2 400? [2]

3.



The figure shows a rectangular piece of wood 62 cm by 118 cm. From its four corners, quadrants of radii 28 cm have been cut off. Find the area and perimeter of the remaining piece of wood. $\left(\text{Take } \pi = \frac{22}{7}.\right)$ [8]

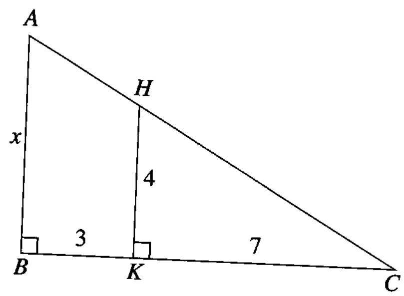
4.



From a solid cylinder whose height is 24 cm and radius 14 cm, a conical cavity of height 12 cm and base radius 10 cm is hollowed out. Find the volume of the remaining solid, giving your answer correct to 4 significant figures. (Take $\pi = 3.142$.) [6]

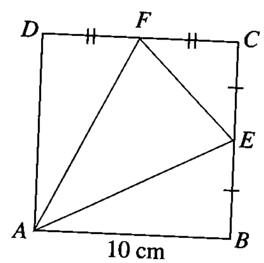
Section B (28 marks)

5.



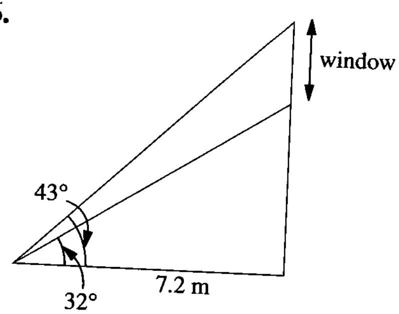
(a) In the figure, $AB = x$ cm, $HK = 4$ cm, $BK = 3$ cm and $KC = 7$ cm. Find the value of x and the angle ACB. [4]

(b)



$ABCD$ is a square of sides 10 cm each. E and F are the midpoints of BC and CD respectively. Find the angle EAF. [3]

6.



The angles of elevation of the top and bottom of a window from a point 7.2 m from its foot are 43° and 32°, respectively. Calculate the height of the window. [6]

7. (a) If $x = 2$ and $y = -5$ satisfy both the equations $ax + by + 1 = 0$ and $(b - 1)x + 5y + 3a = 0$, find the values of a and b. [4]

(b) Using a scale of 1 cm to represent 1 unit on both axes, draw $\triangle ABC$ whose coordinates are $A(2, 1)$, $B(1, 5)$ and $C(4, 2)$. Reflect $\triangle ABC$ using the y-axis as the line of reflection to obtain $\triangle PQR$. Draw $\triangle PQR$. $\triangle PQR$ is enlarged with $O(0, 0)$ as the centre of enlargement and scale factor $\frac{1}{2}$. Find the image of the point R under this enlargement. [4]

8. Copy and complete the following table for $y = 6 + x - 2x^2$. [1]

x	-3	-2	-1	0	1	2	3
y	-15		3		5		-9

Using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = 6 + x - 2x^2$ for $-3 \leqslant x \leqslant 3$. [4]

Use your graph to estimate

(a) the value of y when $x = 0.4$, [1]

(b) the values of x when $y = -6$. [1]

End-of-Year Examination Specimen Paper 3

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** allowed to be used.*

1. Solve the following equations:

(a) $x^2 - 2x - 63 = 0$ [2]

(b) $\sqrt{x - 2} = 4$ [2]

2. Factorise the following:

(a) $x^2 - 45x - 2\,250$ [2]

(b) $15x^2 + 34x - 168$ [2]

(c) $16(x^2 + y^2) + 20(x^2 - y^2)$ [2]

3. How many axes of symmetry does each of the following plane figures have?
   - **(a)** An equilateral triangle [1]
   - **(b)** A regular octagon [1]
   - **(c)** A parallelogram [1]
   - **(d)** A rectangle [1]

4. Given that $p = \frac{1}{2}m(q - 1)$, make q the subject of the formula. [3]

5. Solve the simultaneous equations

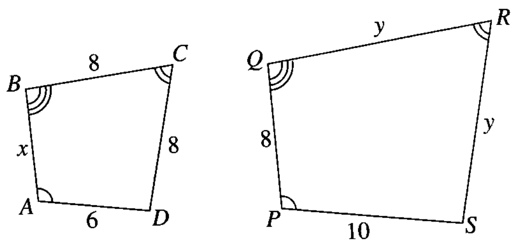
$$\frac{x + y}{5} + \frac{2y - 3}{2} = 3, \quad \frac{x + 2y}{3} - x = 4.$$ [4]

6. Given that $x - y = 7$ and $x^2 - y^2 = 42$, find the value of $x + y$. [3]

7. Two men, John and Peter, can paint a house in 10 days. John alone can paint the house in 15 days. How long will Peter take to paint it himself? [4]

8. x circular discs, each of radius 6 cm and thickness 0.5 cm, are melted to form a circular bar of diameter 2 cm and length 144 cm. Find x. [3]

9. In the diagram, $ABCD$ is similar to $PQRS$. Calculate the values of x and y. [4]



10. Three of the interior angles of a pentagon are 96°, 110° and 126° while the other two angles are each equal to $2x°$. Find the value of x. [3]

11. The sales of three models of a car, A, B and C for the month of February are 45, 72 and x respectively.
   - **(a)** If the total sales amount to 180 cars, find the value of x. [1]
   - **(b)** Express the sales of A as a percentage of the total sales. [1]
   - **(c)** If the sales are represented on a pie chart, calculate the angle of the sector representing the sales of C. [2]

12. **(a)** A translation maps the point (2, 3) onto the point (5, –2) and the point X onto the point (–3, 4). Find the coordinates of X. [2]
   (b) A reflection in the x-axis maps the point P onto the point (–2, –3). Find the coordinates of P. [2]

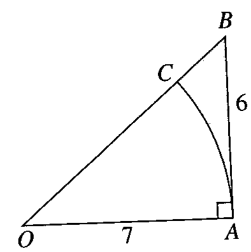
13. If $\sin x = \frac{4}{5}$, where x is an acute angle, find the value of $\sin x + 2\cos x + 3\tan x$. [4]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used.*

Section A (22 marks)

1.



The figure above shows a right-angled triangle OAB. AOC is a minor sector enclosed in the triangle. If OA = 7 cm, AB = 6 cm, calculate the area and perimeter of the shaded region. [5]

2. The length of a diagonal of a rectangular field is 23.7 m and one of its sides is 18.8 m. Find the perimeter of the field. [3]

3. **(a)** State which one of the following plane figures has no axis of symmetry:
   - **(i)** a semicircle [1]
   - **(ii)** an isosceles triangle [1]
   - **(iii)** an equilateral triangle [1]
   - **(iv)** a parallelogram [1]

   (b) Multiply $(2x^3 + 3x^2 - 4x + 1)$ by $(2x - 1)$. [2]
   (c) Divide $(3x^3 + 4x^2 - 7x + 5)$ by $(x - 1)$. [2]

4. If 1 is subtracted from both the numerator and the denominator of a fraction, its value becomes $\frac{1}{6}$. If 3 is added to both the numerator and the denominator, its value becomes $\frac{1}{2}$. Find the fraction. [6]

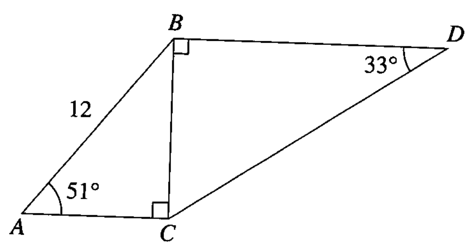
Section B (28 marks)

5. (a) Solve the equation $9x^2 - 1 = 15x + 5$. [2]

(b) Find two numbers whose sum is 10 and whose product is 24. [4]

6. A cone has a base radius of 16 cm and a slant height of 22 cm. Calculate the height of the cone and hence, or otherwise, find its volume. Give your answer correct to 3 significant figures. [6]

7.



In the figure, $A\hat{C}B = C\hat{B}D = 90°$, $B\hat{A}C = 51°$, $B\hat{D}C = 33°$ and $AB = 12$ cm. Calculate, giving your answers correct to 3 significant figures, the length of

(a) BC, [2]

(b) CD, [3]

(c) BD. [3]

8. Copy and complete the following table of values for $y = x^2 - x - 5$. [1]

x	−2	−1	0	1	2	3
y	1		−5	−5		

Plot the graph of $y = x^2 - x - 5$ using a scale of 2 cm to represent 1 unit for both the x- and y-axes. [4]

Use your graph to find

(a) the value of x for which y has the least value, [1]

(b) the value of y when $x = 1.4$, [1]

(c) the values of x when $y = 0$. [1]

End-of-Year Examination Specimen Paper 4

Part I (50 marks) — Time: 1 h

*Answer **all** the questions. Calculators are **not** allowed to be used.*

1. Given that $y = \frac{x}{2a - 3x}$, express x in terms of a and y. Find the value of x when $a = 10$ and $y = 1$. [4]

2. Use factorisation to evaluate each of the following:

(a) $1\,023^2 - 23^2$ [2]

(b) $879^2 + 121 \times 879$ [2]

3. Solve the simultaneous equations

$$2x + 3y = 2,\ 6x - y = 2\frac{2}{3}.$$ [3]

4. Simplify each of the following:

(a) $4(2x - y + z) - 7(5x + 2y - 3z)$ [2]

(b) $(2x - y)(3x + 2y - 1)$ [2]

5. A car travels a distance of 30 km at an average speed of 40 km/h and then travels for 40 minutes at an average speed of 60 km/h. Find

(a) the total distance travelled, [2]

(b) the average speed for the whole journey. [2]

6. Find the mean, mode and median of the following scores obtained after eleven rounds of golf:

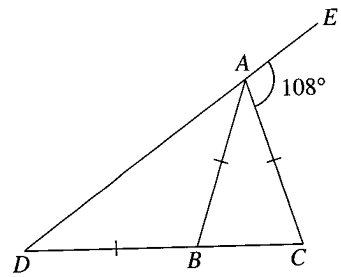
76, 72, 69, 80, 74, 73, 73, 69, 73, 82, 73 [4]

7. Find the volume and surface area of a closed rectangular box which is 16 cm long, 12 cm wide and 7.8 cm high. [5]

8. (a) Each interior angle of a polygon is 140°. How many sides does the polygon have? [2]

(b) Two exterior angles of a hexagon are $2x°$ and $3x°$ while the other 4 exterior angles are each equal to 65°. Find x. [2]

9.



In the figure above, $AB = BD = AC$ and $E\hat{A}C = 108°$. DAE is a straight line. Find the value of

(a) $A\hat{C}B$, [2]

(b) $A\hat{D}B$. [2]

10. The total surface area of a cube is 384 cm^2. Find the volume of the cube. [3]

11. Plot the points $A(1, 0)$, $B(3, 4)$ and $C(4, 2)$ on graph paper. $\triangle ABC$ is mapped onto $\triangle PQR$ by an enlargement with centre $O(0, 0)$ and scale factor 2. Find the coordinates of P, Q and R. [5]

12. A cyclist set off at 11 00 for a destination 50 km away. He cycled at a speed of 15 km/h for 2 hours before he took a rest for 30 minutes. He then completed his remaining journey at a speed of 20 km/h. Using a scale of 2 cm to represent 1 hour on the horizontal axis and a scale of 2 cm to represent 10 km on the vertical axis, draw the distance-time graph for the journey. Find the time of arrival of the cyclist at his destination. [6]

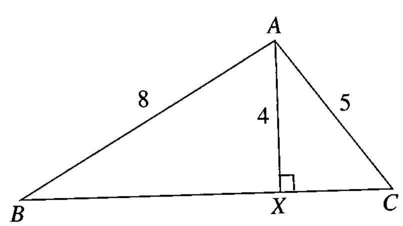
Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used.*

Section A (22 marks)

1. Find the total surface area of a sphere whose volume is 48.72 cm^3, giving your answers correct to 3 significant figures. [3]

2.



In the figure, $AB = 8$ cm, $AC = 5$ cm, $AX = 4$ cm and $A\hat{X}C = 90°$. Calculate the length of BC, giving your answer correct to 3 significant figures. [3]

3. A 13-metre-long ladder leans against a wall. The top of the ladder is 12 m above the ground. Calculate

(a) the distance of the foot of the ladder from the wall, [2]

(b) the angle the ladder makes with the ground. [2]

4. **(a)** Factorise the following:

(i) $(x + 5)^2 + x^2 - 25$ [2]

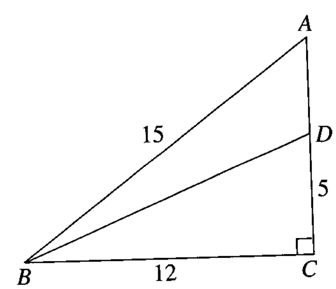
(ii) $(x + 2y)^2 - 2(x + 2y) - 15$ [2]

(b) Given that $x^2 + y^2 = 42$ and $xy = 7$, find the value of $3(x - y)^2$. [3]

5. Four pencils and a ruler cost \$1. Six pencils and three rulers cost \$2.10. Find the cost of three pencils and thirteen rulers. [5]

Section B (28 marks)

6.



ABC is a right-angled triangle with $A\hat{C}B = 90°$, $BC = 12$ cm, $AB = 15$ cm and $CD = 5$ cm. Find the lengths of BD, AD and the angle ABD. [7]

7. Copy and complete the following table of values for $y = 2x^2 + 5x - 3$. [1]

x	−4	−3	−2	−1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$
y	9	0		−6		−3	0		

Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 2 units on the y-axis, plot the graph of $y = 2x^2 + 5x - 3$. [4]

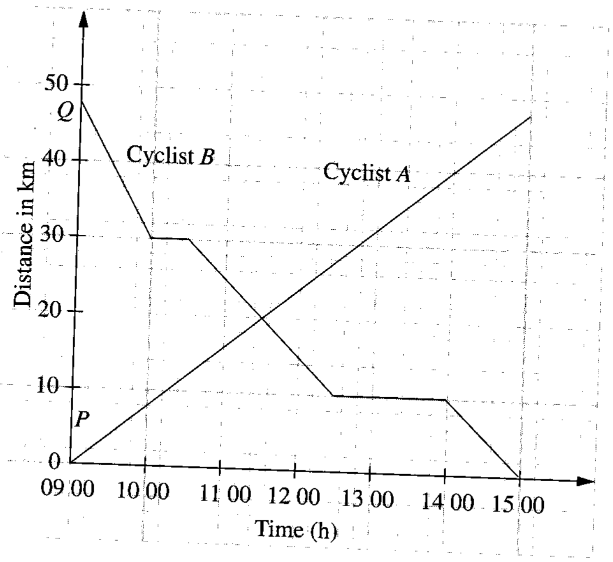
Use your graph to find the value of y when $x = -1.5$. [1]

8. Simplify (a) $\frac{1}{2x} + \frac{3}{4x} - \frac{5}{6x}$, [2]

(b) $\frac{5}{x-y} - \frac{4}{y-x}$, [2]

(c) $\frac{3}{4x-2y} + \frac{5}{y-2x}$. [3]

9. The distance-time graphs show the journeys of two cyclists A and B. A travelled from P to Q while B travelled from Q to P.



Use the graph to answer the following questions:

(a) Find the distance of PQ. [1]

(b) How long did B rest during the journey. [1]

(c) How far from Q did the two cyclists meet? [2]

(d) During which period of time did B travel the fastest? [2]

(e) Find the average speed of cyclist B for the whole journey. [2]

End-of-Year Examination Specimen Paper 5

Part I (50 marks) **Time: 1 h**

*Answer **all** the questions. Calculators are **not** allowed to be used.*

1. A rectangle measures 6 cm by 4 cm. Another rectangle with adjacent sides 8 cm and x cm is geometrically similar to it. Find the two possible values of x. [3]

2. Factorise the following expressions:

(a) $x^2y^2 - 15xy + 56$ [2]

(b) $x^3 + y - x^2y - x$ [2]

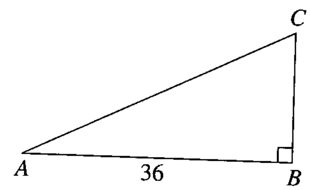
3. Simplify

(a) $(3x - y)^2 - (x + y)(2x - 3y)$, [2]

(b) $3x^2 + 4xy - (x - y)(2x + y - 2)$. [2]

4. Find two positive consecutive odd numbers whose product is 483. [4]

5.



In the right-angled triangle ABC, $\sin \hat{A} = \frac{3}{7}$ and $AB = 36$ cm, find the lengths of AC and BC. [4]

6. Solve the simultaneous equations

$4x + 7y = 29, \quad 2x + 3y = 13.$ [3]

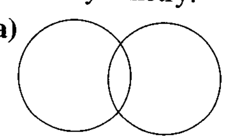
7. A map is drawn to a scale of 1 cm to 250 m.

(a) A straight road has a length of 13.5 km. Find its length on the map. [1]

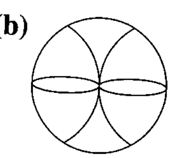
(b) An airport has an area of 240 cm² on the map. Find its actual area in km². [2]

8. Copy the following figures and draw the axis of symmetry.

(a)



(b)



[4]

9. A solid cone of height 8 cm and base radius 4 cm is lowered into a cylindrical jar of radius 6 cm containing water to a height of 12 cm. Find the rise in the water level when the cone is completely submerged. [4]

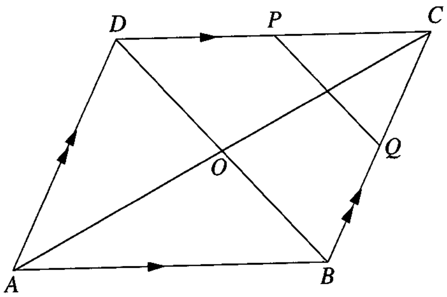
10. Find the mean age and the median age of the following nine children:

12 yr 9 mth, 11 yr 4 mth, 14 yr 3 mth,
15 yr 7 mth, 12 yr 2 mth, 13 yr 8 mth,
10 yr 6 mth, 16 yr 11 mth, 13 yr 7 mth.

[4]

11. A right pyramid has a rectangular base 14 cm long and 8 cm wide. Find the height of the pyramid if the volume is 336 cm^3.

12.

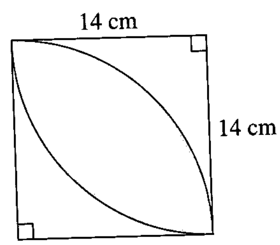


$ABCD$ is a parallelogram whose diagonals intersect at O. P and Q are the midpoints of DC and BC respectively. Describe a single transformation that maps

(a) $\triangle ABC$ onto $\triangle CDA$, [2]

(b) $\triangle CPQ$ onto $\triangle CDB$. [2]

13.



Find the area of the shaded region in the diagram above. $\left(\text{Take } \pi = \frac{22}{7}.\right)$ [3]

14. A cylindrical bar of length 1 m and diameter 2 cm is melted to form a circular disc of thickness 1 cm. Find the diameter of the disc. [3]

Part II (50 marks) Time: 1 h 15 min

*Answer **all** the questions. Calculators may be used.*

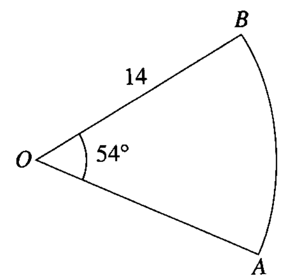
Section A (22 marks)

1. Calculate the length of wire required to make a square of diagonal 10 cm. [3]

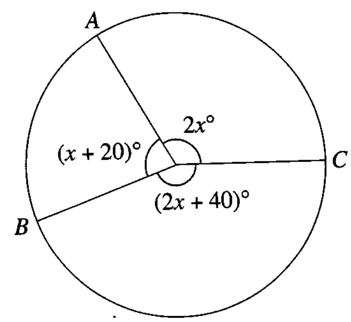
2. A point P is 45 m due south of a tower. If the angle of elevation of the tower from P is 41.4°, find the height of the tower, giving your answer correct to the nearest metre. [3]

3. A cylindrical drum of diameter 24.6 cm can hold 2.46 litres of water. Find the height of the water level, giving your answer correct to the nearest centimetre. [4]

4. The figure below shows a sector of a circle of radius 14 cm. Find its perimeter and area. [4]



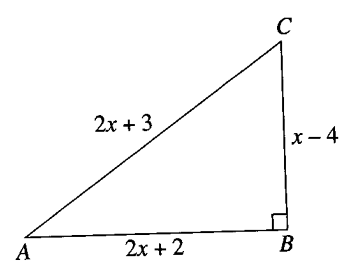
5. Given that the angles of the pie chart are as shown below, find x. If the arc length of AB is 16 cm, find the circumference of the circle. [4]



6. A rectangular tank 32 cm by 24 cm contains water to a height of 15 cm. 72 marbles of diameter 3 cm each are dropped into the tank. Find the rise in the water level, giving your answer correct to the nearest millimetre. [4]

Section B (28 marks)

7.



The figure above shows a right-angled triangle ABC where $AB = (2x + 2)$ cm, $BC = (x - 4)$ cm and $AC = (2x + 3)$ cm.

Form an equation in x and hence, find the lengths of BC, AB and AC. Find also the value of the angle BAC, giving your answer correct to one decimal place. [8]

8. Copy and complete the following table of values for $y = 8 - 2x - x^2$. [2]

x	−5	−4	−3	−2	−1	0	1	2	3
y	−7		5			8	5		−7

Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 2 units on the y-axis, draw the graph of $y = 8 - 2x - x^2$. [4]

Use your graph to estimate

(a) the value of y when $x = -2.5$, [1]
(b) the values of x when $y = 3$. [1]

9. A glass has a mass of 92 g when empty. When 294 cm³ of a liquid is poured into the glass, the total mass becomes 405 g. Find the density of the liquid, giving your answer correct to 3 significant figures. [4]

10. The force F in newtons (N) applied to a pulley to raise a load L kg is given in the table below.

L (kg)	20	40	80	120	160	200
F (N)	70	90	130	170	210	250

Using a scale of 2 cm to 40 kg on the horizontal axis and 2 cm to 40 N on the vertical axis, plot the graph of F (N) against L (kg). [4]

Use your graph to find

(a) the force required to raise a load of
(i) 50 kg, **(ii)** 190 kg, [2]
(b) the initial force required to operate the pulley. [2]

Answers

Exercise 1a *(Pg 4)*

1. 30°, 60°, 120°, 150°
2. 4 hours 3. $54.55
4. 9.22 m/s
5. (a) $56 (b) $20
6. 14 tins
7. (a) 29.648 km/h
   (b) 8.236 m/s
8. 45 days 9. 9 hours
10. 3 hours 11. $39.60
12. 48 sweets
13. (a) (i) 17 00 (ii) 5 400 km
    (b) (i) 5 hours
        (ii) 12 midnight
14. (a) 16 00 (b) 42 km
    (c) 8.4 km/h
15. 32 574 *l*; $36.48

Exercise 1b *(Pg 7)*

1. (a) 4%
   (b) $25 000, $40 000
2. (a) (i) $450 (ii) $540
   (b) 20%
3. (a) $588
   (b) $32 400 000
   (c) $4 536 000, $20 088 000, $5 832 000, $1 944 000
4. (a) 6 days (b) 40%
5. 123 008 tonnes
6. $36 000, $43 200, $88 800, 88.8%
7. (a) $134 640 (b) 19.9%
8. (a) 23.0% (b) 1.97%
9. (c) 88.7% (d) 1998

Exercise 1c *(Pg 11)*

1. $18.80 2. $6 346.15
3. $14 400 4. 13.9%
5. $2 361.40 6. $1 688
7. $2 016
8. (a) $2 095, 5.82%
   (b) $2 635
9. $47, 23.5%
10. US$932
11. $169 85.53
12. No. Only $179.34
13. (a) 25.87%
    (b) $449 million
14. $25.75 million

Exercise 1d *(Pg 17)*

1. (a) 10^{15} (b) 10^{12} (c) 10^{-9}
   (d) 10^{18} (e) 10^{-8} (f) 10^{-13}
   (g) 10^{0} (h) 10^{-15} (i) 10^{35}
   (j) 10^{12} (k) 10^{7} (l) 10^{6}
   (m) 10^{-1} (n) 10^{-9} (o) 10^{12}
2. (a) 2.37×10^2
   (b) 5.6×10^3
   (c) 9.124×10^5
   (d) $6.120\,06 \times 10^5$
   (e) 2.8×10^7
   (f) 4.35×10^{-2}
   (g) 7.7×10^{-4}
   (h) 8.306×10^{-3}
   (i) 2.96×10^{-1}
   (j) 7.48×10^1
   (k) 1.594×10^2
   (l) 7.06×10^4
3. (a) 6 370 (b) 0.004 213
   (c) 0.000 081 (d) 17 290
   (e) 0.382 (f) 9 800 000
   (g) 0.005 09 (h) 247
   (i) 0.141 (j) 840
   (k) 36 (l) 65.5
4. 2.42×10^8
5. 5.1×10^4
6. 9.0×10^{-1}
7. 1.28×10^4
8. (a) 3.48×10^5 (b) 6.0×10^4
   (c) 1.27×10^3 (d) 2.477×10^6
   (e) 7.86×10^3 (f) 2.76×10^6
   (g) 5.55×10^2 (h) 4.0×10^1
   (i) 4.0×10^3 (j) 3.0×10^3
   (k) 1.05×10^3 (l) 7.48×10^1
9. 8.4×10^{-3}
10. 7.6×10^{-3}
11. 7.5×10^{-7}
12. (a) 1.6×10^{14} (b) 6.4×10^{-2}
13. 3.33×10^{-7}
14. 1.76×10^{-6} cm, 2.464×10^{-13} cm²
15. (a) 7.85×10^2 (b) 4.5×10^{-3}
    (c) 1.62×10^6 (d) 6.3×10^1

Exercise 1e *(Pg 19)*

1. 6.721×10^5
2. -1.290×10^4
3. 4.688×10^7
4. 1.840×10^3
5. 2.110×10^1
6. 8.650×10^{-1}
7. 6.022×10^1
8. -1.416×10^9
9. 9.803×10^2
10. 2.291×10^{-1}
11. 8.834×10^0
12. 2.054×10^{18}
13. 5.460×10^9
14. 1.970×10^7
15. 1.164×10^2
16. 2.331×10^{12}
17. 2.904×10^0
18. 4.637×10^4
19. 8.767×10^9
20. 1.229×10^8
21. 2.130×10^{-6}
22. 9.505×10^6
23. 8.539×10^3
24. 1.690×10^{14}
25. 1.985×10^{-1}
26. 3.402×10^7
27. 9.790×10^6

Exercise 1f *(Pg 23)*

1. Break even
2. Less than before the pay cut by $2\frac{1}{4}$%.
3. Its final area was less than its original area by 4%.
4. 80 books
5. $2.70, 125 files
6. $18 500 at *A*, $5 500 at *B*.
7. $10 \times 2^{n-1}$; 1.7×10^8

Review Questions 1 *(Pg 23)*

1. $522 000
2. 10 000
3. $250 000
4. S$79 891.30
5. (a) $6 000, 14.3%
   (b) $5 700, 15.3%
   (c) $5 775
6. (a) $2 898; $2 852, $4 991, $2 139
   (b) *X*: $5 332, *Z*: $3 999; $5 418
7. (a) $2.412\,8 \times 10^9$
   (b) 2.0×10^{-5}

(c) 1.52×10^3
(d) 4.1×10^5
(e) 1.643×10^2
(f) 6.0×10^7
(g) 1.48×10^{-2}
(h) 5.3×10^4
(i) 8.25×10^3
(j) 3.0×10^2
(k) 1.36×10^{-1}
(l) 4.5×10^4
(m) 1.023×10^{-3}
(n) 2.72×10^{-2}

8. (a) 2.08×10^4 **(b)** 3.01×10^8
(c) 4.2×10^7 **(d)** 3.0×10^{-7}
(e) 5.79×10^7 **(f)** 8.25×10^{-3}
(g) 3.456×10^3
(h) 1.8×10^{-2}
(i) 7.5×10^{-4}
(j) 3.12×10^{-4}
(k) 6.43×10^{-2}
(l) 6.78×10^{-5}
(m) 1.5×10^{-4}
(n) 9.02×10^6

9. (a) 8.4×10^5 **(b)** 7.6×10^5
(c) 3.2×10^{10} **(d)** 2.0×10^1
(e) 5.0×10^{-2} **(f)** 2.12×10^6
(g) 1.25×10^{-1} **(h)** 7.0×10^1

10. (a) (ii) \$48 **(b)** 440 km
(c) 900 km **(d)** \$220

11. (a) 9.54×10^7 **(b)** 1.510×10^7
(c) 1.693×10^7, 1.803×10^7

12. (a) 7.84% **(b)** 22.35%

Exercise 2a *(Pg 29)*

1. (a) and (f); (b) and (j); (c) and (e); (d) and (g); (i) and (k)

2. (a) 5 **(b)** *DC*
(c) *AD*, 2 **(d)** *BC*, 5
(e) *ABC*, 90

3. (a) 3.5 **(b)** *VZ*
(c) *WX*, 3.5 **(d)** *ZY*, 2
(e) *YX*, 2 **(f)** *VWX*, 90

4. $\hat{A} = 100°$, $\hat{C} = 75°$
$AB = 3.5$ cm, $CD = 2.4$ cm
$\hat{M} = 65°$, $\hat{O} = 120°$
$MN = 5$ cm, $LO = 3$ cm

Exercise 2b *(Pg 33)*

1. (a) $x = 90°$, $y = 35°$, $z = 55°$
(b) $x = 28°$, $y = 34°$
(c) $x = 7.2$, $y = 10.8$
(d) $x = 9.6$, $y = 5\frac{5}{6}$

2. (a) $x = 9$, $y = 85°$
(b) $x = 95°$, $y = 52°$, $z = 4.8$
(c) $x = 80°$, $y = 10.5$

3. $x = 16$, $z = 1\frac{7}{8}$

4. $x = 270$, $y = 100$, $z = 100°$

Review Questions 2 *(Pg 34)*

1. (a) 70° **(b)** 60°
(c) 50° **(d)** 8 cm

2. (a) 100° **(b)** 70° **(c)** 95°
(d) 95° **(e)** 6 cm

3. (a) 50° **(b)** 68° **(c)** 62°

4. (a) and (h); (b) and (f); (c) and (g); (d) and (e)

5. (a) 60° **(b)** 7.5 cm

6. 48 cm **7.** 4.5 m

Exercise 3a *(Pg 38)*

1. (a) 4.5 m by 3.75 m; 16.875 m²
(b) 3 m by 2.25 m; 6.75 m²
(c) 78.75 m²

2. 58.3 m

3. (a) 80.2 m **(b)** 68.5 m

4. (a) 36 m **(b)** 4 cm

5. (a) $5\frac{2}{3}$ cm **(b)** 14 cm

6. (a) 5.6 cm **(b)** 110 m

7. 1 cm represents 2 m; 12.5 cm

8. (a) 800 km **(b)** 240 km
(c) 1 250 km

9. (a) 5 cm **(b)** 7.6 cm
(c) 2.5 cm **(d)** 0.5 cm

10. 21 cm

11. (a) 5 km **(b)** 1.5 cm

Exercise 3b *(Pg 41)*

1. (a) (i) 1 km **(ii)** 3.75 km
(iii) 300 m **(iv)** 13 km
(b) (i) 8 cm **(ii)** 30 cm
(iii) 0.5 cm
(iv) 3.2 cm

2. (a) (i) 6 km **(ii)** 9 km
(iii) 200 km
(iv) $\frac{1}{2}$ km
(b) (i) 2.5 cm **(ii)** $\frac{1}{4}$ cm
(iii) 4.5 cm **(iv)** $\frac{3}{4}$ cm

3. 4.5 cm

4. 1.1 km

5. 2.5 cm²

6. (a) 1 cm² **(b)** 2 cm²
(c) 5 cm² **(d)** 25 cm²

7. 70 km²

8. 126 km²

9. (a) 1.2 km, 0.8 km
(b) 0.96 km²

10. 3 100 cm²

11. 3 400 ha

12. (a) 2 km **(b)** 56 cm
(c) 3 km²

Review Questions 3 *(Pg 42)*

1. (a) 10.5 km
(b) 1 : 150 000
(c) 36 cm²

2. (a) 6 km **(b)** 14 cm
(c) 1 km²

3. (a) 312 km **(b)** \$82.80
(c) 2 h 42 min
(d) 42 km/h **(e)** \$17.50

4. (a) 11.4 km **(b)** 7 cm
(c) 2 160 ha

5. (a) 98 m **(b)** 25 cm²
(c) 4 ha

Exercise 4a *(Pg 47)*

1. (a) W, M, A, T, H
(b) E, H **(c)** H **(d)** N, S

3. (a) 2 **(b)** 2 **(c)** 2
(d) 1 **(e)** 3 **(f)** 3
(g) 1 **(h)** 1 **(i)** 2
(j) 1 **(k)** 1 **(l)** 2

Exercise 4b *(Pg 49)*

1. (a) (i) 1 **(ii)** 1
(b) (i) 2 **(ii)** 2
(c) (i) 0 **(ii)** 2
(d) (i) 8 **(ii)** 8
(e) (i) 2 **(ii)** 2
(f) (i) 2 **(ii)** 2
(g) (i) 2 **(ii)** 2
(h) (i) 2 **(ii)** 2
(i) (i) 2 **(ii)** 2
(j) (i) 2 **(ii)** 2
(k) (i) 1 **(ii)** 1
(l) (i) 0 **(ii)** 12

Exercise 4c *(Pg 52)*

1. (a) True **(b)** False **(c)** True
(d) True **(e)** True **(f)** True

2. (a) 53° **(b)** 55° **(c)** 100°
(d) 45° **(e)** 30° **(f)** 48°

3. (a) $x = 35°, y = 35°$
(b) $x = 25°, y = 120°$
4. (a) 18° (b) 36° (c) 20°

Exercise 4d *(Pg 55)*

2. (a) True (b) False (c) True
(d) False (e) True (f) False
(g) True (h) False (i) True
(j) True (k) True (l) True
(m) True
3. (a) $x = 27°, y = 63°$
(b) $x = 140°, y = 40°$
(c) $x = 51°, y = 98°$
(d) $x = 59°, y = 31°$
(e) $x = 115°, y = 40°$
(f) $x = 35°, y = 50°$
(g) $x = 72°, y = 54°$
(h) $x = 70°, y = 70°$
(i) $x = 30°, y = 75°$
(j) $x = 50°, y = 40°$

Exercise 4e *(Pg 57)*

2. 9, Yes
3. 4
4. (a) (i) 3 (ii) 3
(b) (i) 9 (ii) 13
(c) (i) 4 (ii) 4
(d) (i) 6 (ii) 6
(e) (i) 6 (ii) 4
(f) (i) infinite (ii) 1
(g) (i) infinite (ii) infinite
(h) (i) infinite (ii) 1
(i) (i) 4 (ii) 1
(j) (i) infinite (ii) infinite

Exercise 4f *(Pg 61)*

1. (a) 1 440° (b) 1 800°
(c) 2 340° (d) 2 880°
2. (a) 108° (b) 162°
(c) 165° (d) 170°
3. (a) 9 (b) 18
(c) 20 (d) 36
4. (a) 8 (b) 10
(c) 12 (d) 30
5. (a) 110° (b) 66°
(c) 86° (d) 104°
(e) 36° (f) 30°
6. 20°; 60°, 80°, 100°, 120°
7. $y = 30$; 90°, 60°, 30°
8. (a) 54° (b) 144°
9. 8
10. (a) 108° (b) 36° (c) 72°

11. (a) 135° (b) 112.5°
(c) 45°
12. (a) 162° (b) 153° (c) 153°

Review Questions 4 *(Pg 62)*

2. (a) (i) 2 (ii) 2
(b) (i) 0 (ii) 4
(c) (i) 8 (ii) 8
(d) (i) 3 (ii) 3
3. (a) 60°
(b) $x = 70°, y = 20°$
(c) 15°
(d) $x = 40°, y = 54°, z = 27°$
4. (a) 1 (b) 2 (c) 1
(d) 2 (e) 2 (f) 1
(g) 2 (h) 2
5. (a) $x = 102°, y = 17\frac{1}{3}°$
(b) $x = 35°$
(c) $x = 30°, y = 90°$
(d) $x = 50°$

Exercise 5a *(Pg 66)*

1. (a) $a^2 + 8a + 16$
(b) $m^2 + 14m + 49$
(c) $p^2 - 4pq + 4q^2$
(d) $16e^2 - 8de + d^2$
(e) $4y^2 - 4y + 1$
(f) $1 - 10c + 25c^2$
(g) $m^2 - \frac{1}{2}m + \frac{1}{16}$
(h) $\frac{9}{16}a^2 - \frac{3}{8}ab + \frac{1}{16}b^2$
(i) $9x^2 + 12xy + 4y^2$
(j) $25x^2 - 2\frac{1}{2}xy + \frac{1}{16}y^2$
(k) $4x^2 - 25y^2$ (l) $9x^2 - 16y^2$
(m) $t^6 - 16t^2$ (n) $x^8 - 9y^2$
(o) $x^6 - 2x^3y + y^2$
(p) $\frac{9}{16}a^2 - 1\frac{1}{2}ac^2 + c^4$
(q) $4t^2 - 16t + 16$
(r) $9x^2 - 4y^2$ (s) $25x^4 - 9y^2$
(t) $25x^2y^4 + 20abxy^2 + 4a^2b^2$
(u) $4a^6b^2 - 12a^3b + 9$
(v) $n^4 - 2n^2t^3 + t^6$
(w) $\frac{25}{64}x^4 - 2\frac{1}{2}x^3y + 4x^2y^2$
(x) $\frac{9}{16}a^2 + ab + \frac{4}{9}b^2$
2. (a) 249 996 (b) 89 975
(c) 9 996 (d) 1 447 209
(e) 81 018 001
(f) 808 201 (g) 795 664
(h) 10 272 025
(i) 4 900 (j) 6 400
(k) 250 000 (l) 90 000

Exercise 5b *(Pg 70)*

1. (a) $a^2 + 10a + 21$
(b) $a^2 - 11a + 30$
(c) $c^2 + 5c - 14$
(d) $x^2 - 8x - 9$
(e) $n^2 + 18n + 81$
(f) $n^2 - 18n + 81$
(g) $x^2 + 2xy - 3y^2$
(h) $b^2 - 16c^2$
(i) $m^2 - mp - 6p^2$
(j) $e^2 + 6ef + 5f^2$
(k) $1 - 49a^2$
(l) $-b^2 + 2bc - c^2$
(m) $1 + 3a - 10a^2$
(n) $4m^2 - 11mn - 3n^2$
(o) $2d^2 + 7de + 3e^2$
(p) $2y^2 + 19y + 9$
(q) $a^2b^2 - 2ab + 1$
(r) $e^2 - 12de + 20d^2$
(s) $x^2 - 4xy + 3y^2$
(t) $a^2 + 2ab - 35b^2$
2. (a) $2x^2 - 62$ (b) $-15y^2$
(c) $6x^2 + 7xy$ (d) $8b^2 - 8a^2$
(e) $7a^2 + 33a - 48$
(f) $-36a^2 + 22a - 7$
(g) $-3p^2 + 4pq + 12q^2$
(h) $p^2 - 20pq - 4q^2$
3. (a) $x^3 + 3x^2 + 3x + 2$
(b) $x^3 - 2x - 1$
(c) $x^3 + x^2 - 3x + 1$
(d) $x^3 - 4x^2 + 7x - 6$
(e) $a^3 + 6a^2 + 7a - 6$
(f) $a^3 - 6a^2 + 13a - 12$
(g) $2a^3 + 7a^2 - 2a - 1$
(h) $3a^3 - 10a^2 - 2a + 4$
(i) $2x^3 + 10x^2 - 3x - 15$
4. 24
5. 4
6. 118

Exercise 5c *(Pg 71)*

1. $c(c + b)$
2. $4(x + 2)$
3. $a(m + p)$
4. $b(b - 3c)$
5. $n^3(n - 1)$
6. $2d(d + 3e^2)$
7. $3x(x - 3y)$
8. $4de(d - 4)$
9. $p^4(1 + p^2)$
10. $y^2(1 + x)$
11. $5d(1 - 5d)$
12. $4(16y - x^2)$
13. no factors
14. $7d^2(d - 4k)$

15. $a^2b^2(2b-1)$ **16.** $d(x-y-d)$
17. $2ab(2ab+1)$ **18.** $5m(m-2n)$
19. $d(c^2d-3e)$ **20.** $5a(a-5b)$
21. $3a^3(3a^2-2)$ **22.** $8x^2(2x-1)$
23. $3bx(5a-3x)$ **24.** $2a^2(a-3b)$
25. $7ab(b^2-a)$ **26.** $2a^2(a-2y)$
27. $x(1+6y-9z)$
28. $7cd(2a+3c-d)$
29. $2ar(r+h)$
30. $8(m-mn-p)$
31. $a(2ab-3b^2c+4cd)$
32. $11pq(p+2p^2q-3q)$
33. $(2a+b)(x+3y)$
34. $(a+b)(2x^2-3y^3)$
35. $(x+9y)(2y+3z)$
36. $3(x-3)(a-2b)$
37. $(m+3n)(5y+2z)$
38. $7(a^2-a^3+1)$
39. $2(x^9-2y^2+4z^3)$
40. $na(na-a+1)$
41. $p^2a^2(1-a-pa)$
42. $3x^2(1+3xy+4y^2)$
43. $p^2z^3(1+pz+p^2z^2)$
44. $abc(1+abc-a^2b^2c^2)$

Exercise 5d *(Pg 72)*

1. $(a+5)(a-5)$
2. $(x+8)(x-8)$
3. $(2c+b)(2c-b)$
4. $(6d+1)(6d-1)$
5. $(n+10m)(n-10m)$
6. $(p+9q^2)(p-9q^2)$
7. $(ab+1)(ab-1)$
8. $(11+y)(11-y)$
9. $(6x+y^4)(6x-y^4)$
10. $(7c+de)(7c-de)$
11. $(c+d+2)(c-d-2)$
12. $(2x+y)^2$ **13.** $(e-3f)^2$
14. $(4n+e)^2$ **15.** $(3f+4g)^2$
16. $4(c-d)^2$ **17.** $(5p-q)^2$
18. $(7y+3z)^2$ **19.** $(3m+2n)^2$
20. $\left(n-\frac{1}{2}\right)^2$
21. $\left(\frac{1}{2}m+n\right)^2$
22. $\left(\frac{3}{2}p+\frac{1}{2}q\right)^2$
23. $\left(4x+\frac{1}{2}y\right)^2$
24. $(3p-2q)^2$ **25.** $(x^2+y)^2$
26. $(-5x-13y)(11x-5y)$
27. $9(a-b)(15a-11b)$
28. $(2mn+5)^2$
29. $(3xy+7z)^2$
30. $6a(a+3)^2$
31. $-(x-y)^2$
32. $-(m^3-n^3)^2$
33. $2x^2y^2(7x-2y)^2$
34. $(1+5xy)(1-5xy)$
35. $(xy+6z)(xy-6z)$
36. $(x+6)(x-6)y^2$
37. $(7x^2y^2+5z^2)(7x^2y^2-5z^2)$
38. $x(15x+13y)(15x-13y)$
39. $(a^2+b^2)(a+b)(a-b)$
40. $\left(\frac{a}{5}+\frac{2}{3}b\right)\left(\frac{a}{5}-\frac{2}{3}b\right)$
41. $2a(5a+2b)(5a-2b)$
42. $\left(\frac{1}{3}x+\frac{1}{4}yz\right)\left(\frac{1}{3}x-\frac{1}{4}yz\right)$
43. $\left(2x+\frac{3}{5}\right)\left(2x-\frac{3}{5}\right)$
44. $3(3a+1)(3a-1)$
45. $2b(4a+b)(4a-b)$
46. $\left(ab+\frac{c^2}{8}\right)\left(ab-\frac{c^2}{8}\right)$
47. 54 **48.** 170
49. 66 000 **50.** 1 800
51. 806 000 **52.** 5 600
53. 795 600
54. 10 230 000 **55.** 4 300
56. 3 700 **57.** 3 600
58. 840 **59.** 920
60. 2 700 **61.** 51.6
62. 326 **63.** 2 300
64. 470

Exercise 5e *(Pg 74)*

1. $(a+y)(x+3)$
2. $(m+b)(m+c)$
3. $(c+d)(a-b)$
4. $(p-q)(p-2)$
5. $(y+z)(1-c)$
6. $(x+y)(a-x)$
7. no factors
8. $(a-5)(a-c)$
9. $(p+q)(4-q)$
10. $(b+c)(7a+1)$
11. $(y+1)(y^2+1)$
12. $(b+c)(a^2+4)$
13. no factors
14. $(1-2c)(2+c^3)$
15. $(x+y)(10-d)$
16. $(c-d)(2+d)$
17. $(p-q)(y-q)$
18. $(e-1)(e+p^2)$
19. $(3b-a)(c-2a)$
20. $(a+1)(a-4)$
21. no factors
22. $(a-q)(1+p)$
23. $(d-c)(a-b)$
24. $(x-y)(a+b)$
25. $(x-y)(p+q)$
26. $(a+b)(x-y)$
27. $(a-b)(p-q)$
28. $(2m+n)(a+b)$
29. $(2p+3q)(x+2y)$
30. $(2x-y)(2a-3b)$
31. $(x+z)(x+y)$
32. $(p+3q)(5p-2r)$
33. $(a-1)(a^2+1)$
34. $(a+b)(p-2)$
35. $(x+a)(ax-3y)$
36. $(8m-a)(3n+b)$
37. $(2q+3r)(4p-q)$
38. $(a+b)(1-c)$
39. $(2b-1)(3a-1)$
40. $(1+pq)(1+p^2)$
41. $(p-r)(pq-2r)$
42. $(7x-1)(7x+a)$
43. $2(a+b)(2x+3y)$
44. $(p+2q)(3p-4r)$
45. $a(2a+3)(2a^2+1)$
46. $xy(x+y)(y-5)$
47. $a(5a-2b+30)$
48. $2x(2x-a-b)$
49. $m^2(3m-1)$
50. $p(2m-5n)$
51. $(2a-b)(3x-4y)$
52. $(3p+q)(x+y)$
53. $(a-3b)(4x-z)$
54. $(3p-2q)(3x+2y)$
55. $(p+4q)(p+4q+3)$
56. $(p-3q)(2p-q)$
57. $(2x+5y)(-x+7y)$
58. $(3x+2y)(3x+2y-2)$
59. $(a-2b)(3+a-2b)$
60. $(x+2y)(2-3x-6y)$
61. $(a+3b)(3a+2b)$

Review Questions 5 *(Pg 75)*

1. (a) $\frac{4}{9}x^2y^2-4xy+9$
(b) $9a^2+4\frac{4}{5}ab+\frac{16}{25}b^2$
(c) $\frac{1}{16}a^2+\frac{1}{12}ab+\frac{1}{36}b^2$
(d) $\frac{1}{16}a^2b^2c^2-\frac{3}{4}abcx^2yz+2\frac{1}{4}x^4y^2z^2$

(e) $\frac{9}{16}x^2y^2 - \frac{1}{9}a^2b^2$
(f) $\frac{1}{4}x^2 - \frac{1}{16}y^2$
(g) $6a^2 + 17ab + 12b^2$
(h) $12a^2 - 23ab + 5b^2$
(i) $5x^2 - 13xy - 6y^2$
(j) $7x^2 + 17xy - 12y^2$

2. (a) $4(x - 2y + 4z)$
(b) $5a(a + 2b + 2c)$
(c) $(a + b)(x - z)$
(d) $(2x + y)(2x + y - 3)$
(e) $(p + q)(3a - 4b)$
(f) $(m - 2n)(5 - m + 2n)$
(g) $(x - y)(a - b)$
(h) $(x - 2y)(x + z)$
(i) $(3a - 2)(a^2 + 1)$
(j) $(x + 1)(x + 2)(x - 2)$
(k) $(3n + 4)(3m + 2)$
(l) $(n - 2)(5m - 3)$

3. (a) $6(p^2 + 2q)(p^2 - 2q)$
(b) $2p^2(p - 3q)(p + 3q)$
(c) $x^2y^2(x + 2y)(x - 2y)$
(d) $2x(4y^2 + x^2)(2y + x)(2y - x)$
(e) $4(4a^2 + b^2)(2a + b)(2a - b)$
(f) $(m^4 + 9)(m^2 + 3)(m^2 - 3)$
(g) $(p - 3q + r)(p + 3q - r)$
(h) $(2a + b + c)(2a - b - c)$

Revision Exercise I No 1 *(Pg 76)*

1. 7%
2. 8, \$2
3. 20
4. (a) 1 : 800 000 **(b)** 9 cm
(c) 7.75 cm^2
5. (a) 8 **(b)** 0 **(c)** 2
6. (a) 6 cm, 4 cm **(b)** 20
7. (a) $3a^2$ **(b)** p^2q^3
(c) $3x$ **(d)** $2ab^3$
8. (a) $2a^3b^4c^2 + 12ab^4c^4 - 10a^3b^3c^2$
(b) $3x^2y^3z^3 + 6x^2y^2z^3 - 12x^3y^2z^3$
(c) $9x^4y - 6x^3y^3 + 24x^2y^4$
(d) $-15x^2 + 17xy + 42y^2$
9. (a) 3.82×10^{11} **(b)** 7.48%
(c) 2.706 times
10. $\frac{5}{6}$

Revision Exercise I No 2 *(Pg 76)*

1. (a) 18 km **(b)** 12 kg
2. 64
3. \$210, \$60
4. (a) $x^2 - 7x - 8$ **(b)** $11x - 2$
5. 117°, 141°, 102°
7. (a) $3a^2 + 23ab + 43b^2$
(b) $17a^2 - 2ab$
8. (a) 45 cm **(b)** 2.88 km^2
9. 72
10. (a) $5xy(x - 3y - 5)$
(b) $(2a - 3b)(x - y)$

Revision Exercise I No 3 *(Pg 77)*

1. (a) \$675 **(b)** \$98
2. \$27
3. 4 h
4. $x = 78°$, $y = 130°$
5. (a) 4.5×10^8 **(b)** 4.0×10^{10}
(c) 8.07×10^5 **(d)** 5.8×10^8
6. (a) $4a^2 - 20ab + 25b^2$
(b) $4a^2 + 20ab + 25b^2$
(c) $4a^2 - 25b^2$
7. (a) 30°, 90° **(b)** 6.5 cm
8. 192 km^2
9. (a) 2 **(b)** 6
10. 6 cm, 9 cm

Revision Exercise I No 4 *(Pg 78)*

1. \$76.50
2. \$2 000
3. (a) $1\frac{1}{7}$ **(b)** $\frac{2}{5}$
4. 12 km/h
5. (a) 6 **(b)** 165
6. (a) 2 cm **(b)** 16 m^2
7. 17.2 m
8. (a) $\frac{1}{5} - \frac{1}{6} = \frac{1}{30}$
(b) $x = 11$
9. (a) 2.88×10^{-3}
(b) 3.84×10^5
(c) 4.174×10^4
(d) 8.5×10^5
10. (a) $-2\frac{1}{2}$ **(b)** -6

Revision Exercise I No 5 *(Pg 78)*

1. (a) $\frac{3}{8}$ **(b)** $1\frac{16}{25}$
(c) $\frac{19}{25}$ **(d)** $\frac{4}{125}$
2. (a) 16.1% **(b)** 87.3%
(c) 69.8%
3. \$9.20
4. 5 years
5. 8.1 cm
6. (a) $2a^4b + 6a^3b^2 - 10a^2b^3$
(b) $-6x^3y^2 + 12x^2y^3 - 21xy^4$
7. 126
8. 24.5 cm, 26.25 cm
9. (a) $8x^3y^3 + 1$ **(b)** $81 - 16a^4$
10. (a) 11.75 cm **(b)** 51.2 km
(c) 51.2 km^2

Exercise 6a *(Pg 82)*

1. $(a + 4)(a + 2)$
2. $(c + 2)(c + 1)$
3. $(m + 8)(m + 1)$
4. $(b + 7)(b + 4)$
5. $(e - 2)^2$
6. $(x - 8)(x - 3)$
7. $(m + 5)(m + 4)$
8. no factors
9. $(a - 7)(a - 2)$
10. $(m - 4)(m - 9)$
11. $(x - 6)(x - 3)$
12. $(p + 4)(p + 3)$
13. $(a - 8)(a - 6)$
14. no factors
15. $(x - 8)(x - 5)$
16. $(m - 3)(m - 5)$
17. $(n - 17)(n - 1)$
18. $c(e + 6)(e - 1)$
19. $(q - 9)(q + 2)$
20. $p(n + 7)(n - 4)$
21. $(y + 8)(y - 3)$
22. no factors
23. $(d - 5)(d + 4)$
24. $y(x - 3)(x + 1)$
25. $a(b - 4)(b + 9)$
26. no factors
27. $(1 + 4b)(1 - 6b)$
28. $(1 + 7x)^2$
29. $(1 - 5k)(1 + 4k)$
30. $\left(p - \frac{1}{2}\right)^2$
31. $(a + 4b)^2$
32. $(x - 5y)^2$
33. $\left(2p + \frac{1}{2}\right)^2$
34. $(2x - 3y)(x + 5y)$
35. $(7m + 3)(2m - 5)$
36. $(7 - 5x)(2 - 3x)$ or $(5x - 7)(3x - 2)$
37. $(3a - 7)(a - 1)$
38. $2(2x - 3)(x - 4)$
39. $(2x + 3)(x + 4)$
40. $2(3x - 2)(2x + 3)$
41. $(6a - 5)(a + 4)$
42. $(5p - 3)(p - 2)$
43. $(5p - 3)(p + 2)$

44. $(3p + 4)(2p - 5)$
45. $(4a + 3)(a + 1)$
46. $(4a - 3)(a - 1)$
47. $(2m + 1)(2m + 3)$
48. $(3p - 4)(2p + 5)$
49. $(5pq + 3)(pq - 2)$
50. $(2m - n)(2m - 3n)$
51. $(3p + 4q)(2p - 5q)$
52. $2(3p - 4q)(2p + 5q)$
53. $(5p + 3q)(p + 2q)$
54. $(6ab + 5)(ab - 4)$
55. $(3xy - 2)(2xy + 3)$
56. $(3x + 2y)(2x - 3y)$
57. $3(2x + 3y)(x + 4y)$
58. $4(2xy - 3)(xy - 4)$
59. $(3a + 7b)(a + b)$
60. $(7 - 5xy)(2 - 3xy)$
61. $(7x + 5y)(2x - 3y)$
62. $(2xy + 3)(xy - 5)$
63. $3(x + 3y)(2x - 9y)$
64. $5(a - b)(2a + 3b)$
65. $(2mn + 5)^2$
66. $(ab + 7)^2$

Exercise 6b *(Pg 83)*

1. **(a)** ± 4 **(b)** $\pm 2\frac{1}{2}$ **(c)** ± 8
(d) ± 1 **(e)** ± 5 **(f)** ± 5
(g) $\pm\frac{1}{2}$ **(h)** $\pm\frac{4}{5}$ **(i)** $\pm 3\frac{1}{3}$
2. **(a)** 0 or –8 **(b)** 0 or $1\frac{1}{3}$
(c) 0 or $\frac{1}{27}$ **(d)** 0 or 4
(e) 0 or –3 **(f)** 0 or –3
3. **(a)** $e = 8$ **(b)** $d = 3$ or -9
(c) $a = -6$
(d) $q = 5$ or -12
(e) $b = 15$ or -8
(f) $a = \frac{1}{2}$ or $-\frac{1}{5}$
(g) $k = 9$ or -7
(h) $p = 2$ or $1\frac{1}{3}$
(i) $m = \frac{1}{2}$ or -3
(j) $b = -2\frac{1}{2}$ or $-1\frac{1}{2}$
(k) $t = 1$ or -4
(l) $s = 5$ or -3
(m) $a = 3$ or 4
(n) $a = 5$ or -1
(o) $p = 2$ or $\frac{1}{2}$
4. $k = -8$, $x = 5$

Exercise 6c *(Pg 87)*

1. 3 **2.** 2
3. 7, 8 **4.** 5, 7
5. 8, 10 **6.** 9, 18
7. 4 m **8.** 5
9. 6, 9 **10.** 34 cm
11. 5, 12 **12.** 4, 9
13. **(a)** $\frac{420}{x}$ h **(b)** $\frac{420}{x+15}$ h
(c) $\frac{420}{x} - \frac{420}{x+15} = \frac{40}{60}$
(d) $x = 90$; 4 h 40 min
14. 48 km/h
15. $\frac{67\,200}{x-5} - \frac{67\,200}{x} = 32$; $x = 105$
16. **(a)** $\frac{5\,800}{x}$ **(b)** $\frac{5\,800}{x+12}$
(c) $\frac{5\,800}{x} - \frac{5\,800}{x+12} = 4\frac{11}{16}$
(d) $x = 116$; 29.3 l

Exercise 6d *(Pg 90)*

1. $x^2 + 2x - 7$
2. $x^3 + 1\frac{1}{2}x - 2$
3. $1\frac{2}{3}x^2 + \frac{2}{3}x - 1\frac{1}{3}$
4. $6x^2 + 1\frac{1}{3}x - 4$
5. $x - 3$
6. $2x - 7$
7. $3x - 1$
8. $5x + 12 + \frac{4}{x-1}$
9. $7x - 20 - \frac{11}{x+3}$
10. $14x + 72 - \frac{8}{x-5}$
11. $4x + 21 - \frac{13}{x-7}$
12. $3x^2 - 3x + 7 - \frac{12}{x+1}$
13. $x^2 + 4x - 16 + \frac{48}{x+3}$
14. $2x^2 - 12x + 76 + \frac{3}{x+6}$
15. $5x^2 + x - 7 + \frac{15}{x+2}$
16. $8x^2 + 2x + 8 - \frac{23}{x+5}$
17. $3x^2 + x + 1 + \frac{4}{3x+1}$
18. $6x^2 + 3x + 2 - \frac{1}{2x+1}$
19. $3x^2 + 9x + 27 + \frac{74}{x-3}$
20. $x^3 - x + 3\frac{1}{2} + \frac{3\frac{1}{2}}{2x-1}$

Review Questions 6 *(Pg 90)*

1. **(a)** $(a - 1)(a + 16)$
(b) $(a - 1)(a - 19)$
(c) $(x + 9)(x + 4)$
(d) $(2p - 3q)^2$
(e) $(5pq + 1)^2$
(f) $(3 + y)^2$
(g) $(1 + 6xy)^2$
(h) $(7 + 2a)(7 - 2a)$
(i) $10(3xy + 1)(3xy - 1)$
2. **(a)** 2 or 10 **(b)** 3 or $-\frac{7}{11}$
(c) 3 or $-\frac{3}{5}$ **(d)** 5 or $\frac{1}{2}$
(e) 3 or $-\frac{1}{2}$ **(f)** –4 or $-1\frac{1}{8}$
(g) $2\frac{2}{3}$ or $-2\frac{1}{5}$
(h) $1\frac{6}{7}$ or $-1\frac{1}{4}$
3. 3 or 12
4. 5
5. **(a)** $(16x + 2)$ cm
(b) $(15x^2 - x - 6)$ cm²
$x = 4$; 66 cm
6. **(a)** $\frac{1}{3}$ or $-\frac{2}{9}$ **(b)** $\frac{1}{2}$ or $-\frac{1}{5}$
(c) 2 or 6 **(d)** 2 or 3
(e) 4 or $-1\frac{1}{3}$ **(f)** 2 or $1\frac{1}{5}$
7. 48
8. 30

Exercise 7a *(Pg 93)*

1. $\frac{3}{4x}$ **2.** $\frac{2x^3}{y}$
3. $\frac{b}{3a}$ **4.** $\frac{a^2b^5}{3c}$
5. $\frac{mp^2}{4n^2}$ **6.** $\frac{c^5}{5a^2b}$
7. $\frac{2b}{3ac^3}$ **8.** $\frac{b^2}{2a}$
9. $\frac{a}{3bc^3}$ **10.** x^5y^5
11. $\frac{-2}{ab^3}$ **12.** $\frac{-z}{8y}$

13. $\frac{9b}{8}$ 14. $\frac{bx}{3}$

15. $\frac{b}{ax^2}$ 16. $\frac{1}{2x}$

17. $\frac{a^2(a+b)}{2b}$ 18. $\frac{a(x+y)}{3b(x-y)}$

19. $\frac{x-3}{2a(x-9)}$ 20. $\frac{x}{a-b}$

21. $\frac{1}{2a}$ 22. $\frac{a^2}{3b}$

23. $\frac{1}{3a^nb^{m+1}}$ 24. $\frac{3x^5y}{5}$

25. $\frac{5}{6(a+b)}$ 26. $\frac{a}{4(b+c)}$

Exercise 7b *(Pg 94)*

1. $\frac{a+2c}{3}$ 2. $\frac{y}{2}$
3. no simpler form
4. no simpler form
5. $\frac{a-b}{3}$ 6. $\frac{e}{d-e}$
7. no simpler form
8. $\frac{1-d}{d}$
9. no simpler form
10. no simpler form
11. $\frac{ab}{3}$ 12. $\frac{3}{2}$
13. $\frac{5}{7}$ 14. $\frac{m}{m-p}$
15. no simpler form
16. $\frac{x+y}{x-y}$ 17. $\frac{c}{d}$
18. $\frac{1}{a-b}$ 19. $p+2$
20. $\frac{4}{9}$ 21. $\frac{m-4}{q}$
22. $\frac{3x-y}{4x-y}$ 23. $\frac{b-2}{3}$
24. $\frac{p+7}{p}$ 25. $\frac{m+3}{m-4}$
26. $\frac{c+2}{c}$ 27. $\frac{q+3}{q+2}$
28. $-\frac{x}{y}$ 29. $\frac{c-a}{a}$
30. $\frac{e+3d}{e-d}$ 31. $-\left(\frac{3p+q}{p+q}\right)$
32. $-\left(\frac{4+m}{2m+1}\right)$ 33. $\frac{e+f-d}{-e+f+d}$

34. $\frac{(x-y)}{(x-3y)}$ 35. $\frac{a-b}{a+b}$
36. $\frac{a-n}{a+n}$ 37. $\frac{a-1}{3}$
38. $-\frac{(b+a)}{2a+3b}$ 39. $\frac{3}{d+3}$
40. $\frac{2y-3}{y+1}$

Exercise 7c *(Pg 95)*

1. $\frac{3}{2b^4}$ 2. $\frac{4a^2p}{b^3}$
3. x 4. $\frac{3b}{4c}$
5. $\frac{m^2}{8}$ 6. $\frac{2n^4}{7mp}$
7. $\frac{36x^2}{7ab^2}$ 8. $\frac{ac^2d^4}{6b}$
9. $\frac{3c^3d^2}{5ab}$ 10. $\frac{2}{3x^2yz^2}$
11. $\frac{6a^2b}{xy}$ 12. $\frac{3}{2x^3y^2}$
13. $\frac{2a^2c^3}{3d^2}$ 14. $\frac{a^2y^3z^2}{2x^2}$
15. $\frac{6a^2d^3}{b^3c^2}$ 16. $\frac{20a^2b^2}{9c}$
17. $\frac{u^2}{100}$ 18. $\frac{27}{64u^3v^3}$
19. $\frac{8}{5x^2y^2}$ 20. $\frac{ac^{2n+1}}{b^3}$
21. $\frac{16a^{2n+5}}{25b^{2n+2}c}$ 22. $\frac{a}{c(a+b)^3}$

Exercise 7d *(Pg 98)*

1. (a) ab (b) $3qr$
(c) $4xy^2z^2$ (d) $7ab$
(e) m (f) $2xy$
(g) $3x^2y^2$ (h) abc
2. (a) a^2bcxy (b) $24x^2y^2z^3$
(c) $48x^4y^3$ (d) $18x^3y^4$
(e) $abcm$ (f) $24x^3y^3$
(g) $60x^5y^4$ (h) $60a^3b^3c^3$
3. (a) ab, $12a^2b^2$
(b) $2ab^2$, $30a^2b^3c^2$
(c) $4xyz$, $12x^2yz^2$
(d) 3, $18a^2b^2c^2$
(e) $2xyz$, $12x^3y^2z^2$
(f) p, $6m^2n^2p^2q^2$
(g) a^2y^3, $6a^4b^2x^2y^4$
(h) $2q$, $40p^2q^3r^3s^2$

Exercise 7e *(Pg 100)*

1. $\frac{3x-1}{4}$ 2. $\frac{y+2}{4}$
3. $\frac{3z-2x-4}{6}$ 4. $\frac{y-x}{3xy}$
5. $\frac{2(2b+c)}{abc}$ 6. $\frac{3m-n+p}{m}$
7. $\frac{x^2}{4}$ 8. $\frac{a^2bc+b^2-c^2}{abc}$
9. 0 10. $\frac{3y+2}{5}$
11. $\frac{5-b}{12}$ 12. $\frac{11a}{24}$
13. $\frac{-7c-18}{15}$ 14. $\frac{11y-18}{12}$
15. $\frac{a+3}{30}$ 16. $\frac{e+1}{5}$
17. $\frac{x-19y}{15}$ 18. $\frac{8}{15x}$
19. $\frac{c+3d}{5}$ 20. $\frac{5}{(c-d)}$
21. $\frac{3y-y^2+x-x^2}{xy}$
22. $\frac{2m-n}{n}$ 23. $\frac{7b+52}{3b}$
24. $\frac{x+1}{12}$ 25. $\frac{a+2x}{3a}$
26. 0 27. $\frac{2}{x-2y}$
28. $\frac{7}{6(e-f)}$ 29. $\frac{9a-4b}{2a-b}$
30. $\frac{-9r}{2q+3r}$ 31. $\frac{2(6c+5d)}{15(2c-d)}$
32. $\frac{3}{2(2e-f)}$ 33. $\frac{m}{6(3m-2)}$
34. $\frac{5a+7}{6(a-4)}$ 35. $\frac{7e-1}{(e-3)(e+2)}$
36. $\frac{5}{2x-5}$ 37. $\frac{2}{2a+1}$
38. $\frac{3c}{4(3c+1)}$ 39. $\frac{2(xz+1)}{2xz-1}$
40. $\frac{p}{r}$ 41. $\frac{mr}{r+m}$
42. $\frac{3a}{2(3b-1)}$ 43. $\frac{2(2d-3)}{3d+4}$
44. $\frac{x+y}{2y}$

Exercise 7f *(Pg 101)*

1. (a) $x=4$ (b) $m=3$
(c) $v=3$ (d) $x=3\frac{1}{2}$

(e) $x = 5\frac{3}{5}$ (f) $x = -1$
(g) $p = 120$ (h) $e = 3\frac{3}{4}$
(i) $u = 1$ (j) $e = 6$
(k) $a = 6$ (l) $x = 30$
(m) $x = 9$ (n) $x = 11\frac{1}{2}$
(o) $x = 2\frac{2}{5}$ (p) $x = 3\frac{1}{2}$
(q) $x = 1\frac{1}{3}$ (r) $x = 15$

2. (a) $m = -3$ (b) $x = 1\frac{1}{3}$
(c) $m = 6$ (d) $a = 5\frac{4}{5}$
(e) $x = \frac{2}{3}$ (f) $x = -\frac{12}{13}$
(g) $a = -\frac{6}{7}$ (h) $n = -1$
(i) $d = 2$ (j) $x = -1$
(k) $x = -\frac{1}{2}$ (l) $x = 1\frac{1}{2}$
(m) $x = \frac{11}{42}$ (n) $x = \frac{3}{8}$

Exercise 7g *(Pg 104)*

1. 5 **2.** 8
3. $\frac{3}{5}$ **4.** 2
5. 7 **6.** 15, 24
7. 24, 45 **8.** 7 yr
9. 27 yr **10.** 22 yr
11. 20 cm **12.** 20 km
13. 120 km **14.** 336 km
15. 252 km **16.** 39 km/h
17. 135 km **18.** \$12.35
19. 288 sweets
20. 672 matches

Exercise 7h *(Pg 107)*

1. (a) $a = \frac{y}{x}$ (b) $a = \frac{q}{p-4}$
(c) $a = \frac{c-by}{x}$ (d) $a = \frac{c}{p} - b$
(e) $a = \frac{7-3m}{2}$
(f) $a = \frac{5b-3c}{2}$
(g) $a = mc - mb = m(c-b)$
(h) $a = \frac{3x-15z}{2} = \frac{3(x-5z)}{2}$
(i) $a = 14p$ (j) $a = \frac{R}{m} - g$
(k) $a = \frac{2b}{x+1}$
(l) $a = \frac{65-2m}{4}$

2. (a) $b = \frac{2A}{h}$ (b) $k = \frac{2Tl}{x^2}$
(c) $t = \frac{m(v-u)}{F}$
(d) $l = \frac{15k-12}{2}$
(e) $a = \frac{bc}{c-b}$
(f) $a = \frac{3}{5}c - \frac{3}{4}b$
(g) $a = \frac{2A}{h} - b$
(h) $l = \frac{2s}{n} - a$
(i) $q = \frac{fp}{p-f}$
(j) $u = \frac{s}{t} - \frac{1}{2}gt$
(k) $p = \frac{q^2}{q+1}$
(l) $a = 2mk - 2m^2l^2$

3. (a) $l = \frac{1}{2}p - b$
(b) $h = \frac{A}{2\pi r} - r$
(c) $c = \frac{3a}{a-b}$ (d) $a = \frac{bc}{x-c}$
(e) $b = \frac{a}{a-1}$
(f) $b = \frac{4}{5}a + 18$
(g) $b = \frac{ac}{a-c}$
(h) $a = \frac{c}{2}(4-b)$
(i) $a = b^2 - b$ (j) $z = \frac{y^2}{y-x}$
(k) $x = \frac{bc}{a+c}$ (l) $y = \frac{3-x}{x-2}$
(m) $r = \frac{wR}{a+w}$ (n) $x = \frac{3-y}{2y+1}$
(o) $c = \frac{4}{1+a}$ (p) $q = \frac{pr-m}{r}$
(q) $p = 3q$
(r) $x = \frac{10-2b}{2+3b}$

Review Questions 7 *(Pg 107)*

1. (a) $\frac{4}{3x}$ (b) $\frac{x-3}{9}$
(c) $\frac{11x+13}{30}$ (d) $\frac{11}{4x}$
(e) $-\frac{2}{3x}$ (f) $\frac{4-3x}{5(2x-1)}$
(g) $\frac{3x^2+2}{x}$ (h) $\frac{9x+18}{5}$
(i) $\frac{4x+5}{(2x-1)(5x+1)}$
(j) $\frac{7x+10}{(x+1)(3x+4)}$
(k) $\frac{8x-17}{(x-2)(3x-7)}$
(l) $\frac{x+5y}{(y+x)(y-x)}$
(m) $\frac{2(8x+1)}{(2x+1)(2x-1)}$
(n) $\frac{3y-x}{x(x-y)}$ (o) $\frac{17x+6y}{4x(x-2y)}$

2. (a) $2\frac{4}{5}$ (b) $14\frac{2}{3}$ (c) 7
(d) $2\frac{1}{7}$ (e) $\frac{1}{10}$ (f) -37
(g) 10 (h) 35 (i) $-2\frac{3}{4}$
(j) $2\frac{1}{7}$ (k) -1 (l) $\frac{7}{20}$

3. (a) $y = \frac{x}{1-x}$ (b) $t = \frac{1-a}{1+a}$
(c) $a = \frac{b}{1-c}$ (d) $a = \frac{b}{x-3}$
(e) $p = \frac{100a}{100+rt}$
(f) $h = \frac{5k}{5+2k}$

4. 16 **5.** 16
6. 4, 6 **7.** 4, 12

Exercise 8a *(Pg 113)*

1. (a) $x = 8, y = 8$
(b) $x = 12, y = 7$
(c) $x = 9, y = 3$
(d) $x = 7, y = 9$
(e) $x = 3, y = 2$
(f) $x = 3, y = 1\frac{1}{2}$
(g) $x = 2, y = 1$
(h) $x = 2, y = -1\frac{1}{2}$
(i) $x = 4, y = 1$

2. (a) $x = -7, y = -13$
(b) $x = 1, y = 2\frac{2}{3}$
(c) $x = 5, y = \frac{2}{5}$
(d) $x = 3\frac{1}{3}, y = -3$
(e) $x = 1\frac{6}{7}, y = 10$
(f) $x = -3, y = 5$
(g) $x = -43\frac{1}{2}, y = -34$
(h) $x = -5, y = 6$

3. (a) $x = 1, y = 1$
(b) $x = 11, y = 9$
(c) $x = 13, y = 5$
(d) $x = 12, y = 15$
(e) $x = 1, y = -4$
(f) $x = 8\frac{1}{5}, y = 2\frac{12}{25}$
(g) $x = -\frac{1}{2}, y = 3$
(h) $x = -8\frac{9}{37}, y = 12\frac{2}{37}$

4. (a) $x = 3, y = 7$
(b) $x = 1, y = -1$
(c) $x = 9, y = 4$
(d) $x = 6, y = -4$
(e) $x = 4, y = 9$
(f) $x = 13, y = 11$
(g) $x = -\frac{1}{2}, y = 3$
(h) $x = 16, y = -4$
(i) $x = 12, y = -2$
(j) $x = -1, y = 1$
(k) $x = 4, y = 1$
(l) $x = 3\frac{6}{11}, y = 1\frac{5}{11}$
(m) $x = 3, y = -4$
(n) $x = 7, y = -4$
(o) $x = 8, y = -7$
(p) $x = 0.5, y = 0.25$

Exercise 8b *(Pg 115)*

1. (a) $x = 6, y = 1$
(b) $x = 3, y = 2$
(c) $x = 1, y = 3$
(d) $x = 0, y = -2$
(e) $x = -1, y = -1$
(f) $x = -3, y = 0$
(g) $x = 3, y = 10$
(h) $x = 2, y = 0$
(i) $x = 1, y = 2$
(j) $x = 5, y = -1$
(k) $x = 7, y = 3$
(l) $x = -4, y = 4$

2. (a) $x = 2, y = 3$
(b) $x = -1, y = -2$
(c) $x = 9, y = 4$
(d) $x = 3, y = 5$
(e) $x = 15, y = -5$
(f) $x = -5, y = 0$
(g) $x = -2, y = 11$
(h) $x = 4, y = -5$
(i) $x = 3, y = 6$
(j) $x = 2\frac{1}{2}, y = 4$
(k) $x = \frac{2}{3}, y = 0$
(l) $x = \frac{1}{5}, y = \frac{2}{5}$
(m) $x = \frac{1}{4}, y = \frac{1}{2}$
(n) $x = 0, y = 1\frac{1}{2}$
(o) $x = -2, y = -\frac{1}{4}$

Exercise 8c *(Pg 118)*

1. 113, 25
2. 146°, 34°
3. $22\frac{1}{2}$, $13\frac{1}{2}$
4. 74°, 46°
5. 1 kg of potatoes costs \$2,
1 kg of carrots costs \$2.40
6. \$3 per stool, \$10 per chair
7. $\frac{7}{9}$
8. \$15 per belt, \$27 per wallet
9. 21 cm
10. $\frac{3}{5}$
11. 40, 8
12. \$32, \$48
13. 30 cm
14. $\frac{47}{7}$, $\frac{48}{7}$
15. (a) 8 cm (b) 32 cm
16. 9 five-cent coins, 6 twenty-cent coins
17. Father: 40 yr
Son: 10 yr
18. 65
19. 15, 5
20. $x = 6, y = 7$, 44 cm
21. 26, 14
22. 20 fifty-cent stamps, 16 ten-cent stamps
23. $L = 20$ cm, $B = 18$ cm
24. Bank A = \$1 300,
Bank B = \$1 200
25. 420 km
26. $x = 14, y = 7$
Area = 875 cm^2

Exercise 8d *(Pg 121)*

1. (17, 6, 77), (18, 24, 58), (19, 42, 39), (20, 60, 20), (21, 78, 1)
2. (5, 42, 53), (10, 24, 66), (15, 6, 79)

Review Questions 8 *(Pg 121)*

1. (a) $x = 2, y = -2$
(b) $x = 2, y = 2.5$
(c) $x = 1, y = -4$
(d) $x = 1\frac{1}{3}, y = -8$
(e) $x = \frac{3}{4}, y = -\frac{1}{4}$
(f) $x = 3, y = -4$
(g) $x = 1, y = 2$
(h) $x = 3, y = 5$
(i) $x = 2, y = 1.5$
2. apple – 40¢.
orange – 35¢
3. 17, 14
4. A = \$11, B = \$13
5. 6, 14
6. pear – 15¢,
mango – 25¢
7. 40, 110
8. $\frac{7}{10}$
9. $\frac{3}{7}$
10. 11 yr; 41 yr
11. 14 kg, 6 kg
12. 68 km/h, 80 km/h
13. 48 km/h, 52 km/h
14. 48

Revision Exercise II No 1 *(Pg 123)*

1. \$125
2. (a) 3 or –12 (b) –3
3. $\frac{1}{250\,000}$
4. (a) $2pq - 4p^2 - 7q^2$
(b) $-2ab^2 - 4a^2 + 3b$

5. **(a)** $5(x-1)(x-4)$
   (b) $(5a-3b)(2a-3b)$
   (c) $(7a+8b)(4a-3b)$
   (d) $(ab+5)(ab-5)$

6. $x = \frac{10ay+4}{15a+3}$; 3

7. $x = 2, y = -3$

8. $a = \frac{3xb+b}{6x-1}$

9. **(a)** $\frac{28a+22b}{15}$ **(b)** $\frac{9+14a}{42a}$

10. $\frac{6}{7}$

Revision Exercise II No 2 *(Pg 123)*

1. 1 min
2. **(a)** a^3+b^3 **(b)** x^3-y^3
3. 10% loss
4. **(a)** $3\frac{3}{5}$ **(b)** $8\frac{1}{2}$
5. **(a)** $5a^4-6a^2+17$
   (b) $11a+4b-1$
6. $x = \frac{3ab}{2a-b}$
7. $x = 3, y = -1$
8. **(a)** $(2x+5)(2x+7)$
   (b) $(5x+3)(x-1)$
   (c) $(2x-3)(3x-7)$
9. **(a)** 1 or $-\frac{1}{3}$ **(b)** 2 or -9
10. **(a)** \$28 **(b)** \$15

Revision Exercise II No 3 *(Pg 124)*

1. $r = \frac{S-a}{S}$; $\frac{1}{10}$
2. 65.71 km/h
3. \$4.80
4. **(a)** $4\frac{1}{2}$ **(b)** $1\frac{1}{10}$
5. 30¢, 25¢
6. $c = \frac{2ab}{3(2b-a)}$; $\frac{5}{6}$
7. 56 cm
8. **(a)** $(x-15)(x-23)$
   (b) $(2x-5)(x+1)$
   (c) $(3x+4y)(3x-4y)$
9. **(a)** 0 or -3
   (b) $-\frac{2}{3}$ or -3
10. **(a)** $5x^2+30x-10$
    (b) $\frac{2x}{(x-1)(x+1)}$

Revision Exercise II No 4 *(Pg 124)*

1. **(a)** $\frac{18}{25}$ **(b)** $\frac{1}{4}$
2. 549
3. $m = \frac{2E}{V^2+2gh}$
5. **(a)** $2(x+2)(x+3)$
   (b) $(x+y+2)(x+y+3)$
   (c) $(a^2+b^2)(a+b)(a-b)$
6. $x = 5, y = 3$
7. **(a)** $x = \frac{abc}{c-a}$ **(b)** $x = \frac{b+d}{c-a}$
8. **(a)** $3pq(p-q)(p-3q)$
   (b) $2xy(x+2y)^2$
   (c) $2(2a-5b)(a-3b)$
9. $\frac{5}{7}$
10. **(b)** 18.8%
    (c) 0.123%, 6.99%
    (d) Asian economic crises

Revision Exercise II No 5 *(Pg 125)*

1. 2.29%
2. 42 kg
3. $k = \frac{A}{R^2-r^2}$; 3.125
4. 67, 9
5. $x = 0, y = -1$
6. 875 cm^2
7. **(a)** $t = \frac{v-u}{a}$, 2.5 **(b)** 15, 16
8. **(a)** $\frac{-x-12}{(x+2)(x-3)}$
   (b) $\frac{-22x}{x^2-36}$ **(c)** $\frac{6-5x}{2x(x+6)}$
9. **(a)** $x = \frac{a}{b+1}$ **(b)** $x = \frac{b+ab}{ac-a}$
10. **(a)** **(i)** $(x^2+2)(x+1)(x-1)$
    (ii) $(x+2)(x-2)(x+3)(x-3)$
    (b) **(i)** 243 **(ii)** 391

Mid-Year Examination Specimen Paper 1 *(Pg 126)*

Part I

1. **(a)** 7 370 **(b)** 97
2. \$200
3. $1\frac{5}{8}x^2$
4. $l = \frac{kx}{T+k}$, 5
5. **(a)** M **(b)** Z, N, H
6. 891 cm^3
7. 7 290 m^3
8. $x = 1, y = 3$
9. $x = 83, 7$
10. **(a)** $5(3x-1)(3x-7)$
    (b) $(3x-1)(3x+1)$
11. **(a)** $4x^2-12x+12$
    (b) $4xy$
12. **(a)** $-2\frac{1}{3}$ or -11
    (b) $1\frac{3}{4}$ or $-7\frac{1}{2}$
13. **(a)** **(i)** 36 **(ii)** 84
    (b) $1\frac{3}{4}$

Part II

1. **(a)** $3x^2-x+7$
   (b) $-4x^2+4xy-9y^2$
2. **(a)** 0 or $-1\frac{2}{5}$ **(b)** $-1\frac{17}{25}$
3. 4.02%
4. 38.3 km/h
5. **(a)** \$60 **(b)** \$10.80
6. **(a)** \$25 400 **(b)** \$100
7. **(a)** 22.5° **(b)** 22
8. **(a)** $8-9x-7x^2$
   (b) 27 yrs, 19 yrs
9. **(a)** US\$635
   (b) **(i)** \$200 **(ii)** 280 km

Mid-Year Examination Specimen Paper 2 *(Pg 127)*

Part I

1. **(a)** 82.8 **(b)** 1.30
   (c) 2.37×10^{-3} **(d)** $\frac{2}{25}$
2. $x = \frac{wa}{2k+w}$
3. \$9 320
4. \$1 080
5. **(a)** $(3x-2)^2$
   (b) $3x(5+3y)(5-3y)$
6. **(a)** 5 or -3 **(b)** 4 or -5
7. 6
8. **(a)** 3.194×10^4
   (b) 7.41×10^5
   (c) 1.8×10^{-8} **(d)** 4×10^9
9. **(a)** 16 cents **(b)** 1.344 kg
   (c) 0.325 **(d)** 40 cm
10. **(a)** $\frac{8a-15b}{10}$ **(b)** $\frac{13x-3y}{6}$
12. 96°
13. $x = 10, y = 6$

Part II

1. 1 000 m; 12 800 m^2
2. **(a)** 4 h 42 min
   (b) 23.4 km/h
3. **(a)** $x = -4, y = 3$ **(b)** $9 322
4. 2 970 cm^3, 12.474 kg
5. 122°
6. **(a)** 53 cm^2, 44 cm
   (b) **(i)** 5 **(ii)** 9
7. 2 cm, 188.4 cm^2
8. **(a)** $x = 65°, y = 70°, z = 65°$
   (b) 4 days **(c)** $10\frac{5}{6}$ km/h

Mid-Year Examination Specimen Paper 3 *(Pg 129)*

Part I

1. $x = \frac{1}{2}, y = 1\frac{1}{2}$
2. 12°
3. **(a)** $(2a - b + 2c)(2a - b - 2c)$
   (b) $2x(3x + 2y)(3x - 2y)$
4. **(a)** $2x^2 + 2xy - 2y^2$
   (b) $3x^2 - 8xy - 3y^2$
5. **(a)** $\frac{5x+3}{8}$ **(b)** $\frac{x^2+3x-2}{x^2-4}$
6. **(a)** 4, 4 **(b)** 3, 3
7. $x = \frac{Tb - la}{T - l}$
8. $x = 15, y = 70$
9. 565.5 l, 28.275 cm
10. 8.16%
11. **(a)** 7 **(b)** 1
12. 40
13. **(a)** 0.054 2
    (b) **(i)** 18.0 **(ii)** 18.05
    (iii) 18.0
14. $5\frac{2}{3}$

Part II

1. **(a)** 4.80×10^{-7}
   (b) 3.72×10^{-7}
2. 11.088 l
3. **(a)** $m = 1, c = 11$
   (b) 46 km/h
4. **(a)** $3(x + 2)(x - 2)$
   (b) 60 km/h
5. **(a)** 2 480 m **(b)** 0.75 cm^2
6. **(a)** $x = \frac{ac - b}{a - 1}$, 4 **(b)** $\frac{1}{2}$ or 2
7. $x = 3, y = 4$; 180 cm^2
8. **(a)** 18 **(b)** $x = 20$
9. 23.79 m^3, $31.88

Mid-Year Examination Specimen Paper 4 *(Pg 131)*

Part I

1. 90 km
2. **(a)** $2a - b$ **(b)** $2x^2 + x - 1$
3. 6, 8 or –8, –6
4. **(a)** 40 **(b)** 3 **(c)** 21
5. 8% increase
6. 10.5 cm, 8.3 cm
7. **(a)** 1 **(b)** $\frac{38}{81}$
8. 10 yrs, 40 yrs
9. $t = \frac{A - P}{Pr}$, 3.5
10. 89.6 km^2
11. **(a)** 8.0×10^7 **(b)** 4.67×10^{10}
    (c) 1.91×10^6
12. $x = 5, y = 1$
13. 17 940 l; $24.40
14. $x = 36$

Part II

1. 4 min
2. **(a)** $(3x + 2)(y - 4)$
   (b) $(x^2 + 4)(x + 2)(x - 2)$
3. 43 or 34
4. **(a)** 17.6 cm, 19.3 cm^2
   (b) $x = 4$
5. **(a)** $x = 1, y = 4$ **(b)** $-\frac{3}{13}$
6. **(a)** 4 **(b)** 100 cm
7. **(a)** 4, 4 **(b)** 6, 6
   (c) 2, 2 **(d)** 2, 2
8. **(a)** $\frac{yz + xz - 2xy}{xyz}$
   (b) $\frac{3x - 7}{(x - 3)(2x - 5)}$

Mid-Year Examination Specimen Paper 5 *(Pg 132)*

Part I

1. 60 cm
2. 44°, 110°
3. $f = \frac{uv}{u + v}$, 0.937 5
4. **(a)** $-8x^2 + 10x - 3$
   (b) $x^3 + 2x^2 - 5x - 10$
   (c) $6 - 5x - 21x^2 - 10x^3$
5. **(a)** $(x - 5)(x + 2)$
   (b) $(x + 2)(x - y)$
   (c) $3(3x + 2y^2)(3x - 2y^2)$
6. $x = 3, y = -2$
7. 6 days
8. $x = 16\frac{2}{3}, y = 20$

9. **(a)** $\frac{47x}{60}$ **(b)** $\frac{3x + 7}{(x + 2)(x + 3)}$
10. $D = \frac{AC}{B - A}$
11. **(a)** 9 or –3 **(b)** 7 or –2
12. 20%
13. 6 yrs, 48 yrs

Part II

1. 1 h 49 min
3. $10.90
4. 100 000 tonnes, 125 000 tonnes
5. $158 000
6. **(a)** 2.06×10^{-2}
   (b) 4.284×10^6
   (c) 2.607×10^1
7. 22
8. 9, 12 or –12, –9
9. **(a)** 4.2 km **(b)** 29.6 cm
   (c) 19.2 cm^2
10. $27 318

Exercise 9a *(Pg 138)*

1. $A(-4, -3)$, $B(-2, 3)$, $C(2, 4)$, $D(4, 2)$, $E(1, 1)$, $F(3, -1)$
2. $A(-4, -6)$, $B(-2, 5)$, $C(1, 0)$, $D(6, 5.5)$, $E(2, -3)$
5. **(a)** rectangle **(b)** rhombus
   (c) isosceles triangle
   (d) quadrilateral
   (e) isosceles triangle
   (f) trapezium
   (g) trapezium
6. (4, 0), (1, 4), (4, 4)
7. The points lie on a straight line.
8. **(c)** A straight line.

Exercise 9b *(Pg 141)*

1. **(a)** 0, 1, 2, 3, 4
   (b) 1, 2, 3, 4, 5
   (c) 2, 0, –2, –4, –6
   (d) –5, –4, –3, –2, –1
3. **(a)** 2, 5, 8
   (c) **(i)** –1, 6.8, 9.5
   (ii) –2, –1.4, –0.7
4. **(a)** –4, 0, 4
   (c) **(i)** –2, 6, 10
   (ii) –0.5, 0.4, 0.9

Exercise 9c *(Pg 144)*

1. **(a)** $y = 2$ **(b)** $y = 9$
   (c) $y = -3$ **(d)** $y = -\frac{1}{2}$
   (e) $y = 0$

2. **(a)** $x = 12$ **(b)** $x = 5$
(c) $x = -4$ **(d)** $x = 0$
(e) $x = -\frac{1}{4}$

3. **(a)** $y = -6$ **(b)** $x = 3$
(c) $x = -10$ **(d)** $y = 8$
(e) $y = -\frac{1}{3}$

4. $y = 15, y = 11, y = 8, y = 3, y = 0, y = -4, y = -6, y = -10, y = -16$

5. $x = -18, x = -13, x = -7, x = -3, x = 0, x = 4, x = 11, x = 14, x = 18$

6. **(a)** $\left(-1, 3\frac{1}{2}\right), \left(0, 3\frac{1}{2}\right), \left(1, 3\frac{1}{2}\right), \left(2, 3\frac{1}{2}\right)$
(b) $(-6, -1), (-6, 0), (-6, 1), (-6, 2)$
(c) $\left(\frac{2}{5}, -1\right), \left(\frac{2}{5}, 0\right), \left(\frac{2}{5}, 1\right), \left(\frac{2}{5}, 2\right)$
(d) $(-1, -7), (0, -7), (1, -7), (2, -7)$
(e) $(-1, 4.2), (0, 4.2), (1, 4.2), (2, 4.2)$
(f) $(-3.3, -1), (-3.3, 0), (-3.3, 1), (-3.3, 2)$
(g) $(20, -1), (20, 0), (20, 1), (20, 2)$
(h) $(-1, -16), (0, -16), (1, -16), (2, -16)$

Exercise 9d *(Pg 149)*

1. **(a)** $y = \frac{1}{2}x + 13, y = \frac{1}{2}x + 10,$
$y = \frac{1}{2}x + 4, y = \frac{1}{2}x,$
$y = \frac{1}{2}x - 5, y = \frac{1}{2}x - 11,$
$y = \frac{1}{2}x - 16$
(b) $y = -\frac{6}{5}x + 15, y = -\frac{6}{5}x + 7,$
$y = -\frac{6}{5}x, y = -\frac{6}{5}x - 6$
$y = -\frac{6}{5}x - 14$

2. $y = 5, y = \frac{2}{5}x + 5, y = \frac{5}{4}x + 5,$
$y = 3x + 5, y = -4x + 5,$
$y = 5, y = -\frac{1}{3}x + 5$

3. **(a)** 08 24 **(b)** 40 km

4. Yes, (2, 2), (4, 2), (4, 6), (6, 6)

5. Square: (4, 2), (2, 4), (6, 4), (4, 6)

6. Rhombus; (6, 2), (12, 4), (0, 4), (6, 6)

7. Kite; (2, 6), (6, 0), (6, 8), (10, 6)

8. Trapezium; (2, 6), (4, 8), (4, 2), (12, 10)

Exercise 9e *(Pg 154)*

1. $x = -1, y = -3$
2. $x = 1, y = -1$
3. $x = -5, y = -2$
4. no solution
5. infinite number of solutions
6. $x = 3, y = 1$
7. $x = 4, y = 2$
8. $x = 0, y = 2$
9. $x = 2, y = -1$
10. $x = 5, y = 3$
11. $x = 2.6, y = -2.6$
12. $x = 3, y = -4$

Exercise 9f *(Pg 156)*

1. **(a)** **(i)** 9.5 cm **(ii)** 15 cm
(b) 5th day

2. **(a)** **(i)** 150 g **(ii)** 240 g
(b) 11.2 m^2

3. **(a)** **(i)** \$70 **(ii)** \$82
(b) **(i)** 3 years
(ii) 11 years

4. **(a)** **(i)** \$1 450 **(ii)** \$550
(b) after 5 years

5. **(a)** **(i)** 40°F **(ii)** 108°F
(iii) 176°F
(b) **(i)** 20°C **(ii)** 38°C
(iii) 82°C

6. **(a)** **(i)** RM24 **(ii)** RM54
(iii) RM78 **(iv)** RM86
(b) **(i)** S\$20 **(ii)** S\$35
(iii) S\$44 **(iv)** S\$59

Exercise 9g *(Pg 161)*

1. **(a)** 4 p.m. **(b)** 30 km
(c) 2 to 2.30 p.m.
(d) 22 km **(e)** $1\frac{1}{2}$ hours

2. **(a)** 30 minutes **(b)** 10 km
(c) 20 km/h **(d)** 40 km/h
(e) 60 km **(f)** 13 30
(g) $27\frac{1}{2}$ km/h

3. **(c)** 80 km/h **(d)** 1 h 12 min
(e) 2 hours **(f)** 85 km/h
(g) 340 km **(h)** $6\frac{3}{5}$ hours
(i) $51\frac{17}{33}$ km/h

4. **(a)** 14 36, 25 km/h
(b) 15 30, $33\frac{1}{3}$ km/h
(c) 44 km
(d) about 16 50
(e) X, 1 hour **(f)** 100 km
(g) 40 km **(h)** 70 km

5. **(a)** $53\frac{1}{3}$ km/h **(b)** 180 km
(c) about 10.43 a.m.
(d) 92 km **(e)** 12 km
(f) Lina, 30 min
(g) 60 km/h

Exercise 9h *(Pg 166)*

2. **(b)** **(i)** 41 km/h **(ii)** 59 km/h
(iii) 104 km/h
(c) **(i)** 2 seconds
(ii) 11 seconds
(iii) 23 seconds

3. **(a)** **(i)** 1 350 **(ii)** 960
(iii) 750 **(iv)** 150
(v) 90
(b) **(i)** 1.5 **(ii)** 17
(iii) 34 **(iv)** 37
(v) 41

4. **(a)** **(i)** \$54.50 **(ii)** \$60.50
(iii) \$67.40
(b) **(i)** 18 **(ii)** 45
(c) No, \$66.20

5. 24, 36, 48, 60, 72, 84, 96, 108
(b) 3.5 litres, 4.7 litres, 7.5 litres
(c) 30 km, 57.6 km, 102 km

6. 8, 12, 16, 20
(b) **(i)** 7 litres **(ii)** 10 litres
(iii) 18 litres
(c) **(i)** 6 seconds
(ii) 26 seconds
(iii) 38 seconds

7. **(a)** **(i)** RM15.60
(ii) RM40.60
(iii) RM78.10
(b) **(i)** S\$19.20 **(ii)** S\$34.60
(iii) S\$56.30

8. **(a)** 29%, 42%, 72%, 87%
(b) 54, 72, 90, 107
(c) 96

9. **(a)** **(i)** \$53 **(ii)** \$90.50 **(iii)** \$43.70 **(iv)** \$114.50
(b) **(i)** 90 **(ii)** 155 **(iii)** 250 **(iv)** 345

Exercise 9i *(Pg 172)*

1. **(a)** 1.04 p.m. **(b)** $15\frac{5}{6}$ km/h
(c) 1.51 p.m.
2. **(a)** 4 **(b)** 5
(c) \$20 **(d)** \$60
3. **(a)** \$50, \$30 **(b)** 150

Review Questions 9 *(Pg 173)*

2. **(a)** $x = 1, y = -1$
(b) $x = 2, y = -1$
(c) $x = 2, y = 1.5$
(d) infinite number of solutions
(e) no solution
3. 19
4. **(a)** **(i)** \$440 **(ii)** \$840
(b) 5
5. **(a)** 28 g **(b)** 65 g
6. 40, 60, 80, 100
(b) **(i)** 32 cm **(ii)** 56 cm **(iii)** 21 cm
7. **(a)** \$20 000 **(b)** \$20 000
(c) \$69 000
8. **(b)** **(i)** 10.50 a.m.
(ii) 12 km/h
9. **(a)** **(i)** 11 30 **(ii)** 11 40
(b) **(ii)** 10 21, 41 km
10. **(a)** **(i)** \$450 **(ii)** \$640 **(iii)** \$190
(b) 4 **(c)** \$380
(d) The business firm suffers a loss of \$120.
11. 150, 200, 250, 300, 350, 400
(b) **(i)** \$140 **(ii)** \$270 **(iii)** \$360
(c) **(i)** 13 **(ii)** 19 **(iii)** 34

Exercise 10a *(Pg 183)*

1. 13, 5, 4, 5, 8, 20, $x = 0$
2. −14, −5, 1, −5, −9, Yes
3. $1\frac{3}{4}$, 0, $-2\frac{1}{4}$, −2, 0
4. **(a)** −2, 2, 4, −8, −16
(c) **(i)** −4.2, 1.2; −5, 2; 2.7
(ii) −4.8, 1.4, −5.4
5. **(a)** 3, 0, 3, 8
(c) **(i)** 0.3, 1.7; −0.7, 2.7; −1.4, 3.4
(ii) 6.8; 0.4; 5.3
(d) $x = 1$
6. **(a)** 6, 0, −4, −6, −6, −4, 6
(b) **(i)** −3.6, 0.61 −4.9; 1.9; −5.3,
(ii) 9.8, −4.8, 3.4

Exercise 10b *(Pg 187)*

1. **(b)** 0 or 4
2. **(b)** 0.7 or 4.3
3. **(b)** −2.5 or 1
4. **(b)** −2.4 or 1.7
5. **(b)** **(i)** 17.2 metres
(ii) 3.8 seconds
6. **(b)** **(i)** 8.5 m/s **(ii)** 2 m/s
(iii) 0.6 s or 3.4 s
7. **(b)** **(i)** 8 handbags
(ii) 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 handbags

Review Questions 10 *(Pg 188)*

1. 4, 6, 4, 0; $x = 2\frac{1}{2}$
2. **(a)** 5, −4, 0, 12, 21
(b) **(i)** −3.8, −1.8, 8.3
(ii) ±1.4, 3.2, 3.5
3. **(a)** 16, 9, 1, 0, 9
(c) **(i)** −0.7, 2.7; −1.8, 3.8; −2.7, 4.7
(ii) 11.6, 0.6, 7.3
4. **(b)** **(i)** −3.9, 2.9; −2.8, 1.8; −2, 1
(ii) 7.4, 5.8, 1.3
5. **(b)** **(i)** −0.7, 2.7; −1.4, 3.4; −1.8, 3.8
(ii) 0.3, −5.4, −3.4
6. **(a)** 2.4, 10.7 **(b)** 8.5
7. 0.2 or 2.2
8. **(a)** −3.3 or 0.8
(b) −1.4 or 2.4
9. −2.4 or 1.1
10. −1.1 or 2.6
11. **(b)** **(i)** 45 items
(ii) \$2 025
12. **(a)** 30.4 metres
(b) 4.5 seconds
(c) 5.6 seconds

Exercise 11a *(Pg 193)*

3. **(a)** (3, −4) **(b)** (2, 3)
(c) (−2, −5) **(d)** (−3, 6)
(e) (7, −1) **(f)** (8, 4)
(g) (−8, 2) **(h)** $(p, -q)$
4. **(a)** (−5, 3) **(b)** (−11, −1)
(c) (4, 9) **(d)** (1, −12)
(e) (−1, 8) **(f)** (4, 5)
(g) (−6, −7) **(h)** $(-p, q)$
5. **(a)** (1, 4) **(b)** (3, 4)
(c) (11, 4) **(d)** (−11, 4)
6. **(a)** (2, 4) **(b)** (2, 10)
(c) (2, −4) **(d)** (2, 15)
7. (3, 2)
8. (3, −10), No
9. (−5, 6)
10. (15, 6), No; combination of reflections is not commutative
11. **(a)** $\triangle COB$ **(b)** $\triangle EOD$
(c) $\triangle AOF$
12. **(a)** $\triangle COB$ **(b)** $\triangle EOD$
(c) $\triangle GOF$ **(d)** $\triangle AOH$
13. **(a)** $\triangle EBC$ **(b)** $\triangle CEF$
(c) *DEIH* **(d)** *CHDB*
(e) $\triangle IEH$

Exercise 11b *(Pg 198)*

1. **(a)** (4, 1) **(b)** (6, 4)
(c) (4, 7) **(d)** (4, 0)
(e) (1, 7) **(f)** (3, 5)
(g) (3, 7) **(h)** (1, 1)
5. $A'(-1, 3)$, $B'(-1, 5)$, $C'(-3, 3)$
6. (3, −5)
7. **(a)** (4, 0) **(b)** (0, 8)
8. **(a)** $\triangle BOC$ **(b)** $\triangle EOF$
(c) $\triangle FOA$ **(d)** $\triangle OED$
(e) $\triangle EDO$ **(f)** $\triangle DOE$
9. **(a)** $\triangle EOF$ **(b)** $\triangle GOH$
(c) $\triangle COD$ **(d)** $\triangle BOC$

Exercise 11c *(Pg 203)*

2. $A'(5, -2)$, $B'(7, -1)$, $C'(8, -3)$
3. (5, −7), (7, −6), (8, −5), (8, −2)
4. (0, 4), (0, 6), (2, 9)
5. 6 units in the negative x-direction and 3 units in the positive y-direction, (−1, 5), 5 units in the negative x-direction and 2 units in the negative y-direction, (1, −5)
6. (6, −6)
7. $h = 11, k = -5$
8. (0, −1)
9. (8, 2), (3, 7)
10. (13, 8), (7, 7), (3, 9)

11. $A'(5, -1)$, $B'(6, 4)$, $C'(10, 2)$, $D'(9, 0)$; $A''(7, 5)$, $B''(8, 10)$, $C''(12, 8)$, $D''(11, 6)$

12. $A'(3, 7)$, $B'(4, 12)$, $C'(8, 10)$, $D'(7, 8)$; $A''(7, 5)$, $B''(8, 10)$, $C''(12, 8)$, $D''(11, 6)$, Yes

13. **(a)** 7 units in the positive y-direction and none in the x-direction
(b) 6 units in the negative x-direction and 1 unit in the negative y-direction

Exercise 11d *(Pg 208)*

6. $P\left(2\frac{1}{2}, 2\frac{1}{2}\right)$, $Q\left(3\frac{1}{2}, 1\frac{1}{2}\right)$, $R(2, 2)$

7. **(a)** (2, 1)
(b) $P(12, 1)$, $Q(6, 7)$

8. $L\left(1\frac{1}{2}, 3\right)$, $M(2, 5)$, $N\left(5, 2\frac{1}{2}\right)$

9. **(a)** $\triangle ABC$ **(b)** $PBQR$

Exercise 11e *(Pg 213)*

1. **(a)** (–3, 7) **(b)** (1, 7)
2. (–2, –5)
3. **(a)** (3, –2) **(b)** (1, 4)
4. (–3, –1)
5. **(a)** (8, 4) **(b)** (6, 3)
6. $A'(-1, -2)$, $B'(-5, -3)$, $C'(0, -5)$
7. $P(-2, 1)$, $Q(-6, 1)$, $R(-6, 3)$, $S(-2, 3)$

10. **(a)** reflection in BE
(b) translation parallel to AB of length 3 cm
(c) reflection in AE followed by reflection in BE
(d) 90° clockwise rotation about E

11. **(a)** reflection in y-axis
(b) 90° clockwise rotation about (0, 0)
(c) reflection in x-axis followed by 90° clockwise rotation about (0, 0) or reflection in y-axis followed by 90° anticlockwise rotation about (0, 0)
(d) reflection in x-axis followed by reflection in y-axis or a 90° clockwise rotation about O followed by another 90° clockwise rotation about O

Review Questions 11 *(Pg 215)*

1. **(a)** (–2, –1) **(b)** (1, 2) **(c)** (3, 4)
2. **(a)** (–3, 1) **(b)** (–3, 1) **(c)** (–1, 3) **(d)** (1, 5)
3. $x = 9$, $y = 0$
5. **(a)** enlargement centre at A and scale factor 2
(b) translation parallel to AE with length AE
(c) 180° rotation about E
(d) reflection in BH followed by 180° rotation about E
(e) reflection in FD followed by 180° rotation about E

Exercise 12a *(Pg 221)*

1. (a), (c), (d), (e), (g), (h)
2. **(a)** 15 cm **(b)** 17 m **(c)** 5 m **(d)** 24 m **(e)** 9 cm **(f)** 29 cm **(g)** 12 cm **(h)** 60 cm **(i)** 45 cm **(j)** 65 cm **(k)** 8.06 m **(l)** 9.22 m **(m)** 6.71 cm **(n)** 8.67 cm **(o)** 8.20 cm **(p)** 12.8 cm
3. **(a)** 9.43 **(b)** 13.1 **(c)** 13.3 **(d)** 4.82 **(e)** 7.07 **(f)** 7.21 **(g)** 14.7 **(h)** 12.3 **(i)** 20.8
4. **(a)** $x = 16$, $y = 9$
(b) $x = 25$, $y = 144$

Exercise 12b *(Pg 224)*

1. 3.80 m **2.** 4.66 m
3. 50.3 m **4.** 20.3 cm
5. 10 cm **6.** 3
7. 28.3 cm **8.** 58.3 m
9. 6.63 m **10.** 11.1 sec
11. 1.73 cm **12.** 10 cm
13. 6.32 cm
14. 9.22 cm, 5.86cm
15. 60 cm^2
16. 24 cm, 168 cm^2

Review Questions 12 *(Pg 225)*

1. **(a)** 11.4 **(b)** 10.3 **(c)** 9.62 **(d)** 10 **(e)** 12 **(f)** 25
2. 28.7 cm
3. 120.2 cm; 903.1 cm^2
4. 92.3 cm, 2 214.3 cm^2
5. 11.2 cm
6. 14.4 cm

Revision Exercise III No 1 *(Pg 227)*

1. **(a)** 4 **(b)** 27 **(c)** $6\frac{1}{2}$ **(d)** $-\frac{1}{5}$
2. 15 yr, 45 yr
3. $d = \frac{L-N}{L}$, $\frac{1}{5}$
5. **(a)** 27, 63
(b) 3 yr 4 mths
6. **(a)** 5 cm **(b)** 30 cm^2
7. 33.6 m
8. 56
9. (3, 2)
10. 5, 0, –3, 12
$x = 1$; 3 or –1

Revision Exercise III No 2 *(Pg 227)*

1. **(a)** –6 **(b)** $-2\frac{1}{2}$ **(c)** 1 **(d)** 3
2. **(a)** $1\frac{2}{5}$ **(b)** \$15
3. $A(6, 0)$, $B(0, 4)$; 18 units^2
4. –5, 0, 3, 0
(a) 4 **(b)** $x = 1$
(c) 3 or –1
(d) 3.45 or –1.45
5. **(a)** $5a(2a + 3)(2a - 3)$
(b) $(a + 3b + 1)(a + 3b - 1)$
7. **(a)** **(i)** S\$5.40 **(ii)** S\$17.10
(b) **(i)** RM93.30
(ii) RM166.70
8. **(a)** 4.9 **(b)** 2.3 or –2.3
9. **(a)** 1 : 75 000
(b) 17.25 km
(c) 13.5 km^2
10. **(a)** **(i)** 3 **(ii)** 1
(b) reflection in the line OP
(c) 120° anticlockwise rotation about O

Revision Exercise III No 3 *(Pg 228)*

1. $c = \frac{3m - b}{2}$
2. \$150
3. $x = -\frac{1}{2}$, $y = 2$
4. **(a)** $\frac{13x - 5}{15}$ **(b)** $\frac{2 - x}{x(x + 2)}$

5. (a) reflection in OB
(b) anticlockwise rotation of 120° about O
6. 12 cm, 9.6 cm
7. 9
8. $289.00
9. (a) $19.30 (b) 80
(c) $13
10. −5, −2, 37
(a) 2.9 (b) 1 or 3.5

Revision Exercise III No 4 *(Pg 229)*

1. (a) $6\frac{2}{9}$ (b) 33
2. (a) $2 + x - 13x^2 + 7x^3 + 3x^4$
(b) $(a + b + c)(a + b - c)$
3. (a) $\frac{x}{21}$ (b) $\frac{x^2 + 2}{x(x - 2)}$
4. $x = 5$, $y = 1$
5. (b) 9.33 cm
6. (a) 125 cm^3 (b) 60 cm
7. (e) $C'(1, 7)$, $R(-4, 4)$, $N(4, 4)$
8. (a) 36.7 (b) 122 (c) 27.8
9. (3, −2)
10. −4, 2, 0
(a) 2.3 (b) 1.3
(c) −0.6 or 3.6

Revision Exercise III No 5 *(Pg 230)*

1. 5.6%
2. (a) $x = 32°$, $y = 48°$
(b) 154 cm^2, 88 cm
3. $x = 3$, $y = -7$
4. (a) $4(x + y)^2$ (b) 11
5. (a) 88 cm (b) 484 cm^2
6. (a) 0, 4 or $-\frac{2}{3}$ (b) −1 or $2\frac{2}{3}$
7. (a) 4.9 m (b) 11.2 cm
8. 11, −5, −4, 4
(a) −4.8 (b) 4.3 or −2.3
9. (a) 6 litres (b) 62.5 km
10. (a) F (b) F (c) T
(d) T (e) F (f) T
(g) F (h) T (i) F

Exercise 13a *(Pg 234)*

1. (a) 28 mm (b) $31\frac{1}{2}$ m
(c) 70 cm
2. (a) 44 cm^2 (b) 0.495 m^2
(c) 401.5 mm^2
3. (a) (i) 154 cm^2 (ii) 88 cm
(b) (i) 42 cm^2 (ii) 44 cm
(c) (i) 1 232 cm^2
(ii) 176 cm
(d) (i) 61.5 cm^2 (ii) 34 cm
4. (a) 672 cm^2 (b) 19.25 cm^2
(c) 462 cm^2
(d) $2\,328\frac{2}{7}$ cm^2
5. (a) 28π m (b) 52π m^2
6. (a) $26\frac{2}{7}$ m^2 (b) $30\frac{4}{7}$ m

Exercise 13b *(Pg 240)*

1. (a) (i) 32.6 cm (ii) 29.7 cm^2
(b) (i) 31.4 cm (ii) 57.1 cm^2
(c) (i) 58.9 cm
(ii) 224.6 cm^2
(d) (i) 188.6 cm
(ii) 1 697.0 cm^2
2. (a) 14.3 cm^2 (b) 101.7 cm^2
(c) 600 mm^2 (d) 230.9 cm^2
3. (a) (i) $17\frac{2}{3}$ cm
(ii) $12\frac{5}{6}$ cm^2
(b) (i) $8\frac{5}{6}$ cm
(ii) $3\frac{5}{24}$ cm^2
(c) (i) $26\frac{2}{3}$ cm
(ii) 44 cm^2
4. (i) (a) 11 cm (b) 77 cm^2
(ii) (a) $14\frac{2}{3}$ cm
(b) $102\frac{2}{3}$ cm^2
(iii) (a) 24.2 cm
(b) 169.4 cm^2
(iv) (a) 30.8 cm
(b) 215.6 cm^2
(v) (a) 46.2 cm
(b) 323.4 cm^2
(vi) (a) 52.8 cm
(b) 369.6 cm^2
(vii) (a) 70.4 cm
(b) 492.8 cm^2
(viii) (a) 83.6 cm
(b) 585.2 cm^2
5. (i) (a) 385 cm^2
(b) 22 cm
(ii) (a) $577\frac{1}{2}$ cm^2
(b) 33 cm
(iii) (a) $898\frac{1}{3}$ cm^2
(b) $51\frac{1}{3}$ cm
(iv) (a) 1 155 cm^2
(b) 66 cm
(v) (a) 1 540 cm^2
(b) 88 cm
(vi) (a) $2\,117\frac{1}{2}$ cm^2
(b) 121 cm
(vii) (a) 2 695 cm^2
(b) 154 cm
(viii) (a) 3 465 cm^2
(b) 198 cm
6. (a) 49° (b) 80°
(c) 263° (d) 346°
7. (a) 60° (b) 166°
(c) 303° (d) 27°
8. (a) 9 cm, 31 cm^2, 23 cm
(b) 108°, 66 mm, 1 155 mm^2
(c) 28 mm, 132 mm, 188 mm
(d) 84 cm, 9 237 cm^2, 388 cm
(e) 225°, 385 m^2, 83 m
(f) 15 cm, 20 cm, 50 cm
9. (a) 9 cm (b) 5 cm
(c) 4 cm (d) 12 cm
(e) 15 cm (f) 6 cm
(g) 12 cm (h) 4 cm
10. (i) (a) 242 cm^2
(b) 138 cm^2
(ii) (a) 12.5 cm^2
(b) 7.14 cm^2
11. (a) $\frac{m}{360} \times 2\pi r + \frac{m}{360} \times 2\pi R + 2(R - r)$
(b) (i) 117.8 cm^2
(ii) 40.85 cm^2
(iii) 1 642 cm^2
12. 191°
13. 25.7 cm
14. 1.18 m
15. 24.1 m^2
16. (a) 105° (b) 38.5 cm, 21 cm
17. (a) 100° (b) 385 cm^2, 21 cm
18. 344 cm
19. 84 cm^2
20. 38.5 cm^2

Exercise 13c *(Pg 248)*

1. (a) 245 cm^3 (b) 192 cm^3
(c) 16 cm^3 (d) 231 cm^3
2. 3 125 000 m^3

3. 9 m
4. 5 cm
5. $7\frac{1}{2}$ cm
6. $32\frac{1}{2}$ cm
7. 960 cm^3
8. **(a)** 48 cm^3 **(b)** 96 cm^2
9. $\frac{3}{4}$ cm
10. 2 640 g

Exercise 13d *(Pg 254)*

1. **(a)** 527.8 cm^3 **(b)** 25 cm^3
(c) 179.6 cm^3
(d) 12 941.2 mm^3
2. **(a)** 88 cm^2
(b) 1 319.5 mm^2
(c) 1 650 cm^2 **(d)** 174.6 m^2
3. **(a)** 154 cm^2, 1 232 cm^3, 550 cm^2, 704 cm^2
(b) 8 cm, $113\frac{1}{7}$ cm^2, $188\frac{4}{7}$ cm^2, $301\frac{5}{7}$ cm^2
(c) 8 cm, $1\,005\frac{5}{7}$ cm^3, $427\frac{3}{7}$ cm^2, $628\frac{4}{7}$ cm^2
(d) 20 cm, 25 cm, $707\frac{1}{7}$ cm^2, $1\,178\frac{4}{7}$ cm^2
4. **(a)** 2.5 cm **(b)** 9.8 cm^2
5. **(a)** 603.2 cm^3 **(b)** 377 cm^2
6. 8 192
7. **(a)** 351.86 m^2 **(b)** 552.9 m^3
8. 13.14 cm
9. 28.7 cm
10. 1 555 cm^3

Exercise 13e *(Pg 259)*

1. **(a)** 179.6 cm^3, 154.0 cm^2
(b) 4 189.3 m^3, 1 256.8 cm^2
(c) 14.1 mm^3, 28.3 mm^2
2. **(a)** 6 cm, 288π cm^3
(b) 18 mm, $7\,776\pi$ mm^3
(c) 4 m, $85\frac{1}{3}\pi$ m^3
(d) 15 cm, $4\,500\pi$ cm^3
3. 5.88 m, 433.9 m^2
4. 12 cm, 7 238.2 cm^3
5. **(a)** 179.6 cm^3 **(b)** 381.7 cm^3
6. **(a)** $2\frac{1}{4}$ cm **(b)** 4 m

7. **(a)** $2\,258\frac{2}{3}$ cm^3, 902 cm^2
(b) 32 340 cm^3, 5 082 cm^2
8. 3 cm
9. 713
10. 805.8 g
11. **(a)** 227 m^2 **(b)** 477 m^3
12. 620.5 kg
13. 1.28 cm
14. 272.7 m^3
15. **(a)** 182 cm^2 **(b)** 23 mm
16. 20.3 cm
17. 14.9 cm
18. **(a)** $5\,088\pi$ **(b)** $1\,104\pi$
19. **(a)** 15 317.25 cm^3
(b) 3 208.56 cm^2
20. **(a)** 44 400 cm^3
(b) 7 605.89 cm^2

Review Questions 13 *(Pg 260)*

1. **(a)** 115.8 cm^2, 50.3 cm
(b) 42.6 cm^2, 37.8 cm
(c) 58.1 cm^2, 33.7 cm
(d) 11 766 cm^2, 396.2 cm
2. $\frac{240}{360} \times 2\pi \times 12$
(a) 8 cm, 301.6 cm^2
(b) 596.5 cm^3
3. **(a)** 70 695 mm^3, 14 655.6 mm^2
(b) 89 808.8 cm^3, 11 547 cm^2
(c) 333.6 m^3, 241.9 m^2
(d) 2.6 m^3, 11.9 m^2
(e) 23 200 cm^3, 5 400 cm^2
(f) 13 069.5 cm^3, 3 596.1 cm^2
4. 1 017 cm^2
5. 3 : 1 : 2
6. 5.3 cm, 3.4 kg
7. **(a)** 400
(b) $\sqrt{\frac{3V}{4\pi}}$, 1.44 mm
8. 12 cm
10. $141.75
11. $235.64
12. 829.4 cm^3
13. $10\,111\frac{1}{9}$ kg
14. **(a)** 794.4 g
(b) 10 525.8 kg/m^3
15. 220 cm
16. 8 cm
17. **(a)** 0.6 cm
(b) 0.622 cm^3, 0.679 cm^3
18. 8 062.65 m^3, 117 252
19. 113.15 m^3, 43 985

Exercise 14a *(Pg 269)*

1. **(a)** 6 **(b)** 65 **(c)** 11.3
(d) 151 **(e)** 110.7
2. 38.1
3. 29.6
4. 112.15
5. $35.86
6. **(a)** 246 **(b)** 47
7. 11
8. **(a)** 27 cm **(b)** 42 cm
9. **(a)** $8 400 **(b)** $770
10. 168 cm
11. **(a)** 5 **(b)** 63
(c) 0.5 **(d)** 99
12. 2
13. 4, 5

Exercise 14b *(Pg 274)*

1. **(a)** 12.7, 12, 11
(b) 12, 13, 14
(c) 7.125, 7, 7
(d) 96.1, 98, 98
(e) 3.81, 4, 5 **(f)** 6.54, 6, 6
2. **(a)** $829.63 **(b)** $850
(c) $760
3. 6.4, 6.5, 7
4. **(a)** 2.58 **(b)** 2 **(c)** 2
5. **(a)** 5 **(b)** 10
6. **(a)** 5, 4.67 **(b)** Peter, No
(c) 3, 4 **(d)** 2, 6
(e) mode
7. **(a)** No **(b)** 7, 7 **(c)** mode
8. **(c)** $x = 20$, $y = 24$ **(d)** 3, 3

Review Qustions 14 *(Pg 276)*

1. **(a)** 8 **(b)** 1.2
(c) **(i)** 1 **(ii)** 1
2. **(a)** 0.9 **(b)** 1 **(c)** 0
3. **(a)** 34 **(c)** $x = 13$, $y = 21$
(d) 2
4. **(a)** 2 **(b)** 2.15 **(c)** 2
5. **(a)** 2 **(b)** 3.2, 3
6. $207, $194.50, $197.18
7. **(a)** 44, 48, 50, 54, 54, 56, 58, 59, 60, 61, 63, 65, 67, 67, 67, 69, 71, 72, 75, 78
(i) 62 kg **(ii)** 67 kg
(iii) 61.9 kg
(b) 146, 148, 150, 154, 157, 157, 160, 161, 162, 164, 165, 167, 168, 169, 171, 171, 171, 171, 175, 175

(i) 164.5 cm
(ii) 171 cm
(iii) 163.1 cm

8. **(a)** $81\frac{1}{3}$ **(b)** 183%
(d) mean = US$3.23
median = US$2.59

Exercise 15a *(Pg 281)*

1. **(a)** **(i)** *AC* **(ii)** *BC* **(iii)** *AB*
(b) **(i)** *LM* **(ii)** *MN* **(iii)** *LN*
(c) **(i)** *PQ* **(ii)** *QR* **(iii)** *PR*
(d) **(i)** *XY* **(ii)** *XZ* **(iii)** *YZ*

2. **(a)** **(i)** $\frac{x}{z}$ **(ii)** $\frac{y}{z}$ **(iii)** $\frac{x}{y}$ **(iv)** $\frac{y}{z}$ **(v)** $\frac{x}{z}$ **(vi)** $\frac{y}{x}$
(b) **(i)** $\frac{y}{z}$ **(ii)** $\frac{x}{z}$ **(iii)** $\frac{y}{x}$ **(iv)** $\frac{x}{z}$ **(v)** $\frac{y}{z}$ **(vi)** $\frac{x}{y}$
(c) **(i)** $\frac{z}{x}$ **(ii)** $\frac{y}{x}$ **(iii)** $\frac{z}{y}$ **(iv)** $\frac{y}{x}$ **(v)** $\frac{z}{x}$ **(vi)** $\frac{y}{z}$
(d) **(i)** $\frac{x}{y}$ **(ii)** $\frac{z}{y}$ **(iii)** $\frac{x}{z}$ **(iv)** $\frac{z}{y}$ **(v)** $\frac{x}{y}$ **(vi)** $\frac{z}{x}$
(e) **(i)** $\frac{y}{x}$ **(ii)** $\frac{z}{x}$ **(iii)** $\frac{y}{z}$ **(iv)** $\frac{z}{x}$ **(v)** $\frac{y}{x}$ **(vi)** $\frac{z}{y}$
(f) **(i)** $\frac{z}{y}$ **(ii)** $\frac{x}{y}$ **(iii)** $\frac{z}{x}$ **(iv)** $\frac{x}{y}$ **(v)** $\frac{z}{y}$ **(vi)** $\frac{x}{z}$

3. **(a)** **(i)** $\frac{4}{5}$ **(ii)** $\frac{3}{5}$ **(iii)** $\frac{4}{3}$ **(iv)** $\frac{3}{5}$ **(v)** $\frac{4}{5}$ **(vi)** $\frac{3}{4}$
(b) **(i)** $\frac{5}{13}$ **(ii)** $\frac{12}{13}$ **(iii)** $\frac{5}{12}$ **(iv)** $\frac{12}{13}$ **(v)** $\frac{5}{13}$ **(vi)** $\frac{12}{5}$
(c) **(i)** $\frac{8}{17}$ **(ii)** $\frac{15}{17}$ **(iii)** $\frac{8}{15}$ **(iv)** $\frac{15}{17}$ **(v)** $\frac{8}{17}$ **(vi)** $\frac{15}{8}$
(d) **(i)** $\frac{24}{25}$ **(ii)** $\frac{7}{25}$ **(iii)** $\frac{24}{7}$ **(iv)** $\frac{7}{25}$ **(v)** $\frac{24}{25}$ **(vi)** $\frac{7}{24}$

Exercise 15b *(Pg 284)*

1. **(a)** 1.036 **(b)** 0.707 1
(c) 0.848 0 **(d)** 0.477 0
(e) 0.967 3 **(f)** 0.509 0
(g) 2.079 **(h)** 0.998 2
(i) 0.217 3 **(j)** 0.124 2
(k) 0.028 62 **(l)** 0.041 18

2. **(a)** 1.230 **(b)** 2.346
(c) 13.86 **(d)** 0.284 7
(e) 4.665 **(f)** 1.376
(g) 1.141 **(h)** 0.863 6
(i) 1.059 **(j)** –1.392
(k) 0.719 0 **(l)** 0.652 9
(m) 1.985 **(n)** 0.439 1

Exercise 15c *(Pg 286)*

1. **(a)** 7.444 **(b)** 35.07
(c) 9.706 **(d)** 13.81
(e) 25.80 **(f)** 7.571
(g) 5.470 **(h)** 19.94
(i) 16.87 **(j)** 26.74

2. **(a)** $x = 6.710$, $y = 9.948$
(b) $x = 11.83$, $y = 10.77$
(c) $x = 6.807$, $y = 9.764$
(d) $x = 16.22$, $y = 20.17$
(e) $x = 10.71$, $y = 19.01$
(f) $x = 12.29$, $y = 22.45$
(g) $x = 22.59$, $y = 24.28$
(h) $x = 147.5$, $y = 89.67$
(i) $x = 26.60$, $y = 70.77$
(j) $x = 19.27$, $y = 34.74$
(k) $x = 12.82$, $y = 8.422$
(l) $x = 35.88$, $y = 15.95$
(m) $x = 17.51$, $y = 9.380$
(n) $x = 29.50$, $y = 44.00$
(o) $x = 29.19$, $y = 73.45$
(p) $x = 24.68$, $y = 40.70$
(q) $x = 22.31$, $y = 58.08$
(r) $x = 10.35$, $y = 36.94$

Exercise 15d *(Pg 290)*

1. **(a)** 50° **(b)** 70° **(c)** 44°
(d) 14° **(e)** 70° **(f)** 52°

2. **(a)** 50.9° **(b)** 68.7° **(c)** 51.3°
(d) 31.8° **(e)** 69.4° **(f)** 43.5°
(g) 38.2° **(h)** 51.9° **(i)** 27.9°
(j) 60.1° **(k)** 40.1° **(l)** 22.5°
(m) 51.0° **(n)** 64.9° **(o)** 25.0°
(p) 40.8° **(q)** 24.2° **(r)** 35.7°

Exercise 15e *(Pg 292)*

1. **(a)** 42.71° **(b)** 22.17°
(c) 28.74° **(d)** 58.53°
(e) 41.36° **(f)** 25.50°
(g) 51.90° **(h)** 42.11°
(i) 49.59° **(j)** 54.71°
(k) 67.01° **(l)** 52.67°

2. **(a)** $x = 69.51°$, $y = 11.24$
(b) $x = 27.32°$, $y = 9.210$
(c) $x = 27.40°$, $y = 4.767$, $z = 17.24°$
(d) $x = 23.72°$, $y = 28.28°$, $z = 7.809$
(e) $x = 5.295$, $y = 6.102$, $z = 9.119$
(f) $x = 71.77°$, $y = 3.790$

Exercise 15f *(Pg 296)*

1. 28.2° **2.** 7.27 cm
3. 35.6 m **4.** 12.2 cm
5. 33.7 m **6.** 24.9 m
7. 57.1° **8.** 62.7 m
9. 73.7° **10.** 52.9°
11. 5.74 m, 3.05 m **12.** 36.3 m
13. 17.0° **14.** 105.2 m
15. 50.0° **16.** 10.1 cm
17. $x = 8.92$ cm, $\theta = 48.0°$

Exercise 15g *(Pg 300)*

1. 11.25 m **2.** 15.44 m
3. 105.6 m, 89.0 m
4. 11.43 m **5.** 52.09 m
6. 40.26 m **7.** 4.213 cm
8. **(a)** 43.3 cm **(b)** 25 cm
(c) 15 cm **(d)** 70.9°
9. **(a)** 21.65 cm **(b)** 37.5 cm
(c) 35.36 cm
10. $0.7x$, 19.3°
11. $x = 43.2°$, 0.728 7 m
12. **(a)** 89.1 cm **(b)** 45.4 cm
(c) 45°
13. 7
14. 1.533 cm
15. 13.24 m

Review Questions 15 *(Pg 301)*

1. **(a)** 17.6 **(b)** 12.5 **(c)** 7.58
(d) $x = 14.5$, $y = 4.59$
(e) $x = 1.61$, $y = 61.5$
(f) $x = 12.5$, $y = 129$
2. **(a)** 4 **(b)** $2\sqrt{3}$
3. $h = -3$, $k = 3$
4. **(a)** 40 **(b)** 32 **(c)** $2\frac{2}{15}$
5. $BD = 21.0$ cm, $AD = 34.1$ cm
6. 34.82 cm, 23.49 cm

7. 0.707 m
8. 15 cm, 14.1 cm
9. 123.9 m
10. 3.1 m
11. 133.9 m
12. 121.2 m

Revision Exercise IV No 1 *(Pg 304)*

1. $k = \frac{c}{x+y}$, $1\frac{1}{4}$
2. **(a)** 72 km/h **(b)** 20 m/s
3. **(a)** $(x-1)(x-20)$
(b) $(x+4)(x+9)$
(c) $(1+3x)(1-5k)$
(d) $(2x+y)(6x-5y)$
4. **(a)** 0.251 cm
(b) 25 cm, 20 cm
5. **(a)** 6 **(b)** 5 **(c)** 4.7
6. **(a)** $\frac{17-2x}{12}$ **(b)** $-5a-b$
(c) $\frac{x^2+2x+3}{x^3}$
(d) $\frac{a^2-b^2}{ab}$
7. **(a)** 112 cm³ **(b)** 15 cm
8. **(a)** 32 cm **(b)** 320 cm²
9. **(a)** **(i)** 9.90 cm
(ii) 141.2 cm³
(b) 480.0 g
10. 3, –5, –5, 3
(a) –3.8 **(b)** –6

Revision Exercise IV No 2 *(Pg 305)*

1. **(a)** $\frac{x^2+y^2+z^2}{xy}$
(b) $\frac{7x-8y-6}{6}$
2. **(a)** 1 **(b)** 1
(c) 15 **(d)** $16\frac{4}{7}$
3. **(a)** 10 cm **(b)** 4.8 cm
4. **(a)** $1\frac{12}{13}$ **(b)** $\frac{5}{13}$
5. **(a)** **(i)** 48.2° **(ii)** 8.94 m
(b) 38.7°
6. **(a)** 405 cm³ **(b)** 4.33 cm
7. **(a)** 6 283 cm³, 1 885 cm²
(b) 1 150 cm³, 531 cm²
8. **(a)** 64π cm² **(b)** $6\frac{2}{3}\pi$ cm
(c) $26\frac{2}{3}\pi$ cm²
9. 20, 60

10. 11, 3, 3, 6, 18
(a) 4.3; 3.6 or –1.6
(b) $x = 1$

Revision Exercise IV No 3 *(Pg 305)*

1. $1 440
2. **(a)** $b = \frac{cx-ay}{x-y}$, $6\frac{1}{2}$
(b) 22
3. $12, $10.50
4. **(a)** 8 cm **(b)** 33.7°
5. 24 cm, 134.8°
6. **(a)** 160.8 cm³, 217.0 cm²
(b) 3 176.25 cm²
7. **(a)** 31.4 cm² **(b)** 31 *g*
8. $8\frac{2}{7}$
9. **(a)** 13 200 cm³
(b) 9 240 *g*
10. **(a)** 13.5 cm **(b)** 2 464 cm²

Revision Exercise IV No 4 *(Pg 307)*

1. **(a)** $300 **(b)** $800
2. **(a)** 88 cm **(b)** 308 cm²
3. **(a)** 51.3 cm³ **(b)** 5.32 cm
4. 1 m
5. 2.6 m
6. **(a)** 17.5 cm **(b)** 74.5°
7. 188.8 cm²
8. **(a)** 13 **(b)** 13 **(c)** 12.7
9. 12 cm, 30 cm²
10. **(a)** 11 cm **(b)** 121 cm²

Revision Exercise IV No 5 *(Pg 307)*

1. 60.7 km/h
2. $168
3. $a = \frac{2s-2ut}{t^2}$, $1\frac{1}{3}$
4. $7, $2.50
5. **(a)** 42.7 cm³ **(b)** 70.5°
6. **(a)** 4.27 cm **(b)** 28.3 cm
7. $x = 1.6$, $y = 5.2$
8. 32.3 cm, 6.32 cm
10. **(a)** 11 **(b)** 12.5
(c) 13 **(d)** 15

End-of-Year Examination Specimen Paper 1 *(Pg 309)*

Part I

1. **(a)** 1.53×10^4 **(b)** 7.14×10^5
2. 308 cm²
3. $x = \frac{yk}{n+yh}$

4. **(a)** –26.5 **(b)** 16
5. $86.25
6. **(a)** $(x+y)(p-q)$
(b) $(2c-3d)(c-d)$
7. **(a)** $\frac{1}{2}$ or –3 **(b)** $\frac{1}{4}$ or $1\frac{3}{4}$
8. 100 000
9. 189 cm³
10. 10
11. **(a)** B, E, T **(b)** S
12. **(a)** 11 **(b)** 9 **(c)** $8\frac{7}{9}$
13. 189
14. **(a)** **(i)** reflection in the line *GC*
(ii) a 180° rotation about *O*
(iii) an enlargement, scale factor 2, centre at *A*
(iv) a translation of 3 cm parallel to *AG*
(b) a reflection in *DF* followed by a 180° rotation about *O*

Part II

1. 8.78 cm
2. **(a)** $A'(1, -5)$, $B'(4, -2)$
(b) $A''(-5, 1)$, $B''(-2, 4)$
3. **(a)** 226.8 cm³ **(b)** 11.03 cm
4. 42.6 cm, 2 300 kg
5. 209.2 m
6. –1, –4, –1
(a) –3 **(b)** 2.7 or –0.7
7. **(a)** $13\frac{1}{3}$ km/h
(b) 10 36, 21.5 km from *Q*
(c) 10 00; 1 h
(d) 11.5 km
8. **(a)** 18.04%
(b) $5 741 million
(c) $7 479 **(d)** 587 500

End-of-Year Examination Specimen Paper 2 *(Pg 311)*

Part I

1. $s = \frac{v^2-u^2}{2a}$, 5
2. 40, 54
3. $a = 3$, $b = -4$
4. 10 cm
6. **(a)** and **(b)**, an enlargement of scale factor 2, centre at *A*
7. **(a)** 4 **(b)** $4\frac{1}{6}$ **(c)** 40°

8. (a) $\frac{x+y}{xy}$
(b) $\frac{1}{(x+2)(x+3)}$
(c) $\frac{5x+y}{(x+y)(x-y)}$ or $\frac{5x+y}{x^2-y^2}$

9. (a) $6x^2 - 7xy - 5y^2$
(b) $4x^2 - 4xy - 8x - 3y^2 + 12y$

10. 0, 2 or 3

11. (a) $7\frac{1}{5}$ (b) $1\frac{11}{13}$

12. 50 cm

13. (a) $(6x + 7y)(6x - 7y)$
(b) $(12a + 5)(a - 3)$
(c) $(h + 6k)(h - 9k)$

Part II

1. 32.61 cm, 521.8 cm^2
2. (a) RM450 (b) S$107
(c) RM5 400
3. 4 852 cm^2, 312 cm
4. 13 520 cm^3
5. (a) $x = 5\frac{5}{7}$ cm, 29.7°
(b) 36.9°
6. 2.215 m
7. (a) 7, 3 (b) (2, 1)
8. −4, 6, 0
(a) 6.1 (b) 2.7 or −2.2

End-of-Year Examination Specimen Paper 3 *(Pg 313)*

Part I

1. (a) 9 or −7 (b) 18
2. (a) $(x - 75)(x + 30)$
(b) $(3x + 14)(5x - 12)$
(c) $4(3x + y)(3x - y)$
3. (a) 3 (b) 8
(c) 0 (d) 2
4. $q = \frac{2p+m}{m}$
5. $x = -1\frac{13}{14}$, $y = 4\frac{1}{14}$
6. 6
7. 30 days
8. $x = 8$
9. $x = 4.8$, $y = 13\frac{1}{3}$
10. 52
11. (a) $x = 63$ (b) 25%
(c) 126°
12. (a) (−6, 9) (b) (−2, 3)
13. 6

Part II

1. 3.64 cm^2, 13.18 cm
2. 66.46 cm
3. (a) (iv)
(b) $4x^4 + 4x^3 - 11x^2 + 6x - 1$
(c) $3x^2 + 7x + \frac{5}{x-1}$
4. $\frac{2}{7}$
5. (a) 2 or $-\frac{1}{3}$
(b) 4, 6
6. 15.1 cm, 4 050 cm^3
7. (a) 9.33 cm
(b) 17.1 cm
(c) 14.4 cm
8. −3, −3, 1
(a) 0.5 (b) −4.4
(c) 2.8 or 1.8

End-of-Year Examination Specimen Paper 4 *(Pg 315)*

Part I

1. $x = \frac{2ay}{1+3y}$, 5
2. (a) 1 046 000
(b) 879 000
3. $x = \frac{1}{2}$, $y = \frac{1}{3}$
4. (a) $-27x - 18y + 25z$
(b) $6x^2 + xy - 2y^2 - 2x + y$
5. (a) 70 km
(b) 49.4 km/h
6. mean = 74, mode = 73, median = 73
7. 1 497.6 cm^3, 820.8 cm^2
8. (a) 9 (b) 20
9. (a) 72° (b) 36°
10. 512 cm^3
11. $P(2, 0)$, $Q(6, 8)$, $R(8, 4)$
12. 14 30

Part II

1. 64.5 cm^2
2. 9.93 cm
3. (a) 5 m
(b) 67.4°
4. (a) (i) $2x(x + 5)$
(ii) $(x + 2y - 5)(x + 2y + 3)$
(b) 84
5. $5.65
6. BD = 13 cm, AD = 4 cm; 14.3°
7. −5, −5, 4, 9; $y = -6$

8. (a) $\frac{5}{12x}$
(b) $\frac{9}{x-y}$
(c) $\frac{-7}{2(2x-y)}$
9. (a) 48 km
(b) 2 h
(c) 28 km
(d) 09 00 to 10 00
(e) 8 km/h

End-of-Year Examination Specimen Paper 5 *(Pg 317)*

Part I

1. $5\frac{1}{3}$ or 12
2. (a) $(xy - 7)(xy - 8)$
(b) $(x - y)(x + 1)(x - 1)$
3. (a) $7x^2 - 5xy + 4y^2$
(b) $x^2 + 5xy + y^2 + 2x - 2y$
4. 21, 23
5. 39.8 cm, 17.1 cm
6. $x = 2$, $y = 3$
7. (a) 54 cm
(b) 15 km^2
9. 1.185 cm
10. mean = 13 yr 5 mth, median = 13 yr 7 mth
11. 9 cm
12. (a) a 180° rotation about O
(b) an enlargement, scale factor 2, centre at C
13. 112 cm^2
14. 20 cm

Part II

1. 28.28 cm
2. 40 m
3. 5 cm
4. 41.2 cm, 92.4 cm^2
5. $x = 60$, 72 cm
6. 13 mm
7. $x = 11$, 7 cm, 24 cm, 25 cm, 16.3°
8. 0, 8, 9, 0
(a) 6.8
(b) 1.4 or −3.4
9. 1.06 g/cm^3
10. (a) (i) 100 N
(ii) 240 N
(b) 50 N